Molecular states of $D^*D^*\bar{K}^*$ and $B^*B^*K^*$ nature

Natsumi Ikeno
(Tottori University)

Melahat Bayar, Luis Roca, Eulogio Oset

N. Ikeno, M. Bayar and E. Oset, Phys. Rev. D 107, 034006 (2023)
M. Bayar, N. Ikeno and L. Roca, Phys. Rev. D 107, 054042 (2023)

鳥取大学
Tottori University

Exotic hadrons

Many exotics have been observed since the discovery of X(3872)

- $Z_c(3900)$: close to $D\bar{D}^*$, $c\bar{q}q\bar{c}$ $(q = u, d)$
- $Z_{cs}(4000)$: close to $\bar{D}_s^* D / \bar{D}_s D^*$, $c\bar{q}s\bar{c}$
- $X_0(2900)$, $X_1(2900)$ [$T_{cs0}(2900)$, $T_{cs1}(2900)$] : close $D^* \bar{K}^*$, $c\bar{q}s\bar{q}$
- $T_{cc}(3875)$: close to DD^*, $c\bar{q}c\bar{q}$

clearly exotic mesonic structure

....

=> Cannot be explained as the ordinary mesons $q\bar{q}$

Many possible types of hadronic structures were proposed:
Tetraquarks, Meson-meson molecules, ...

These findings open the door to the formation of few-body systems with several open heavy quarks like ccs, ccc, bbs, etc.

Our study: $D^*D^*\bar{K}^*$ and $\bar{B}^*\bar{B}^*\bar{K}^*$ states based on the molecular picture

Molecular states of $D^* D^* \bar{K}^*$ and $\bar{B}^* \bar{B}^* \bar{K}^*$ nature

$D^* D^* \bar{K}^*$ system [$c\bar{q}c\bar{q}s\bar{q}$] : Very Exotic system with **ccs** open quarks

- $D^* \bar{K}^*$ interaction: R. Molina, E. Oset, PLB811 (2020)
  The $J^P = 0^+$ bound state is identified with the $X_0(2900)$

- $D^* D^*$ interaction: L. R. Dai, R. Molina, E. Oset, PRD 105 (2022)
  Bound state in I = 0 and $J^P = 1^+$
  (using the same q_{max} of D^*D interaction fixed by the $T_{cc}(3875)$ data
  A. Feijoo, W. H. Liang, E. Oset, PRD104 (2021).)

$\bar{B}^* \bar{B}^* \bar{K}^*$ system [$b\bar{q}b\bar{q}s\bar{q}$] : Very Exotic system with **bbs** open quarks

- $\bar{B}^* \bar{K}^*$ interaction: E. Oset and L. Roca, EPJ.C 82 (2022)

- $\bar{B}^* \bar{B}^*$ interaction: L. R. Dai, E. Oset, A. Feijoo, R. Molina, L. Roca, A. M. Torres and K. P. Khemchandani, PRD 105 (2022)

=> A search for possible bound states of the three-body system

Recent studies of three body systems of molecular nature

- DDK T. W. Wu et al., PRD100(2019): A. Martinez Torres, et al., PRD99(2019): Y. Huang et al., PRD101(2020).
- $D\bar{D}^*K$ X. L. Ren et al., PLB785, 112 (2018).
- DD^*K L. Ma et al., , Chin. Phys. C43, 014102 (2019).
- $D\bar{D}K$ T. W. Wu, M. Z. Liu, L.S. Geng, et al., PRD103, L031501 (2021): X. Wei, Q. H. Shen, J. J. Xie, EPJC82 (2022)
- $D^*D^*D^*$ S. Q. Luo, W. Tian-Wei, M. Z. Liu, L. S. Geng, X. Liu, PRD105 (2022): M. Bayar, A. Martinez Torres, K. P. Khemchandani, R. Molina, E. Oset, EPJC83 (2023)
- $\bar{K}B^*B^*$ M. P. Valderrama, PRD98, 014022 (2018)
- $\bar{K}^{(*)}B^{(*)}\bar{B}^{(*)}$ X. L. Ren and Z. F. Sun, PRD 99, 094041 (2019)
- T_{bbb} as BBB H. Garcilazo, A. Valcarce, PLB784, 169(2018)

$\Rightarrow$ Contain $cc\bar{s}$ or $c\bar{c}\bar{s}$

Method to solve the three-body system:

- Gaussian expansion method
- **Fixed center approximation (FCA) to the Faddeev equation**

=> Both methods gave similar results in the case of $D\bar{D}K$

Also in the case of $D^*D^*D^*$

Fixed Center Approximation (FCA) to the Faddeev equation

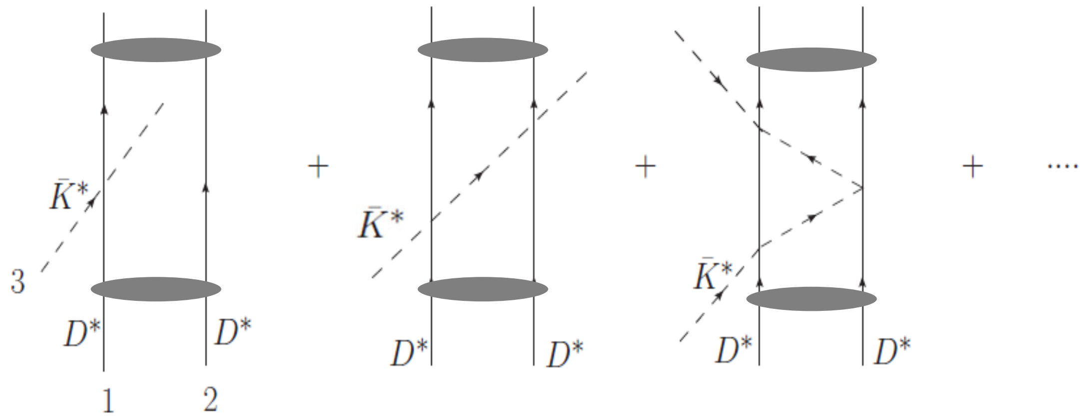
There is a cluster of two bound particles D^*D^* and the third one ($\bar{K}^*$) collides with the components of this cluster without modifying the D^*D^* wave function.

L. Roca and E. Oset, PRD82, 054013 (2010)

Total three-body scattering amplitude T

$$T \equiv T_1 + T_2$$

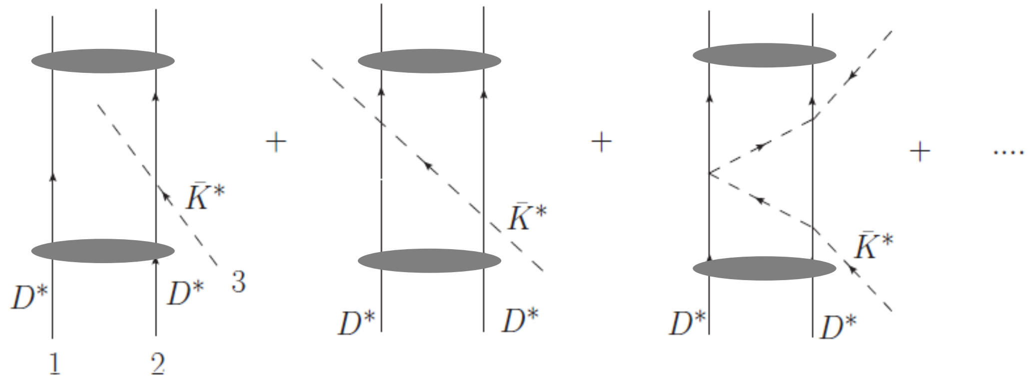
$$T_1 = t_1 + t_1 G_0 T_2,$$
$$T_2 = t_2 + t_2 G_0 T_1,$$



t_i is the scattering amplitude for $D^*(i)\bar{K}^*$

G_0 is the $\bar{K}^*$ propagator folded with the cluster wave function

$$G_0 = \frac{1}{2m_C}\int \frac{d^3q}{(2\pi)^3} F(\vec{q}) \frac{1}{q^{02} - \vec{q}^{\,2} - m_{\bar{K}^*}^2 + i\epsilon}$$



The form factor F(q) encodes the information about the D^*D^* bound state:

$$F(\vec{q}) = \int d^3\vec{r}\, e^{-i\vec{q}\cdot\vec{r}}\Psi_c^2(\vec{r}) = \frac{1}{N}\int_{\substack{p<q_{\max} \\ |\vec{p}-\vec{q}|<q_{\max}}} d^3p \frac{1}{m_C - \sqrt{m_{D^*}^2 + \vec{p}^{\,2}} - \sqrt{m_{D^*}^2 + \vec{p}^{\,2}}} \, \frac{1}{m_C - \sqrt{m_{D^*}^2 + (\vec{p}-\vec{q})^2} - \sqrt{m_{D^*}^2 + (\vec{p}-\vec{q})^2}}$$

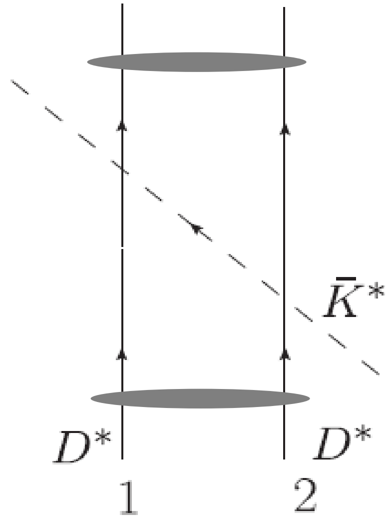
Normalization of the amplitudes

L. Roca and E. Oset, PRD82, 054013 (2010)

- S matrix in the diagram of double scattering:

$$S^{(2)} = -i(2\pi)^4\delta^4(p_{\rm fin}-p_{\rm in})\frac{1}{\mathcal{V}^2}\frac{1}{\sqrt{2\omega_{\bar K^*}}}\frac{1}{\sqrt{2\omega_{\bar K^*}}}\frac{1}{\sqrt{2\omega_{D^*}}}\frac{1}{\sqrt{2\omega_{D^*}}}\frac{1}{\sqrt{2\omega_{D^*}}}\frac{1}{\sqrt{2\omega_{D^*}}}t_1t_2$$
$$\int\frac{d^3q}{(2\pi)^3}F(\vec q)\frac{1}{q^{0^2}-\vec q^{\,2}-m^2_{\bar K^*}+i\epsilon}$$

where F(q) is the form factor of the cluster



- Macroscopic perspective of $(D^*(1)D^*(2))_c\ \bar K^*$

$$S^{(2)} = -i(2\pi)^4\delta^4(p_{\rm fin}-p_{\rm in})\frac{1}{\mathcal{V}^2}\frac{1}{\sqrt{2\omega_{\bar K^*}}}\frac{1}{\sqrt{2\omega_{\bar K^*}}}\frac{1}{\sqrt{2\omega_C}}\frac{1}{\sqrt{2\omega_C}}T^{(2)}$$

$$=> \quad T^{(2)} = \frac{2\omega_C}{2\omega_{D^*}}\frac{2\omega_C}{2\omega_{D^*}}\frac{1}{2\omega_C}t_1t_2\int\frac{d^3q}{(2\pi)^3}F(\vec q)\frac{1}{q^{0^2}-\vec q^{\,2}-m^2_{\bar K^*}+i\epsilon}$$

It is convenient to write the partition functions suited to the macroscopic formalism as

$$\tilde T_1 = \tilde t_1 + \tilde t_1\tilde G_0\tilde T_2$$
$$\tilde T_2 = \tilde t_2 + \tilde t_2\tilde G_0\tilde T_1$$

by defining $\tilde t_1 = \frac{2m_C}{2m_{D^*}}t_1 \qquad \tilde t_2 = \frac{2m_C}{2m_{D^*}}t_2$

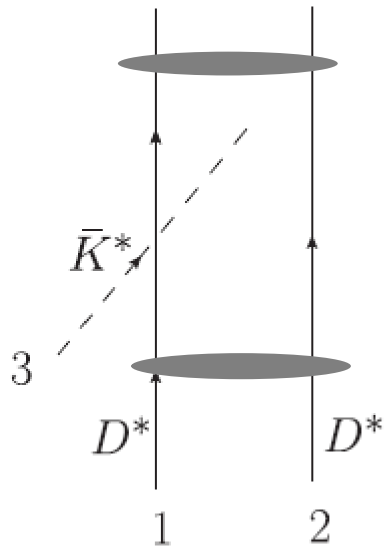
In this case, $t_1=t_2$, then $T_1=T_2$

$$\tilde T_1 = \tilde t_1 + \tilde t_1\tilde G_0\tilde T_1;\quad \tilde T_1 = \frac{1}{\tilde t_1^{\,-1}-\tilde G_0};\qquad \tilde T = \tilde T_1+\tilde T_2 = 2\tilde T_1$$

Consideration of the isospin and spin of the $D^*\bar{K}^*$ amplitudes t_1

Cluster: D^*D^* bound state in I = 0 and $J^P = 1^+$ L. R. Dai, R. Molina, E. Oset, PRD 105 (2022)

$$|D^*D^*, I=0\rangle = -\frac{1}{\sqrt{2}}(D^{*+}D^{*0} - D^{*0}D^{*+})$$



- Isospin considerations:

To make a connection with the $D^*\bar{K}^*$ isospin amplitudes,
we combine the third component of $D^*(1)$ with the one of $\bar{K}^*$

$|I(D^*(1)\bar{K}^*), I_3(D^*(1)\bar{K}^*)\rangle|I_3(D^*(2))\rangle$ with I_3= 1/2 of $\bar{K}^*$

$$\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\left(\left\langle D^*D^*,\frac{1}{2},-\frac{1}{2}\right| - \left\langle D^*D^*,-\frac{1}{2},\frac{1}{2}\right|\right)\left\langle \bar{K}^*,\frac{1}{2}\right|t_1\left(\left|D^*D^*,\frac{1}{2},-\frac{1}{2}\right\rangle - \left|D^*D^*,-\frac{1}{2},\frac{1}{2}\right\rangle\right)\left|\bar{K}^*,\frac{1}{2}\right\rangle$$

$$t_1 = \frac{3}{4}t^{I=1}_{D^*\bar{K}^*} + \frac{1}{4}t^{I=0}_{D^*\bar{K}^*}$$

- Spin consideration: three total spins J=0, 1, 2 for $D^*D^*\bar{K}^*$

For J=0 $\quad t_1 = t^{j=1}_{D^*\bar{K}^*}$

For J=1 $\quad t_1 = \frac{1}{4}\left(\frac{4}{3}t^{j=0}_{D^*\bar{K}^*} + t^{j=1}_{D^*\bar{K}^*} + \frac{5}{3}t^{j=2}_{D^*\bar{K}^*}\right).$

For J=2 $\quad t_1 = \frac{1}{4}t^{j=1}_{D^*\bar{K}^*} + \frac{3}{4}t^{j=2}_{D^*\bar{K}^*}$

Consideration of the isospin and spin of the $D^*\bar{K}^*$ amplitudes

Combining the isospin and the spin decomposition of the amplitudes, we find the final contributions

For J=0 $$t_1 = \frac{3}{4} t^{I=1,\ j=1} + \frac{1}{4} t^{I=0,\ j=1}$$

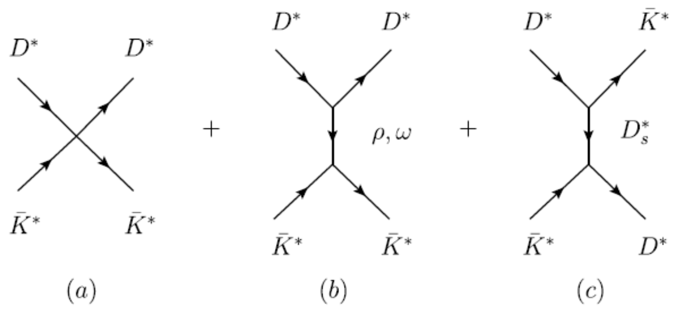
For J=1 $$t_1 = \frac{1}{16}\left\{5t^{I=1,\ j=2} + 3t^{I=1,\ j=1} + 4t^{I=1,\ j=0} + \frac{5}{3}t^{I=0,\ j=2} + t^{I=0,\ j=1} + \frac{4}{3}t^{I=0,\ j=0}\right\}$$

For J=2 $$t_1 = \frac{1}{16}\left\{9t^{I=1,\ j=2} + 3t^{I=1,\ j=1} + 3t^{I=0,\ j=2} + t^{I=0,\ j=1}\right\}$$

- The $D^*\bar{K}^*$ amplitude t for the different I, j states

R. Molina, T. Branz, and E. Oset, PRD82(2010) 014010
R. Molina, E. Oset, PLB811 (2020)

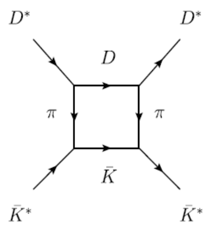
Bethe-Salpeter eq. $$t_1 = \frac{1}{V^{-1} - G_{D^*\bar{K}^*}}$$



The interaction V of $D^*\bar{K}^*$ in I=0 is attractive

$I(J^P)$	M[MeV]	Γ[MeV]	Coupled channels	state
$0(2^+)$	2775	38	$D^*\bar{K}^*$	?
$0(1^+)$	2861	20	$D^*\bar{K}^*$	?
$0(0^+)$	2866	57	$D^*\bar{K}^*$	$X_0(2866)$

Decay width

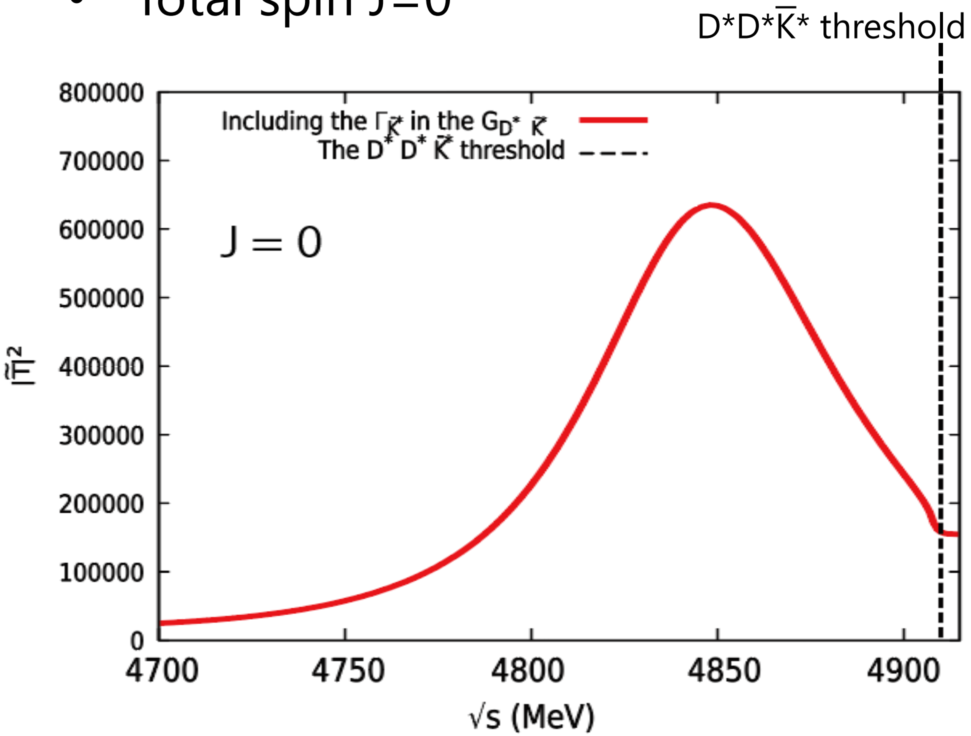


In I = 1, V is repulsive=> No bound state

Bound states of $D^* D^* \bar{K}^*$

N. Ikeno, M. Bayar, E. Oset,
PRD107, 034006 (2023)

- Total spin J=0



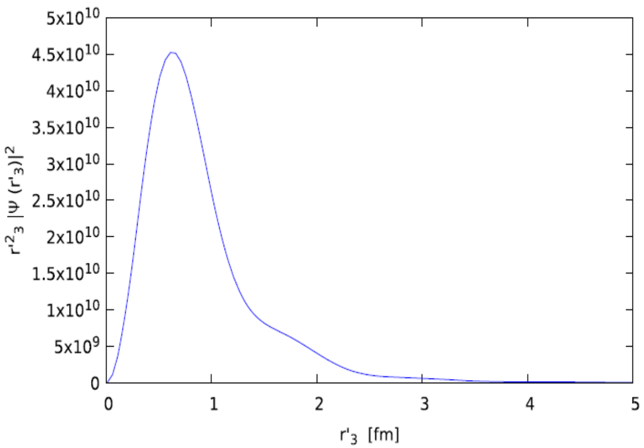
- A clear peak around 4845 MeV, about 61 MeV below the $D^*D^*\bar{K}^*$ threshold

  D*D* is bound by about 4-6 MeV
  D*K̄* is bound by about 30 MeV
  => The interaction of K̄* with two D* would give rise to a binding about twice as big as the one of D*K̄*

- The width is around 80 MeV

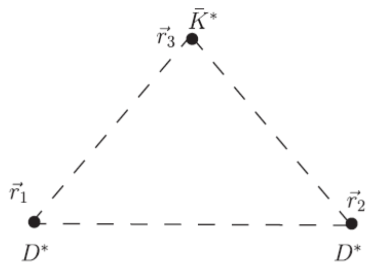
- Wave function for the K̄* in the D*D*K̄* system at rest.

D. Gamermann, J. Nieves, E. Oset,
E. Ruiz Arriola, PRD 81(2010)014029



$$|\Psi(r_3')|^2 = \int d^3r_1 d^3r_2 (|\phi(\vec{r}_{31})|^2 + |\phi(\vec{r}_{32})|^2)|\phi'(\vec{r}_{12})|^2 \times \delta^3(m_{D^*}\vec{r}_1 + m_{D^*}\vec{r}_2 + m_{\bar{K}^*}\vec{r}_3),$$

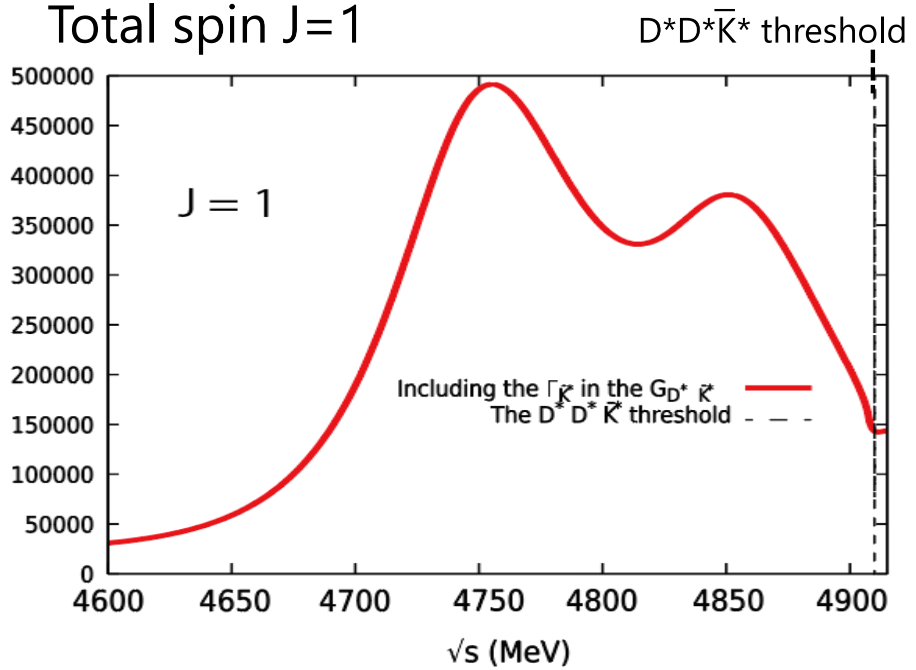
- A peak around 0.7 fm
- The mean square radius ~ 1 fm
  Bigger than that of the proton (0.84 fm),
  Smaller than that of the deuteron (2.1 fm)



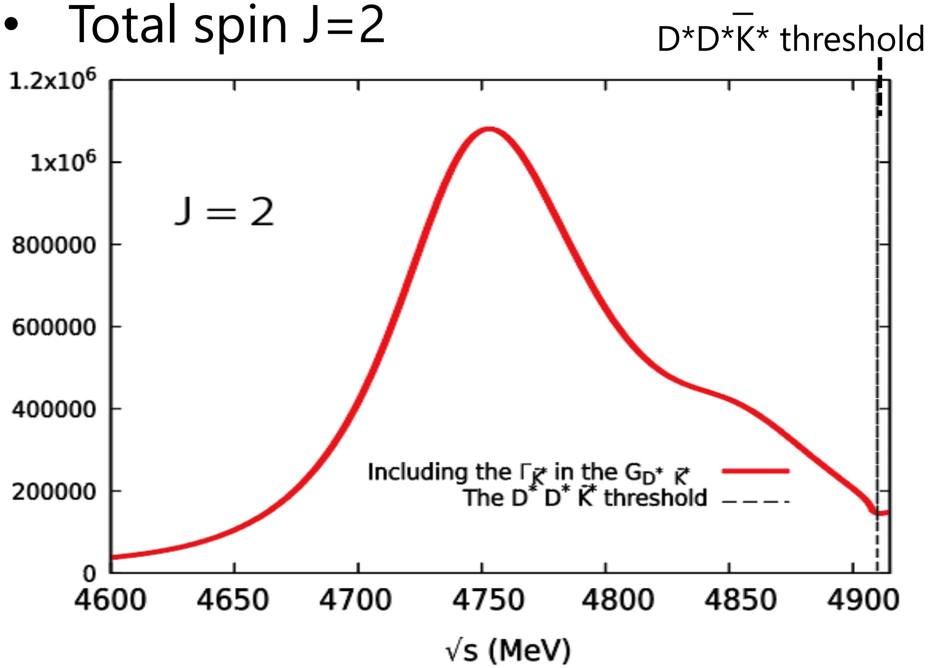
Bound states of $D^* D^* \bar{K}^*$

N. Ikeno, M. Bayar, E. Oset,
PRD107, 034006 (2023)

- Total spin J=1



- Total spin J=2



We see two peaks, indicating
two states

=>Easy to trace the origin of the peaks

- Total spin J=1 case
  First peak (higher energy) is due to $t^{I=0,j=0,1}$
  Second peak is due to $t^{I=0,j=2}$
- Total spin J=2 case
  First peak (higher energy) is due $t^{I=0,j=1}$
  Second peak is due to $t^{I=0,j=2}$

Spin consideration:

For J=1 $t_1 = \frac{1}{4}\left(\frac{4}{3}t^{j=0}_{D^*\bar{K}^*} + t^{j=1}_{D^*\bar{K}^*} + \frac{5}{3}t^{j=2}_{D^*\bar{K}^*}\right).$

For J=2 $t_1 = \frac{1}{4}t^{j=1}_{D^*\bar{K}^*} + \frac{3}{4}t^{j=2}_{D^*\bar{K}^*}$

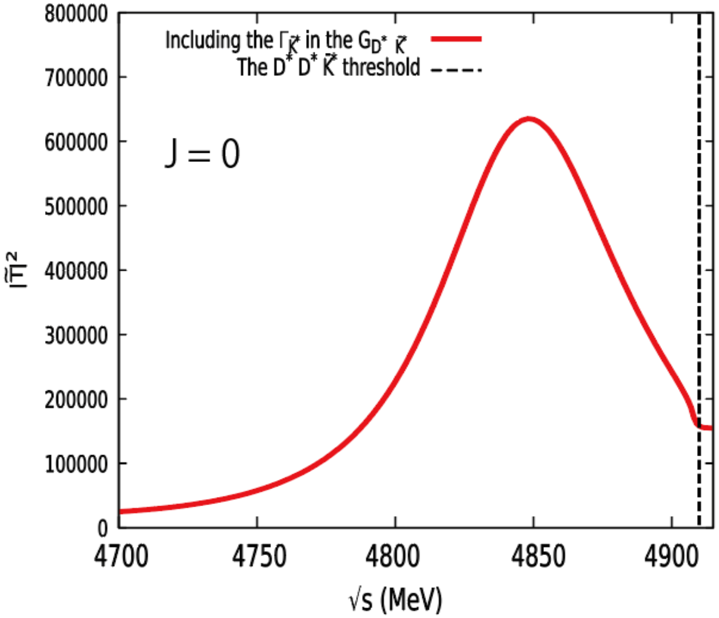
$I(J^P)$	M[MeV]	Γ[MeV]	Coupled channels	state
$0(2^+)$	2775	38	$D^*\bar{K}^*$	?
$0(1^+)$	2861	20	$D^*\bar{K}^*$	?
$0(0^+)$	2866	57	$D^*\bar{K}^*$	$X_0(2866)$

R. Molina, E. Oset, PLB811 (2020)

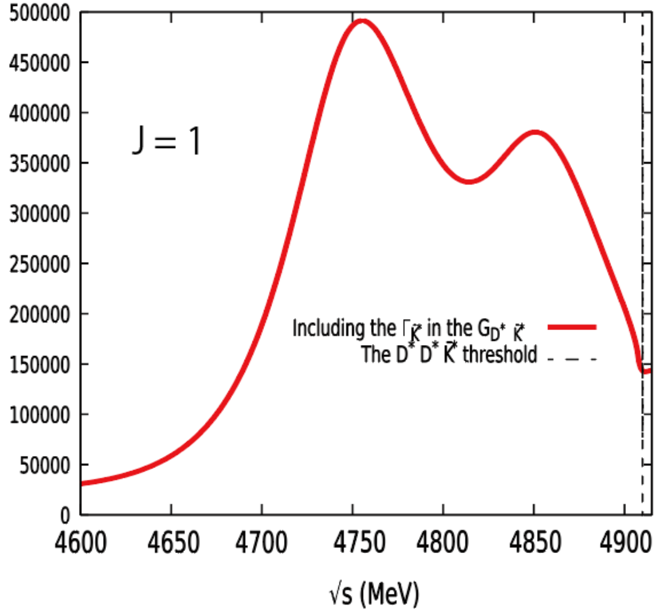
Bound states of $D^* D^* \bar{K}^*$

N. Ikeno, M. Bayar, E. Oset,
PRD107, 034006 (2023)

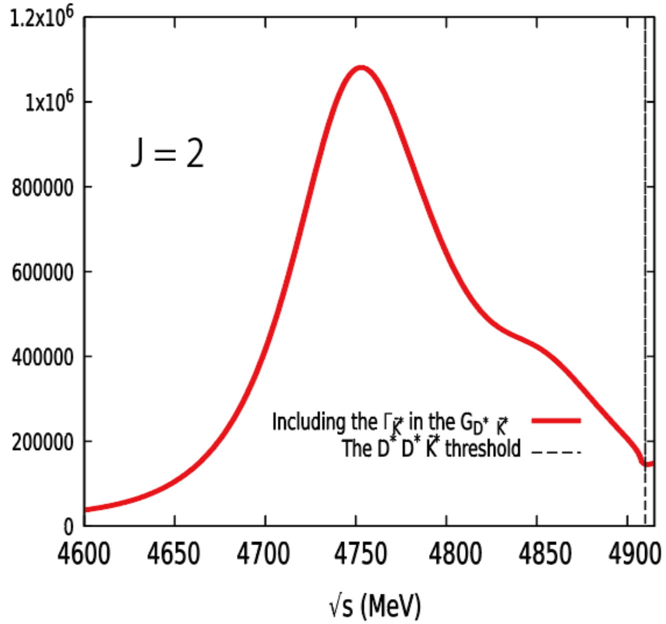
- Total spin J=0



- Total spin J=1



- Total spin J=2



J		M [MeV]	B [MeV]	Γ [MeV]	Main decay mode
0	(State I)	4845	61	80	$D^* D^* \bar{K}$
1	(State I)	4850	56	94	$D^* D \bar{K}$, $D^* D^* \bar{K}$
1	(State II)	4754	152	100	$D^* D \bar{K}$, $D^* D^* \bar{K}$
2	(State I)	4840	66	85	$D^* D^* \bar{K}$
2	(State II)	4755	151	100	$D^* D \bar{K}$, $D^* D^* \bar{K}$

Bound states obtained:
One state for J = 0
two states for J = 1, 2

BE = 56 MeV to 152 MeV
Γ = 80 MeV to 100 MeV

Bound states of $\bar{B}^*\bar{B}^*\bar{K}^*$

M. Bayar, N. Ikeno and L. Roca,
PRD107, 054042 (2023)

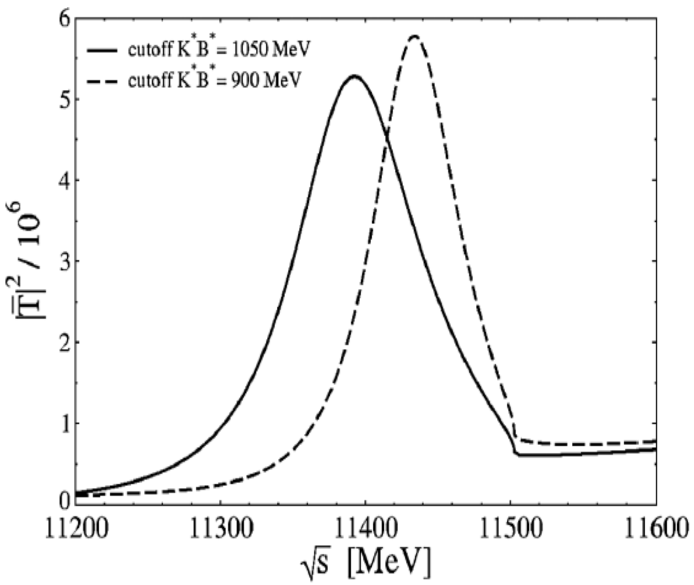
- Similar calculations of $D^*D^*\bar{K}^*$ except the interaction of $\bar{B}^*\bar{B}^*\bar{K}^*$

  $\bar{B}^*\bar{K}^*$ interaction is strongly attractive E. Oset and L. Roca, EPJ.C 82 (2022)

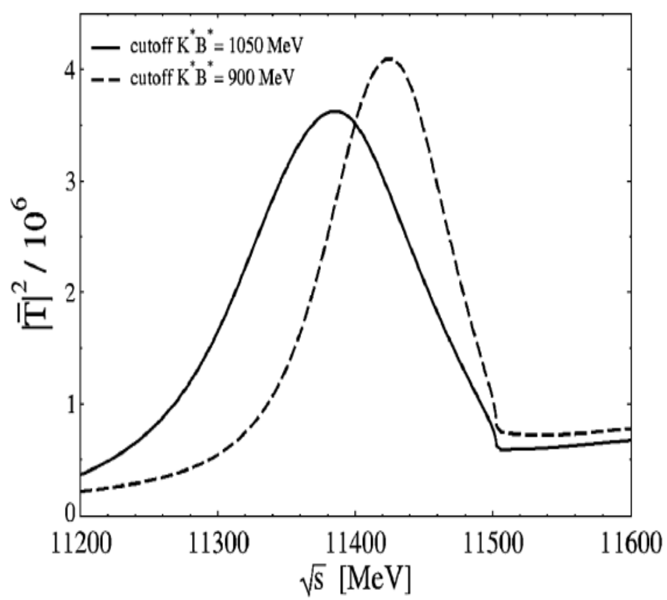
  $\bar{B}^*\bar{B}^*$ in $I(J^P) = 0(1^+)$ is bound with BE about 40 MeV

  L. R. Dai, E. Oset, A. Feijoo, R. Molina, L. Roca, A. M. Torres and K. P. Khemchandani, PRD 105 (2022)

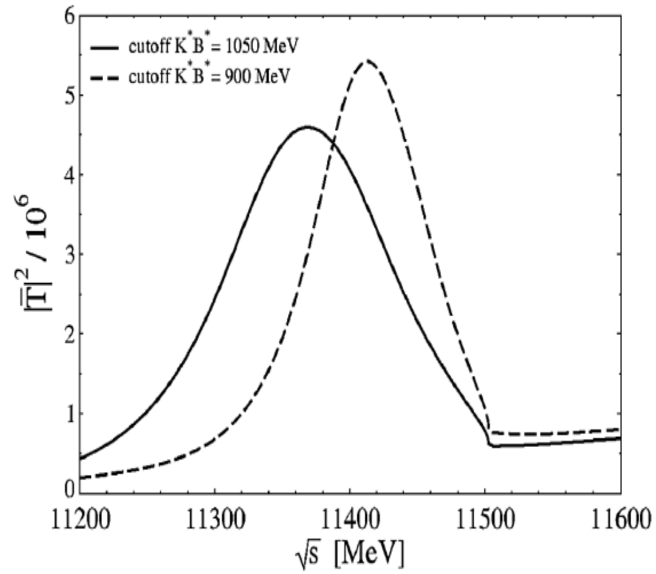
- Total spin J=0



- Total spin J=1



- Total spin J=2



different cutoff parameter of K*B*
q_{max}=900, 1050 MeV

One bound state is obtained for each J

Bound states of $\bar{B}^*\bar{B}^*\bar{K}^*$

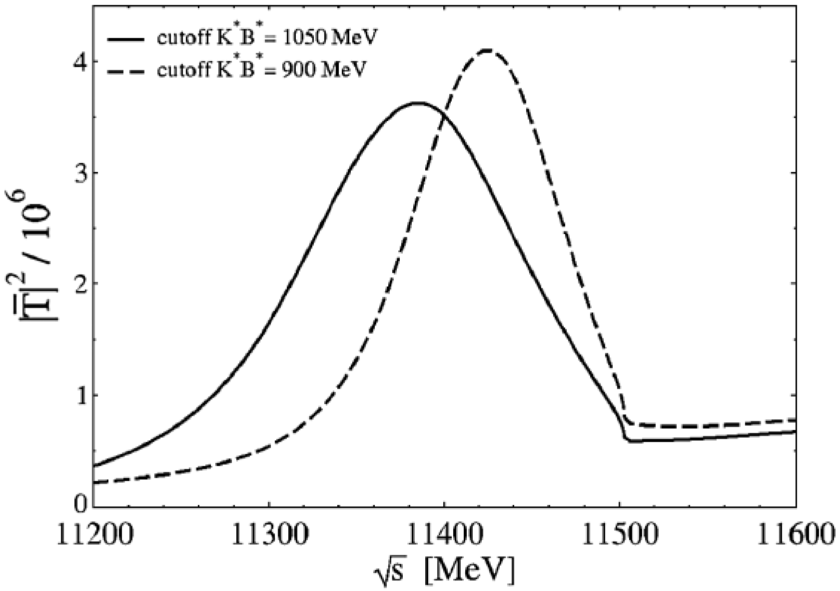
M. Bayar, N. Ikeno and L. Roca,
PRD107, 054042 (2023)

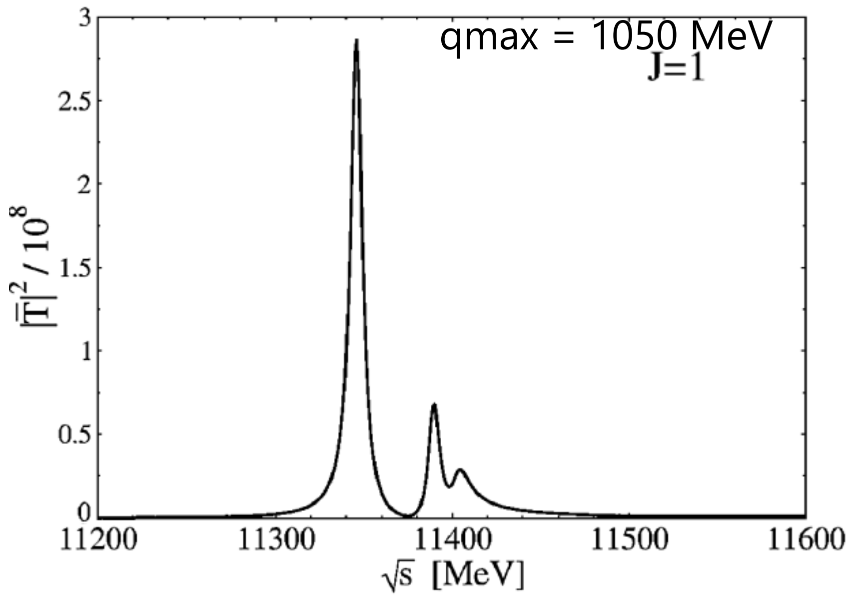
J	E_B [MeV]	Γ [MeV]
0	109–150	72–104
1	118–158	106–153
2	130–174	103–149

BE and Γ are larger than those of $D^*D^*\bar{K}^*$

$D^*D^*\bar{K}^*$ case:
BE = 56 -152 MeV, Γ = 80-100 MeV

- We artificially reduced the dominant source of the imaginary part of the K*B* amplitude to 5% of its true value.
  We can see three clear narrow peaks in the three-body amplitudes





Summary

Molecular states of $D^* D^* \bar{K}^*$ and $\bar{B}^* \bar{B}^* \bar{K}^*$ nature

- Very exotic hadron contains ccs and bbs open quarks, respectively

- We searched the bound state of three-body system
  - Both D*D* and B*B* have a bound state in I=0.
  - Both D*$\bar{K}$* and B*K* Interactions are attractive.

- We used FCA to the Faddeev equations

- We obtained bound states with total spin J = 0, 1, 2

- We hope that these super-exotic mesons, with open strange and double-charm(bottom) flavor, can be experimentally found in a near future

Wave function

The unitary approach that we use to obtain the D*D*bound states can be easily visualized as coming from the use of a separable potential of the type

$$V(\vec{q},\vec{q}\,') = V\theta(q_{\max} - |\vec{q}|)\theta(q_{\max} - |\vec{q}\,'|)$$

one can easily deduce **the wave function in momentum space** as

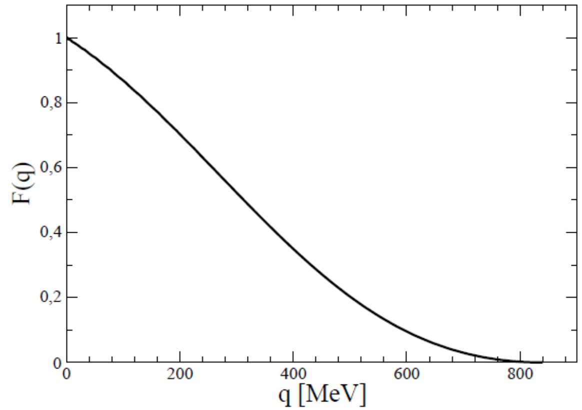
$$\Psi(\vec{p}) = g_R \frac{\theta(q_{\max} - |\vec{p}|)}{E - \omega_1(\vec{p}) - \omega_2(\vec{p})}$$

where g_R is the coupling of the state to the two components of the state

The form factor F(q) encodes the information about the D^*D^* bound state:

$$F(\vec{q}) = \int d^3\vec{r}\, e^{-i\vec{q}\cdot\vec{r}}\Psi_c^2(\vec{r})$$

$$= \frac{1}{N}\int_{\substack{p<q_{\max} \\ |\vec{p}-\vec{q}|<q_{\max}}} d^3p \frac{1}{m_C - \sqrt{m_{D^*}^2 + \vec{p}^2} - \sqrt{m_{D^*}^2 + \vec{p}^2}} \, \frac{1}{m_C - \sqrt{m_{D^*}^2 + (\vec{p}-\vec{q})^2} - \sqrt{m_{D^*}^2 + (\vec{p}-\vec{q})^2}}$$



To obtain the wave function in coordinate space we write

$$\phi(\vec{r}) \equiv \langle \vec{r}|\phi\rangle = \int \frac{d^3q}{(2\pi)^{3/2}} e^{i\vec{q}\cdot\vec{r}}\phi(\vec{q}) \qquad e^{i\vec{q}\cdot\vec{r}} = 4\pi \sum_{\ell} i^{\ell} j_{\ell}(qr) \sum_{m} Y^*_{\ell m}(\hat{q}) Y_{\ell m}(\hat{r}).$$

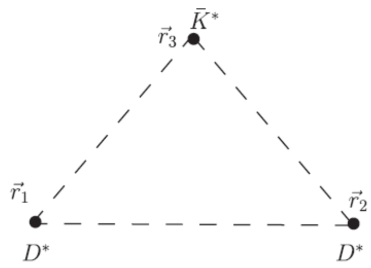
$$\phi(\vec{r}) = \frac{2\pi}{(2\pi)^{3/2}} g_R \frac{2}{r} \int_0^{q_{\max}} q dq \frac{\sin(qr)}{E - \omega_1(q) - \omega_2(q)}$$

Wave function for the K* in the D*D*K* system at rest.

- we have a K*bar orbiting around the cluster of D*D*. The K*bar will orbit around one D* and sometimes around the other D*.

- In this picture we can have the K*bar distribution given

$$|\Psi(r_3')|^2 = \int d^3r_1 d^3r_2 (|\phi(\vec{r}_{31})|^2 + |\phi(\vec{r}_{32})|^2)|\phi'(\vec{r}_{12})|^2 \times \delta^3(m_{D^*}\vec{r}_1 + m_{D^*}\vec{r}_2 + m_{\bar{K}^*}\vec{r}_3),$$



$D^*\bar{K}^*$ amplitudes

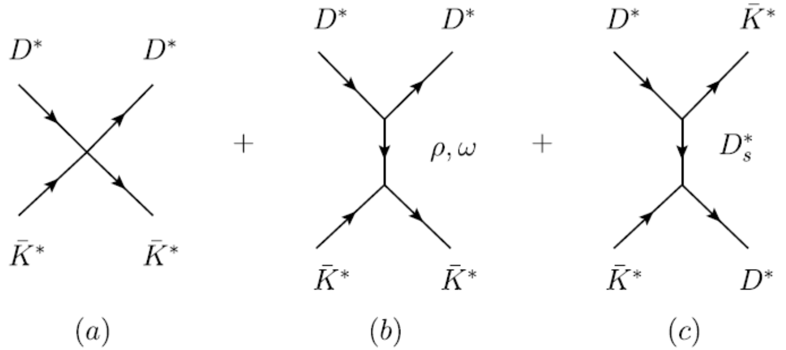


Fig. 1. Feynman diagrams for the terms of the hidden local gauge approach contributing to the $D^*\bar{K}^* \to D^*\bar{K}^*$ interaction at the tree level; (a) contact term; (b) exchange of light vectors; (c) exchange of a heavy vector.

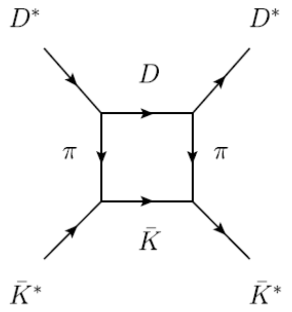


Fig. 2. Box diagram accounting for the width of the $\bar{D}^*\bar{K}^*$ state decaying to $D\bar{K}$.

Tree level amplitudes for $D^*\bar{K}^*$ in $I=0$. The last column shows the value of V at threshold.

J	Amplitude	Contact	V-exchange	$\sim$ Total
0	$D^*\bar{K}^* \to D^*\bar{K}^*$	$4g^2$	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D_s^*}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} - \frac{3}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$-9.9g^2$
1	$D^*\bar{K}^* \to D^*\bar{K}^*$	0	$\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D_s^*}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} - \frac{3}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$-10.2g^2$
2	$D^*\bar{K}^* \to D^*\bar{K}^*$	$-2g^2$	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D_s^*}} + \frac{1}{2}g^2(\frac{1}{m^2_\omega} - \frac{3}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$-15.9g^2$

TABLE XII. Amplitudes for $C=1$, $S=-1$ and $I=1$.

J	Amplitude	Contact	V exchange	$\sim$Total
0	$D^*\bar{K}^* \to D^*\bar{K}^*$	$-4g^2$	$\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D_s^*}} + \frac{g^2}{2}(\frac{1}{m^2_\omega} + \frac{1}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$9.7g^2$
1	$D^*\bar{K}^* \to D^*\bar{K}^*$	0	$-\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D_s^*}} + \frac{g^2}{2}(\frac{1}{m^2_\omega} + \frac{1}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$9.9g^2$
2	$D^*\bar{K}^* \to D^*\bar{K}^*$	$2g^2$	$\frac{g^2(p_1+p_4).(p_2+p_3)}{m^2_{D_s^*}} + \frac{g^2}{2}(\frac{1}{m^2_\omega} + \frac{1}{m^2_\rho})(p_1+p_3).(p_2+p_4)$	$15.7g^2$