The anomaly-driven breaking of chiral symmetry in the NJL model and the instanton liquid model

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Introduction

- Motivation
  - **Role of $\mathrm{U}_A(1)$ anomaly in dynamical chiral symmetry breaking**

    - $\mathrm{U}_A(1)$ anomaly assists dynamical chiral symmetry breaking[1]

    - Sigma mass is suppressed [1]

- Contents
  - Anomaly-driven breaking of chiral symmetry in chiral effective models [1]
  - Generalization of anomaly-driven breaking
  - Application to instanton liquid model
  - Summary

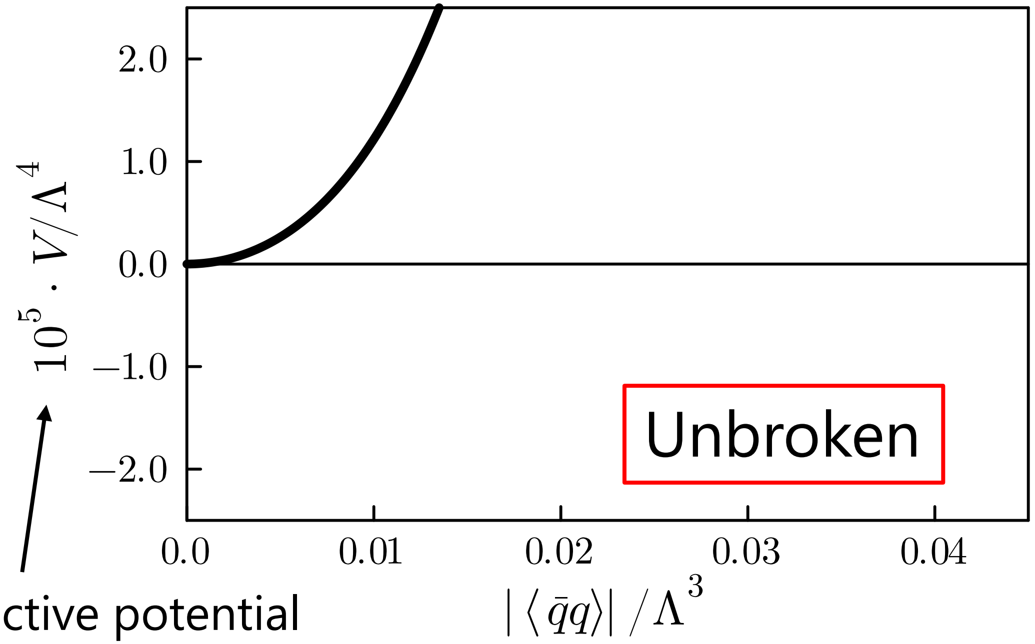
[1] S. Kono, D. Jido, Y. Kuroda and M. Harada, PTEP **2021**, 093D02 (2021)

Anomaly-driven breaking of chiral symmetry in chiral effective models

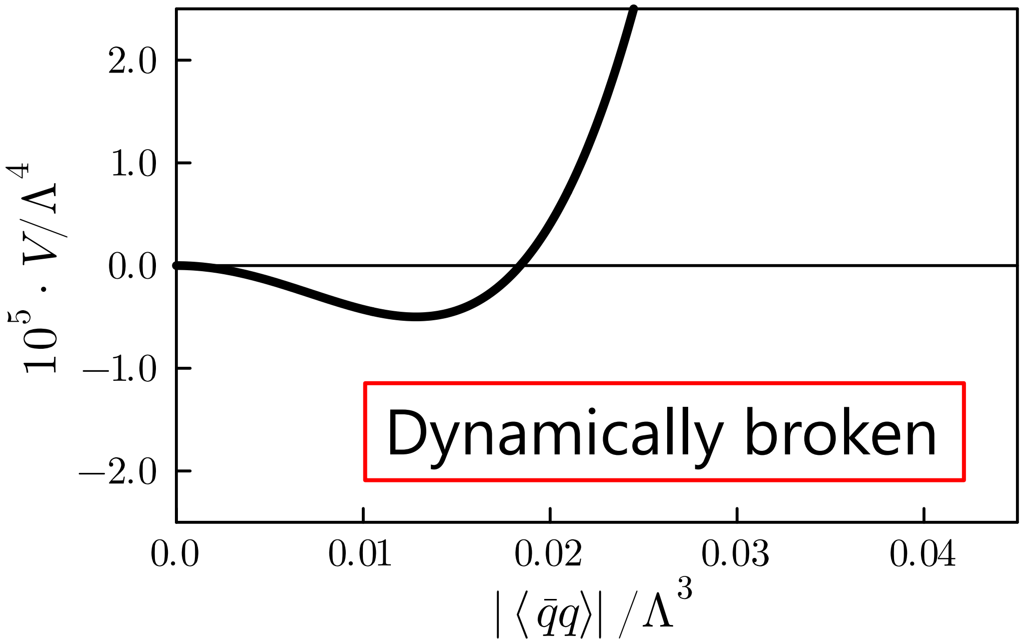
Reference: S. Kono, D. Jido, Y. Kuroda and M. Harada, PTEP **2021**, 093D02 (2021)

Anomaly-driven breaking of chiral symmetry

- Chiral symmetry breaking is described by chiral effective models



Effective potential



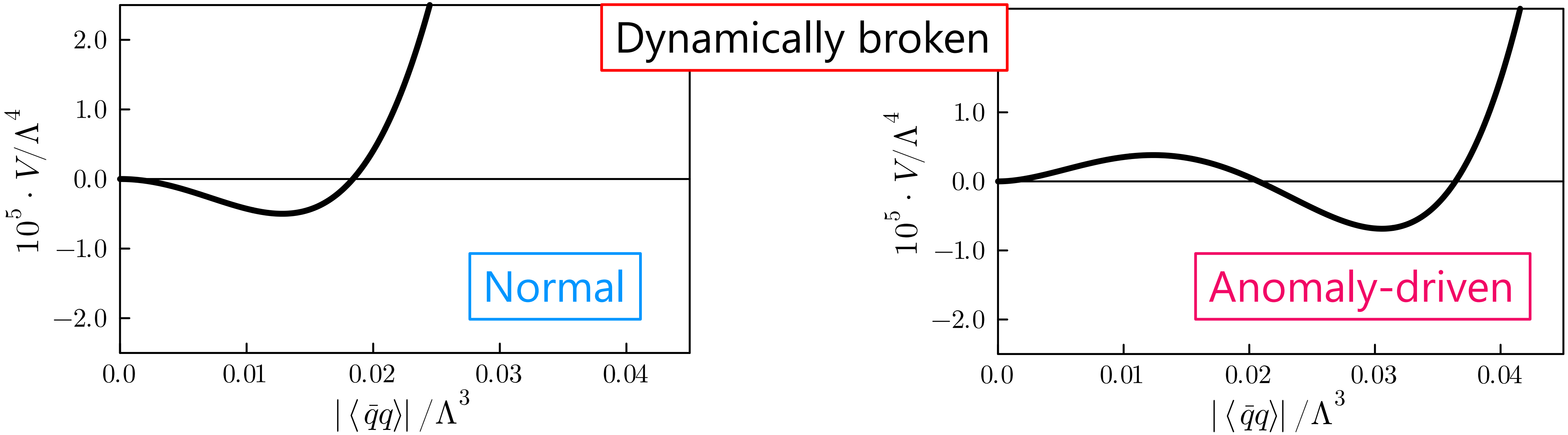
Model	Interaction	Unbroken	Dynamically Broken
NJL [2]	$\mathcal{L}_{\text{int}} = \sum_{a=0}^{8} \frac{g_S}{2} [(\bar{q}\lambda_a q)^2 + (\bar{q} i \lambda_a \gamma_5 q)^2]$	$g_S < g_S^{\text{crit}}$	$g_S > g_S^{\text{crit}}$

[2] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **112** (1961) 345

$q = (u\ d\ s)^T$: Quark flavor triplet

Anomaly-driven breaking of chiral symmetry

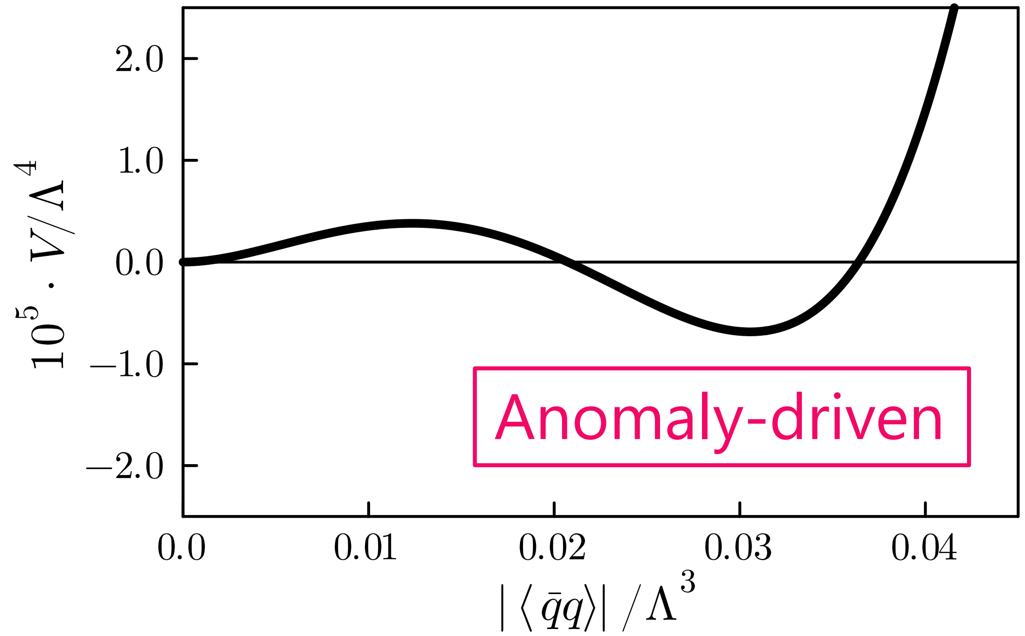
- $U_A(1)$ anomaly term assists dynamical chiral symmetry breaking



Model	**Interaction**	**Dynamically broken**	
		Normal	**Anomaly-driven**
NJL	$\mathcal{L}_{\text{int}} = \sum_{a=0}^{8} \frac{g_S}{2}[(\bar{q}\lambda_a q)^2 + (\bar{q}i\lambda_a\gamma_5 q)^2]$ $+g_D[\det(\bar{q}_i(1-\gamma_5)q_j) + \mathrm{H.c.}]$	$g_S > g_S^{\text{crit}}$	$g_S < g_S^{\text{crit}}, g_D < 0$

What characterizes anomaly-driven breaking?

- Strength of coupling g_S and existence of non-trivial vacuum



$$\begin{cases} g_S < g_S^{\text{crit}}, g_D < 0 \\ \min\big(V(\langle\bar{q}q\rangle)\big) = V(\langle\bar{q}q\rangle \neq 0) \end{cases}$$

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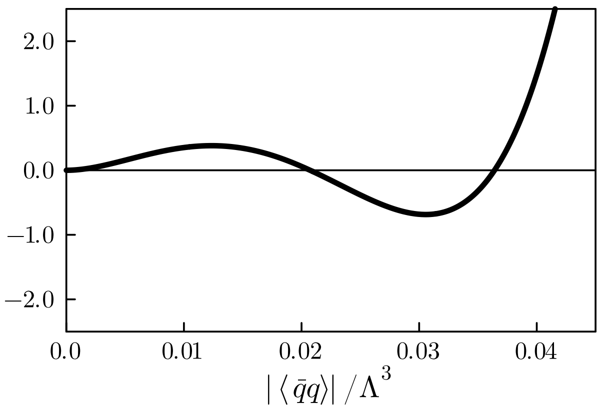
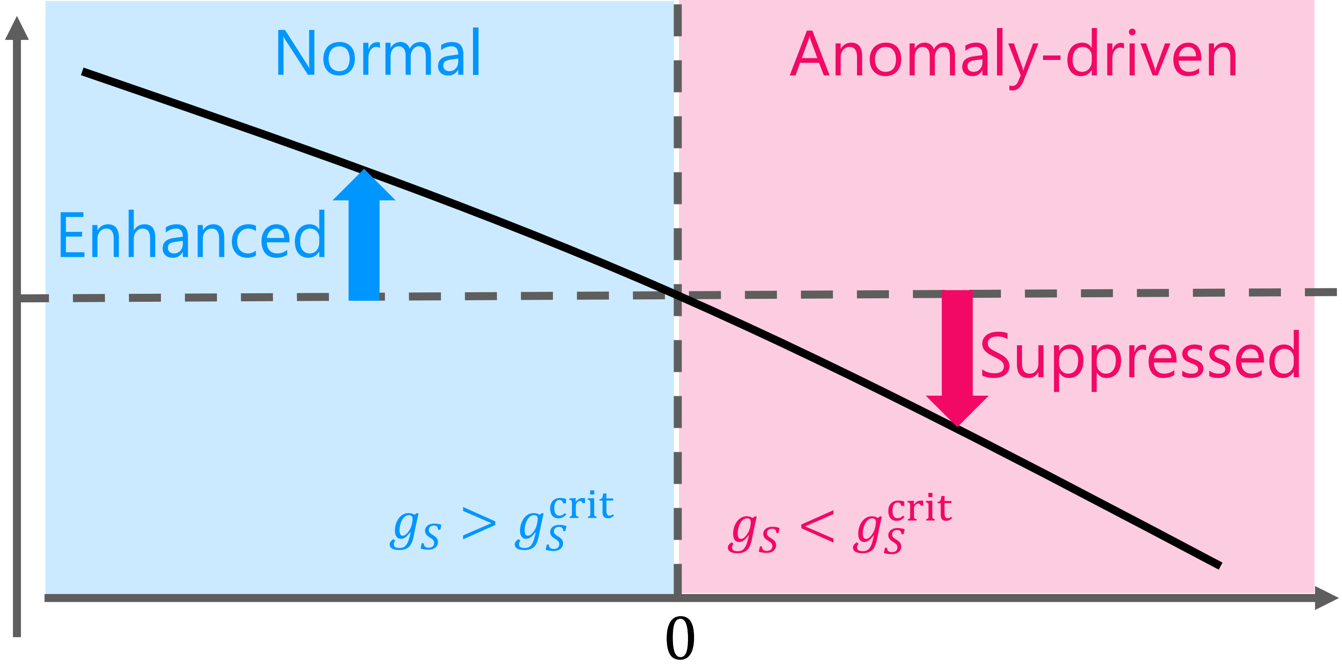
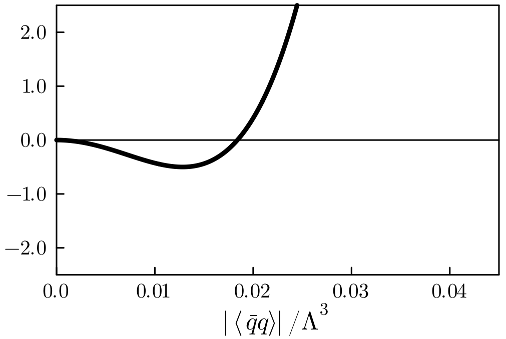
Relationships of properties of $f_0(500)$

- Mass of $\sigma/f_0(500)$ is physical test ground







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Generalization of anomaly-driven breaking

Generalization of anomaly-driven breaking

- From coupling constant to curvature of effective potential at origin

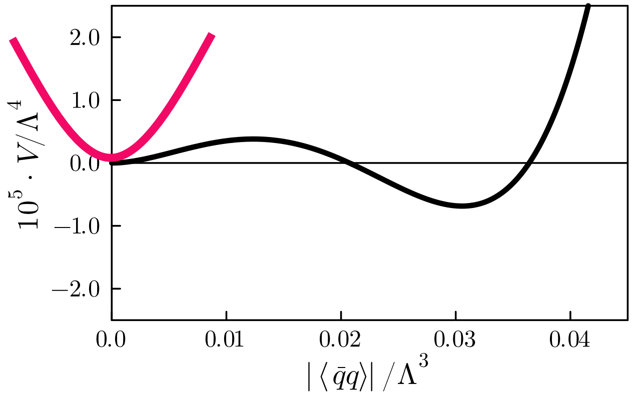
Definition in the NJL model

$$g_S < g_S^{\mathrm{crit}}, g_D < 0 \quad \Leftrightarrow \quad \left.\frac{\partial^2 V}{\partial \langle \bar{q}q \rangle^2}\right|_{\langle \bar{q}q \rangle = 0} = g_S^{\mathrm{crit}} - g_S > 0$$

after some calculations

Definition in our work

Generalized definition: $\left.\frac{\partial^2 V}{\partial \langle \bar{q}q \rangle^2}\right|_{\langle \bar{q}q \rangle = 0} > 0$



- Generalized definition is equivalent to original one in the NJL model
- One can determine anomaly-driven breaking or not by evaluating the **curvature** of effective potential (equivalent to vacuum energy density) **at origin** of quark condensate.

Application to instanton liquid model

Application to instanton liquid model

- Interacting Instanton liquid model (IILM) [3]

[3] E. Shuryak, Nucl. Phys. B **203** (1982) 93; 116

  - What?

    - QCD vacuum $\underset{\substack{\uparrow \\ \text{Low energy region}}}{\approx}$ Liquid of instantons

$$Z_{\text{IILM}} = \frac{1}{N_+!N_-!}\int \left(\prod_{i=1}^{N_+ + N_-} \underbrace{d\Omega_i}_{\text{Collective coordinates of instantons}} \underbrace{d(\rho_i)}_{\text{Semiclassical Instanton amplitude}} \right) \exp(-\overbrace{S_{\text{int}}}^{\text{Instanton-instanton interaction}}) \prod_{f=1}^{N_f} \overbrace{\text{Det}(\gamma_\mu D_\mu + m_f)}^{\text{Instanton-quark interaction}}$$

  - Why?

    - IILM includes chiral symmetry breaking and instantons relate to $\text{U}_A(1)$ anomaly

  - How?

    - Compute vacuum energy density and quark condensate
    - Evaluate curvature by fitting

Application to instanton liquid model

- Computation of IILM

Arrange instantons in four-volume (with PBC)

▼

Calculate forces acting on instantons according to IILM action

▼

Calculate movement of instantons (HMC & Metropolis method)

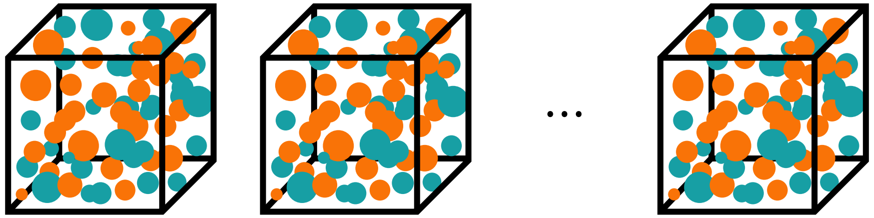
▼

Obtain a configuration of instantons

repeat

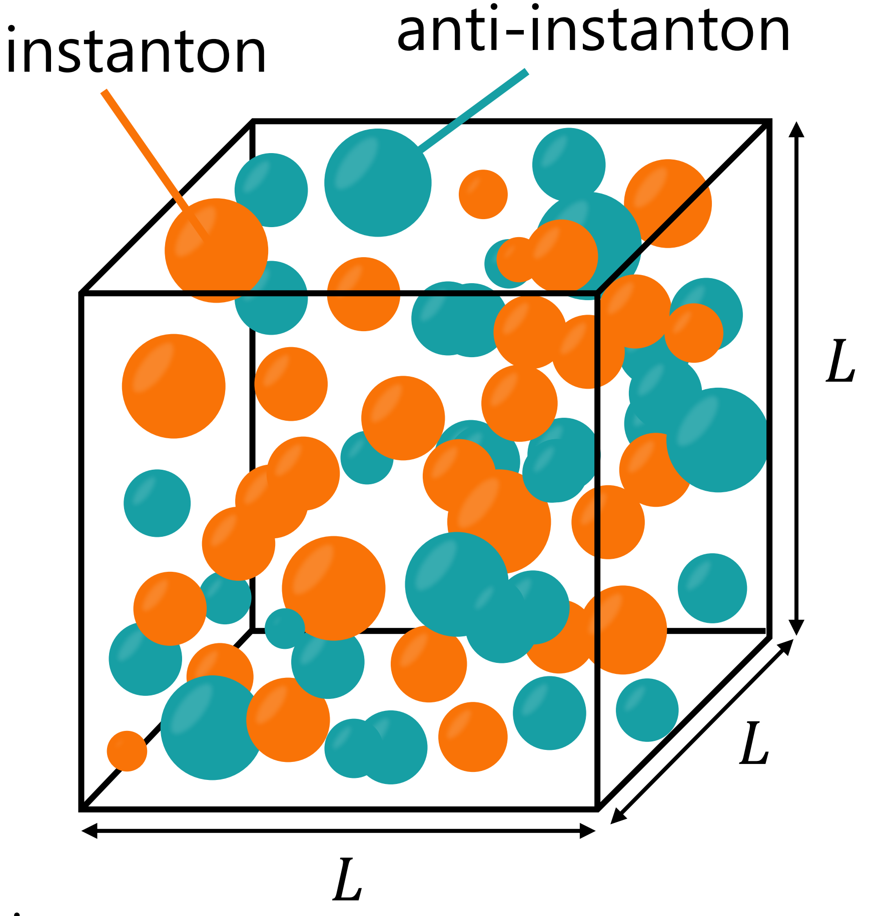
▼ Pick up

Generate configurations



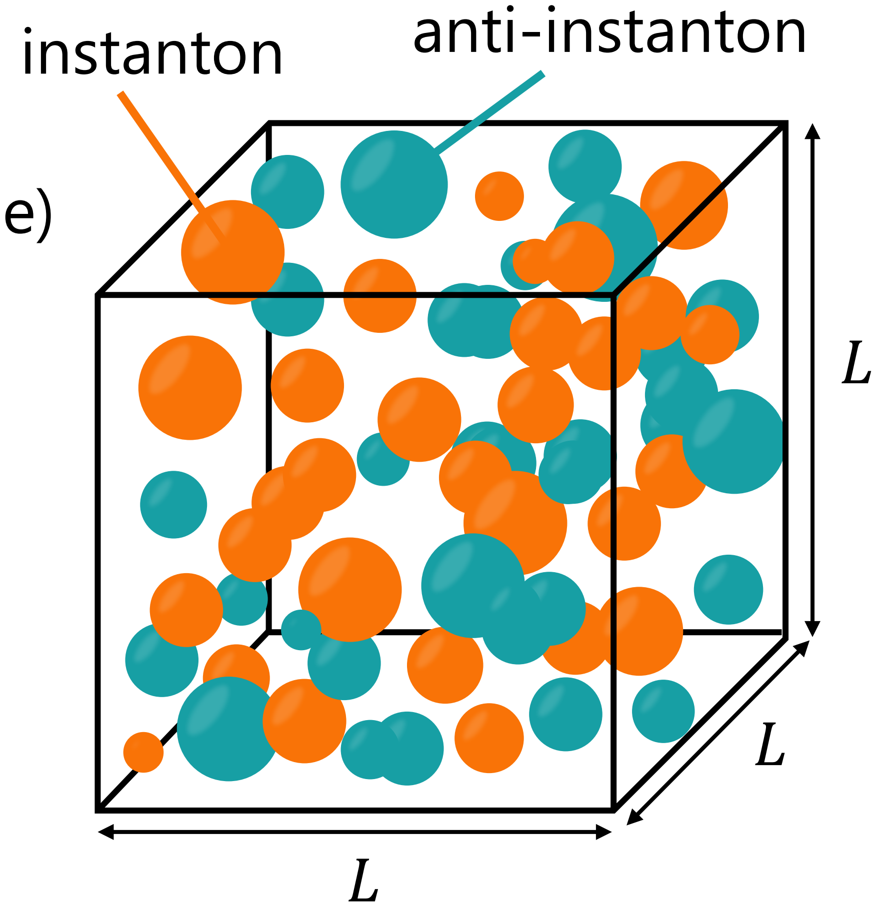
▶ Compute physical quantities

- Vacuum energy density
- Quark condensate



Application to instanton liquid model

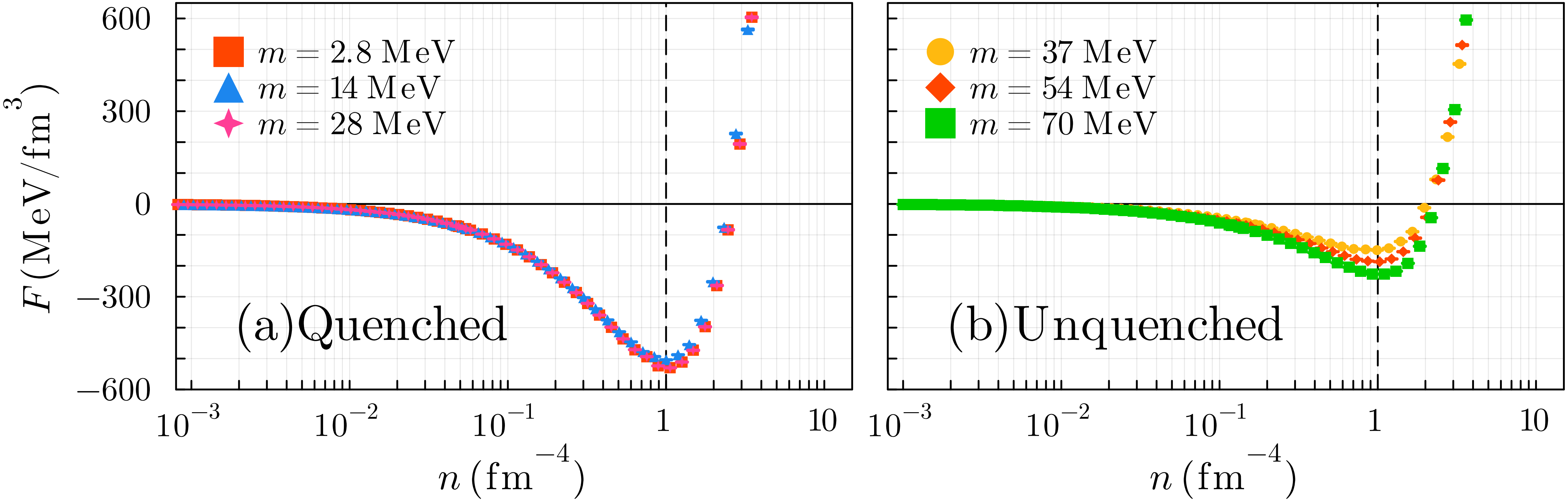
- Setups
  - Parameters
    - Instanton density $n = \frac{N}{V}$ (N: fixed, $V = L^4$: variable)
    - Current quark mass m (in quark propagator)
  - Types
    - Quenched approximation
    - Unquenched (full)



$$Z_{\mathrm{IILM}} = \frac{1}{N_+!N_-!} \int \left(\prod_{i=1}^{N_+ + N_-} d\Omega_i d(\rho_i) \right) \exp(-S_{\mathrm{int}}) \underbrace{\prod_{f=1}^{N_f} \mathrm{Det}(\gamma_\mu D_\mu + m_f)}_{=1}$$

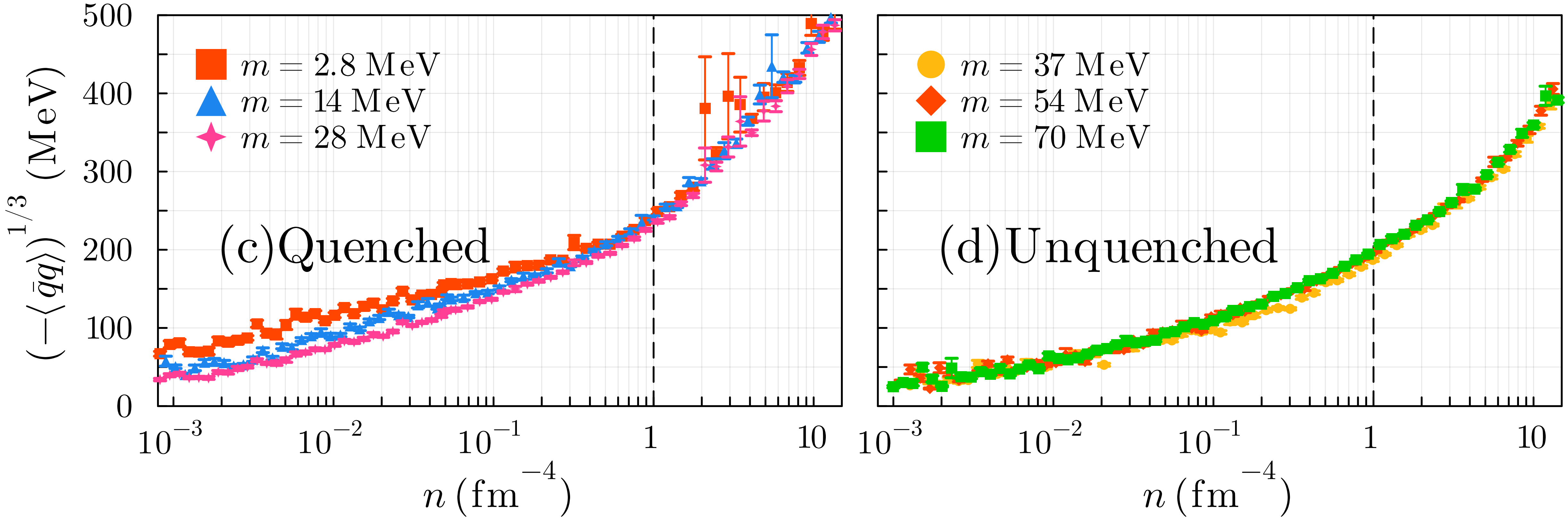
Results: Vacuum energy density

- Instanton density dependence of vacuum energy density
  - Parameters: instanton density n, current quark mass m



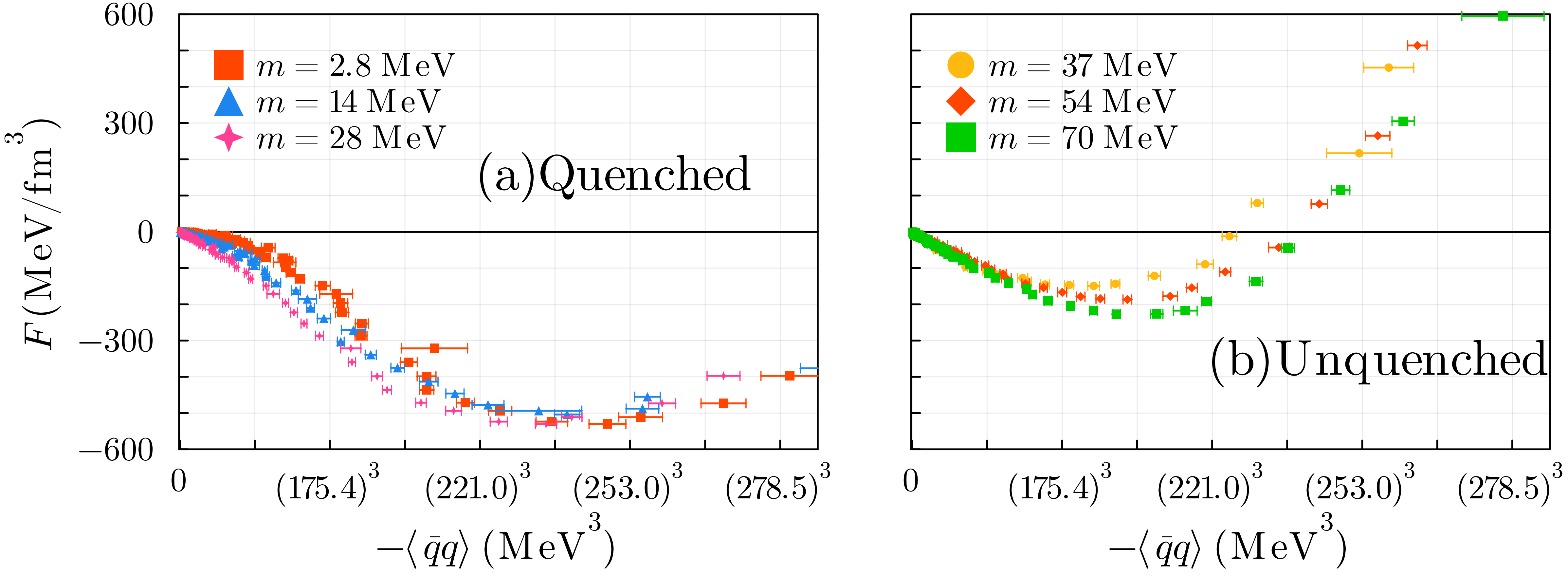
Results: Quark condensate

- Instanton density dependence of quark condensate
  - Parameters: instanton density n, current quark mass m

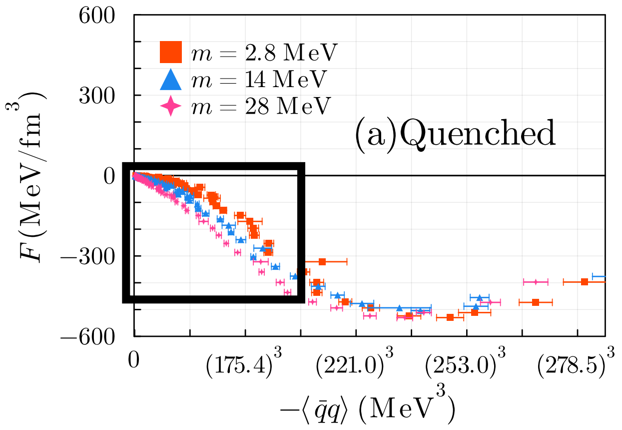


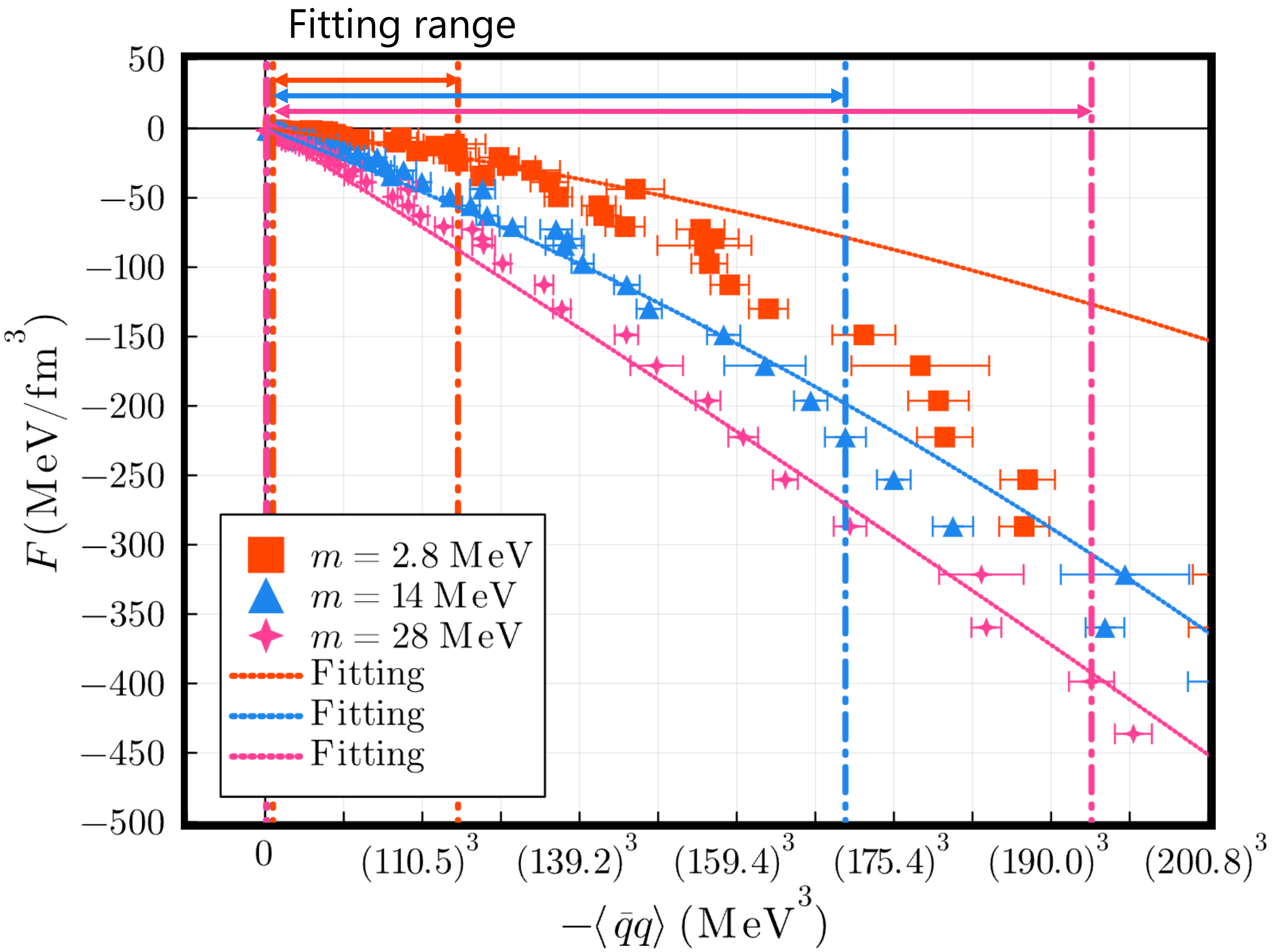
Results: Vacuum energy vs. Quark condensate

- Quark condensate dependence of vacuum energy density
  - Combining two calculations through the instanton density n



Results: Curvature in quenched calculation





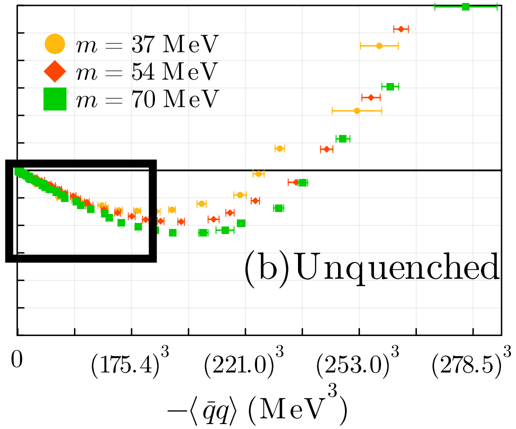
- Fitting results

Quenched			
m (MeV)	C_1 (MeV)	C_2 ($10^{-5}\mathrm{MeV}^{-2}$)	χ^2/d.o.f.
2.8	83(2)	−0.76(22)	6.65
14	244(3)	−1.24(1)	12.4
28	401(3)	−0.34(7)	18.2

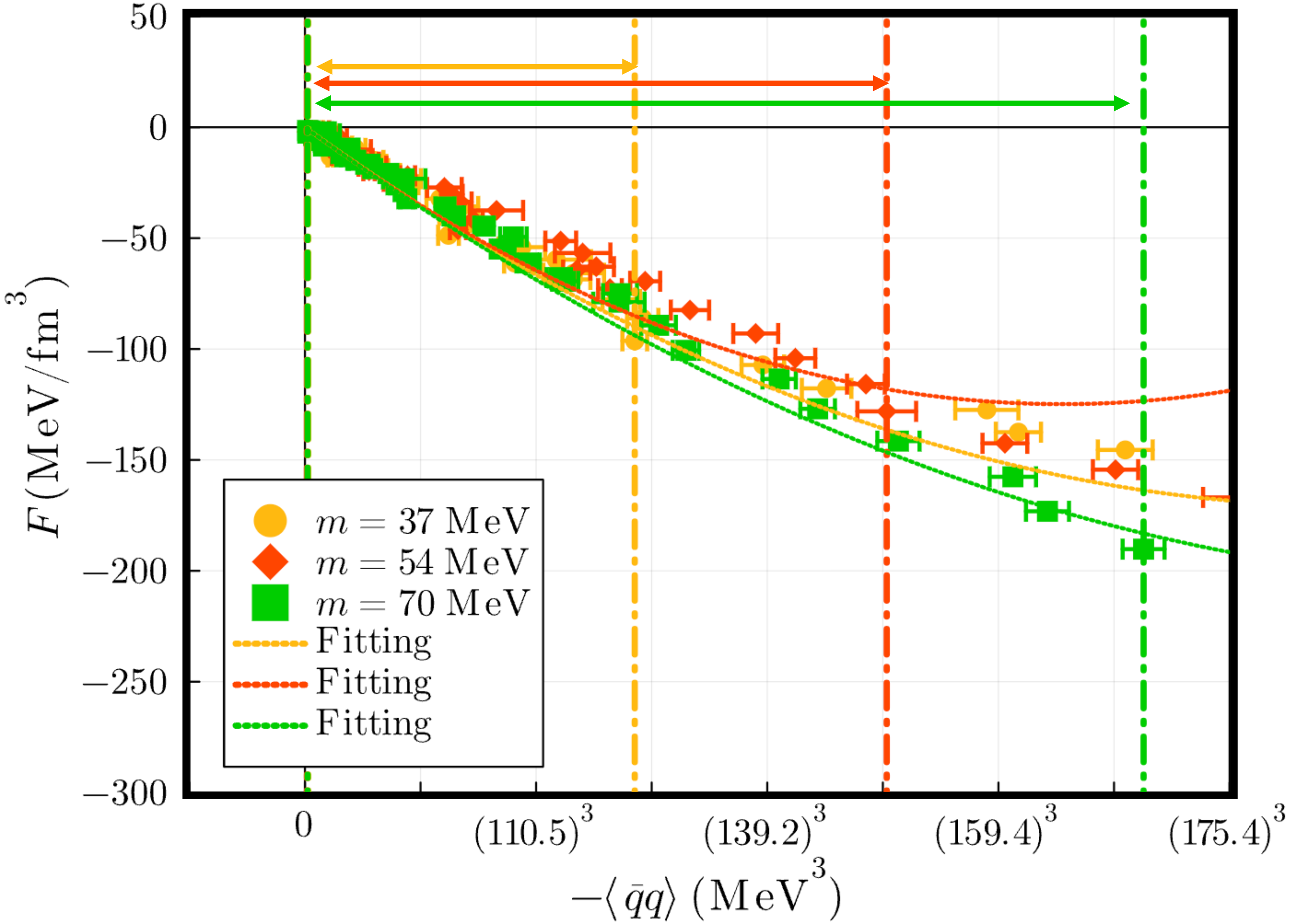
- Fitting model

$$F = C_1 \langle\bar{q}q\rangle + C_2 \langle\bar{q}q\rangle^2$$

Results: Curvature in unquenched calculation



Fitting range



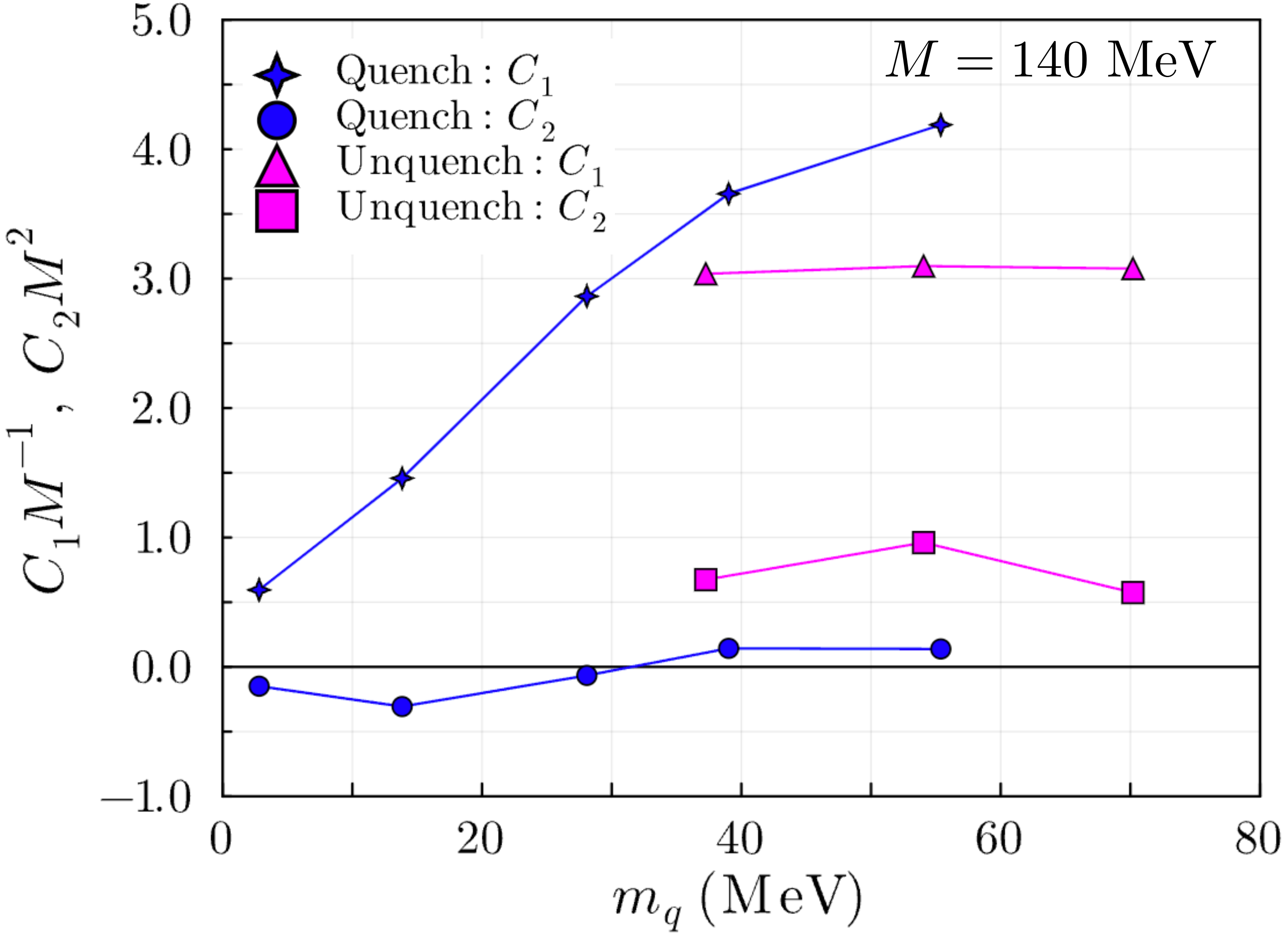
- Fitting results

Unquenched			
m (MeV)	C_1 (MeV)	C_2 (10^{-4}MeV^{-2})	χ^2/d.o.f.
37	425(6)	3.44(39)	4.78
54	434(2)	4.90(7)	23.0
70	431(2)	2.93(5)	12.5

- Fitting model

$$F = C_1 \langle\bar{q}q\rangle + C_2 \langle\bar{q}q\rangle^2$$

Results: Quark mass dependence of curvature



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- Curvature < 0 : **Quench**
→ **Normal breaking**

- Curvature > 0 : **Unquench**
→ **Anomaly-driven breaking**

●Summary

- We study the role of $\mathrm{U}_A(1)$ anomaly in dynamical chiral symmetry breaking
- Anomaly-driven breaking has been studied in the chiral effective models
- It may be seen as change of hadron properties, such as sigma and η' mass
- To see anomaly-driven breaking in other system, we generalize its definition using sign of curvature of effective potential at origin of quark condensate
- In the IILM, the curvature is positive (negative) for quenched (unquenched) calculation
- That implies the anomaly-driven breaking is taken place in unquenched IILM

●Next works

- Compute the mass of mesons, for example $\sigma/f_0(500), \eta'$, etc.
- Identify the difference between the quenched and the unquenched calculations
- What other evidence that anomaly-driven breaking is realizing?

Thank you for your careful attention!