Gravitational Form Factors in holographic QCD

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- AdS/CFT approach
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Form factor

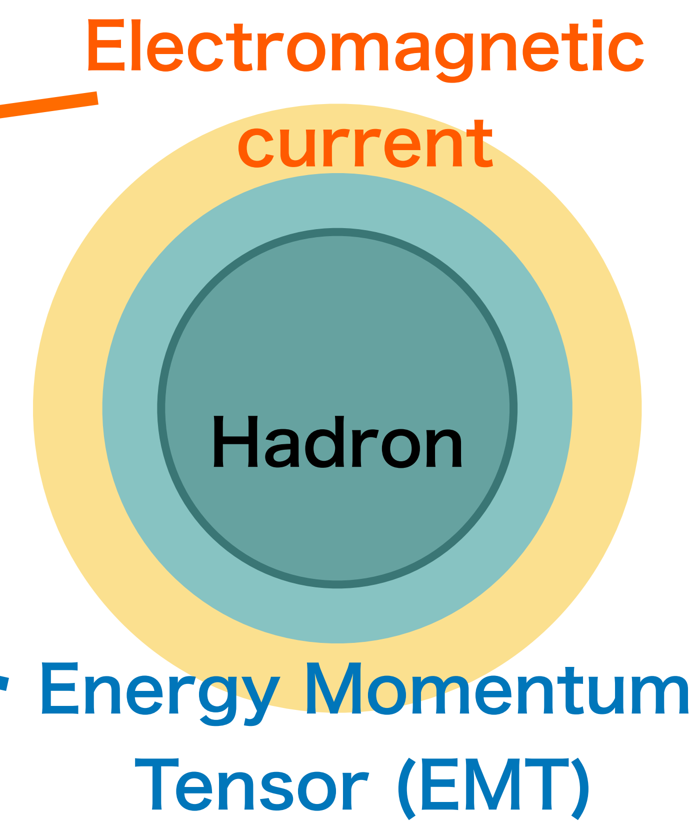
Form factor・・・**Fourier** transf. of **distributions**

～ Characterize the **current matrix element** of the particle with **internal structure**

▶ **Some distributions**

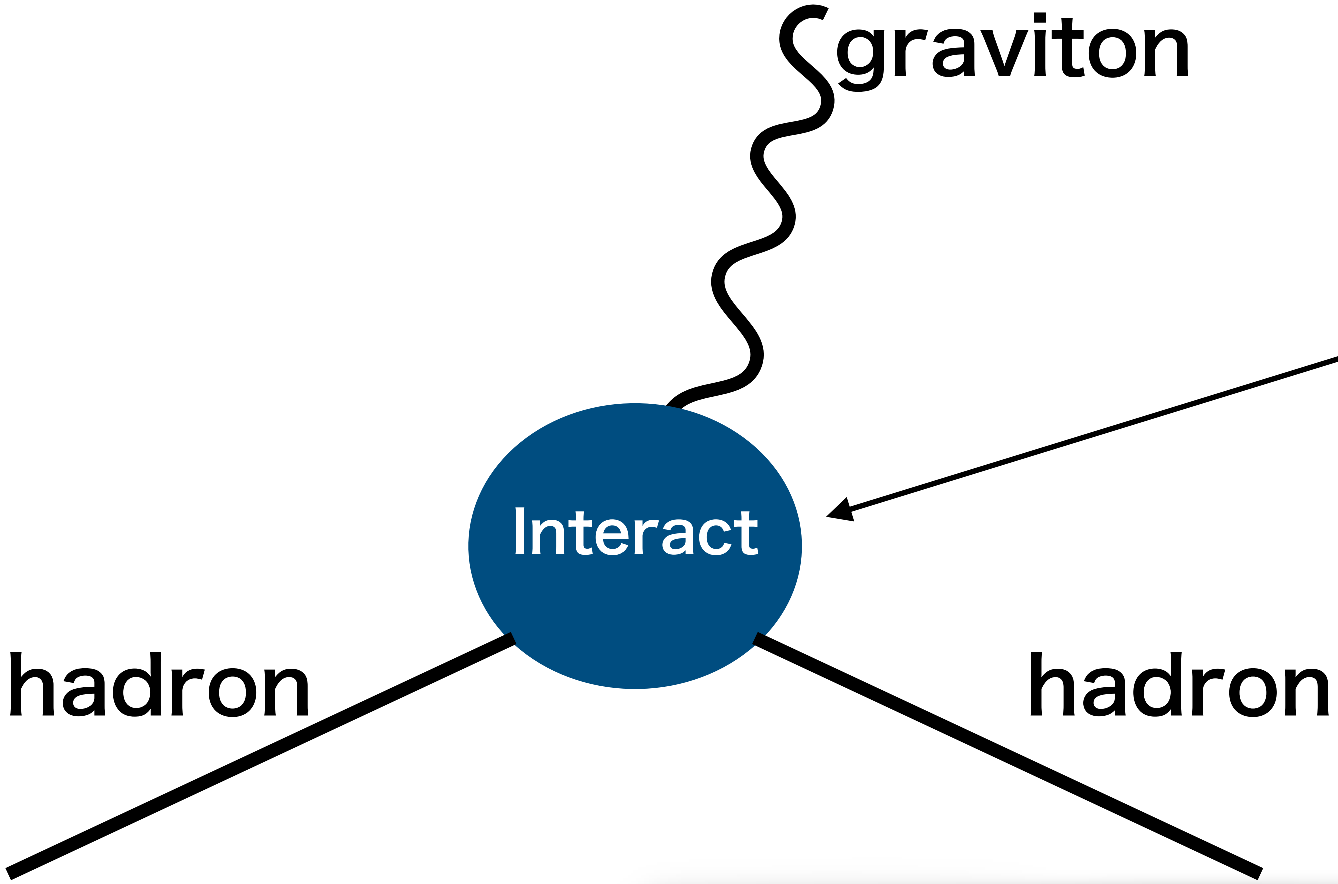
- **Electric** Charge distribution
- **Magnetic** distribution

- **Mass** distribution
- **Momentum** distribution
- **Stress** distribution (**Pressure** and **Shear** force)



Stress distribution **confines** quarks and gluon into hadrons

Gravitational Form Factors (GFFs)



GFFs

~Matrix element of EMT

Fourier Transf. →

- **Mass** distribution
- **Momentum** distribution
- **Spin** distribution
- **Stress** distribution (**pressure** and **shear**)

~Form factor $D(t)$

EMT and Stress distribution

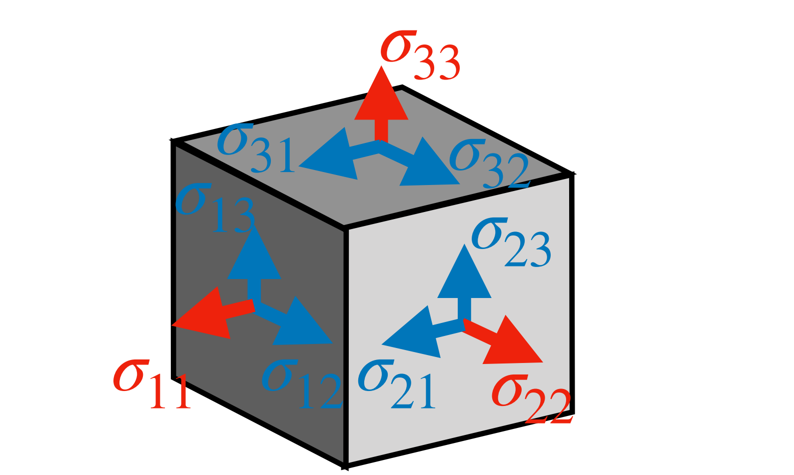
- Spatial components of EMT → Stress distribution

$$T_{\mu\nu} = \begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{pmatrix}$$

Energy, Momentum

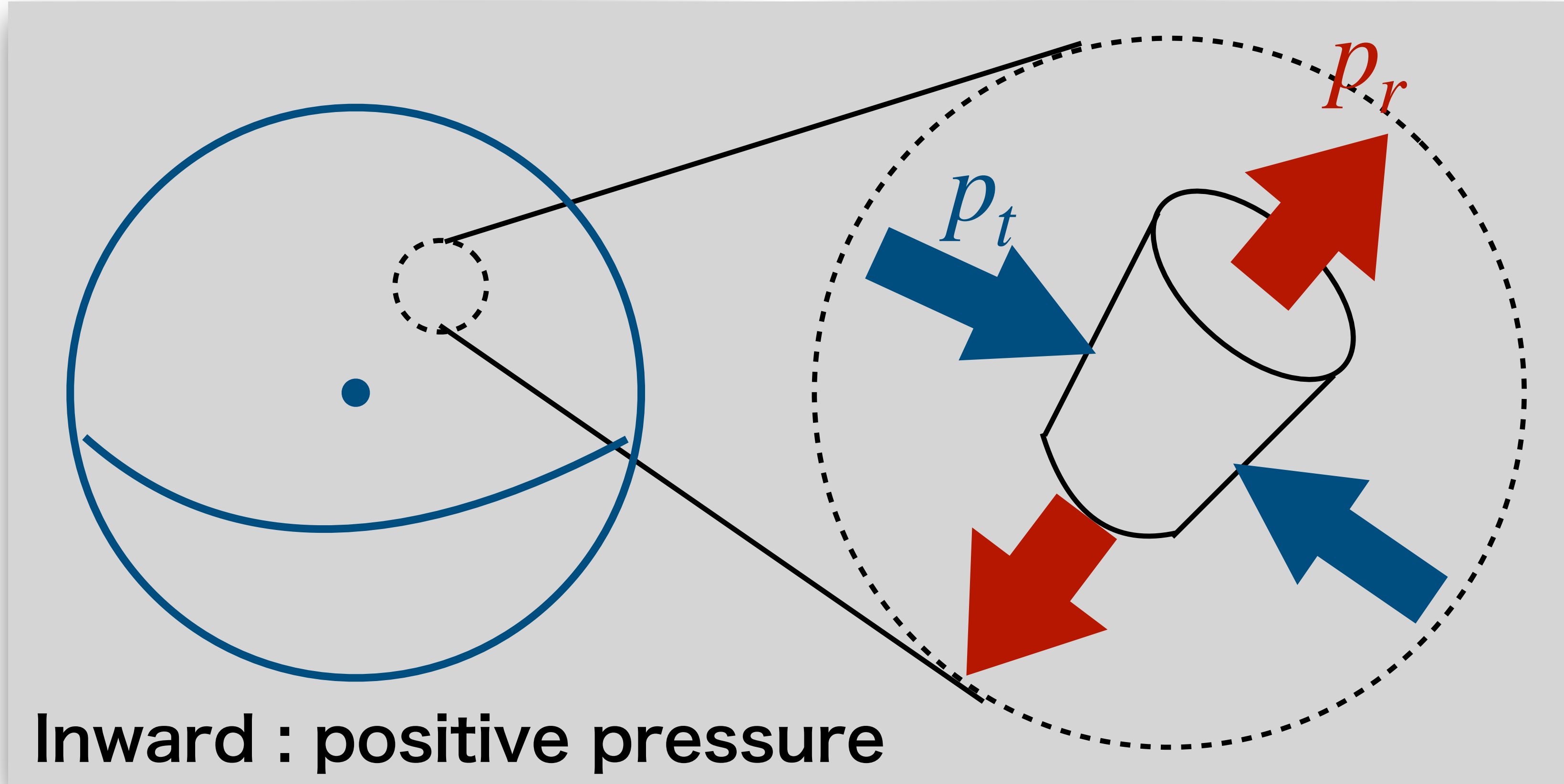
Shear force $s(r)$

Pressure $p(r)$



$$T_{ij} = -\sigma_{ij}$$

- Spherical system



$$T_{ij} = \begin{pmatrix} p_r & 0 & 0 \\ 0 & p_t & 0 \\ 0 & 0 & p_t \end{pmatrix}$$

p_r : Radial pressure

p_t : Tangential pressure

Stability condition

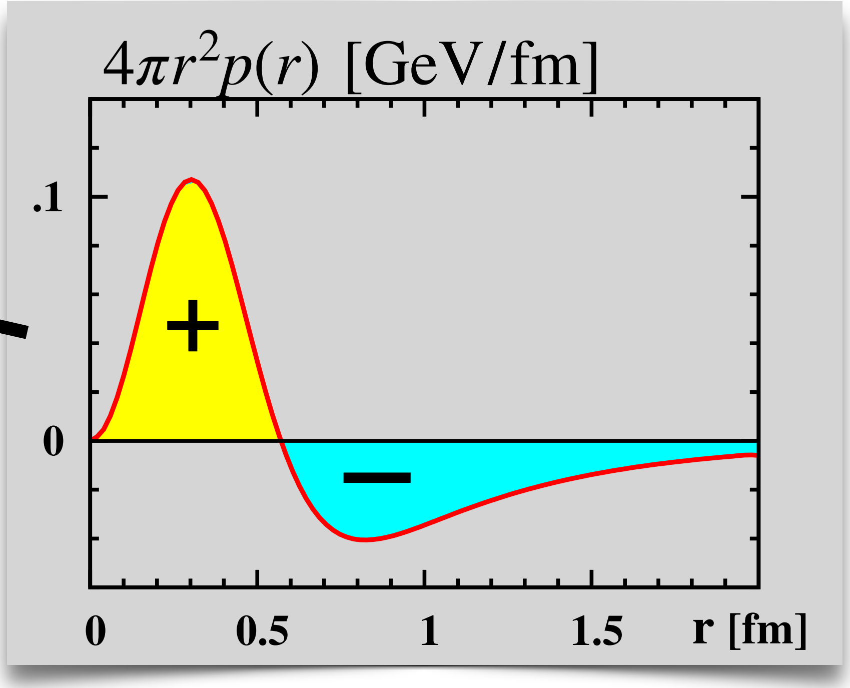
- Static EMT conservation $\nabla_i T^{ij} = 0$

→ $\int dr r^2 p(r) = 0$

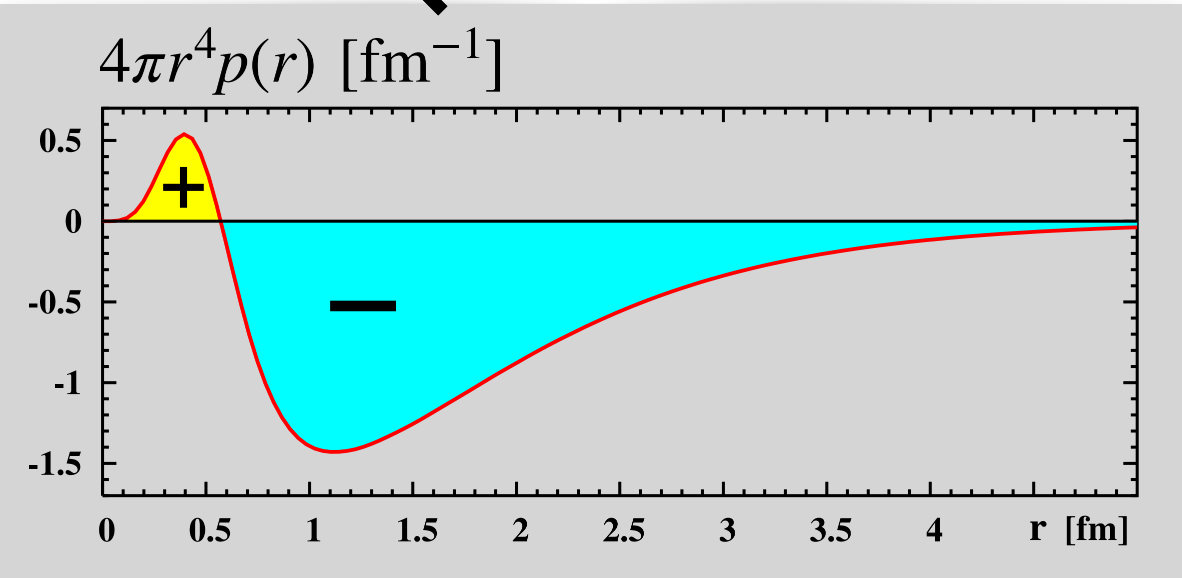
→ $D = D(0) \propto \int dr r^4 p(r) < 0$

$$= - \int dr r^4 s(r) < 0$$

→ $s(r) > 0$



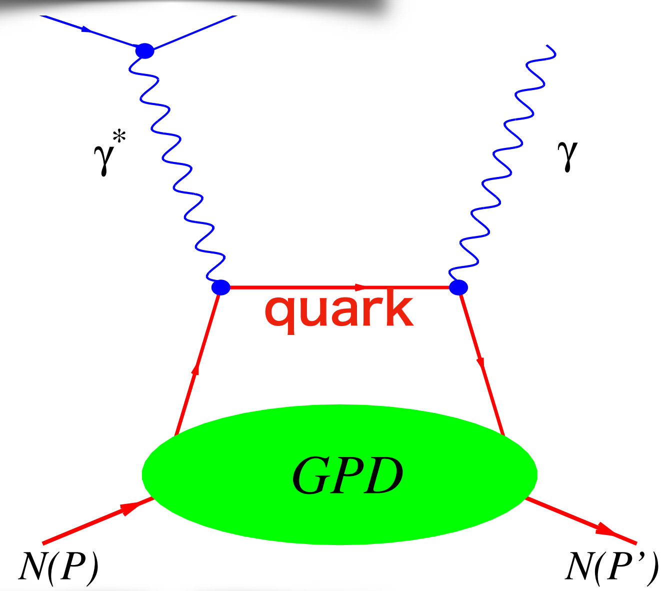
[Polyakov, Schweitzer (2018)]

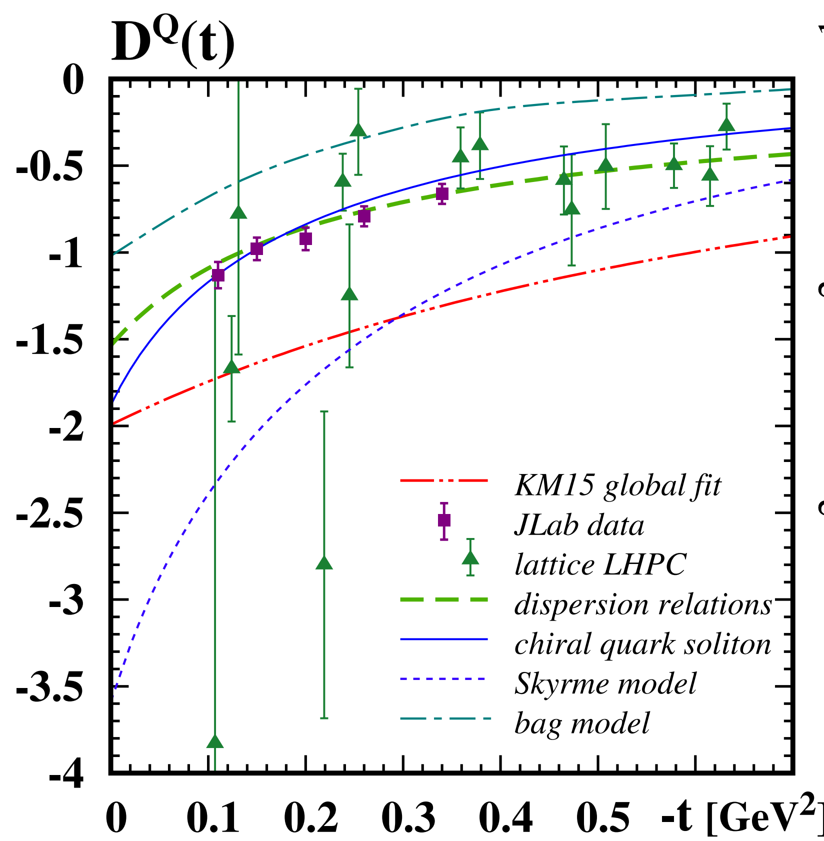


Experiment

Deeply Virtual Compton Scattering

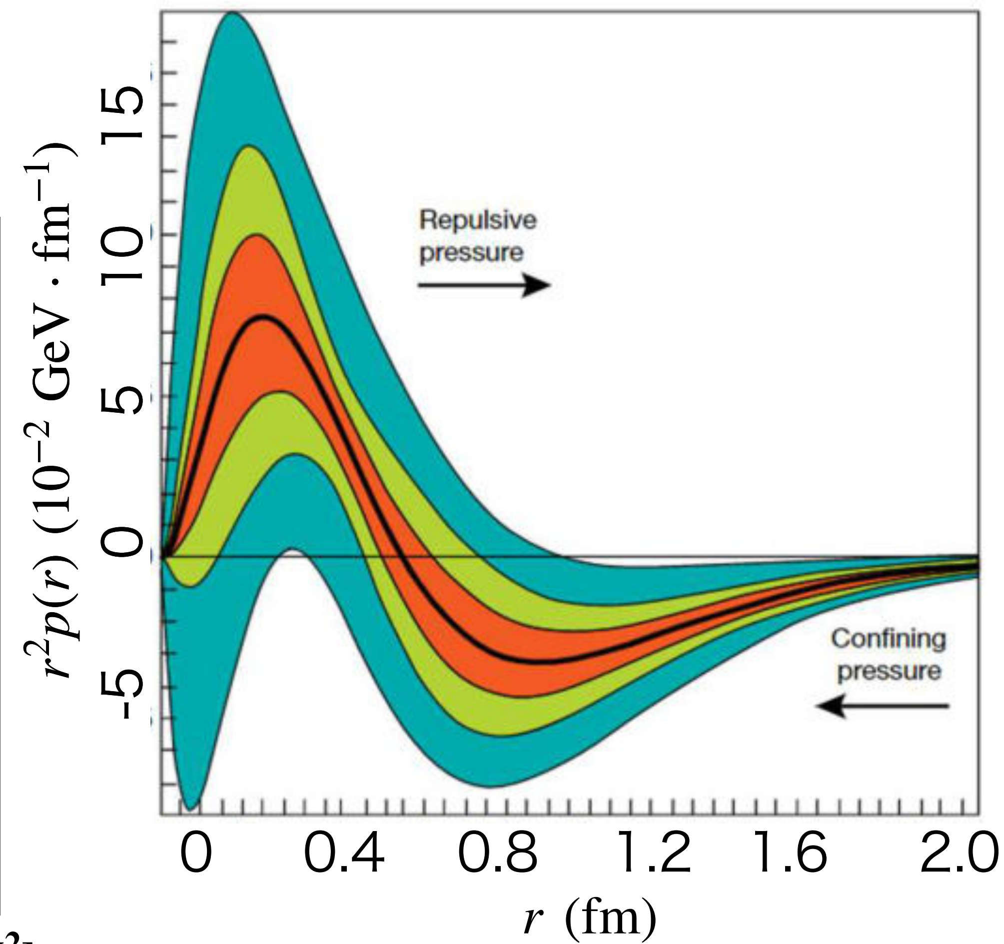
Nucleon



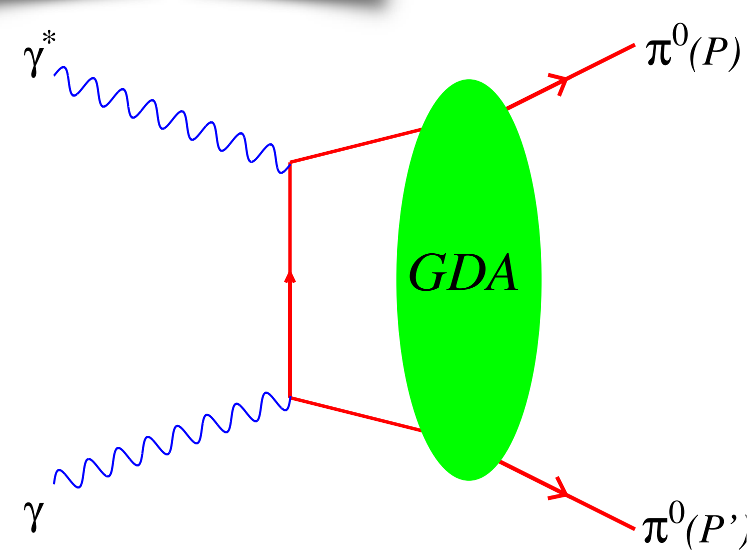


[Polyakov, Schweitzer (2018)]

[V. D. Burkert et al. (2018)]



Pion



▶ **Two GFFs for pion from GDA**

[Kumano et al. (2018)]

$$A^q(0) = 0.70$$

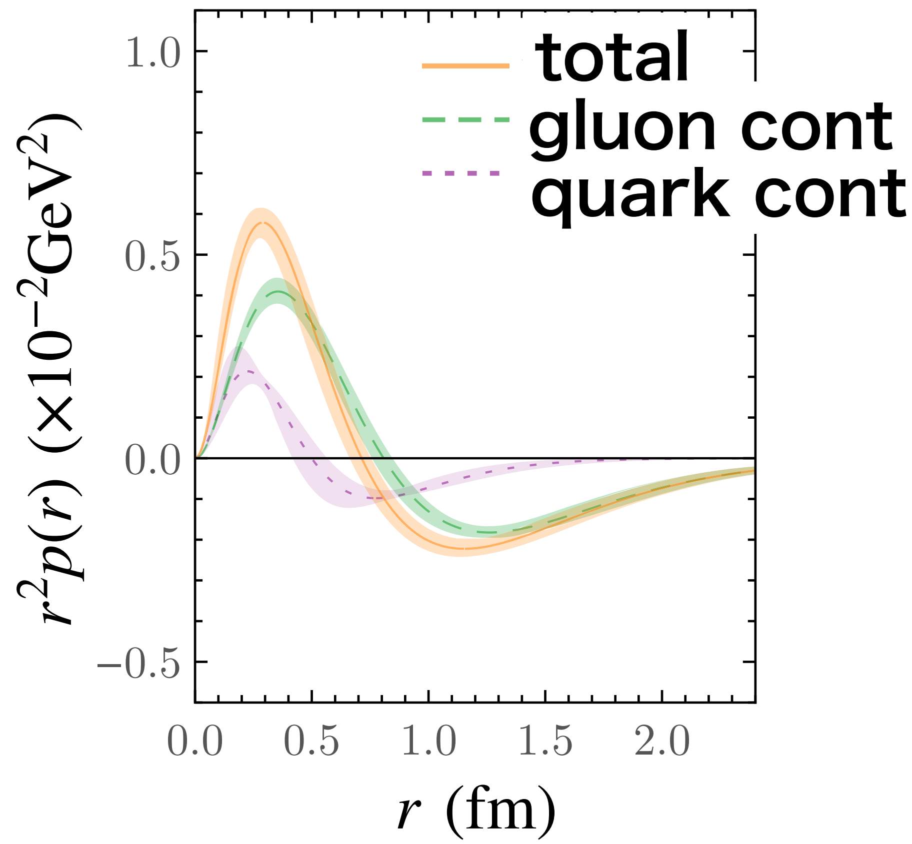
$$D^q(0) = -0.75 < 0$$

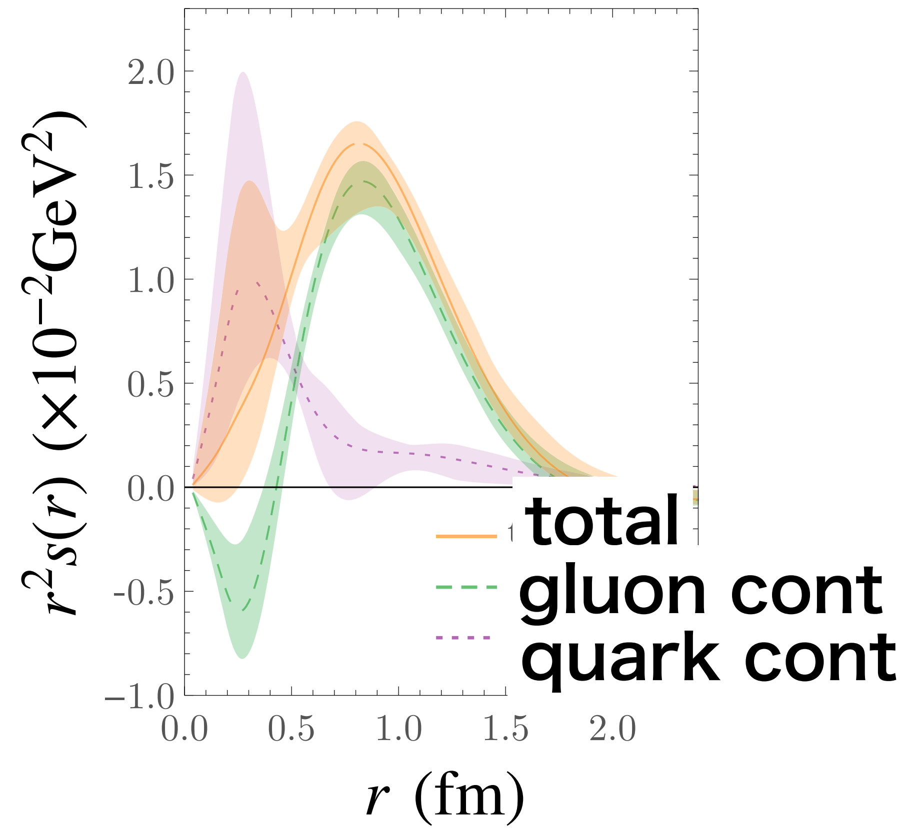
Stability

Lattice

Nucleon

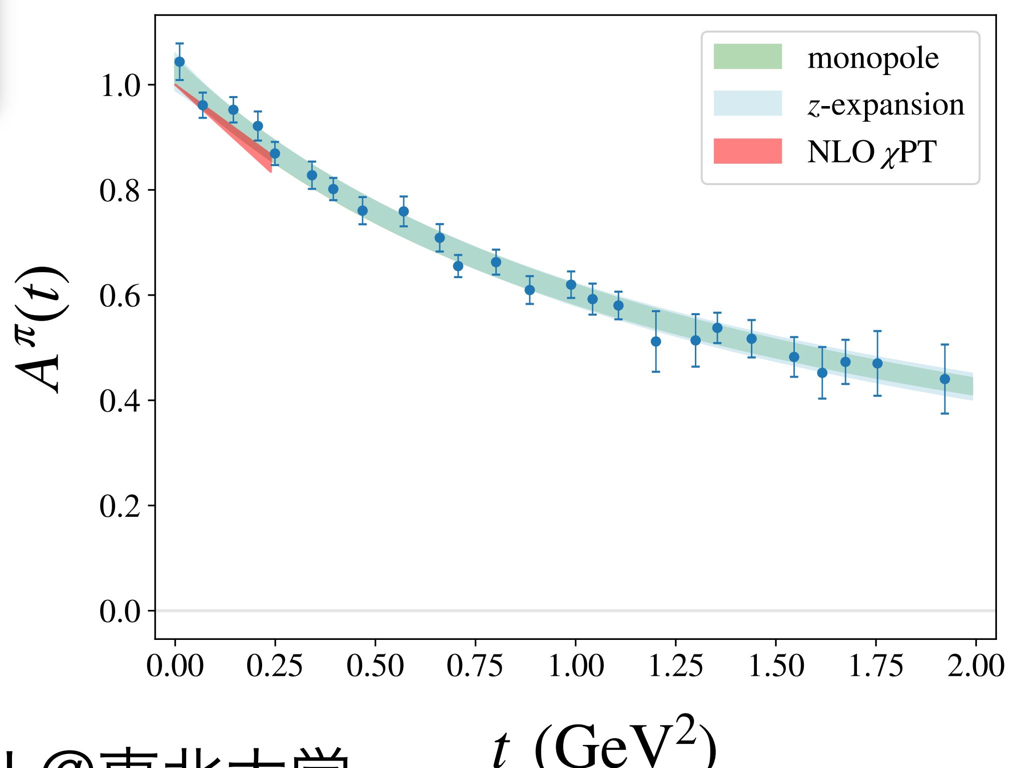
[Shanahan, Detmold (2019)]

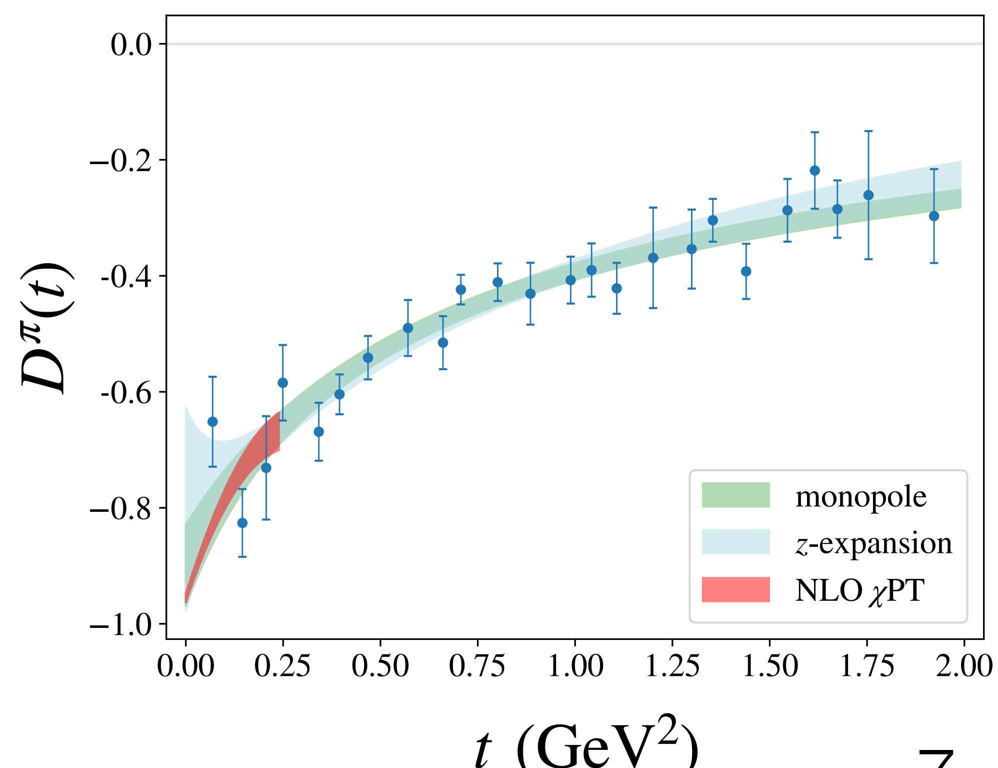




Pion

[Hackett, et al. (2023)]





Motivation

- Study **GFFs** for the **Vector meson** from **Holographic QCD** (Top down approach)
- The **First analysis** of GFFs for the vector meson from the **Top down approach**
- Reveal the relation between the **Stability condition** and **Hadron physics**

Investigate some aspects of **Confinement** and the role of **Chiral symmetry breaking**.

AdS/CFT approach

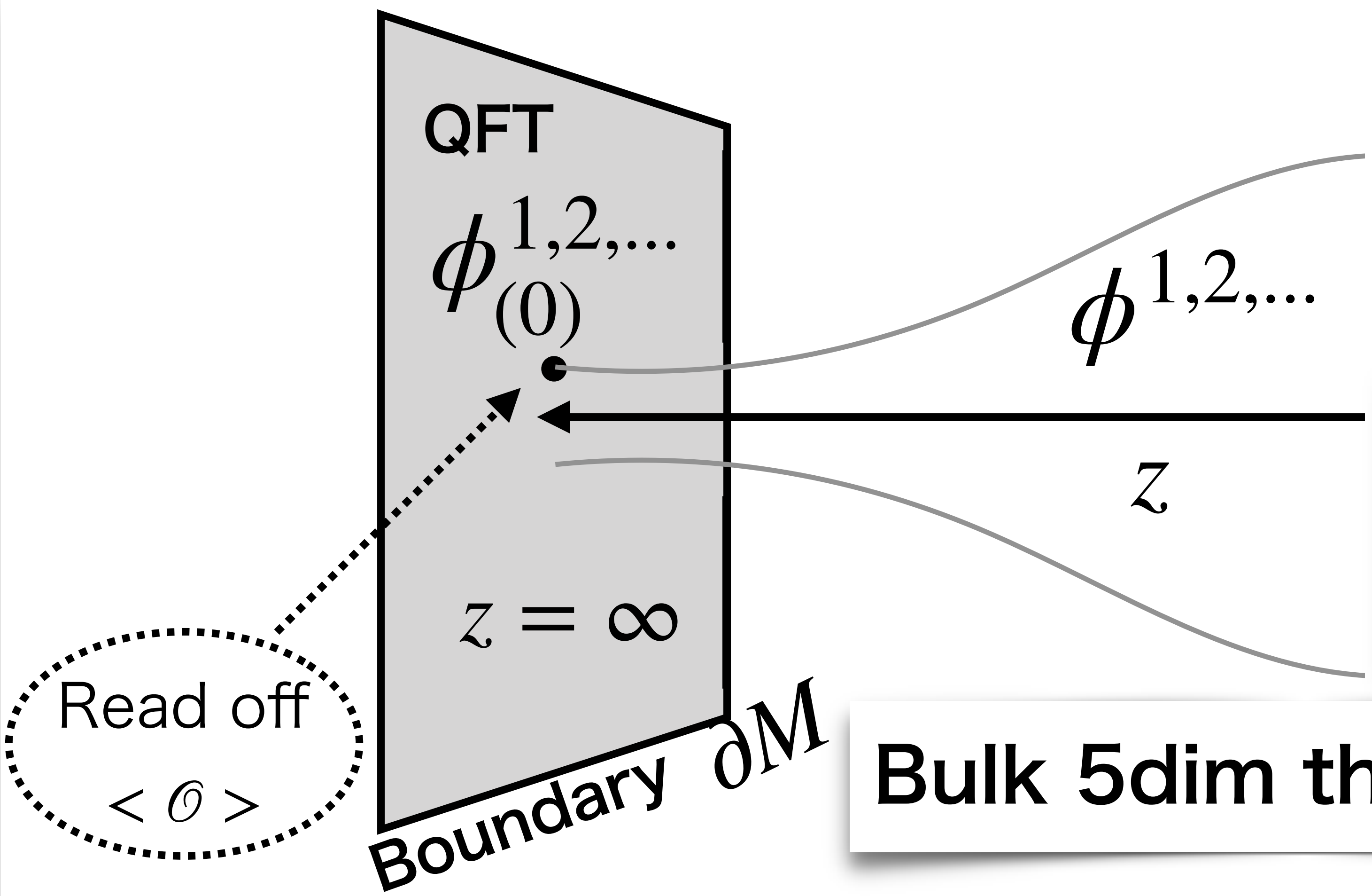
AdS/CFT correspondence

10d → 5d
Compactify

$$Z^{\rm 4d}_{\rm QFT} = Z^{\rm 10d}_{\rm string} = \exp\left(- \underline{S}^{\rm 5d}_{\rm (gravity+matter)} \right)$$

String length → 0
Classical limit

On-shell action



- **The n point function**

$$\langle \mathcal{O}^1 \mathcal{O}^2 \ldots \rangle = \frac{\delta \underline{S}^{\rm 5d}_{\rm On-shell}[\phi^1, \phi^2, \ldots]}{\delta \phi^1_{(0)} \delta \phi^2_{(0)} \ldots}$$

Bulk 5dim theory (M)

Sakai-Sugimoto model

- **All the contents of QCD are included**

  but contains redundant heavy matter fields (~1GeV)

Bulk action

$$S = -C \int \sqrt{-g}\, g^{MP} g^{NQ} \mathrm{Tr}\left(F_{MN} F_{PQ}\right)$$

$M = x^0, x^1, \ldots\ldots x^9$

$\mu = x^0, x^1, x^3, x^4$

$$= -\kappa \int \mathrm{Tr}\left(k(z)^{-1/3} F_{\mu\nu} F_{\mu\nu} + k(z) F_{\mu z} F_{\mu z}\right)$$

g_{MN} : Gravitational field $k(z) = 1 + z^2$: Metric **Only two parameter**

F_{MN} : Field strength of $SU(N_f)$ gauge fields

- **Two point function**

Decay constants for vector mesons

←Determined by **Metric**

$$i \int dx^4 e^{iq\cdot x} \langle 0 | \mathcal{T}(J_\mu J_\nu) | 0 \rangle = \sum_n \frac{g_n \psi_n(z)}{q^2 + m_n^2} \left(\eta_{\mu\nu} - q_\mu q_\nu / q^2\right)$$

Eigenfunctions

$\mathcal{T}$: Time ordering **q : momentum transfer**

J_μ : Chiral current

Mass of vector meson =

Eigenvalues of **bulk EoM**

Results

Matrix elements for EMT

$$\langle 0 | J_V^\alpha T_{\mu\nu} J_V^\beta | 0 \rangle \longrightarrow \textbf{GFFs}$$

J_V^α : Vector currents $T_{\mu\nu}$: EMT

- Form factor $D(t)$

Coupling constants of **glueball-vector meson**×2

Decay constants of n's **glueball**

$$D(q) = -A(m_V, q) \sum_n^\infty \frac{\alpha_n g_n^{gVV}}{q^2 + M_n^2}$$

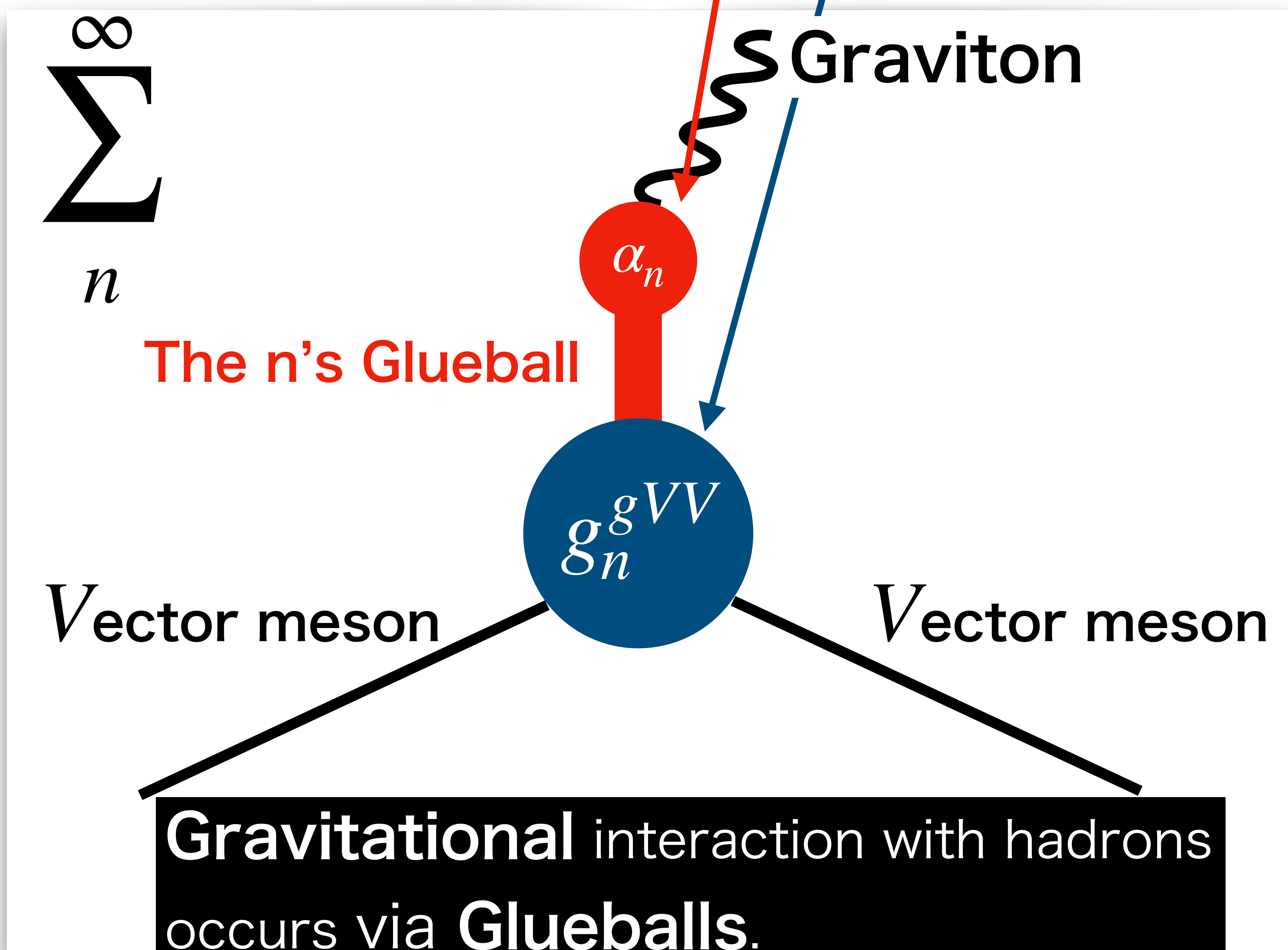
All values are determined by the **metric**

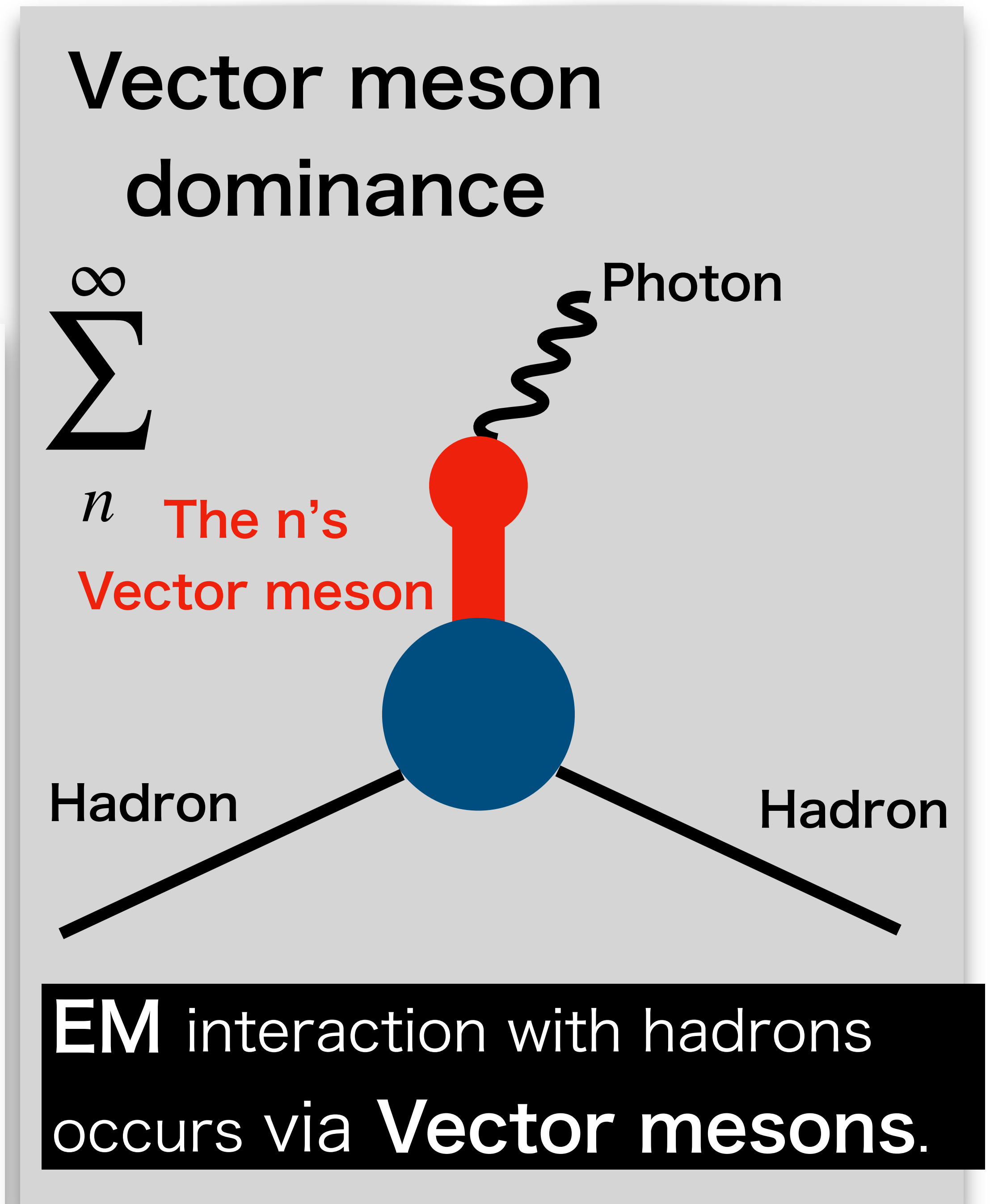
The n's glueball mass

m_V : Vector meson mass
q : momentum transfer

Glueball dominance

$$(\text{GFFs}) \propto \sum_{n}^{\infty} \frac{\alpha_n g_n^{gVV}}{q^2 + M_n^2}$$





D-term and Sum rule

- **D-term**

$$D = D(0) = -A(m_\rho, q=0)\sum_n^\infty \frac{\alpha_n g_n^{gVV}}{M_n^2}$$

$$= -A(m_\rho, q=0) \times C < 0$$

Stability condition

- **Sum rule**

$$\sum_n^\infty \frac{\alpha_n g_n^{gVV}}{M_n^2} = C \text{ (const.)}$$

We connect the **stability** and **hadron spectra**

Summary and Outlook

- To approach the understanding of **Confinement** from the **Onset mechanism** of **Stress distribution** inside hadrons
- **The first attempt** to determine the **GFFs** of a **Meson** using the **Top down approach**
- **Gravitational interaction** with hadrons is **via Glueballs** (Glueball dominance)
- We find a relation between **hadron spectra** and **stability**.

■ Further calculation

■ Axial sector

■ More relation of hadron physics

Thank you for your attention