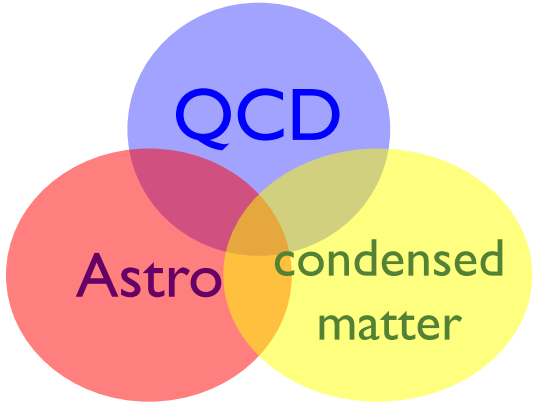
From hadrons to quarks in neutron stars:

-- *why the quark substructure of hadrons is important*



Toru Kojo

(**Tohoku Univ.**)

Refs) Baym-Hatsuda-TK-Powell-Song-Takatsuka, "QHC", review on neutron stars (2018)

TK, "Stiffening of matter in quark-hadron continuity" PRD (2021)

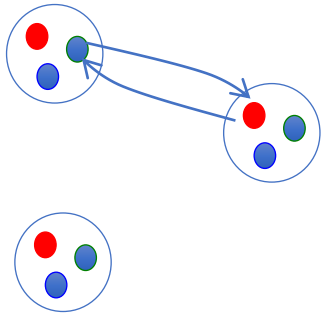
Fujimoto-TK-McLerran, "IdylliQ matter model" arXiv: 2306.04304 [nucl-th]

State of matter: overview

($n_0 = 0.16$ fm^{-3})

[Masuda+ '12; TK+ '14]

- few meson exchange
- nucleons only



ab-initio nuclear cal.

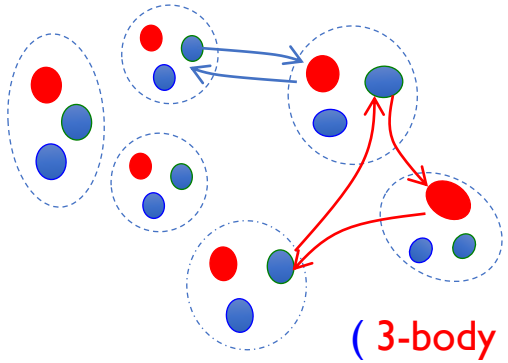
laboratory experiments

steady progress

~ 1.4 $M_\odot$

~ $2n_0$

- many-quark exchange
- structural change,...
- hyperons, Δ, ...



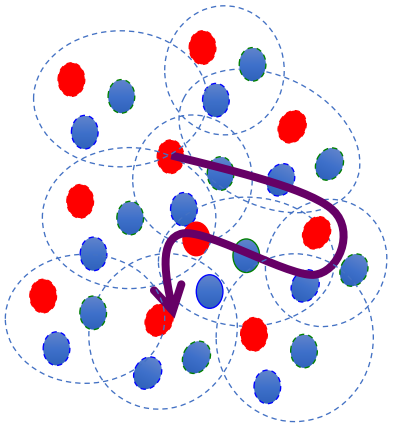
(3-body)

most difficult

(d.o.f ??)

Hints from NS

- Baryons overlap
- Quark Fermi sea



strongly correlated

(d.o.f : quasi-particles??)

not explored well

~ 2 $M_\odot$

~ $5n_0$

(pQCD)

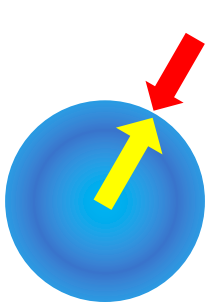
[Freedman-McLerran, Kurkela+, Fujimoto+...]

~ $40n_0$

n_B

EoS & Neutron Star M-R relation

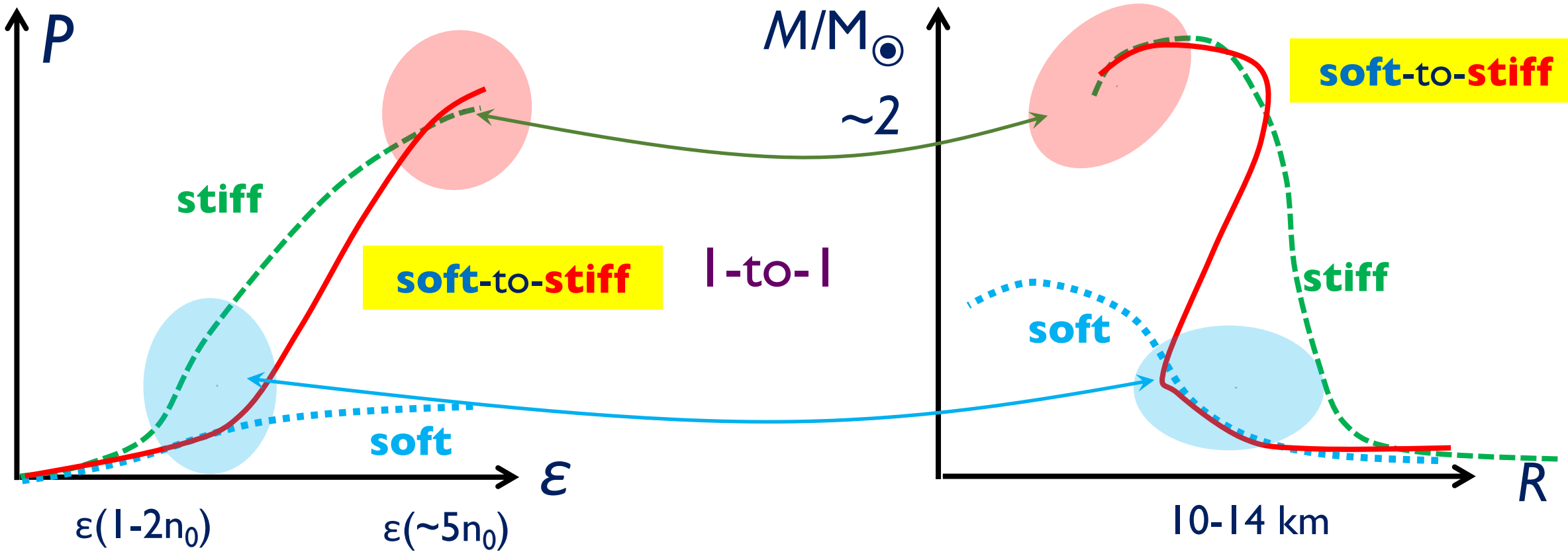
Einstein eq. : $G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ ..... QCD (+EW) EoS



gravity

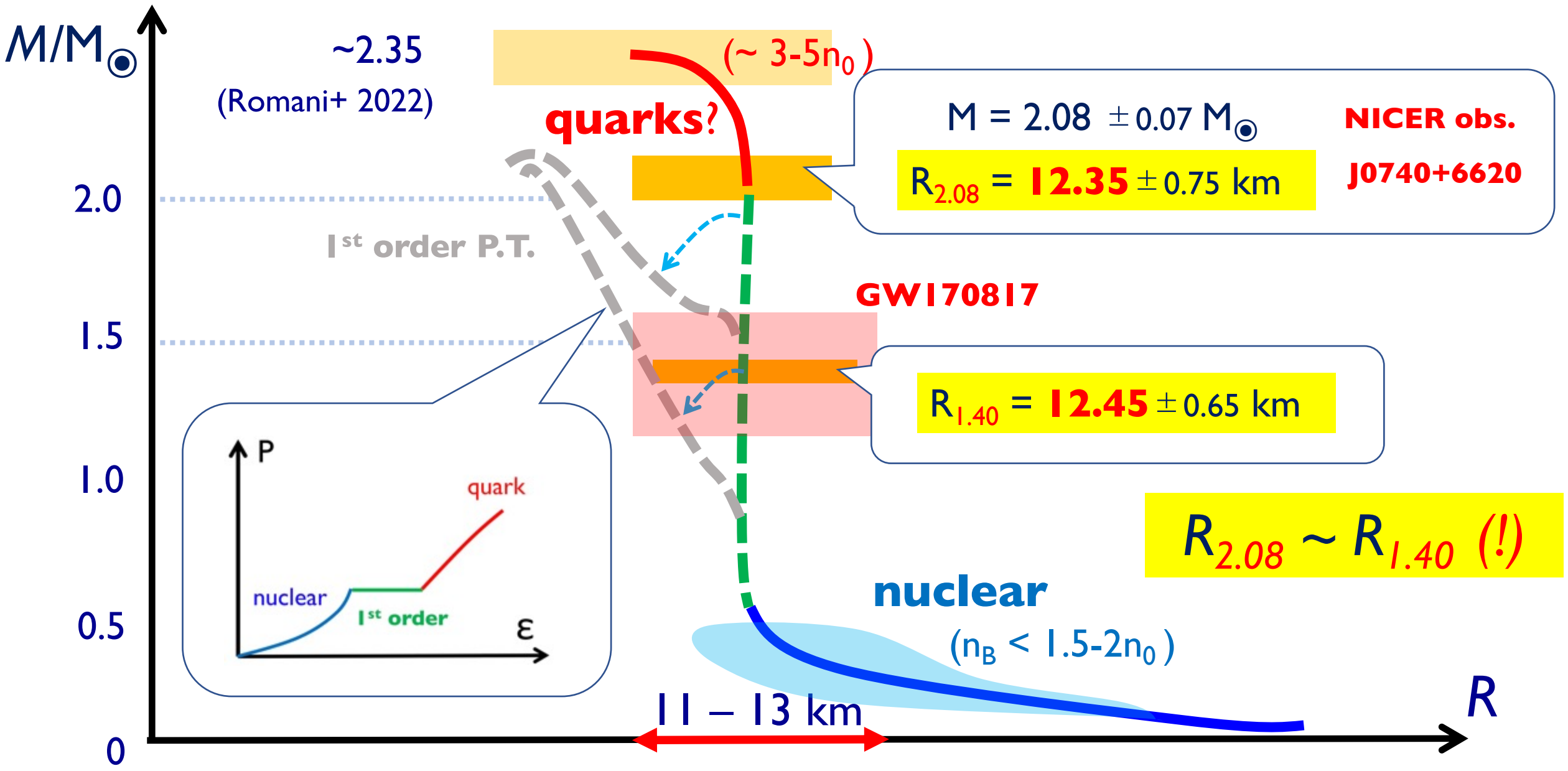
(source: ε_{QCD})

QCD pressure



Observations: (NICER, GW170817, nuclear)

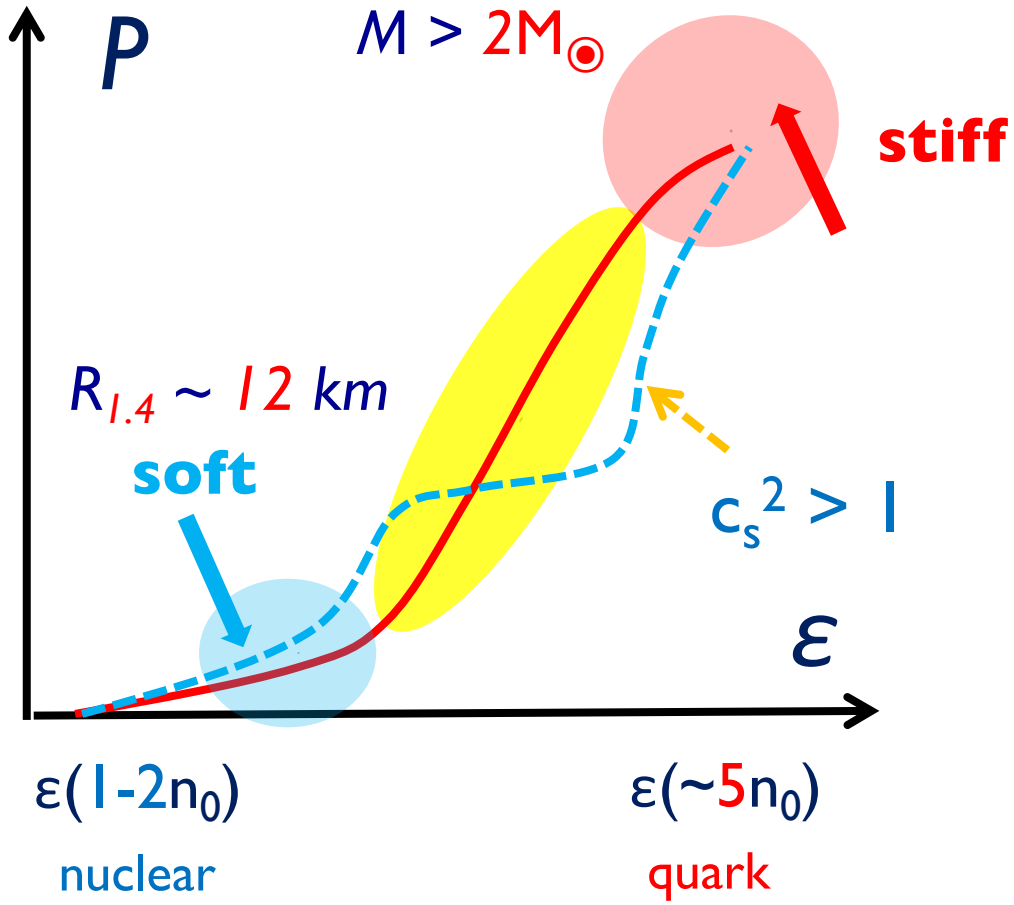
[e.g., Miller+ '21]

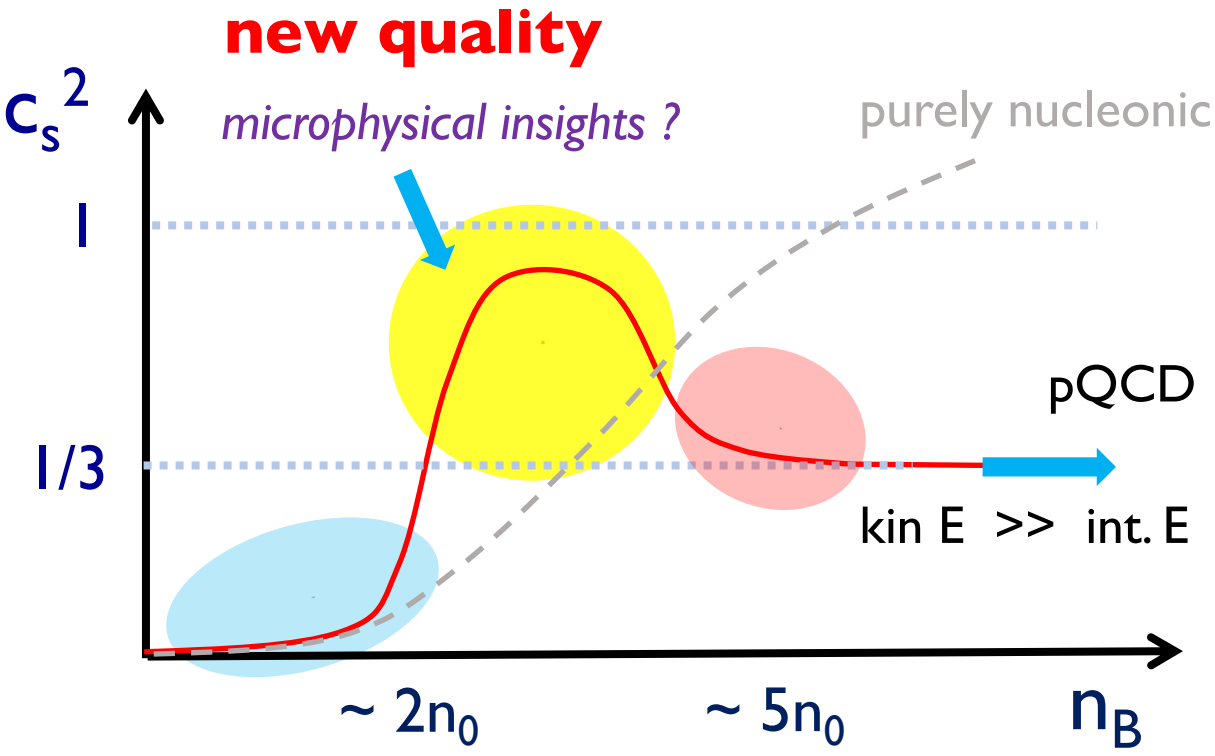


Soft to stiff is challenging:

sound velocity: $c_s^2 = dP/d\varepsilon < 1$ (*causality*)

nuclear & *quark* physics constrain each other

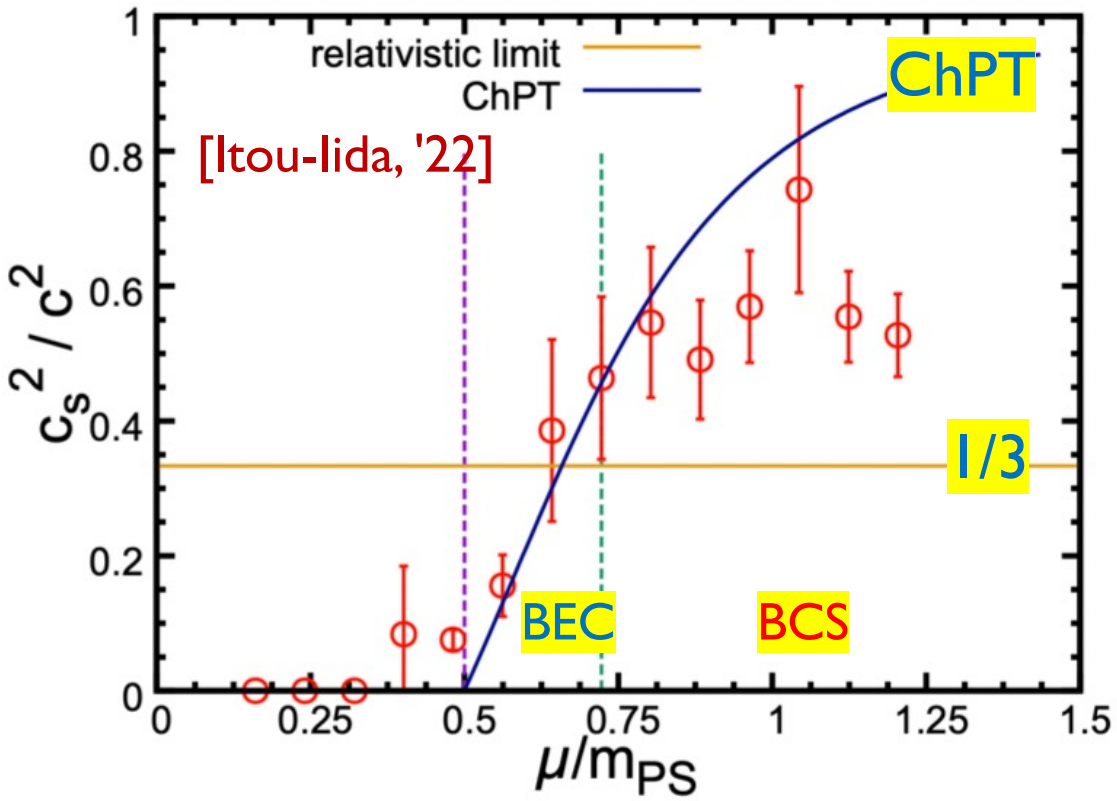




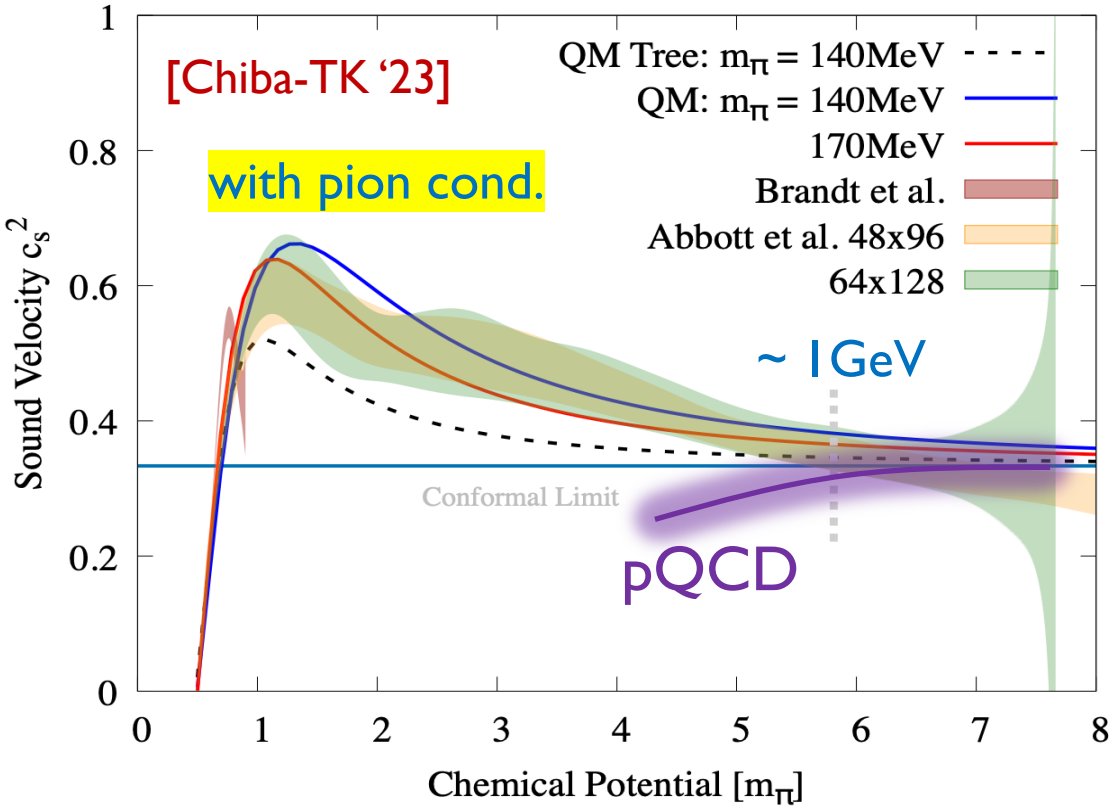
baseline: *quark-hadron continuity* **(QHC)**

Peak in c_s^2 on the lattice (QCD-like theories)

two-color QCD



isospin QCD



c_s^2 found (!), while relevant d.o.f. at low μ are bosons, differing from QCD at finite μ_B....

Contents

pressure from $\varepsilon(n_B)$

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\left(\frac{\varepsilon}{n_B}\right)$$

energy per particle

e.g.) gas of heavy particles (massive limit)

$$\varepsilon(n_B) = m_N n_B \quad \Rightarrow \quad \varepsilon/n_B = m_N \quad \Rightarrow \quad P = 0$$

gas of relativistic particles (massless limit)

$$\varepsilon(n_B) = a n_B^{4/3} \quad \Rightarrow \quad \varepsilon/n_B = a n_B^{1/3} \quad \Rightarrow \quad P = \frac{\varepsilon}{3}$$

c_s^2 in purely nucleonic models

$$\varepsilon(n_B) = \underbrace{m_N n_B}_{\textbf{large (!)}} + \underbrace{a\frac{n_B^{5/3}}{m_N}}_{\textbf{small (!)}} + bn_B^{\alpha}$$

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\left(\frac{\varepsilon}{n_B}\right)$$

$$P = \underbrace{\frac{2}{3}a\frac{n_B^{5/3}}{m_N}}_{\textbf{small (!)}} + b(\alpha-1)n_B^{\alpha}$$

➡ at LO: p << ε (!)

if interactions dominate (at large n_B):

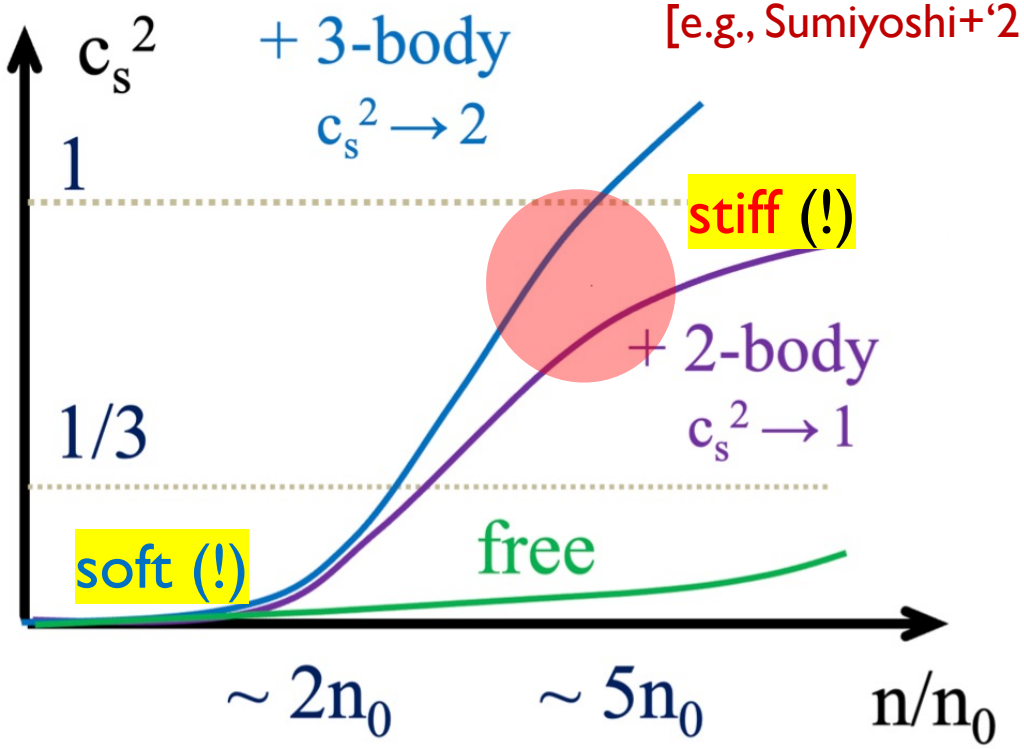
$$P \sim (\alpha-1)\varepsilon \;\rightarrow\; c_s^2 \sim (\alpha-1)$$

2-body int. → α = 2 (contact type)

3-body int. → α = 3

causality & convergence ??

[e.g., Sumiyoshi+'21]



alternative **baseline**: quark EOS

e.g.) free massless quarks

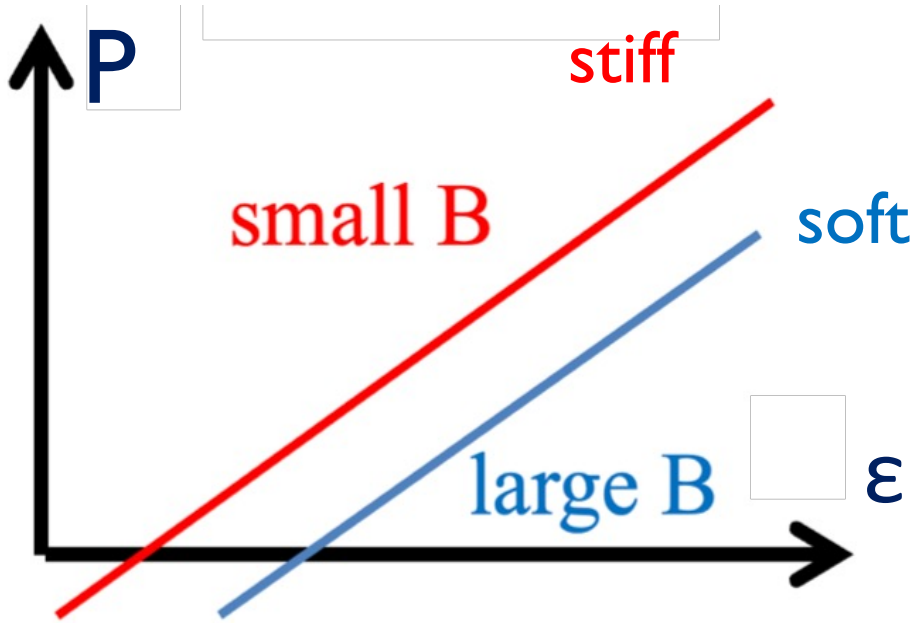
$c_s^2 = 1/3$

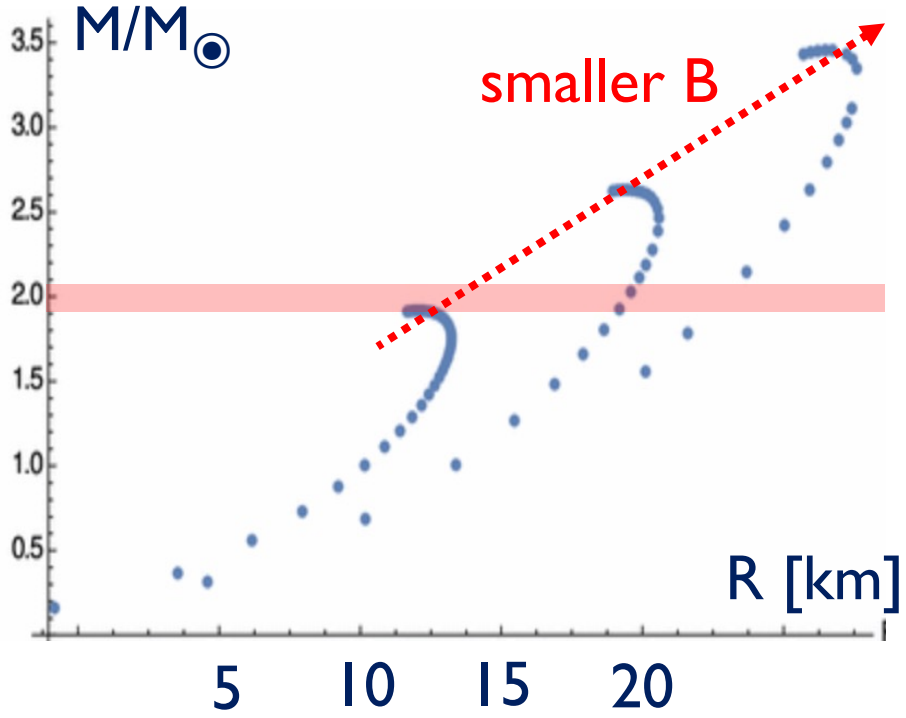
$$P = \frac{\varepsilon}{3} - B'$$

normalization

quark kin. E ~ N_c^2 x nucl. kin. E

~ N_c x p_F^2/M_q ~ $p_F^2/N_c M_q$





relativistic pressure → stiff EOS

can be a good starting point!?

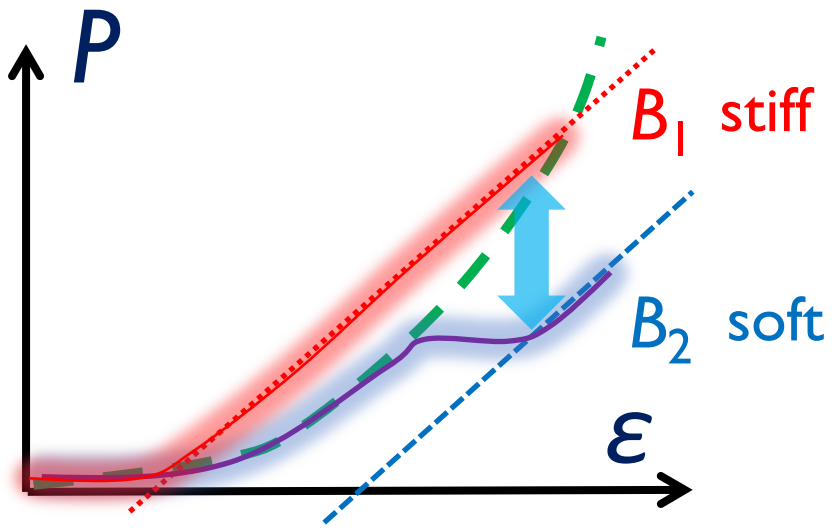
Hadron-to-quark transitions?

Confusing point:

- *Switching from baryonic to quark bases*

→ *a source of confusions in hybrid models*

(e.g. normalization of energy)



Strategy:

Keep track of quark states from nuclear to quark matter

(within a *single* model, e.g., percolation model, Fukushima-TK-Weise '20)

Contents

Sum rules for occupation probabilities

cf) [TK '21, TK-Suenaga '21]

occupation **probability** of **quark** state with p

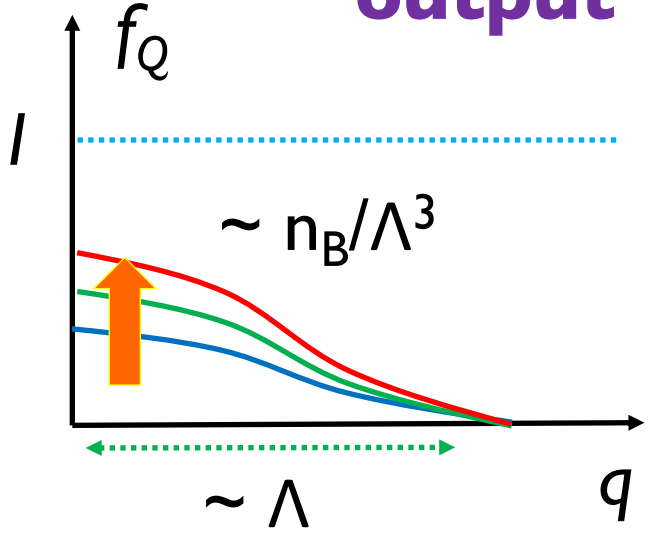
occupation **probability** of **baryon** state with P_B

quark mom. distribution **in a baryon**

$$f_{\mathrm{Q}}(\boldsymbol{q}; n_{\mathrm{B}}) = \int_{\boldsymbol{P}_{\mathrm{B}}} f_{\mathrm{B}}(\boldsymbol{P}_{\mathrm{B}}; n_{\mathrm{B}}) \varphi_{\mathrm{Q}}^{\mathrm{B}}(\boldsymbol{q}; \boldsymbol{P}_{\mathrm{B}})$$

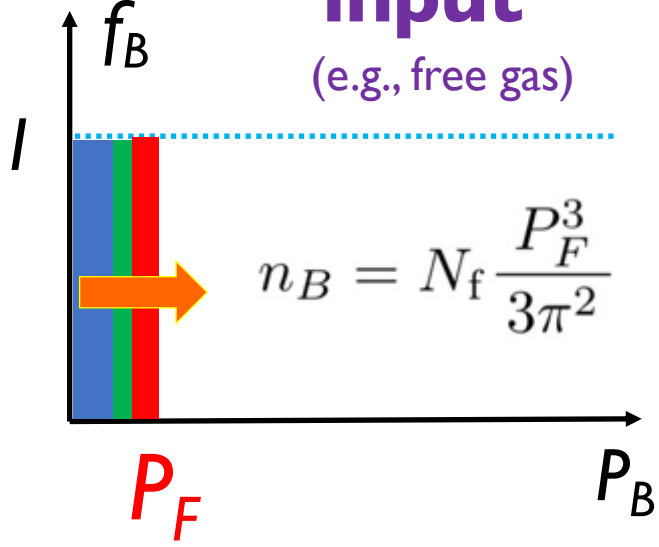
e.g.) in **ideal** baryonic matter

output



input

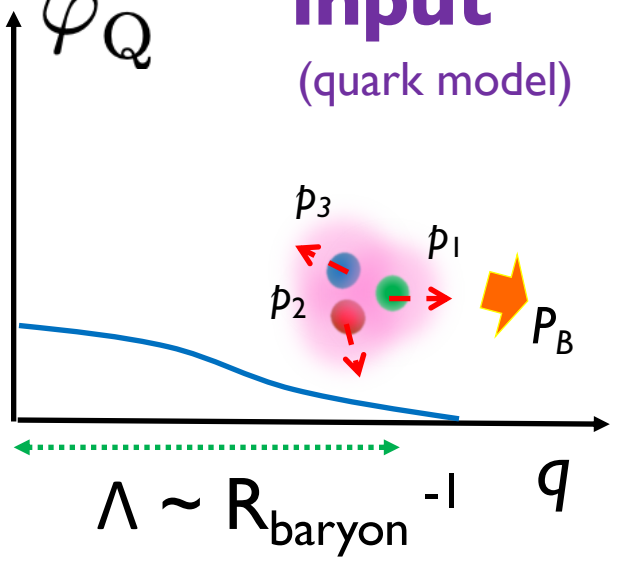
(e.g., free gas)



⊗

input

(quark model)



An ideal model

[Fujimoto-TK-McLerran, '23]

1) neglect interactions *except* confining forces

e.g.) 2-flavor hamiltonian:

$$\varepsilon_{\rm B}[f_{\rm B}] = 4\int_k E_{\rm B}(k) f_{\rm B}(k)$$

isospin, spin

2) quark distributions in a baryon remains the same (confinement persists)

3) use a special quark distribution → models become analytically *solvable*

$$\varphi_{\rm 3d}(\boldsymbol{q}) = \frac{2\pi^2}{\Lambda^3}\frac{e^{-q/\Lambda}}{q/\Lambda} \qquad \hat{L} = -\boldsymbol{\nabla}^2 + \frac{1}{\Lambda^2} \qquad \hat{L}[\varphi(\boldsymbol{p}-\boldsymbol{q})] = \frac{(2\pi)^3}{\Lambda^2}\delta(\boldsymbol{p}-\boldsymbol{q})$$

nontrivial output

$$f_{\rm Q}(\mathrm{q}) = \int_{\boldsymbol{P}_B} f_{\rm B}(\boldsymbol{P}_B)\varphi_Q^B(\mathrm{q}-\boldsymbol{P}_B/N_{\rm c})$$

*natural at **low** density*

⇄

nontrivial output

$$f_{\rm B}(N_{\rm c}\boldsymbol{q}) = \frac{\Lambda^2}{N_{\rm c}^3}\hat{L}\left[f_{\rm Q}(\boldsymbol{q})\right]$$

*natural at **high** density*

useful for studies of the *transient regime* (d.o.f are not clear-cut)

Variational problem with sum rule constraints

[Fujimoto-TK-McLerran, '23]

Lagrange multiplier

$$\tilde{\varepsilon} = \varepsilon_{\rm B}[f_{\rm B}] - \lambda_{\rm B} n_{\rm B}$$

$$E_{\rm B}(k) = \sqrt{M_{\rm B}^2 + k^2} \qquad n_{\rm B} = 4\int_k f_{\rm B}(k)$$

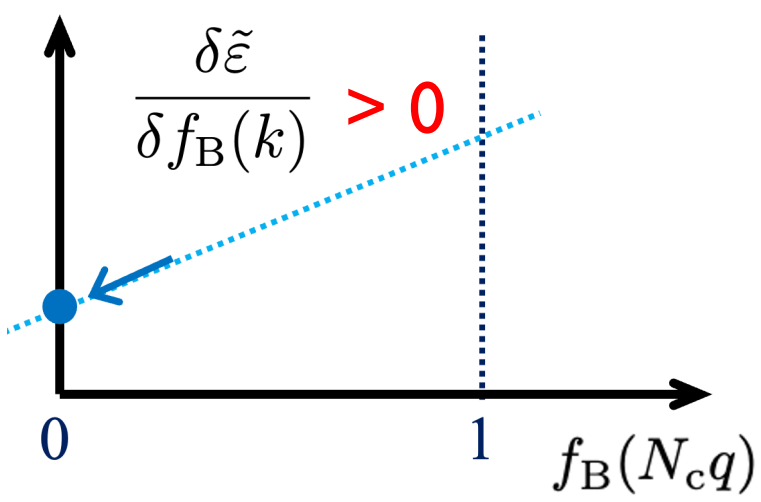
optimization: $\dfrac{\delta\tilde{\varepsilon}}{\delta f_{\rm B}(k)} = E_{\rm B}(k) - \lambda_{\rm B}$ *at a given k*

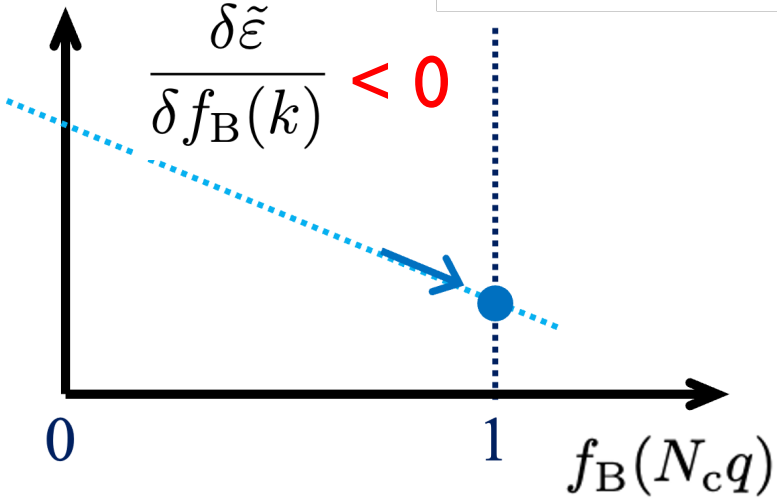
$E_B(k) > \lambda_B$

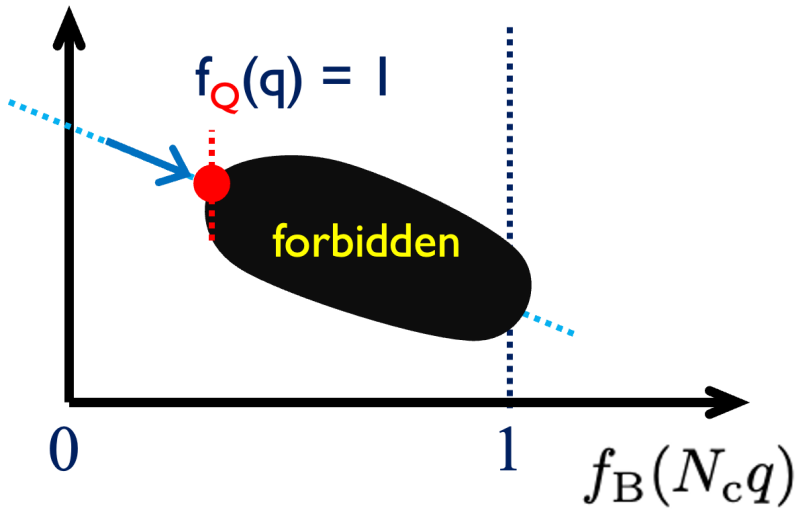
$E_B(k) < \lambda_B$

w.o. saturation

with saturation

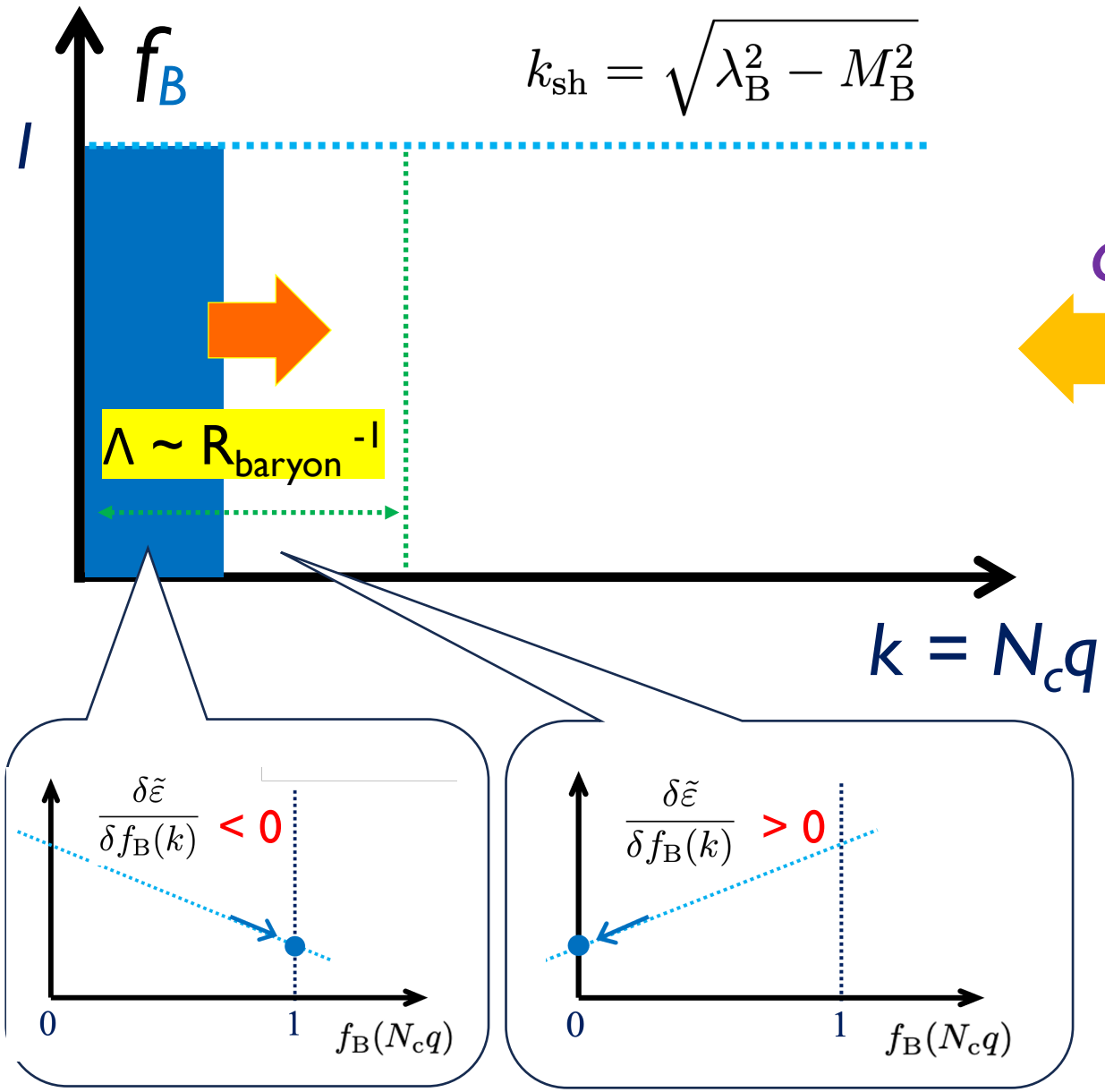




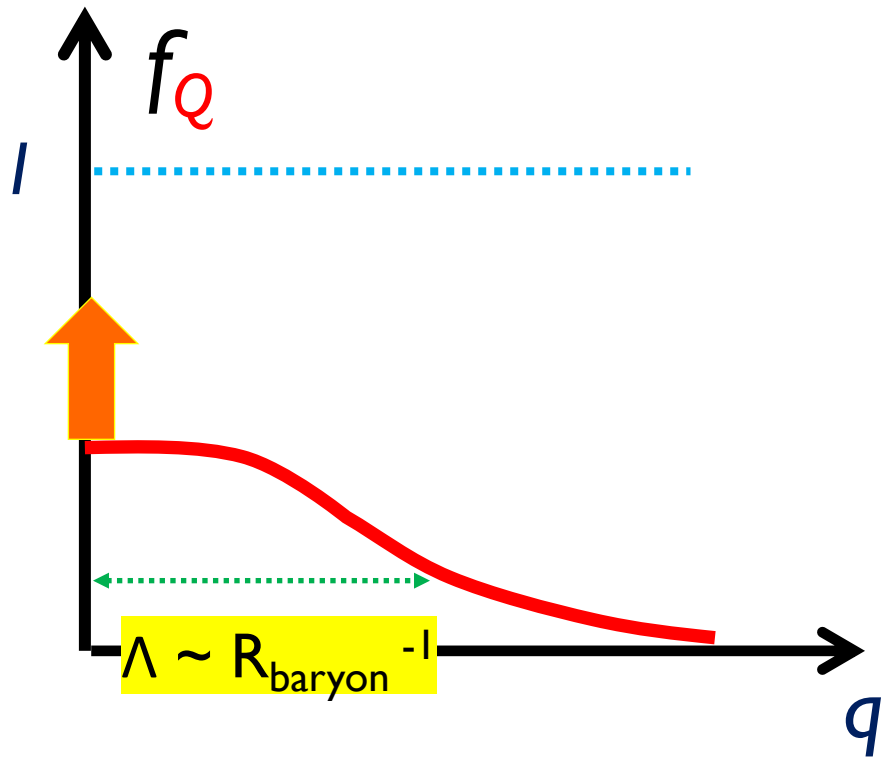


Solution (dilute regime)

[Fujimoto-TK-McLerran, '23]

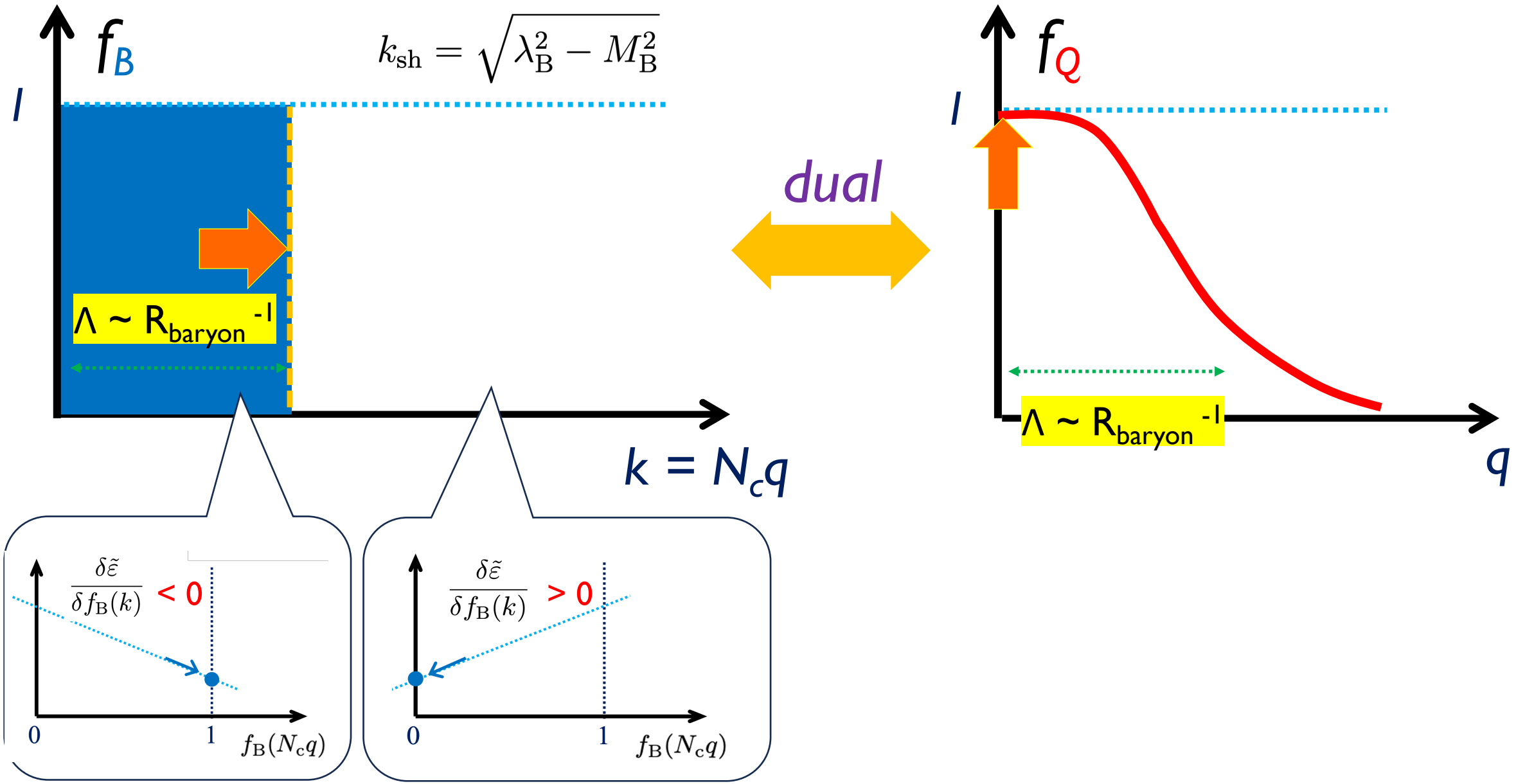


dual



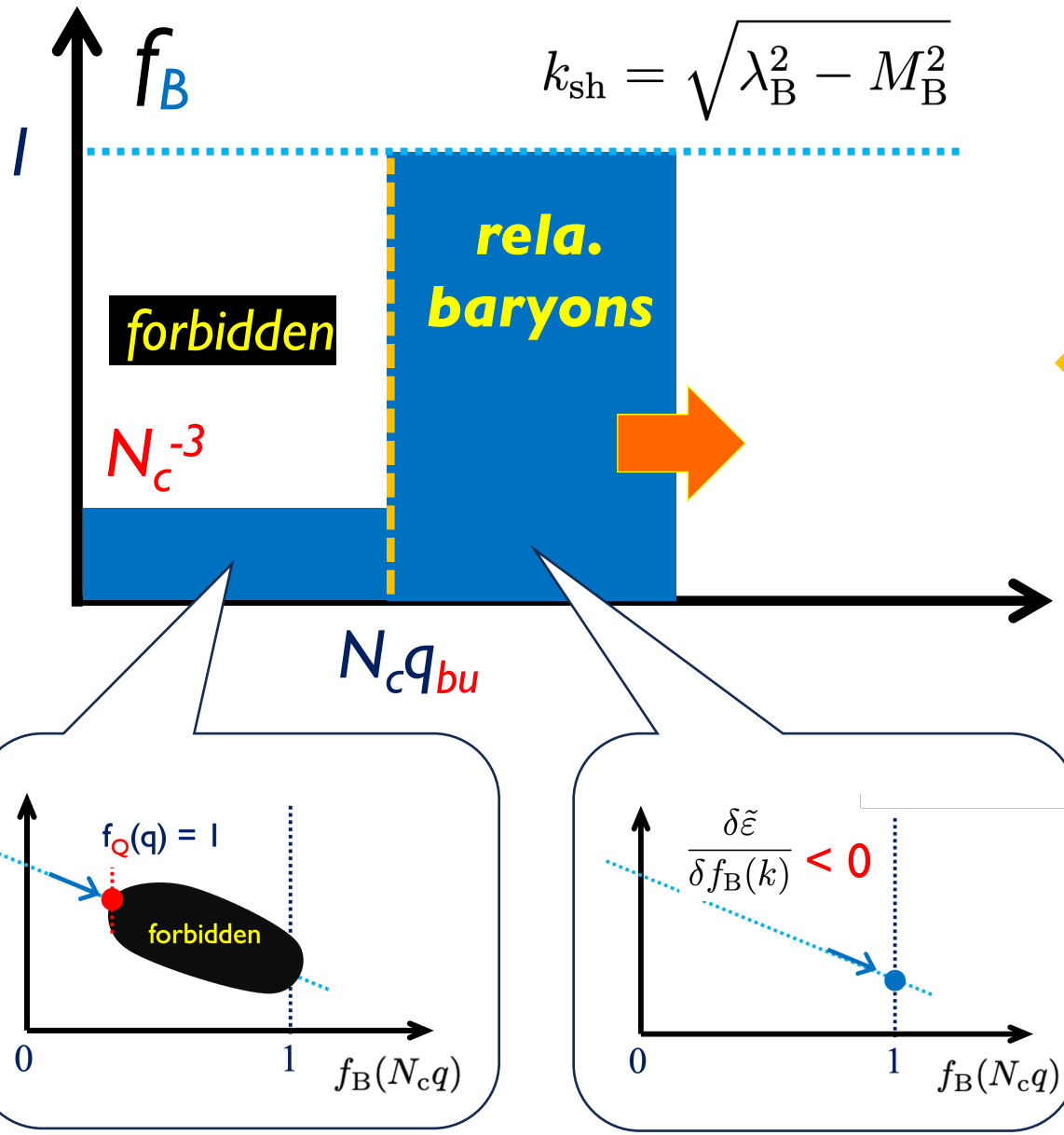
Solution (at saturation)

[Fujimoto-TK-McLerran, '23]

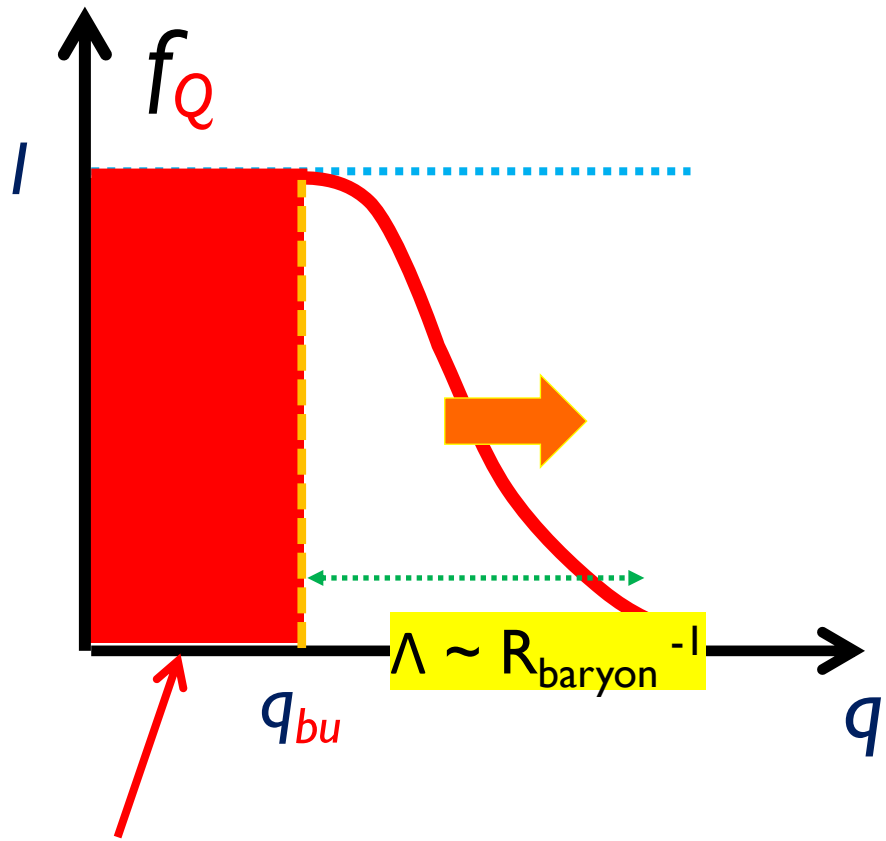


Solution (post saturation)

[Fujimoto-TK-McLerran, '23]

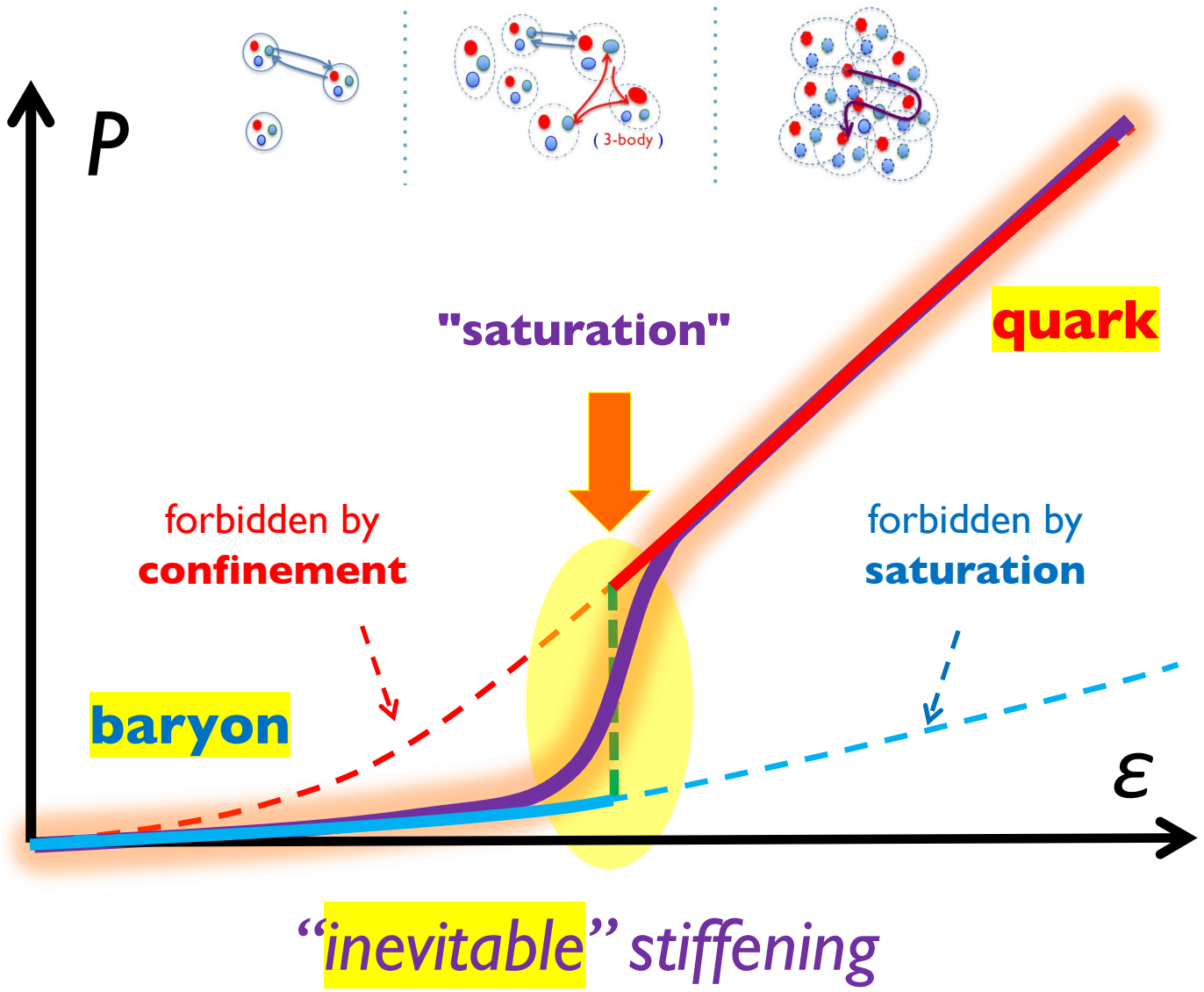


dual

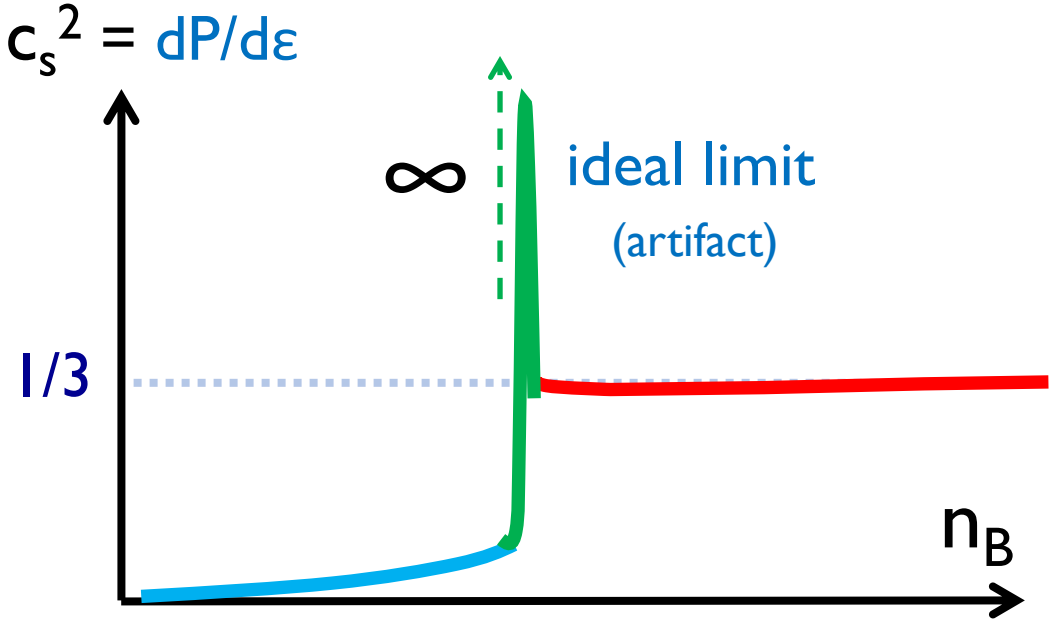


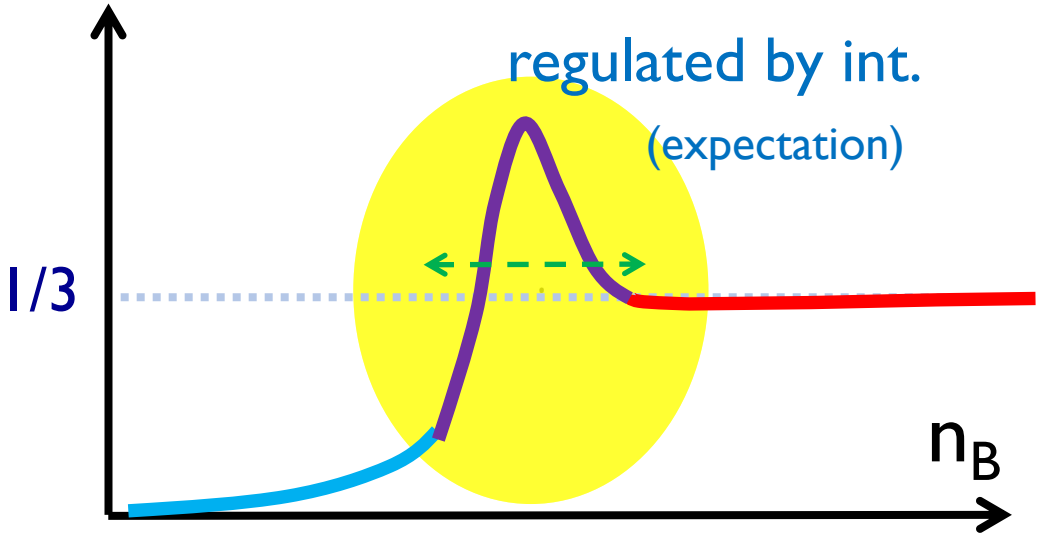
"inevitable" formation of the quark Fermi sea

Peak in sound velocity



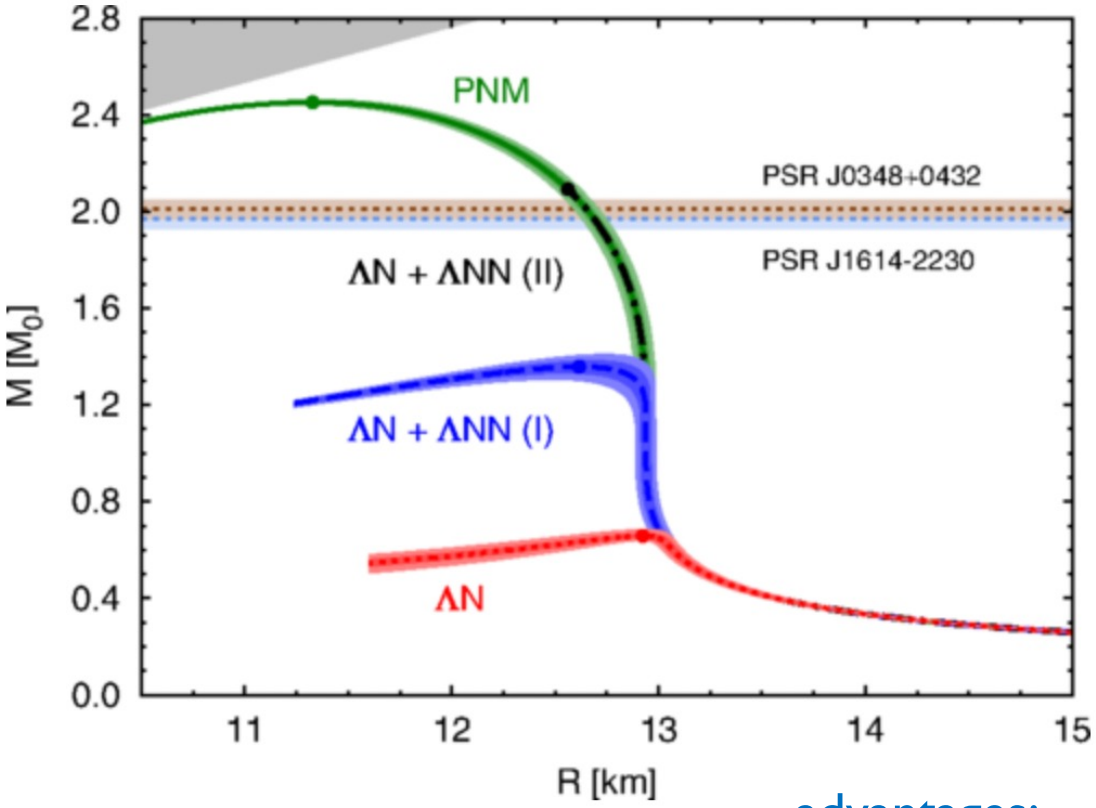
"inevitable" stiffening





Contents

Hyperon Puzzle



the appearance of hyperons (or Δ,...):

- adds large energy but small pressure

  ➔ softening of EOS (at 2-3n_0)

- many species; 3Σ, 2Ξ, Λ (octet),...

often used approach to pass 2Msun:

introduce
strong YN, YY, YNN,.. repulsion

... convergence or regularity??

advantages:

1) statistical repulsion (Pauli blocking)
2) more repulsive at higher density
3) no double counting of quarks from different baryons

alternative idea:

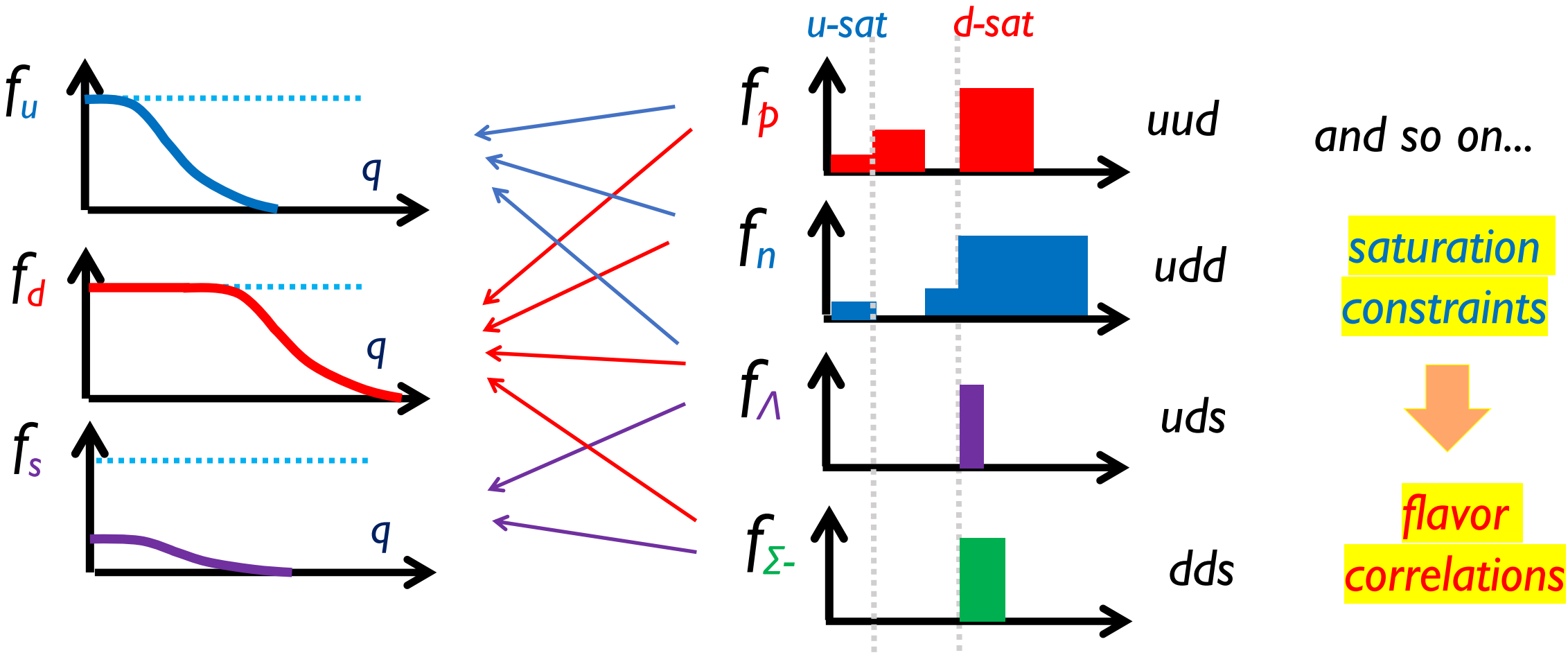
quark saturation

Multi-flavor extension

[Fujimoto-TK-McLerran, in prep.]

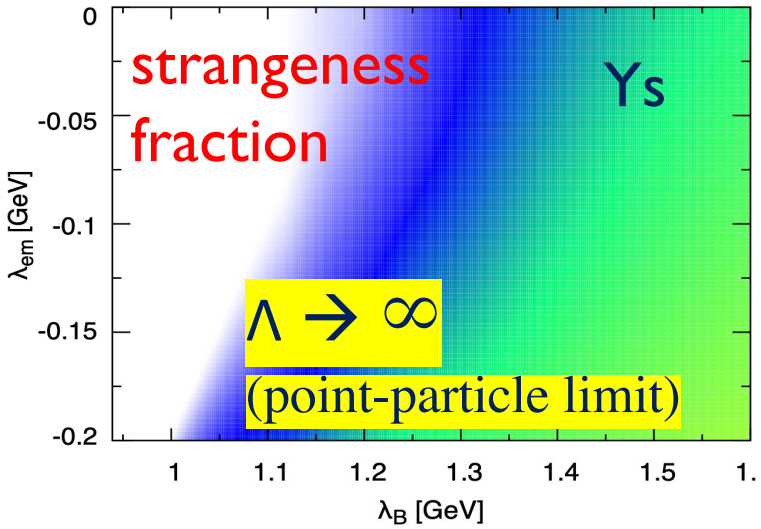
$$f_{\mathrm{Q}}(\boldsymbol{q}) = \sum_{B=p,n,\Sigma,\cdots} N_{\mathrm{Q}}^{\mathrm{B}} \int_{\boldsymbol{k}} f_{\mathrm{B}}(\boldsymbol{k}) \varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}}{N_{\mathrm{c}}}\right)$$

Q = u, d, s



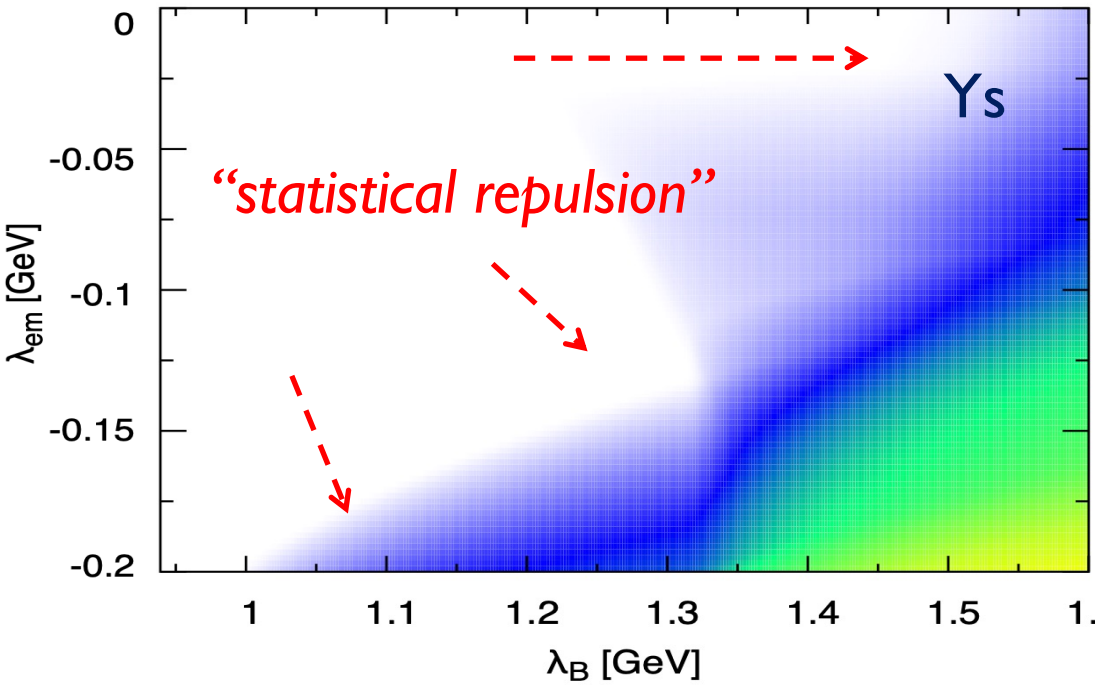
Hyperon suppressions

[Fujimoto-TK-McLerran, in prep.]



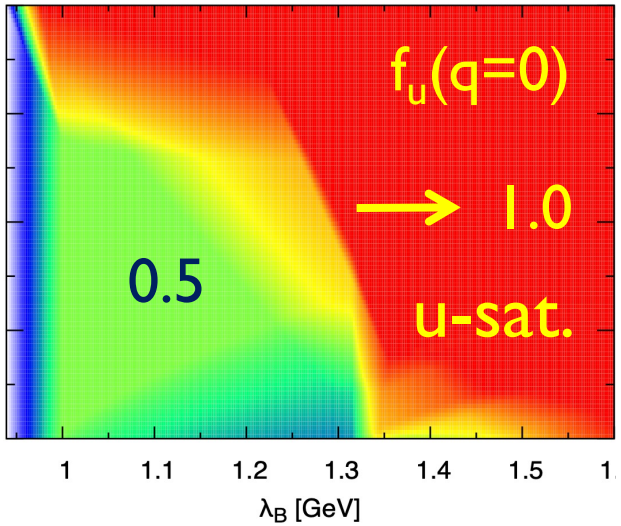
finite size

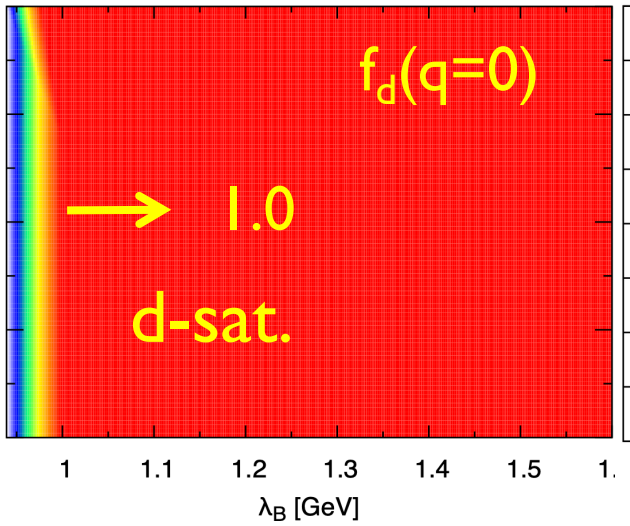
Λ = 0.3 GeV

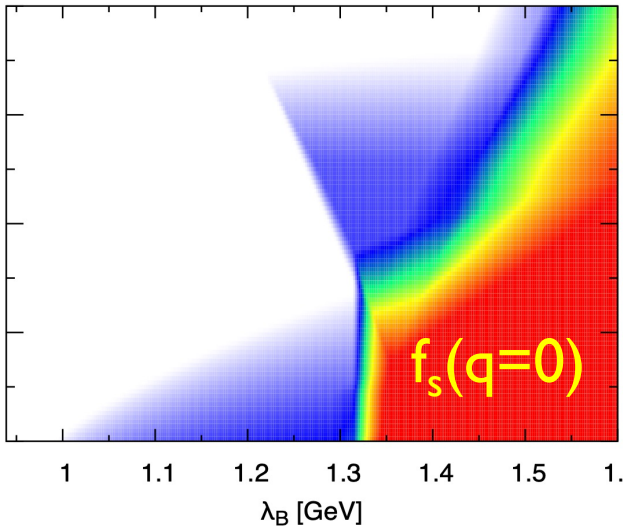


u, d-saturation

-> statistical repulsion on hyperons







Summary & Outlook

- For soft-to-stiff EOS: QHC is a good baseline
- Quark saturation likely occurs at ~1-3n_0 ➔ *statistical* repulsion

  quantitative estimates need info of quark w.f. of hadrons in medium

- Location of cs2 peak ➔ info. about the quark substructure of hadrons

  a new context: a link between NS EOS and hadron spectroscopy is being built.

- How do quarks in exchanging mesons become *real*?

  need quark descriptions of hadron-hadron interactions & hadron structures