Analysis of T_{cc} and T_{bb} based on the hadronic molecular model and their spin structures

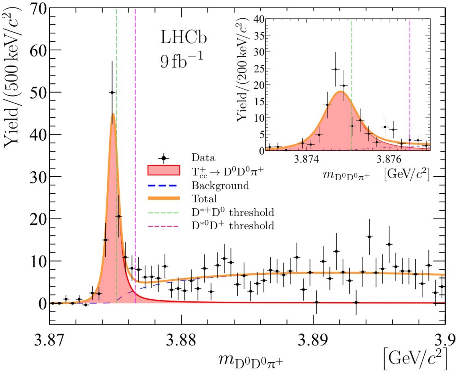
Manato Sakai

in collaboration with
Yasuhiro Yamaguchi

Nagoya University

November 8, 2023

$T_{cc}(cc\bar{u}\bar{d})^+$



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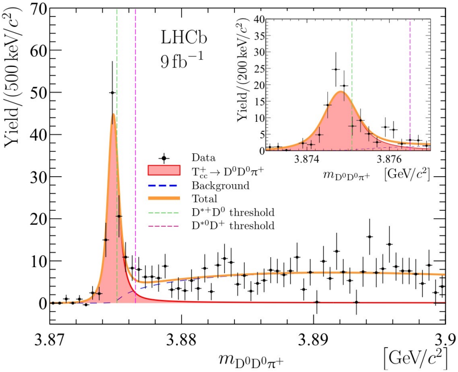

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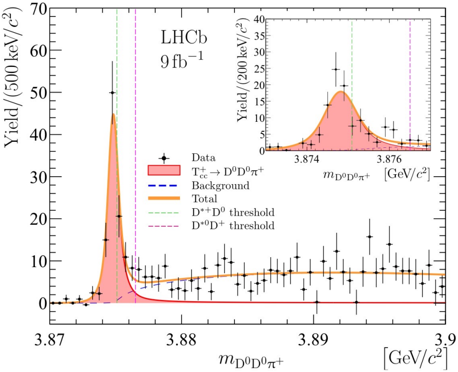

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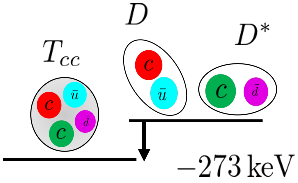
- mass difference from the $D^{*+}D^0$ threshold
  - Breit-Wigner
    $\delta m_{\rm BW} = -273 \pm 61\,{\rm keV}/c^2$
  - pole
    $\delta m_{\rm pole} = -360 \pm 40\,{\rm keV}/c^2$



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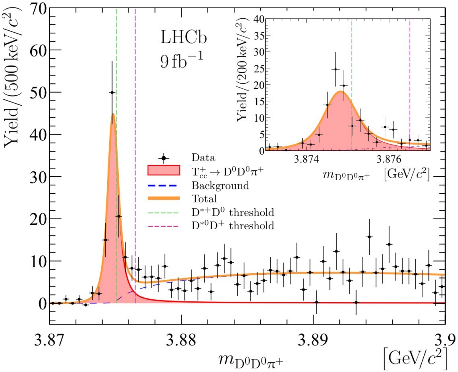

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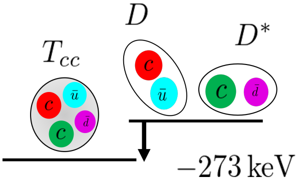


- T_{cc} exists slightly below the $D^{*+}D^0$ threshold.
  $\Rightarrow$ T_{cc} can be considered as a **hadronic molecule** like a deuteron!!

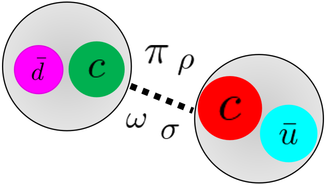
Theoretical researches on T_{cc}

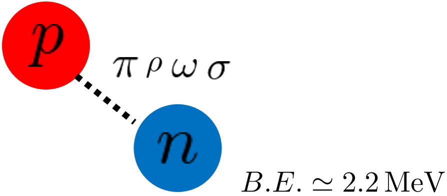
- non-relativistic quark model
  - J.I.Ballot and J.M.Richard, Phys.Lett. B **123**, 449 (1983)
  - S.Zouzou *et al*, Z. Phys. C **30**,457 (1986)
  - Q. Meng *et al*, Phys. Lett. B **814**, 136095 (2019)
- Hadronic molecule
  - S.Ohkoda et al, Phys. Rev. D **86**, 034019 (2012)
    bound and resonant states of T_{cc} and T_{bb} based on a hadronic molecule.
    Interaction: one pion exchange potential and one π, ρ and ω exchange potential.
    We follow this study.
- Lattice QCD
  - Y. Ikeda *et al*, Phys. Lett. B **729**, 85 (2014)
  - M. Padmanath and S. Prelovsek, Phys. Rev. Lett. **129**, 032002 (2022)
  - Y. Lyu *et al*, arXiv:2302.04505 [hep-lat] (2023)

The studies of T_{cc} are summarized in [1]

[1] Hua-Xing Chen *et al*, Rept. Prog. Phys. **86**, 026201 (2023)

Hadronic Molecule in our study





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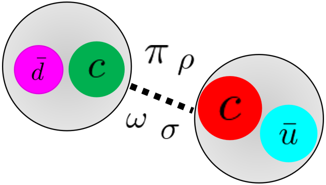

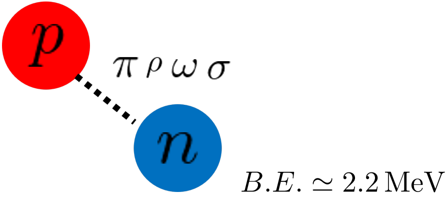

- Structure
  Loosely bound state of two mesons
  comparable with the deuteron.

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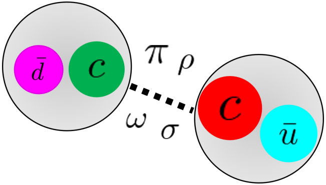

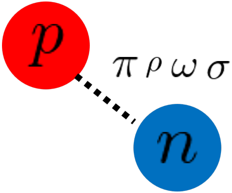

- Structure
  Loosely bound state of two mesons
  comparable with the deuteron.
- Interaction
  One boson exchange potential (OBEP) $\pi, \rho, \omega,\ \sigma$
  This interaction respects the chiral symmetry and the heavy quark symmetry

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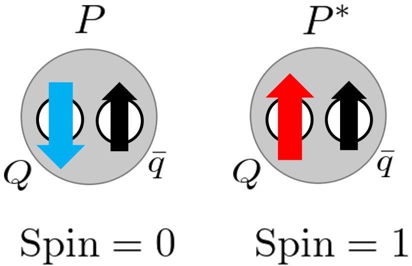
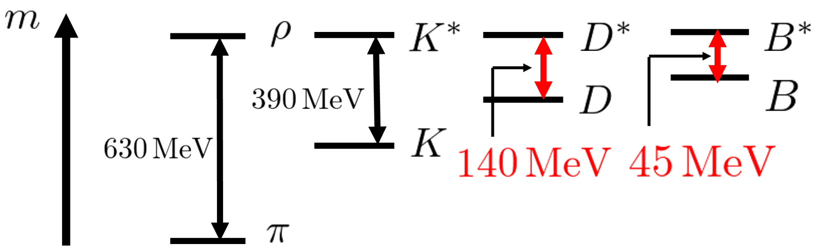
(e.g.)
$Q\bar{q}$ meson



$\Rightarrow$ The pseudoscalar P and the vector meson P^* are degenerate!!
This is called the **HQS doublet**.

HQS doublet

- Mass difference between the pseudoscalar and the vector mesons.



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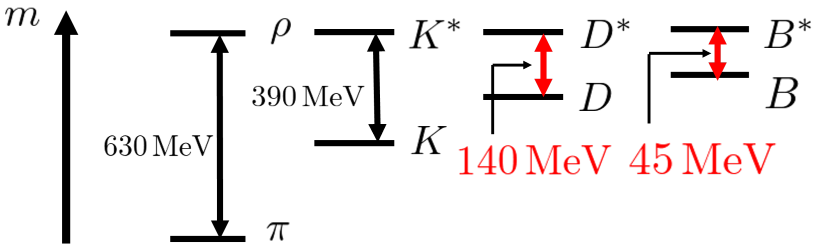
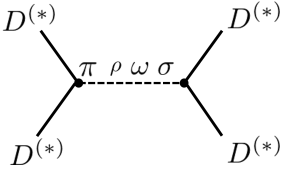


- The mass difference of the pseudoscalar meson and vector meson decreases as the mass of a meson increases.

$\Rightarrow$ P and P^* are degenerate ($P = D, B,\ P^* = D^*, B^*$).

$\Rightarrow$ In the hadronic molecule with heavy quarks, PP, PP^* and P^*P^* are coupled!!

Potential



- Form factor (attached to each vertex)

$$F(\vec{q}; m) = \frac{\Lambda^2 - m^2}{\Lambda^2 + \vec{q}^{\,2}}, \quad \Lambda\text{: cut off}$$

- pion exchange

$$V_\pi = \pm\frac{1}{3}\left(\frac{g_\pi}{2f_\pi}\right)^2 [\vec{\mathcal{O}}_1 \cdot \vec{\mathcal{O}}_2 C(r; m_\pi) + S_{\mathcal{O}_1\mathcal{O}_2} T(r; m_\pi)]\vec{\tau}_1 \cdot \vec{\tau}_2$$

- vector exchange (ρ, ω)

$$V_v^\lambda = \pm\frac{1}{3}(\lambda g_V)^2 [2\vec{\mathcal{O}}_1 \cdot \vec{\mathcal{O}}_2 C(r; m_v) - S_{\mathcal{O}_1\mathcal{O}_2} T(r; m_v)]\vec{\tau}_1 \cdot \vec{\tau}_2$$

$$V_v^\beta = \left(\frac{\beta g_V}{2m_v}\right)^2 C(r; m_v)\vec{\tau}_1 \cdot \vec{\tau}_2$$ ($\vec{\tau}_1 \cdot \vec{\tau}_2$ is removed for ω mesons.)

- σ exchange

$$V_\sigma = -\left(\frac{g_\sigma}{m_\sigma}\right)^2 C(r; m_\sigma)$$

Parameter

parameter	value	
g_π	0.59	$D^* \to D\pi$
β	0.9	Lattice QCD
λ	$0.56\,\mathrm{GeV}^{-1}$	B decay
g_σ	3.4	$g_{\sigma NN}/3$

All parameters expect Λ are fixed.

S. Ahmed et al., CLEO Collaboration, Phys. Rev. Lett. 87, 251801 (2001)
Ming-Zhu Liu et al., Phys. Rev. D 99 094018 (2019)
C. Isola, Phys. Rev. D 68, 114001(2003)

Numerical calculation

- Possible channel of T_{cc} with $0(1^+)$

$$\psi^{\mathrm{HM}}_{0(1^+)} = \begin{pmatrix} |[DD^*]_-(^3S_1)\rangle \\ |[DD^*]_-(^3D_1)\rangle \\ |D^*D^*(^3S_1)\rangle \\ |D^*D^*(^3D_1)\rangle \end{pmatrix}$$

$$[DD^*]_\pm = \frac{1}{\sqrt{2}}(DD^* \pm D^*D)$$

- Solving the Schrödinger e.q. by using the Gaussian expansion method.

E. Hiyama *et al*, Prog.Part.Nucl.Phys., 51 (2003) 223-307

E. Hiyama *et al*, Front.Phys.(Beijing) 13 (2018) 6, 132106

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  $\rightarrow$ One boson exchange is important.

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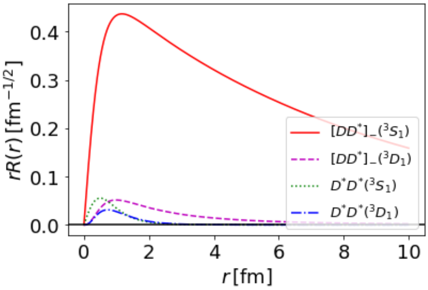

B.E.	$0.273\,\mathrm{MeV}$
$[DD^*]\_(^3S_1)$	$99.2\,\%$
$[DD^*]\_(^3D_1)$	$0.467\,\%$
$D^*D^*(^3S_1)$	$0.229\,\%$
$D^*D^*(^3D_1)$	$0.0854\,\%$
$\sqrt{\langle r^2 \rangle}$	$6.43\,\mathrm{fm}$

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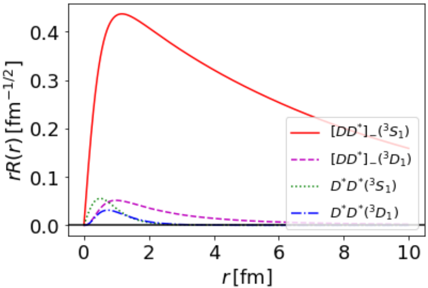

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▶ $[\mathbf{DD^*}]\_(^3\mathbf{S_1})$ is the dominant channel.

Potential Expectation Value

	$[DD^*]_-(^3S_1)$	$[DD^*]_-(^3D_1)$	$D^*D^*(^3S_1)$	$D^*D^*(^3D_1)$
$[DD^*]_-$ $(^3S_1)$	$-4.7(\sigma)$	$-0.55(\pi)$	$-0.31(\rho)$	$-0.34(\pi)$
$[DD^*]_-$ $(^3D_1)$	$-0.55(\pi)$	-0.0048	-0.061	0.01
D^*D^* $(^3S_1)$	$-0.31(\rho)$	-0.061	-0.11	-0.042
D^*D^* $(^3D_1)$	$-0.34(\pi)$	0.01	-0.042	-0.0051

The bosons written in () are ones which mainly contribute to each component.

- σ exchange force is the most important (1,1).
- Tensor force of π exchange is also important (1,2), (2,1), (1,4), (4,1).

T_{cc} with other quantum numbers with g_σ dependence

g_σ	3.06	3.4	3.74	
Λ[MeV]	1147.1	1069.8	1001.3	
$0(0^-)$	-	-	-	
$0(1^+)$	-0.273	-0.273	-0.273	$\leftarrow$ input
$0(1^-)$	-	-	-	
$1(0^+)$	-	-	-	
$1(0^-)$	-	-	-	
$1(1^+)$	-	-	-	
$1(1^-)$	-	-	-	

The value is given in units of MeV.

g_σ is a coupling constant of σ and Λ is determined to reproduce T_{cc} data.

- There is no bound state with $I(J^P)$ other than $0(1^+)$.

T_{bb} with $0(1^+)$ (OBEP)

- T_{bb} is composed of $bb\bar{q}\bar{q}$.

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- T_{bb} is composed of $bb\bar{q}\bar{q}$.
- $m_{B^{(*)}} > m_{D^{(*)}} \rightarrow T_{bb}$ is more likely to be bound than T_{cc}.

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- $m_{B^{(*)}} > m_{D^{(*)}} \to T_{bb}$ is more likely to be bound than T_{cc}.
- OBEP: Fix $\Lambda = 1069.8\,\mathrm{MeV}$, reproducing the T_{cc} data.

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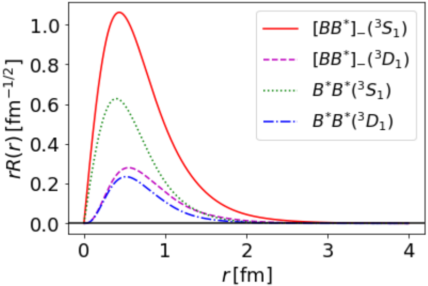

B.E.	$46.0\,\mathrm{MeV}$
$[BB^*]\_(^3S_1)$	$70.7\,\%$
$[BB^*]\_(^3D_1)$	$4.71\,\%$
$B^*B^*(^3S_1)$	$21.6\,\%$
$B^*B^*(^3D_1)$	$3.00\,\%$
$\sqrt{\langle r^2 \rangle}$	$0.620\,\mathrm{fm}$

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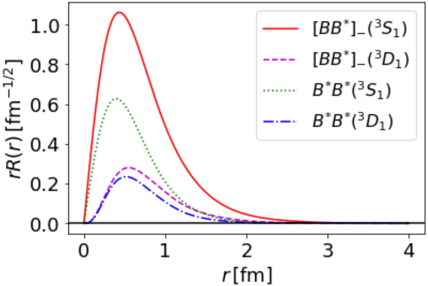

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▶ $[\mathbf{BB^*}]\_(^3\mathbf{S}_1)$ is dominant and $\mathbf{B^*B^*}(^3\mathbf{S}_1)$ is important.
(D^*D^* $(^3S_1)$ is not important channel for T_{cc}.)

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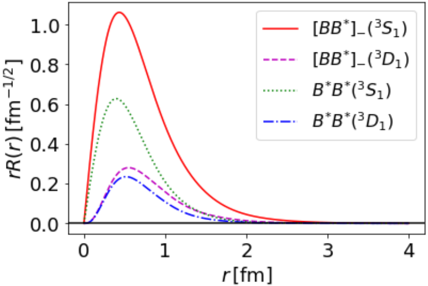

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- $\mathbf{[BB^*]\_(^3S_1)}$ is dominant and $\mathbf{B^*B^*(^3S_1)}$ is important.
  (D^*D^* $(^3S_1)$ is not important channel for T_{cc}.)
  $\because\ \Delta m_B = m_{B^*} - m_B < \Delta m_D = m_{D^*} - m_D$
  $\Rightarrow [BB^*]\_$ and B^*B^* are more coupled than $[DD^*]\_$ and D^*D^*.

Potential Expectation Value

	$[BB^*]_-({}^3S_1)$	$[BB^*]_-({}^3D_1)$	$B^*B^*({}^3S_1)$	$B^*B^*({}^3D_1)$
$[BB^*]_-$ $({}^3S_1)$	$-40(\sigma)$	$-7.3(\pi)$	$-11(\rho)$	$-6.2(\pi)$
$[BB^*]_-$ $({}^3D_1)$	$-7.3(\pi)$	-0.67	-4.1	0.33
B^*B^* $({}^3S_1)$	$-11(\rho)$	-4.1	$-14\ (\sigma)$	-3.5
B^*B^* $({}^3D_1)$	$-6.2(\pi)$	0.33	-3.5	-0.51

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- σ exchange and tensor force of π exchange are important to bind T_{bb}.

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- σ exchange and tensor force of π exchange are important to bind T_{bb}.
- (1,3), (3,1) and (3,3) are also important while they are not important for T_{cc}.$\rightarrow$ $B^*B^*({}^3S_1)$ is also important channel.

T_{bb} with other quantum numbers with g_σ dependence

g_σ	3.06	3.4	3.74
Λ[MeV]	1147.1	1069.8	1001.3
$0(0^-)$	-30.7	-24.4	-19.2
$0(1^+)$	-56.2	-46.0	-37.9
$0(1^-)$	-	-	-
$1(0^+)$	-3.70	-7.23	-10.8
$1(0^-)$	-	-	-
$1(1^+)$	-0.0254	-2.46	-6.98
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- Many bound states exist (Only $0(1^+)$ state is found for T_{cc}).
- As g_σ increases, $T_{bb}(I=0)$ becomes shallower, while $T_{bb}(I=1)$ becomes deeper. $\rightarrow$ π is important for $I=0$, σ is dominant for $I=1$.

Light-Cloud Basis in the heavy quark limit

- Hadronic-Molecule Basis (HMB) → Light-Cloud Basis (LCB)

$$\left[L\left[\left[S_{Q_1}S_{q_1}\right]_{S_1}\left[S_{Q_2}S_{q_2}\right]_{S_2}\right]_S\right]_J \to \left[\left[S_{Q_1}S_{Q_2}\right]_{S_Q}\left[L\left[S_{q_1}S_{q_2}\right]_{S_q}\right]_{J_l}\right]_J$$

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- In the light-cloud basis, the spin wave function of the hadronic molecule is decomposed into the sector of the **heavy quark spin** and the **light cloud**.
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- We can see the **spin multiplets** of the origins of the doubly heavy tetraquarks obtained by our analyses.

T_{QQ} with $0(1^+)$ in the heavy quark limit

$$V^{\mathrm{HM}}_{\pi,0(1^+)} = \begin{pmatrix} -C_\pi & \sqrt{2}T_\pi & 2C_\pi & \sqrt{2}T_\pi \\ \sqrt{2}T_\pi & -C_\pi - T_\pi & \sqrt{2}T_\pi & 2C_\pi - T_\pi \\ 2C_\pi & \sqrt{2}T_\pi & -C_\pi & \sqrt{2}T_\pi \\ \sqrt{2}T_\pi & 2C_\pi - T_\pi & \sqrt{2}T_\pi & -C_\pi - T_\pi \end{pmatrix}$$

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$$V^{\mathrm{LC}}_{\pi,0(1^+)} = \left(\begin{array}{c|cc|c} -3C_\pi & 0 & 0 & 0 \\ \hline 0 & C_\pi & 2\sqrt{2}T_\pi & 0 \\ 0 & 2\sqrt{2}T_\pi & C_\pi - 2T_\pi & 0 \\ \hline 0 & 0 & 0 & -3C_\pi \end{array}\right)$$

- (1,1) and (3,3) components
  $S_Q = 1,\ S_q = 0$
- (2,2) component
  $S_Q = 0,\ S_q = 1$

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  The ground state of T_{QQ} is the origin of T_{cc} and T_{bb} with $0(1^+)$.

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$\Rightarrow$ heavy diquark spin $S_Q = 0$, anti light diquark spin $S_q = 1$.

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⇒ heavy diquark spin $S_Q = 0$, anti light diquark spin $S_q = 1$.
⇒ T_{QQ} with $0(1^+)$ in the heavy quark limit belongs to the **HQS singlet**!!

The example of the heavy quark multiplet

We consider $0(0^-)$ and $0(1^-)$ states.

- $\psi^{\rm HM}_{0(0^-)} \to \psi^{\rm LC}_{0(0^-)}$

$$\left(\left|[PP^*]_+(^3P_0)\right\rangle\right) \to \left(-\left|\left[[QQ]_1\ \left[P\ [\bar{q}\bar{q}]_1\right]_1\right]_0\right\rangle\right)$$

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$$V^{\rm LC}_{\pi,0(0^-)} = (C_\pi + 2T_\pi)$$

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$\to T_{QQ}$ with $0(0^-)$ and $0(1^-)$ belong to the same **HQS multiplet**!!

T_{QQ} in the heavy quark limit

- $0(0^-)$ and $0(1^-)$ (Take $m_P = m_{P^*} = 5m_{B^*}$)

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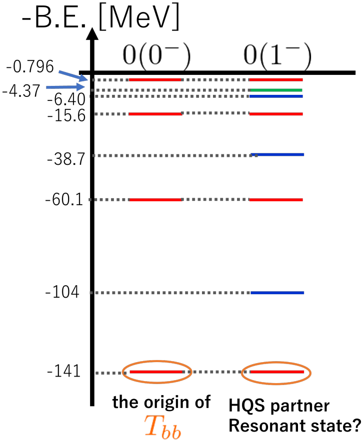

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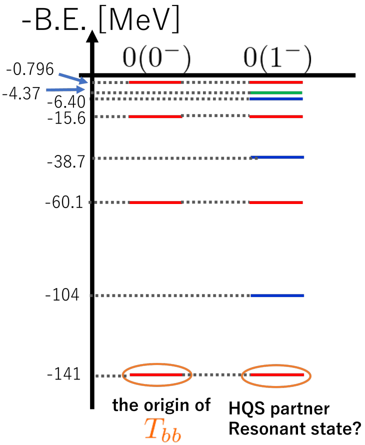

▶ Every bound state of T_{QQ} with $0(0^-)$ is degenerate with a certain bound state of T_{QQ} with $0(1^-)$.

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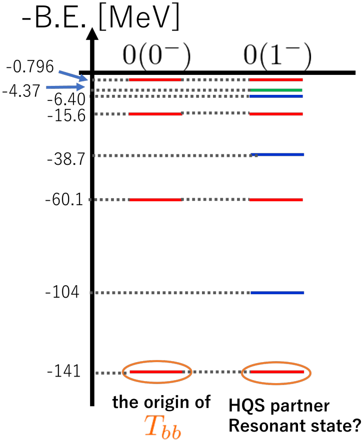

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- red line
  $S_Q = 1, S_q = 1, J_l = 1$
- blue line
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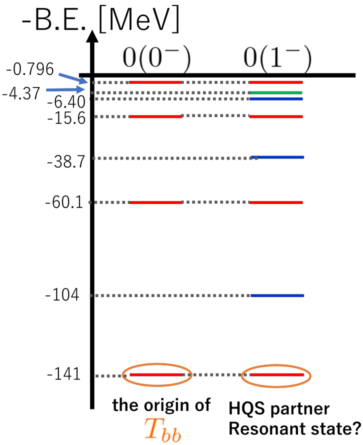

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- The origin of T_{bb} with $0(0^-)$ obtained by our analysis has the spin structure $(\mathbf{S_Q}, \mathbf{S_q}, \mathbf{J_l}) = (\mathbf{1}, \mathbf{1}, \mathbf{1})$.
  $\rightarrow$ The HQS partner with $0(1^-)$ may exist.

Summary

- We analyze T_{cc} as a hadronic molecule.
  - $I(J^P) = 0(1^+)$
    We determine the cut off parameter $\Lambda = 1069.8$ MeV.
    $[DD^*]$ $(^3S_1)$ is the dominant channel.
  - Other quantum numbers
    No bound state exists other than $0(1^+)$.
- We analyze T_{bb} as a hadronic molecule.
  - $I(J^P) = 0(1^+)$
    The binding energy of T_{bb} with $0(1^+)$ is $46.0\,\mathrm{MeV}$.
    $[BB^*]\_(^3S_1)$ is dominant and $B^*B^*(^3S_1)$ is also important.
  - Other quantum numbers
    Many bound states exist.
    g_σ dependence of the binding energy varies with the differences in the isospin.
- We analyze T_{QQ} in the heavy quark limit.
  The HQS partner for T_{QQ} with $0(1^+)$ does not exist.
  The HQS partner with $0(1^-)$ may exist.