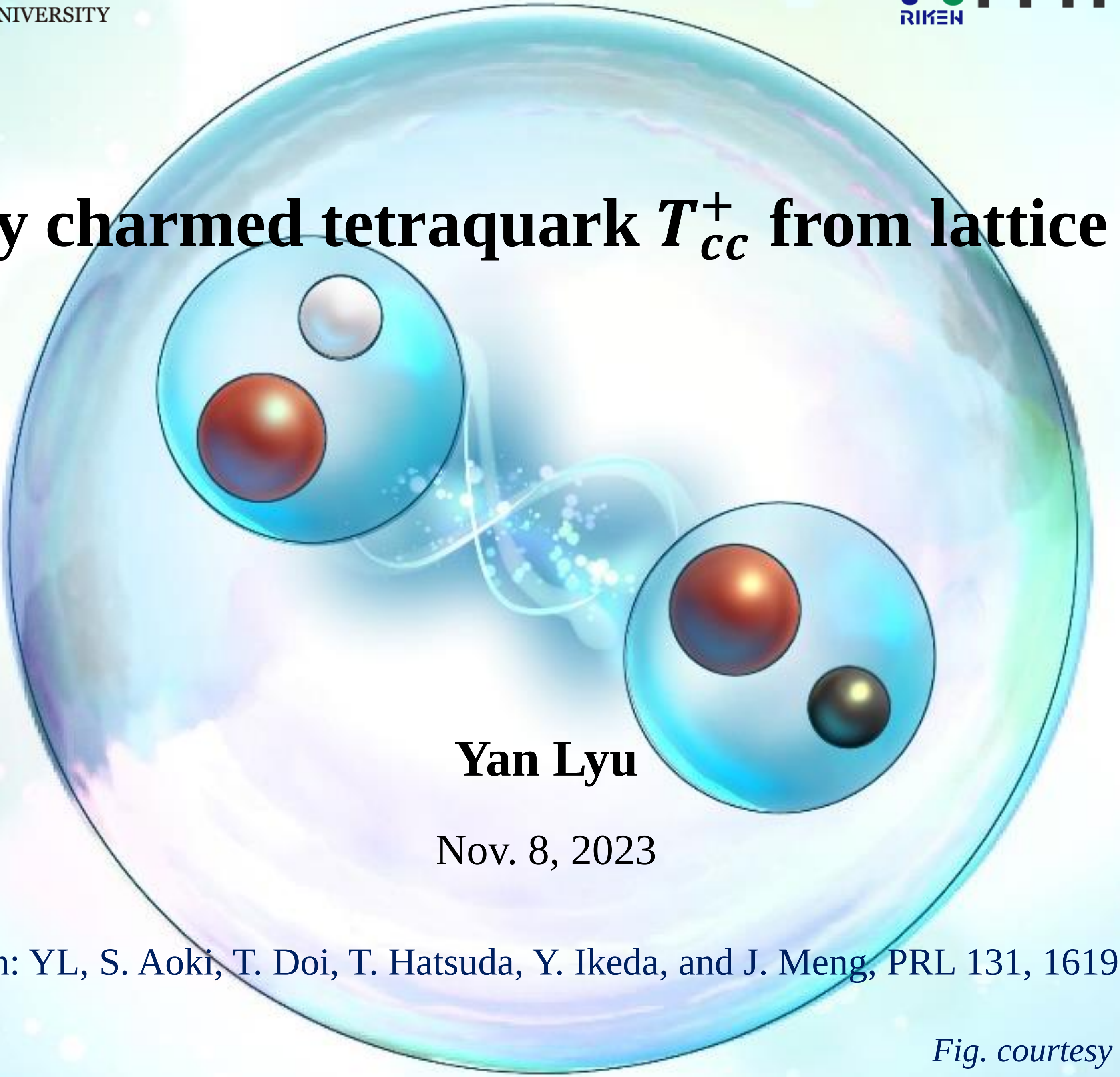
# Doubly charmed tetraquark $T_{cc}^{+}$ from lattice QCD



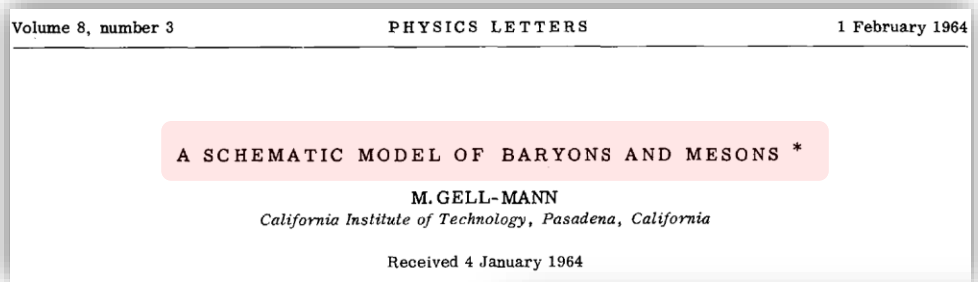
**Yan Lyu**

Nov. 8, 2023

Based on: YL, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, and J. Meng, PRL 131, 161901 (2023)

*Fig. courtesy of K. Murano*

# Conventional hadrons

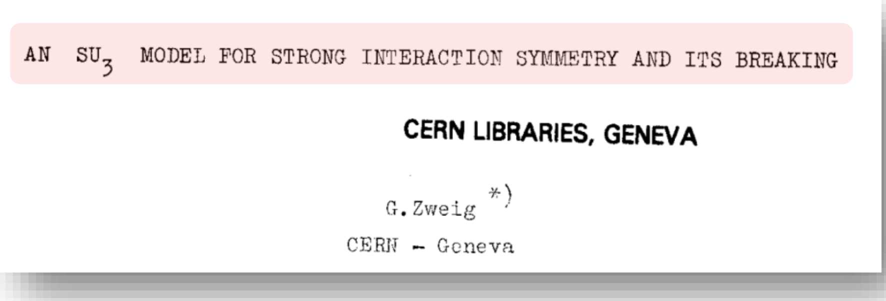


Volume 8, number 3 PHYSICS LETTERS 1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
California Institute of Technology, Pasadena, California

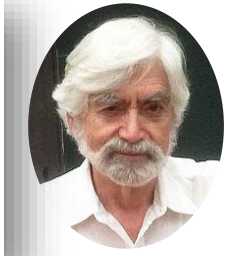
Received 4 January 1964

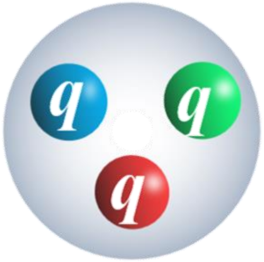


AN $SU_3$ MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

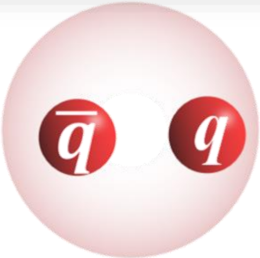
CERN LIBRARIES, GENEVA

G. Zweig *)
CERN - Geneva

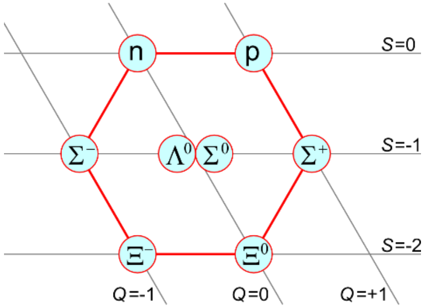


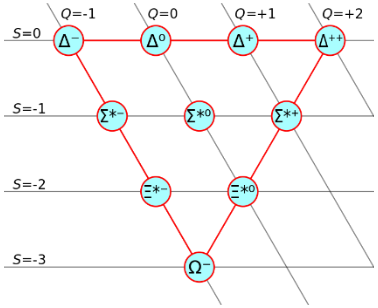


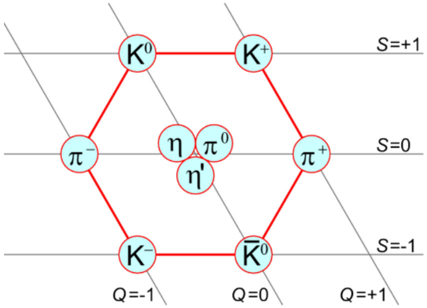
baryon

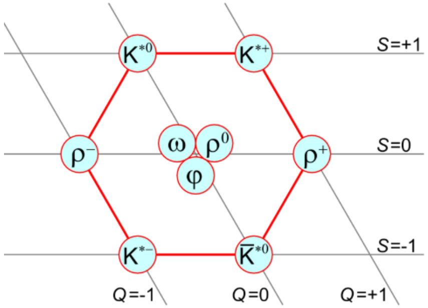


meson

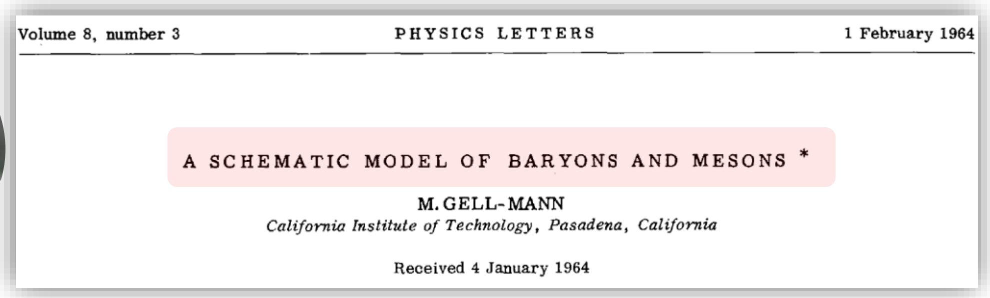








# Exotic hadrons



Volume 8, number 3 PHYSICS LETTERS 1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
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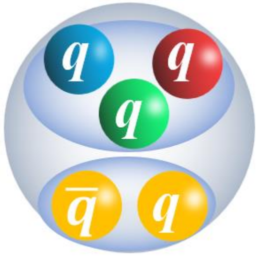
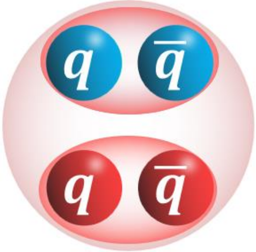
Received 4 January 1964

*"Baryons can now be constructed from quarks by using the combinations* $(qqq)$, $\boldsymbol{(qqqq\bar{q})}$, *etc., while mesons are made out of* $(q\bar{q})$, $\boldsymbol{(qq\bar{q}\bar{q})}$, *etc."*



pentaquark



tetraquark

## Hidden-charm exotic candidates

● 2003
$\boldsymbol{\chi_{c1}(3872)}$

$\psi(4360)$
$\psi(4660)$

$Z_c(4430)$
$X(4050)$
$X(4250)$

$\chi_{c1}(4140)$

$\chi_{c1}(4274)$

● 2013
$\boldsymbol{Z_c(3900)}$

$X(4020)$
$Z_c(4200)$
$R_{c0}(4240)$

● 2015
$\boldsymbol{P_c(4380)}$
$\boldsymbol{P_c(4440)}$

$X(4055)$

$\chi_{c0}(4700)$
$\chi_{c0}(4500)$

$P_c(4312)$
$P_c(4440)$
$P_c(4457)$

● 2020
$\boldsymbol{X(6900)}$

$Z_{cs}(3985)$
$Z_{cs}(4220)$
$\chi_{c1}(4685)$
$X(4630)$

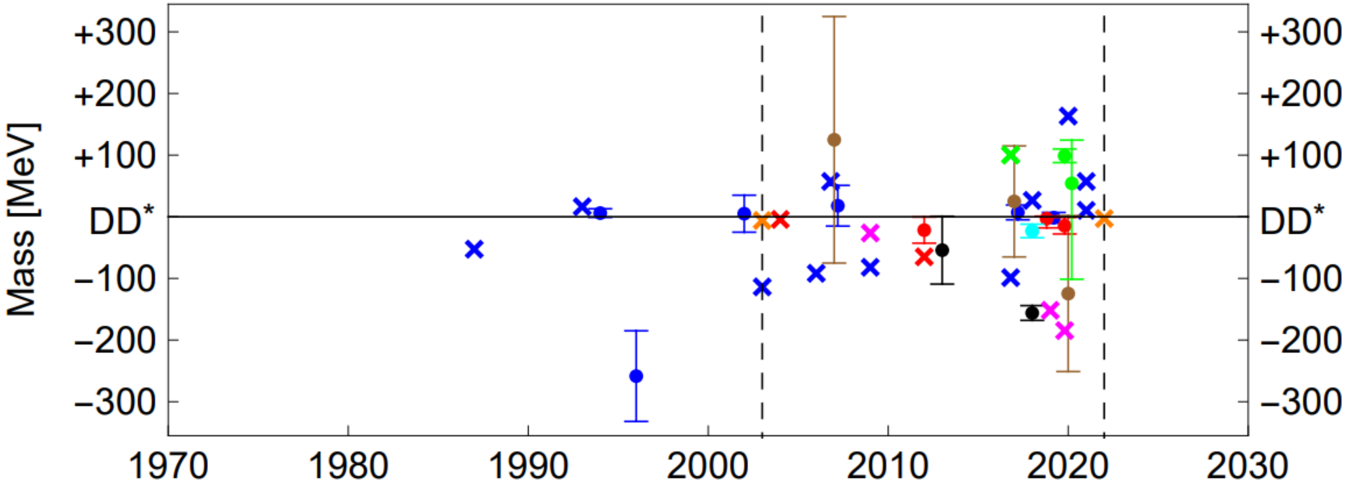
# $QQ\bar{q}\overline{q'}$ extotics

- Intriguing aspects on $QQ\bar{q}\overline{q'}$
  - Open flavor, once observed its minimal quark content contains four quarks
  - Likely to be bound in the limit of $m_Q \to \infty$

A. Manohar and M. Wise, Nucl. Phys. B 339, 17 (1993)

**$bb\bar{q}\overline{q'}$ (√)** **$cc\bar{q}\overline{q'}$ (?)** **$ss\bar{q}\overline{q'}$ (×)**

- A long history of theoretical prediction on $cc\bar{u}\bar{d}$ $(IJ^P = 01^+)$



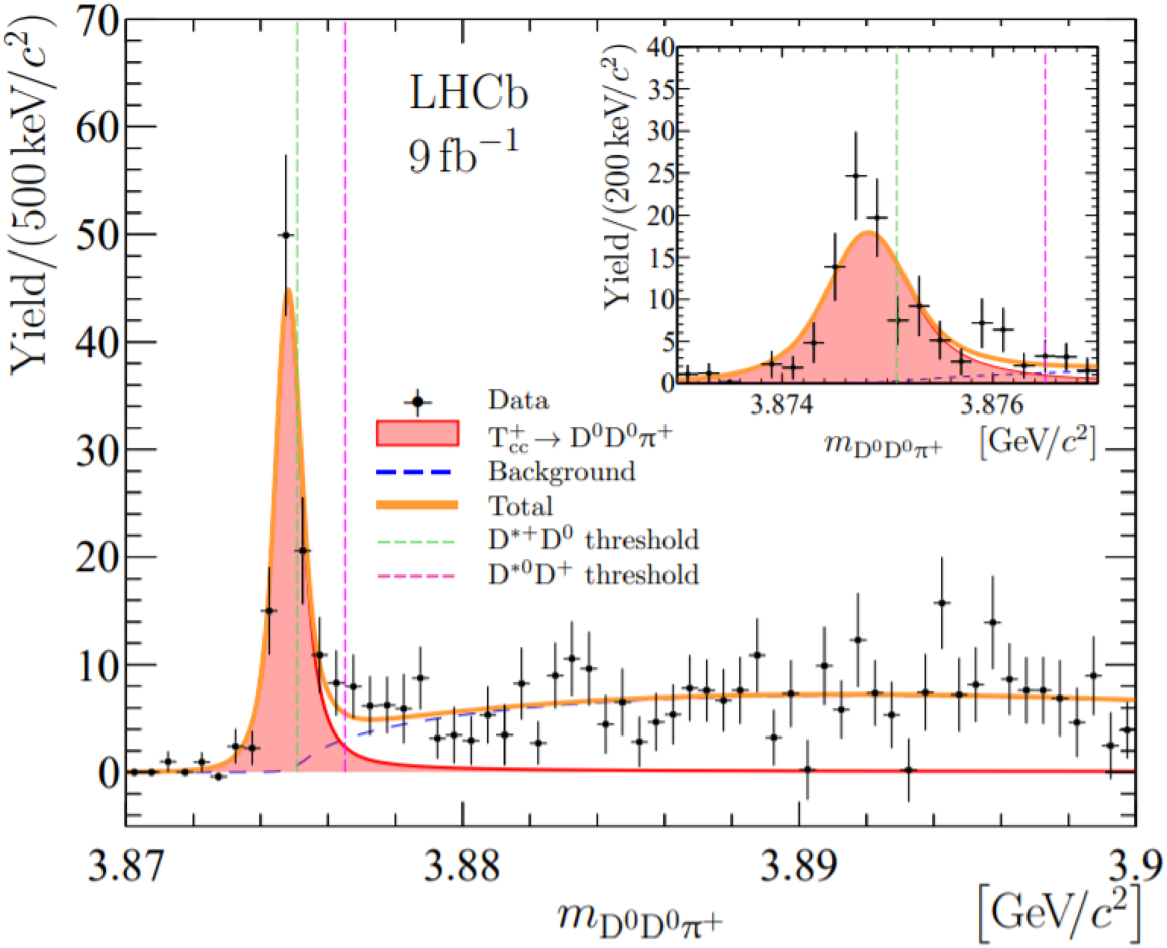
H.-X. Chen *et al.*, Rept. Prog. Phys. 86, 026201 (2023)

  - Mass varies within ±300 MeV with respect to $m_{D^*} + m_D$

# First doubly charmed tetraquark $T_{cc}^{+}$

- 2022, LHCb discovered $T_{cc}^{+}$ in the $D^0 D^0 \pi^+$ spectrum

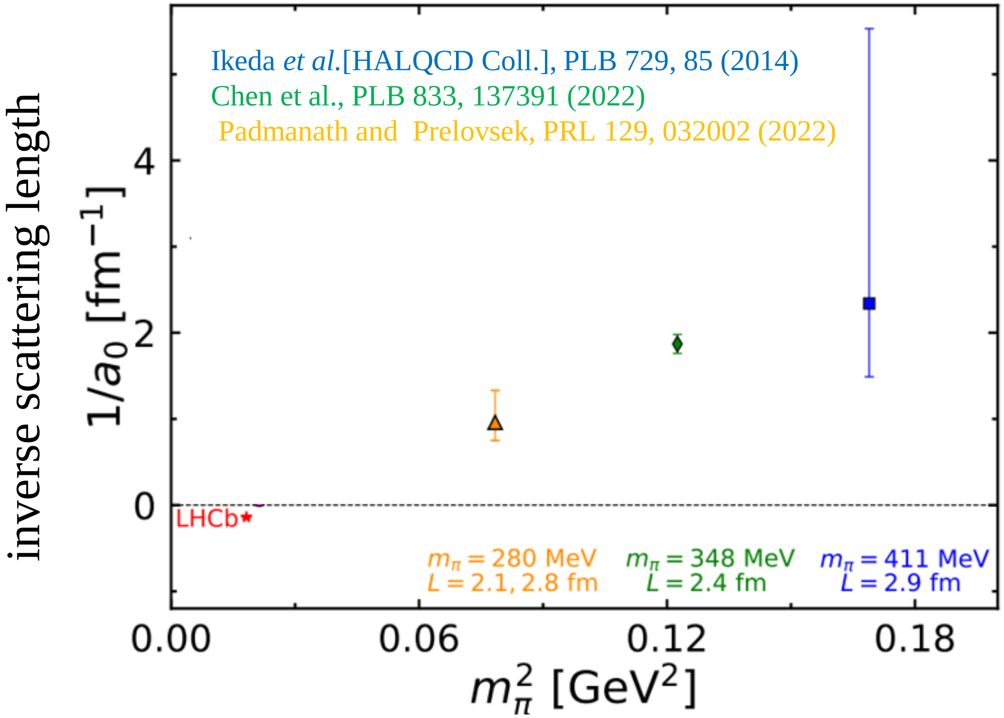
LHCb Coll., Nature Phys. 18, 751 (2022); Nature Comm. 13, 3351 (2022)

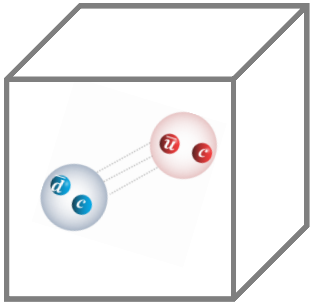


| $I(J^P)$ | $\delta m_{\text{pole}}$ | $\Gamma_{\text{pole}}$ | $\text{Re}(a_0)$ | $\text{Im}(a_0)$ |
|---|---|---|---|---|
| $0(1^+)$ | -360±40 keV | 48±2 keV | 7.16 ± 0.51 fm | -1.85 ± 0.28 fm |

# $T_{cc}^+$ from first-principle lattice QCD

- Limited to heavy quark masses ( $m_\pi \geq 280$ MeV)



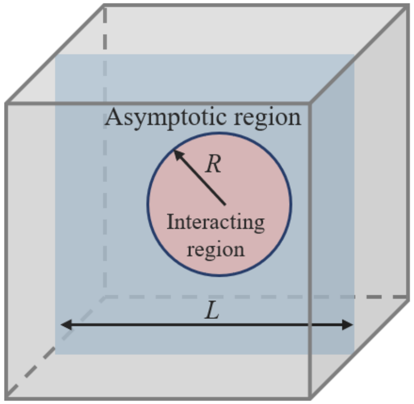


- Purpose of this talk
  1. present the latest lattice results with (nearly) physical quark masses
  2. directly compare theoretical and experimental $DD\pi$ mass spectrum

# HAL QCD method

- Nambu-Bethe-Salpeter (NBS) amplitude

$$\boxed{\psi^k(\boldsymbol{r})e^{-Et} = \langle 0|\hat{D}^*(\boldsymbol{r},t)\hat{D}(\boldsymbol{0},t)|D^*(\boldsymbol{k})D(-\boldsymbol{k});E\rangle}$$



  - Asymptotic region: $\psi^k(r) \simeq A\,\frac{\sin(kr-l\pi/2+\delta(k))}{kr}$
  - Interacting region: define potential

$$\boxed{(\nabla^2+k^2)\psi^k(\boldsymbol{r}) = 2\mu\int d\boldsymbol{r}'\,U(\boldsymbol{r},\boldsymbol{r}')\psi^k(\boldsymbol{r}')}$$

- $R$ correlator (superposition of NBS amplitudes) $R(\boldsymbol{r},t) = \sum_n a_n\psi_n(\boldsymbol{r})e^{-\Delta E_n t}$

$$\boxed{\left(\frac{1}{8\mu}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu}\right)R(\boldsymbol{r},t) = \int d\boldsymbol{r}'\,U(\boldsymbol{r},\boldsymbol{r}')R(\boldsymbol{r}',t)}$$

  - Derivative expansion: $U(\boldsymbol{r},\boldsymbol{r}') = \sum V_i(\boldsymbol{r})\,\nabla^i\delta(\boldsymbol{r}-\boldsymbol{r}')$

$$\boxed{V(r) = R(\boldsymbol{r},t)^{-1}\left(\frac{1}{8\mu}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu}\right)R(\boldsymbol{r},t)}$$

N. Ishii, S. Aoki and T. Hatsuda, Phys. Rev. Lett. 99, 022001 (2007)
N. Ishii, *et al.* [HAL QCD Coll.], Phys. Lett. B 712, 437 (2012)

# Lattice setup

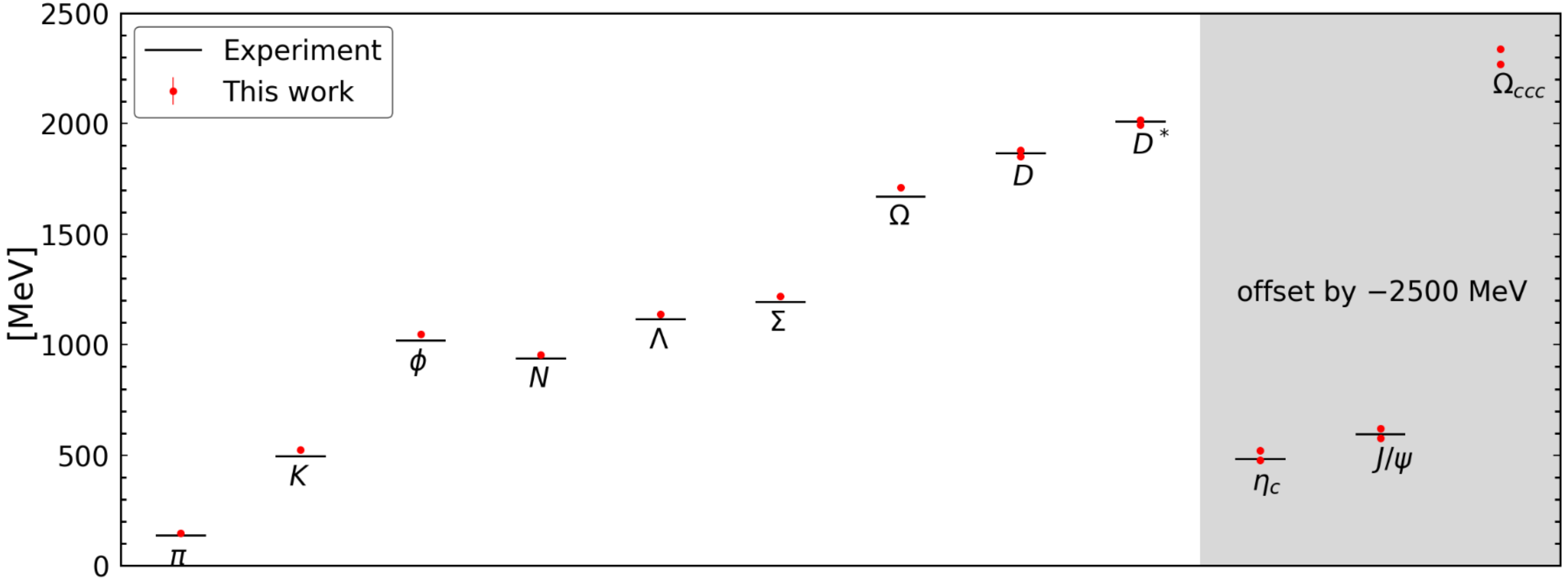
Fugaku supercomputer (440 PFlops)

- (2+1)-flavor configuration
  - Iwasaki gauge action
  - $O(a)$-improved Wilson quark action for *uds* quark
  - Relativistic heavy quark action for $c$ quark

K.-I. Ishikawa *et al.* [PACS Coll.], *Proc. Sci.*, LATTICE2015 075 (2016)
Y. Namekawa *et al.* [PACS Coll.], *Proc. Sci.*, LATTICE2016 125 (2017)

| $L^3 \times T$ | $a$ [fm] | $La$ [fm] | $m_\pi$ [MeV] | $m_K$ [MeV] |
|---|---|---|---|---|
| $96^3 \times 96$ | 0.0846 | 8.1 | 146 | 525 |

- Hadron mass



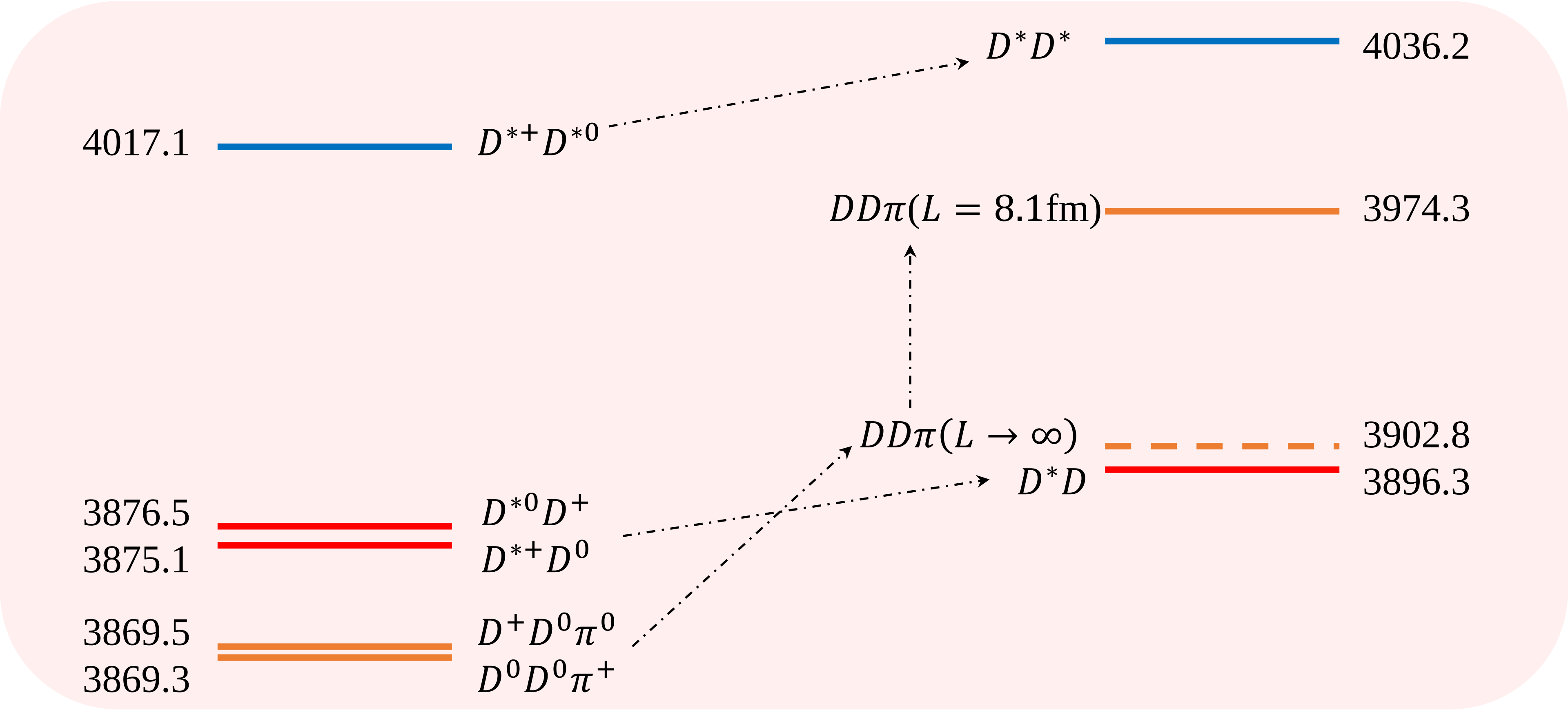
# Energy levels

**Nature**

$\pi^0(134.98)$ $\pi^+(139.57)$
$D^0(1864.84)$ $D^+(1869.66)$
$D^{*0}(2006.85)$ $D^{*+}(2010.26)$

**Lattice** [MeV]

$\pi(146.4)$
$D(1878.2)$
$D^*(2018.1)$



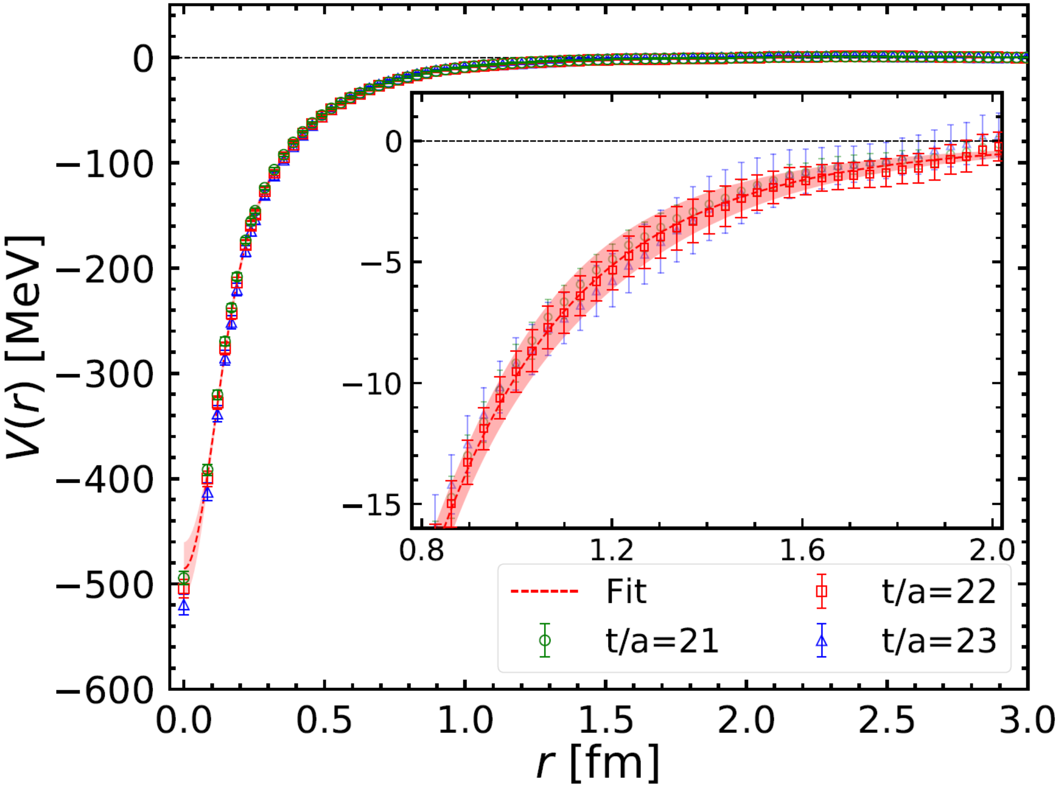
4017.1 $D^{*+}D^{*0}$
3876.5 $D^{*0}D^+$
3875.1 $D^{*+}D^0$
3869.5 $D^+D^0\pi^0$
3869.3 $D^0D^0\pi^+$

$D^*D^*$ 4036.2
$DD\pi(L = 8.1\text{fm})$ 3974.3
$DD\pi(L \to \infty)$ 3902.8
$D^*D$ 3896.3

- The lowest energy level of $DD\pi$ ($D^*D^*$) is around 78 (140) MeV above on the lattice

# $D^*D$ interaction

- $D^*D$ potential in the $(I, J^P) = (0, 1^+)$ channel



- Short range: antidiquark-diquark $\left[\bar{u}\bar{d}\right]_{3_c, I=J=0}[cc]_{\bar{3}_c, J=1}$

R. Jaffe and F. Wilczek, Phys. Rev. Lett. 91 232003 (2003)

- Long range: attraction from pion-exchange interaction

# Long-range potential

- One-pion exchange

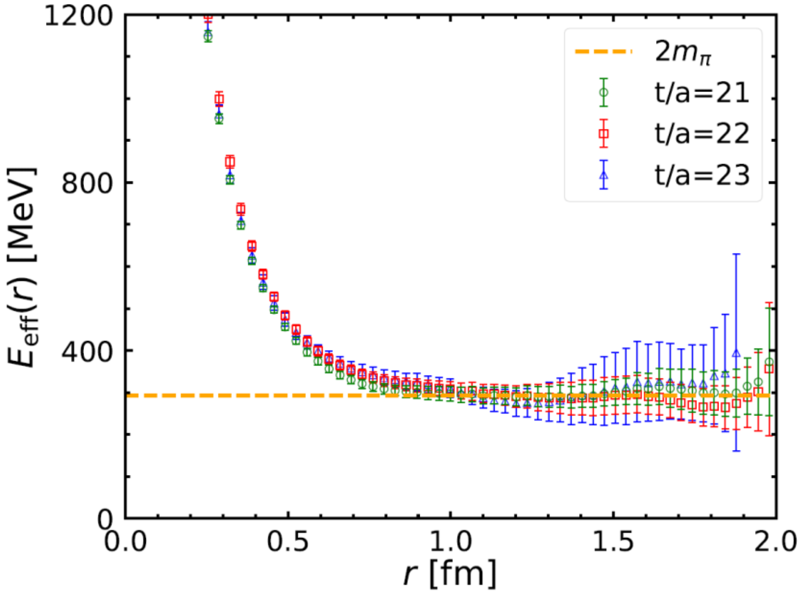
S. Ohkoda *et. al.*, Phys. Rev. D 86, 034019 (2012)
N. Li, *et. al.*, Phys. Rev. D 88, 114008 (2013)

$$\boxed{V(r) = -\alpha\frac{e^{-\mu r}}{r}, \quad \mu = m_\pi \text{ or } \sqrt{(m_{D^*} - m_D)^2 - m_\pi^2}}$$

  - Fail to describe long-range potential (why?)

- Two-pion exchange

$$\boxed{V(r) = -\alpha\frac{e^{-2m_\pi r}}{r^2}} \Longrightarrow \boxed{E_{\text{eff}}(r) = -\frac{\ln\left[-V(r)r^2/\alpha\right]}{r}}$$



  - Long range potential ($1 \leq r \leq 2$ fm) is consistent with two-pion exchange

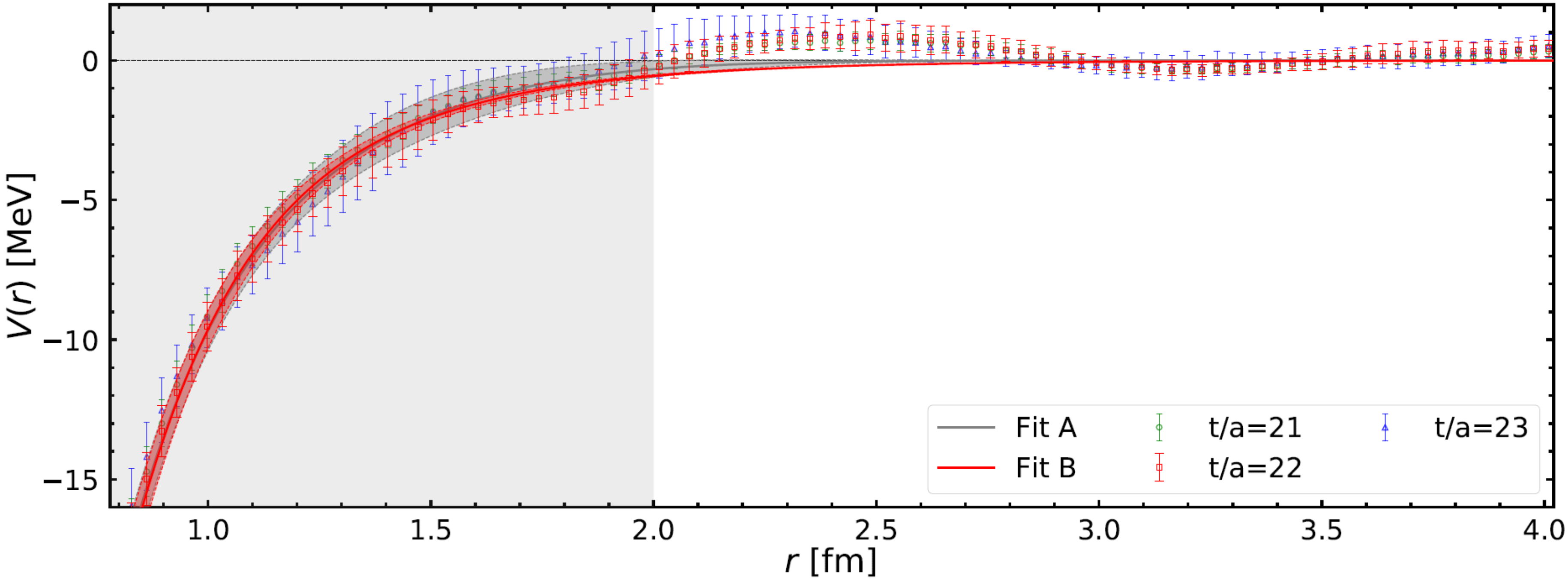
# Fit

- Fit A: purely phenomenological fit ($\chi^2/\mathrm{d.o.f.} = 1.01$)

$$\boxed{V_{\mathrm{fit}}(r) = \sum_{i=1,\cdots,4} a_i e^{-(r/b_i)^2}}$$

- Fit B: TPE-motivated fit ($\chi^2/\mathrm{d.o.f.} = 0.96$)

$$\boxed{V_{\mathrm{fit}}(r; m_\pi) = \sum_{i=1,2} a_i e^{-(r/b_i)^2} + a_3 (1 - e^{-(r/b_3)^2})^2 \frac{e^{-2m_\pi r}}{r^2}}$$

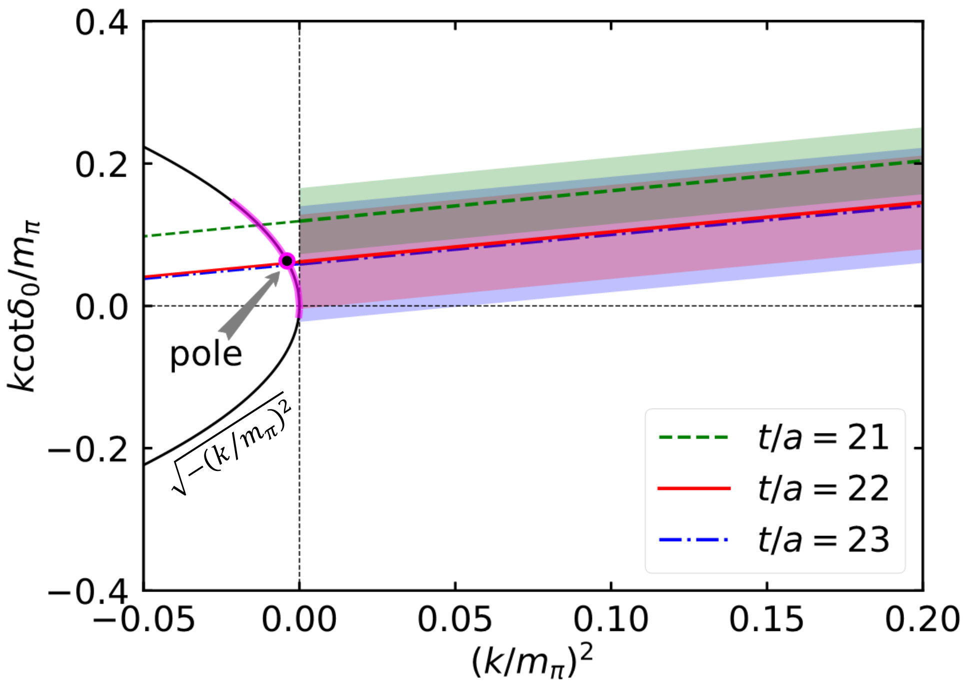


# Scattering properties

- Scattering phase shift

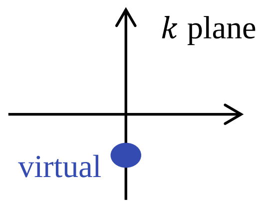
$$k\cot\delta_0 = \frac{1}{a_0} + \frac{1}{2} r_{\mathrm{eff}} k^2$$

$$f^{-1}(k) \sim k\cot\delta_0 - ik$$



- Scattering parameters and pole singularities

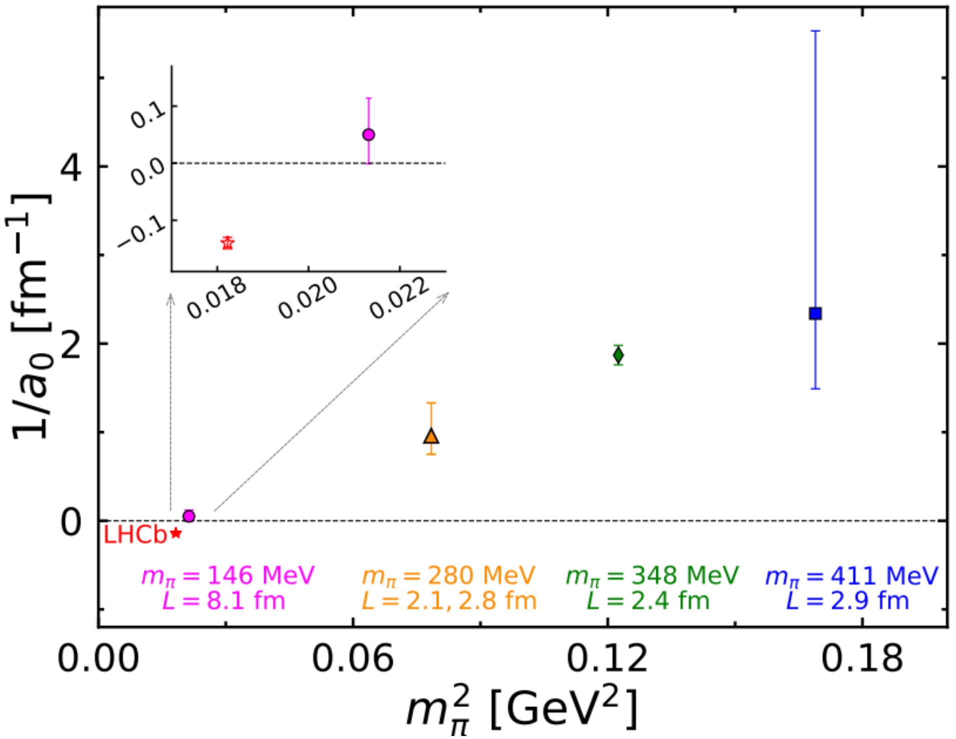
| | $m_\pi$ (MeV) | 146.4 |
|---|---|---|
| | $1/a_0$ (fm$^{-1}$) | $0.05(5)\left(^{+2}_{-2}\right)$ |
| | $r_{\mathrm{eff}}$ (fm) | $1.12(3)\left(^{+3}_{-8}\right)$ |
| $k = i\kappa_{\mathrm{pole}}$ | $\kappa_{\mathrm{pole}}$ (MeV) | $-8(8)\left(^{+3}_{-5}\right)$ |
| | $E_{\mathrm{pole}}$ (keV) | $-59\left(^{+53}_{-99}\right)\left(^{+2}_{-67}\right)$ |



- $T_{cc}^{+}$ appears as a virtual state at $m_\pi = 146.4$ MeV

# Comparison

- $1/a_0$



Ikeda *et al.*[HALQCD Coll.], Phys. Lett. B 729, 85 (2014)
Chen et al., Phys. Lett. B 833, 137391 (2022)
Padmanath and Prelovsek, Phys. Rev. Lett. 129, 032002 (2022)

- As $m_\pi$ decreases, LQCD results approach to the experimental data

# Extrapolate to physical point based on TPE

## Extrapolation

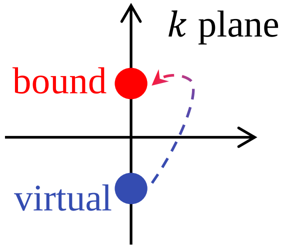
- Extrapolate TPE interaction to physical point

$$\boxed{V_{\text{fit}}(r; m_\pi = 146 \rightarrow 135\ \text{MeV})}$$

- Adopt physical values for $m_{D^{*+}}$ and $m_{D^0}$
- Do NOT consider isospin breaking nor opening of $DD\pi$ channel

## Scattering parameters and pole singularities

| $m_\pi$ (MeV) | 146.4 | 135.0 |
|---|---|---|
| $1/a_0$ (fm$^{-1}$) | $0.05(5)(^{+2}_{-2})$ | $-0.03(4)$ |
| $r_{\text{eff}}$ (fm) | $1.12(3)(^{+3}_{-8})$ | $1.12(3)$ |
| $k = i\kappa_{\text{pole}}$ $\kappa_{\text{pole}}$ (MeV) | $-8(8)(^{+3}_{-5})$ | $+5(8)$ |
| $E_{\text{pole}}$ (keV) | $-59(^{+53}_{-99})(^{+2}_{-67})$ | $-45(^{+41}_{-78})$ |



- $m_\pi$ =146 → 135 MeV, $T_{cc}^+$ evolves from a virtual state into a bound state

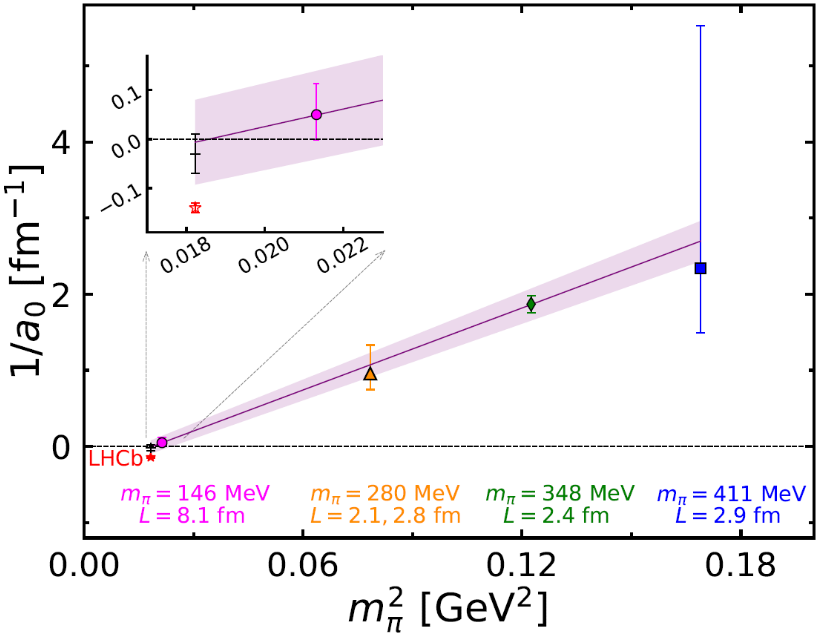
# Extrapolation to physical point based on $a_0$

- Extrapolation
  - A linear fit to four $1/a_0$s from different $m_\pi$

$$\boxed{1/a_0(m_\pi) = c + d m_\pi^2}$$

  - Four data from different calculations and posses different systematics

- $1/a_0$



  - $1/a_0$ from two extrapolations are consistent with each other

# Construction of $D^0D^0\pi^+$ spetrum

- Production amplitude of $D^{*+}D^0$ from a source function $P$

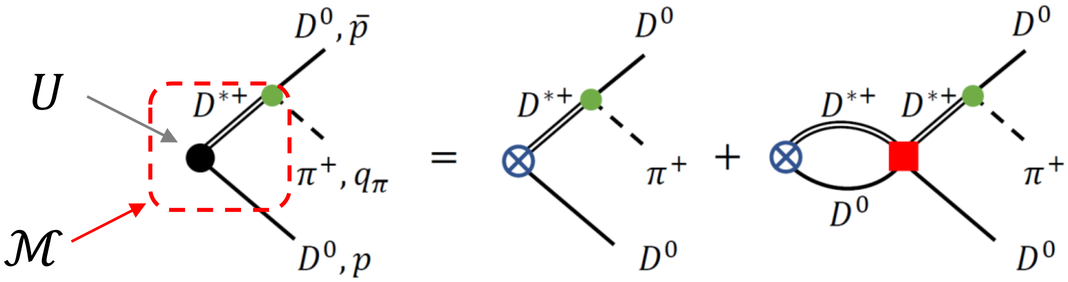
$$\boxed{U(M,p) = P + \int \frac{d^3q}{(2\pi)^3} T(M,p,q)G(M,q)P}$$

PHYSICAL REVIEW D **105**, 014024 (2022)

**Coupled-channel approach to $T_{cc}^+$ including three-body effects**

Meng-Lin Du,1,* Vadim Baru,2,3,† Xiang-Kun Dong,4,5,‡ Arseniy Filin,2 Feng-Kun Guo,4,5,§ Christoph Hanhart,6,‖ Alexey Nefediev,7,8,¶ Juan Nieves,1,** and Qian Wang9,10,11,††

- For simplicity, consider a pointlike source (constant in $p$-space, $P = \mathcal{N}$)
- Only $S$-wave production at low energies



- Adopt experimental values for $m_{D^{*+}, D^0, \pi^+}$ and $\Gamma_{D^{*+}}$ in the kinematics to keep the same phase space with the experiment

- Three-body mass spectrum for $D^0D^0\pi^+$

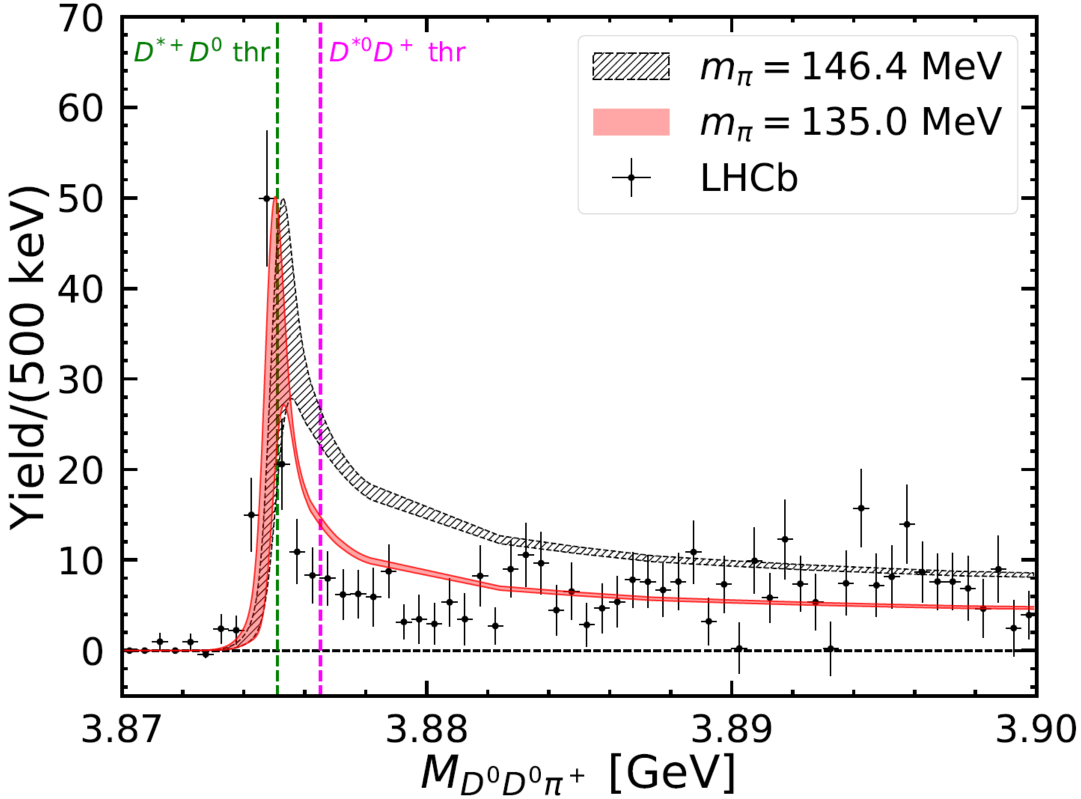
$$\boxed{\begin{aligned}\mathcal{M}(U \to D^0D^0\pi^+) &= U(M,p)G(M,p)q_\pi + U(M,\bar{p})G(M,\bar{p})\bar{q}_\pi \\ \frac{d\mathrm{Br}}{dM} &= \mathcal{N}' \int_0^{p_{\max}} pdp \int_{\bar{p}_{\min}}^{\bar{p}_{\max}} \bar{p}d\bar{p} |\mathcal{M}(U \to D^0D^0\pi^+)|^2\end{aligned}}$$

- A known energy resolution function needs to considered for comparison w/ exp. data

LHCb Coll., Nature Comm. 13, 3351 (2022)

# $D^0 D^0 \pi^+$ spetrum

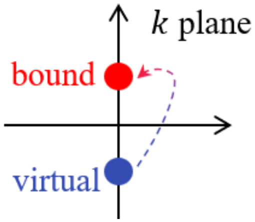
- Results at different $m_\pi$



- A peak around $D^{*+}D^0$ threshold
- $m_\pi$ =146 MeV → 135 MeV, peak position shifts to the left, better description to LHCb data

# Summary & Outlook

- Summary: present the scattering properties of $D^*D$ system from LQCD with nearly physical $m_\pi$=146 MeV
  - Attractive potential with two-pion exchange interaction at long distances
  - The potential leads to a near-threshold virtual state
  - Extrapolate the potential to physical point ($m_\pi$ =146→135 MeV)
    - ✓ virtual state→bound state
    - ✓ better description to the $DD\pi$ spectrum of LHCb experiment



- Outlook
  - Physical-point simulation
    - ✓ opening of three-body channel
    - ✓ isospin breaking effect (i.e., coupled channel calculation)
  - Study one-pion exchange interaction and associated left-hand-cut from LQCD

**Thanks!**

*Fig. courtesy of K. Murano*