Hadron effective masses from lattice simulation in 2-color QCD at finite density

(格子計算による有限密度2カラーQCDにおけるハドロン有効質量の研究)

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based on

KM, D. Suenaga, K. Iida and E. Itou, **“Measurement of hadron masses in 2-color finite density QCD,”** PoS LATTICE2022, 154 (2023) [arXiv: 2211.13472 [hep-lat]]

ELPH研究会C035「実験、反応・構造計算、格子QCD で解き明かすハドロン分光」

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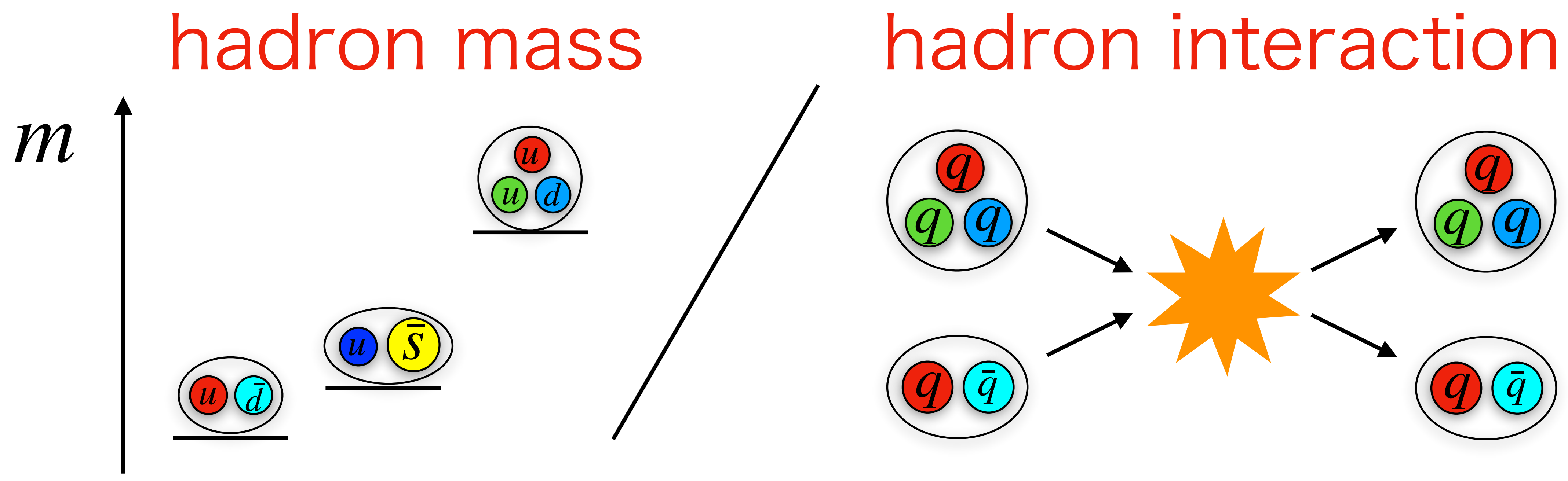
Introduction

- studying dense QCD is one of the most important problems in physics
    - connected to understanding of neutron stars, structure of nuclei, etc.

- **hadronic properties in medium** is interesting subject from various aspects
    - key to understand **the origin of the thermodynamic properties** in dense QCD

      e.g., equation of state
    - signatures of chiral-symmetry restoration in medium
    - strongly related to nuclear physics and studies of the nuclear exotic states

      e.g., kaon mass in medium ➡ kaonic nuclei

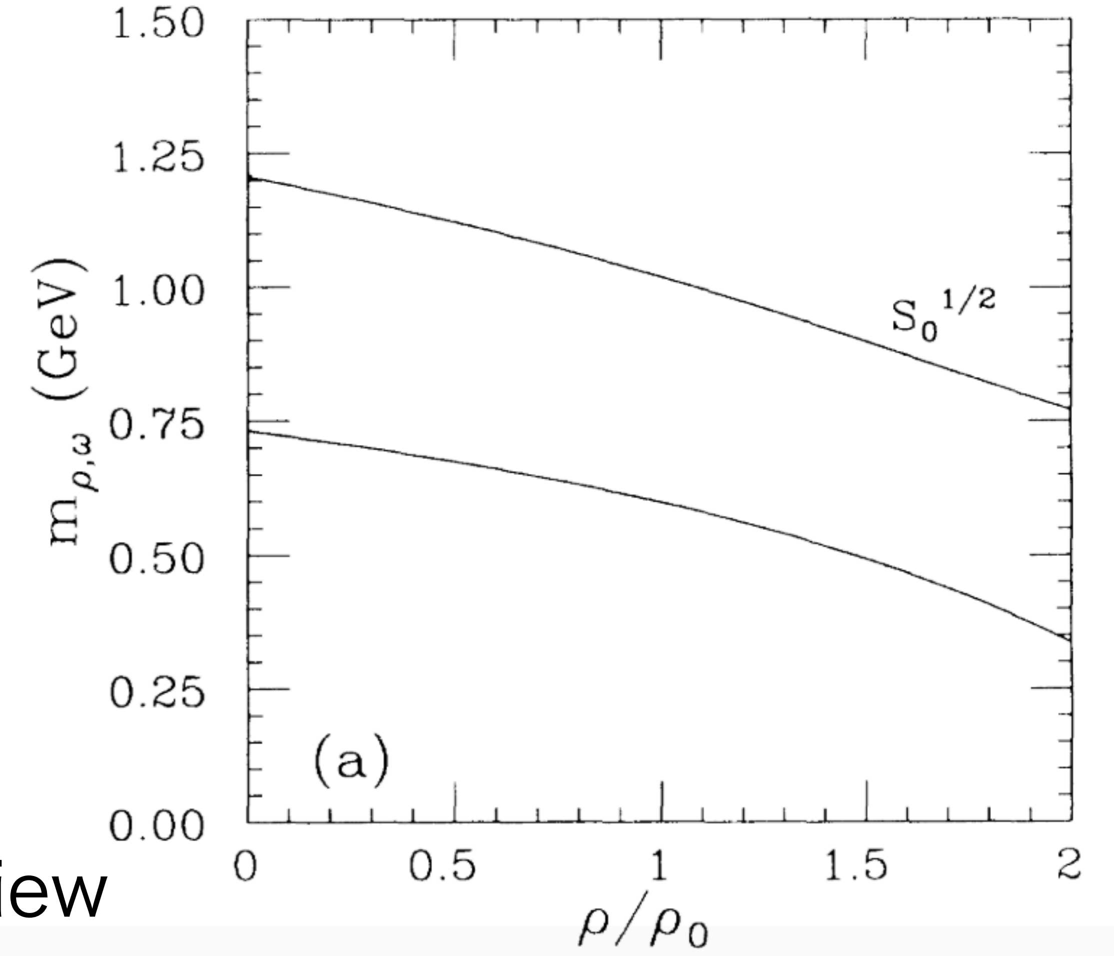


Hadron masses in medium

- we consider hadron mass in low T and finite μ
- zero density:
  - **QCD inequality: pion is the lightest meson**
  - a lot of studies for a long time
- finite density:
  - **QCD inequality does not hold**
  - attract both theoretical and experimental attention for several decades

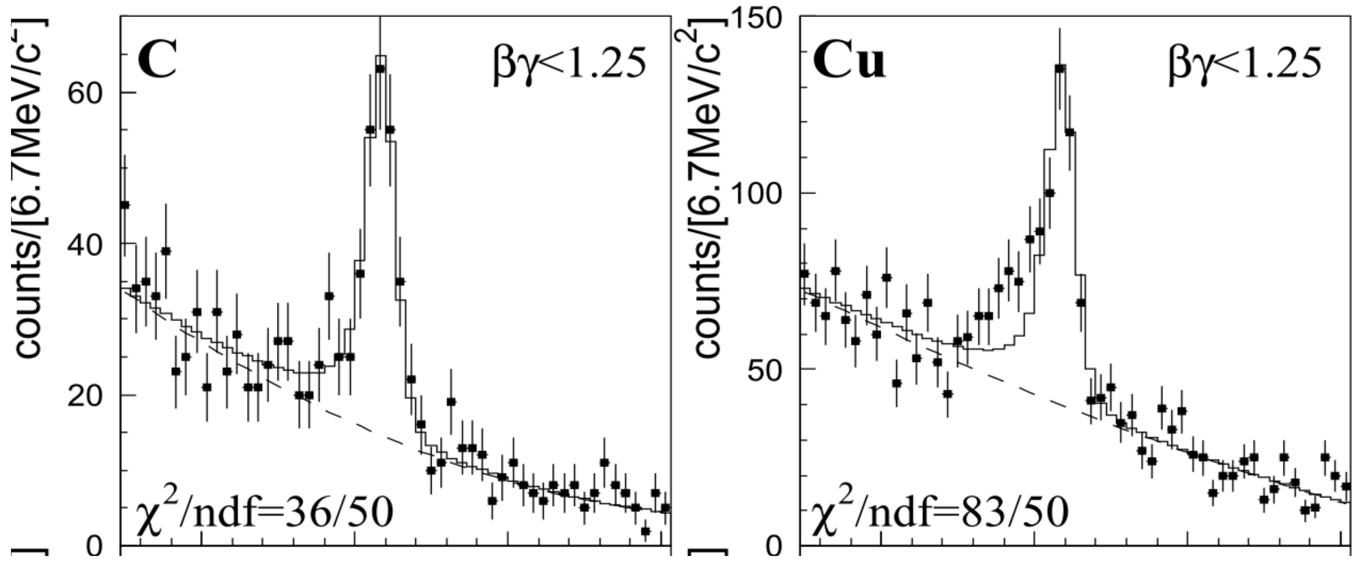
[Gubler, Satow, 2019] for review
  - lattice QCD: **difficult due to sign problem**

m_ρ v.s. density from QCD sum rule



(T. Hatsuda and S. H. Lee, Phys. Rev. C **46**, no.1, R34 (1992))

ϕ meson mass spectra from p+C and p+Cu



(KEK-PS E325, Phys. Rev. Lett. 98, 042501 (2007))

- we study hadron masses in **2-color QCD at finite density**

2-color QCD in finite density

- 2-color QCD: QCD with SU(2) gauge field
- **lattice simulation is available for** $\mu \neq 0$

C : charge conjugation operator

τ_i : Pauli matrix on 2-color space

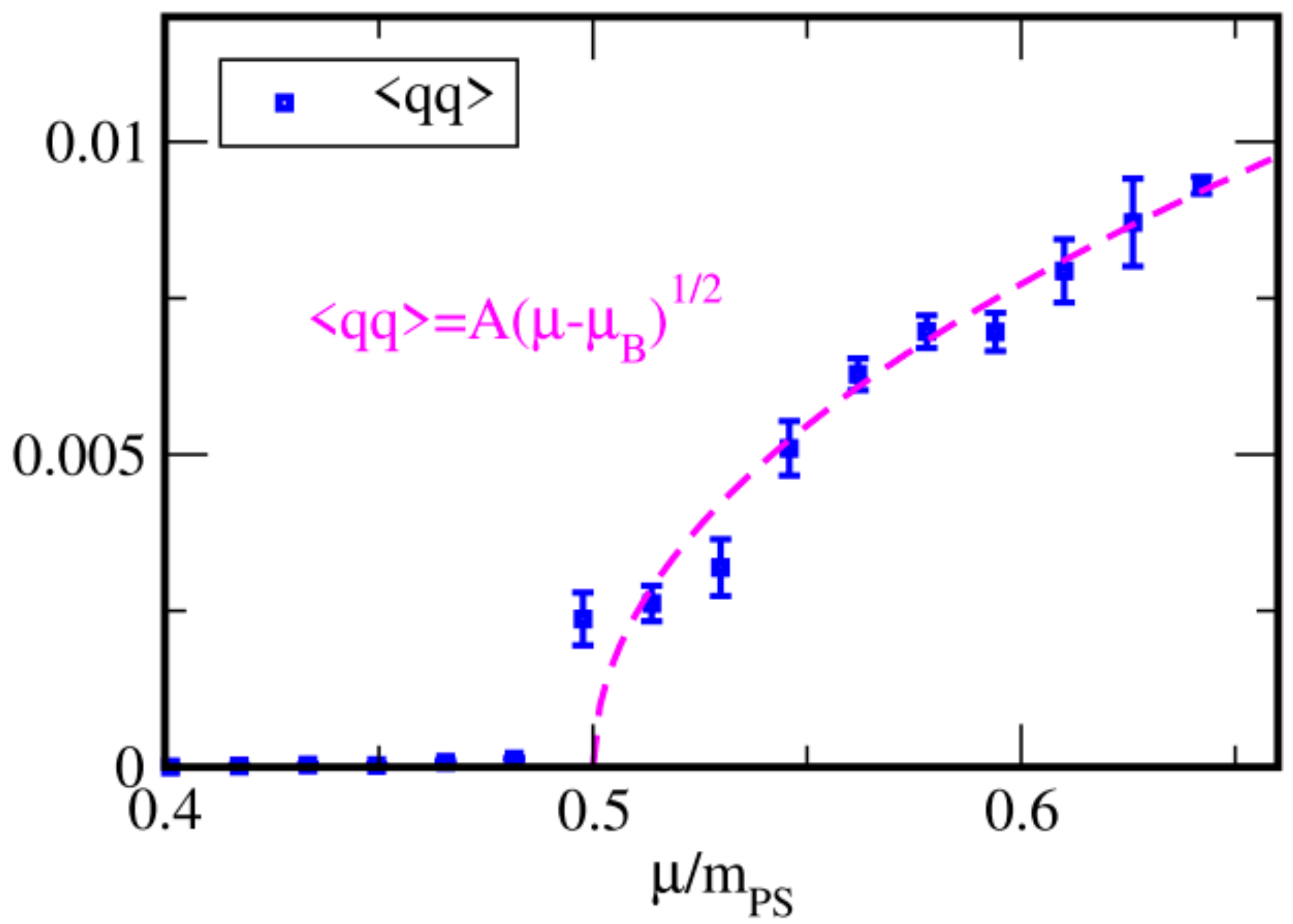
$$(\det D(\mu))^* = \det D^*(\mu) = \det[\gamma_5 \underline{C\tau_2} D(\mu)\tau_2 C^{-1}\gamma_5] = \underline{\det D(\mu)}$$

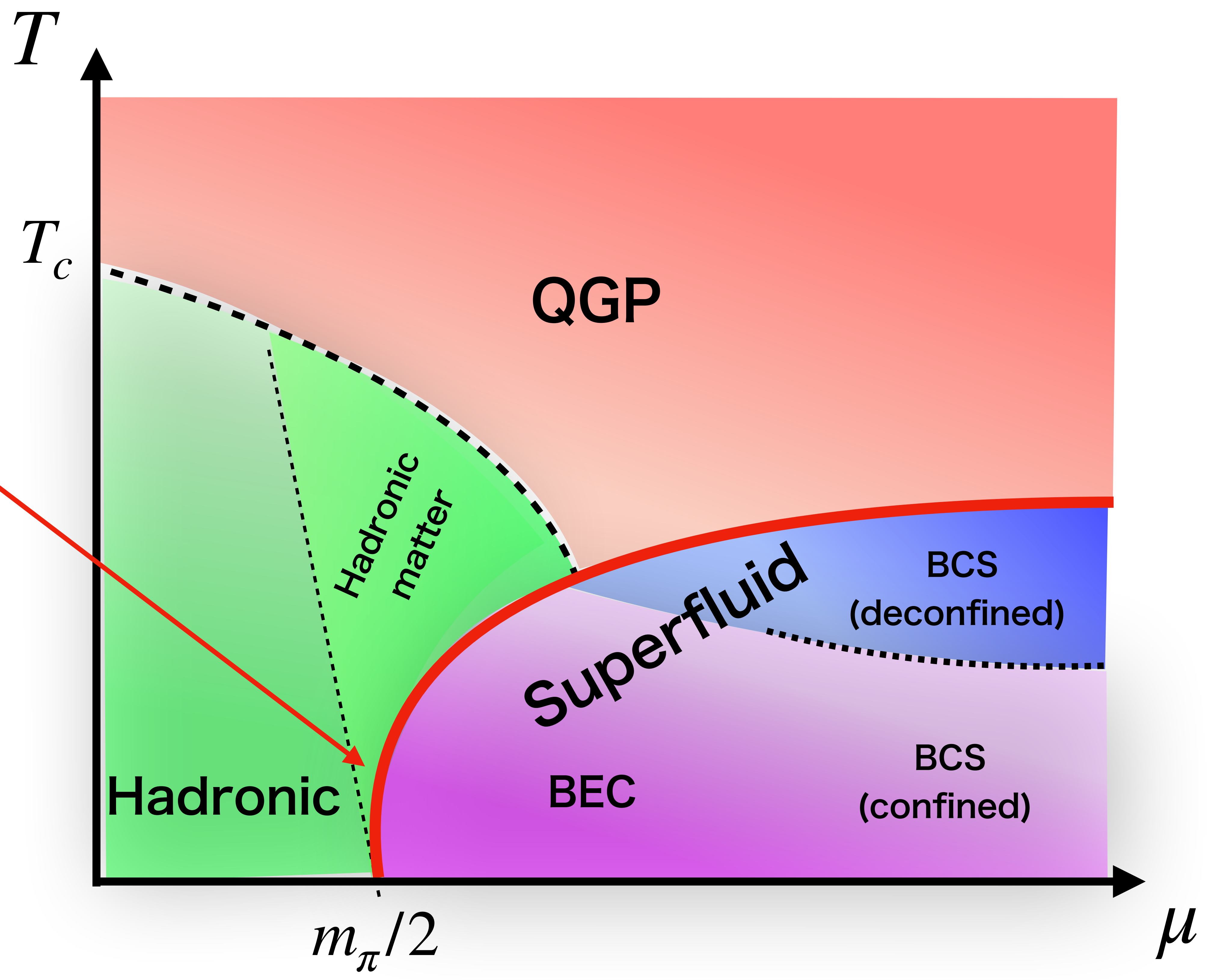
→ **real**

- phase diagrams in 2-color QCD:

2nd-order phase transition
due to diquark condensation

(K. Iida, E. Itou and T. -G. Lee,
JHEP **01**, 181 (2020))





Hadrons in 2-color QCD

- (main) hadrons in 2-color QCD: mesons and **diquarks**

$$\bar{\psi}_a \Gamma \psi_a \qquad \epsilon_{ab}\psi_a (C\gamma_5)\Gamma\psi_b$$

- flavor symmetry in low T ($N_f = 2$):

Classical (Lagrangian) level at $\mu = 0$, $m = 0$

$$SU(4) \times \cancel{U(1)_A}$$ (cf., 3-color QCD Lagrangian: $SU(2)_V \times SU(2)_A \times U(1)_B \times \cancel{U(1)_A}$)

anomaly

$\langle \bar{\psi}\psi \rangle \neq 0$, $m \neq 0$

$$Sp(4) \ (\sim SO(5))$$

5 (pseudo-)NG bosons

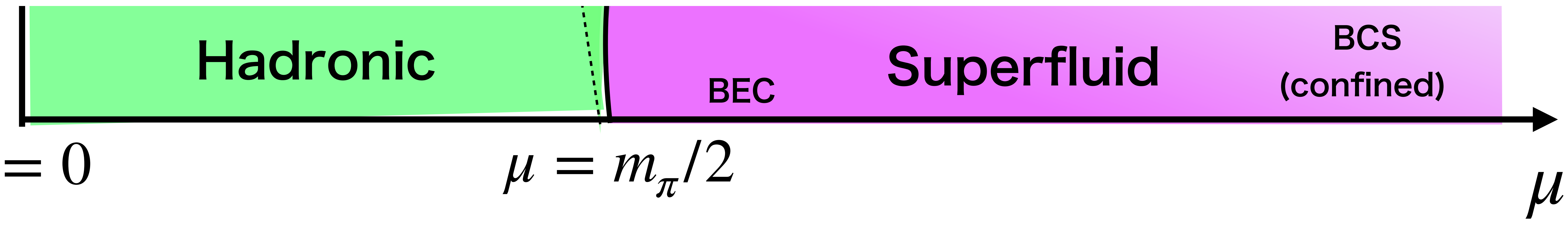
- pion ($\pi^{\pm}$, π^0)
- scalar diquark (qq)
- scalar antidiquark ($\bar{q}\bar{q}$)

$U(1)_B \times SU(2)_V$

mass shift for qq, $\bar{q}\bar{q}$

$Sp(2)_V \ (\sim SU(2)_V)$

no $U(1)_B$, NG mode by $\langle \psi\psi \rangle \neq 0$

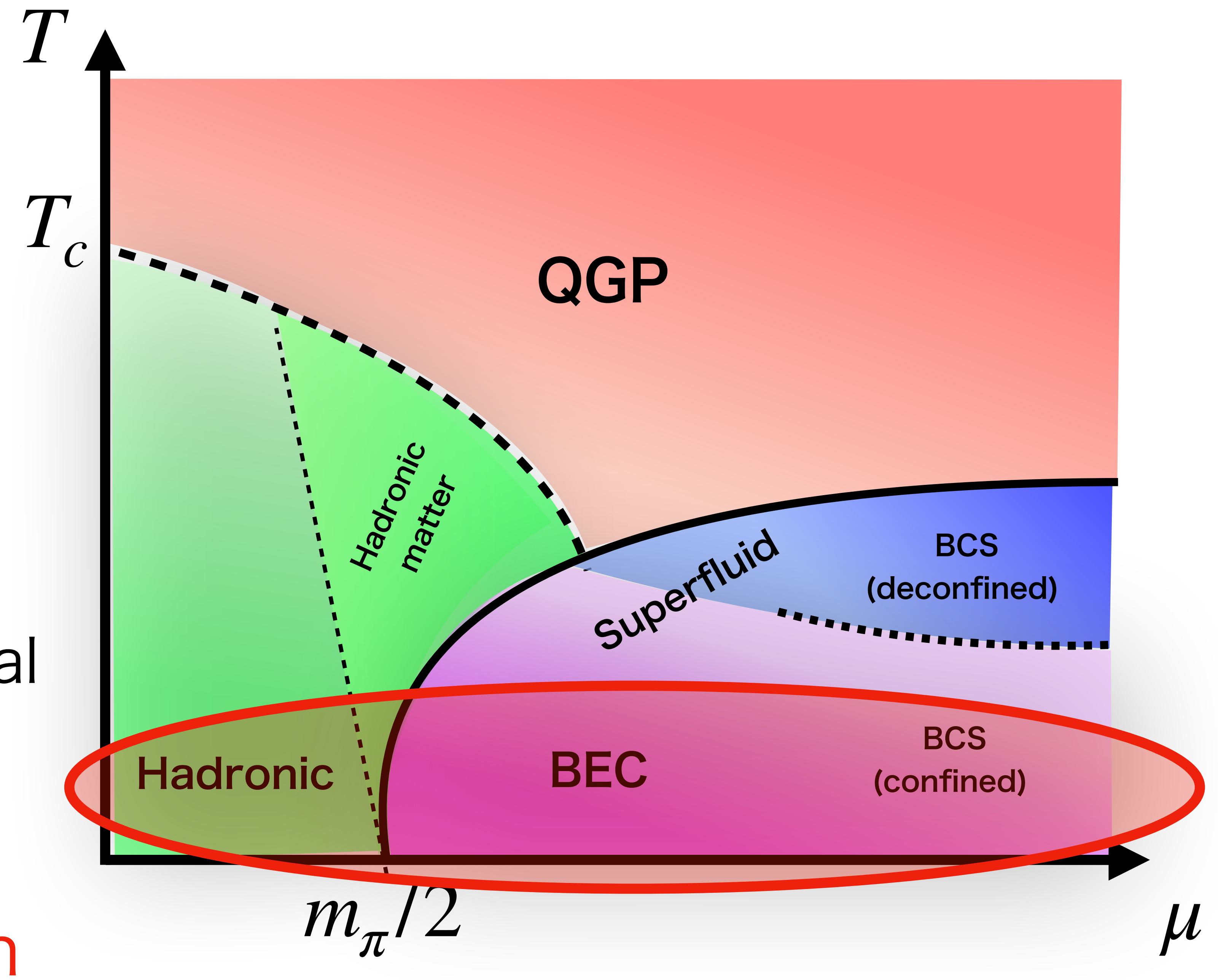


Hadron masses in 2-color QCD in finite density

- we focus on hadron masses in low T and $\mu \neq 0$

- hadronic phase: **μ-dependence of single hadron energy is trivial at** $T = 0$ (for any #color)

(MK, Itou, Iida, submitting to journal [arXiv:2309.08143 [hep-lat]])



$$E = \sqrt{\mathbf{p}^2 + \underline{m}^2} - \mu \underline{n}$$

hadron mass at $\mu = 0$

quark number of the hadron (meson: $n = 0$, diquark: $n = +2$)

- superfluid phase: **non-trivial μ-dependence** has been observed in several previous lattice studies [Muroya, Nakamura, Nonaka, 2003] [Hands, Sitch, Skullerud, 2008] [Wilhelm et al., 2019]
  - flipping spectral ordering of π and ρ meson
  - NG mode generated by the diquark condensation

Lattice simulation in superfluid phase

- **another problem in superfluid phase:** lattice simulation cannot be done due to zero eigenvalue of the Dirac operator (**onset problem**)

caused by gapless mode (NG mode) for diquark condensation

- cf., in the case of χSSB at $\mu = 0 \cdots$

NG bosons generated by χSSB (pion) become massless at $m = 0$

→ zero eigenvalues of Dirac operator (lattice simulation is impossible) at $m = 0$

- to avoid the **onset problem**, we add **diquark source term** to the quark action

"mass term" for the gapless mode

$$S_F = \int d^4x\ \bar{\psi}_f(x)(\gamma_\mu D_\mu + m + \mu\gamma_0)\psi_f(x) + j\hat{D} \quad (f = 1, 2)$$

$$\hat{D} = -\frac{1}{2}\int d^4x\ [\bar{\psi}_1(x)(C\gamma_5)\tau_2\bar{\psi}_2^T(x) - \psi_2^T(x)(C\gamma_5)\tau_2\psi_1(x)]$$: diquark source term

Quark propagators for 2-color QCD in finite density

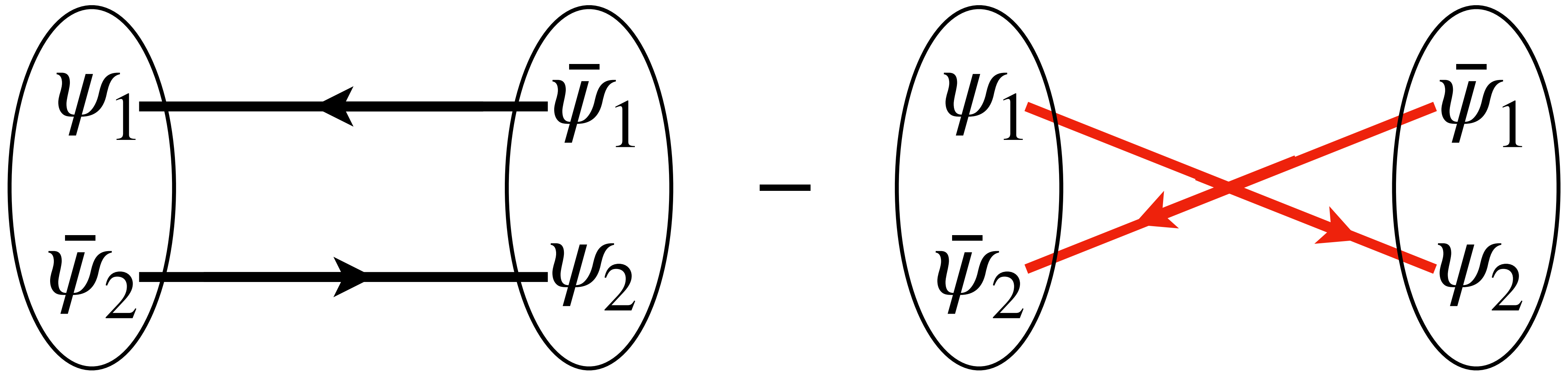
- quark action with diquark source term $S_F = \int d^4x\, \bar{\psi}_f(x)(\gamma_\mu D_\mu + m + \mu\gamma_0)\psi_f(x) + j\hat{D}$

➡ **3 types of quark propagators**

- $\psi_f \bar{\psi}_f$: normal propagator
- $\psi_2 \psi_1^T$: anomalous propagator (quark ➡ antiquark) ($\propto j$)
- $\bar{\psi}_2^T \bar{\psi}_1$: anomalous propagator (antiquark ➡ quark) ($\propto j$)

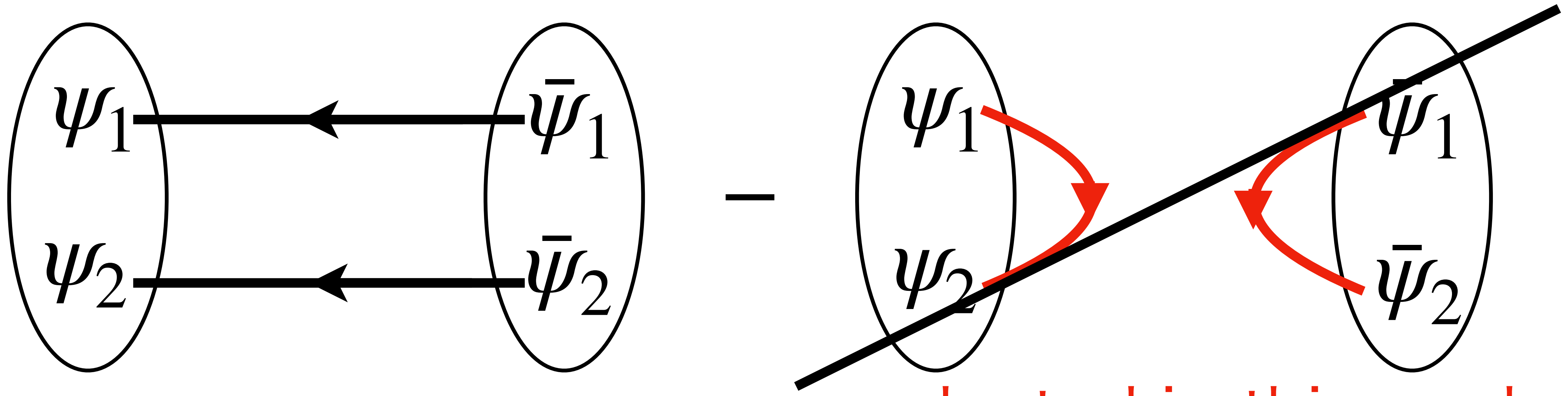
- examples of the quark contractions
  - pion 2pt function

$$\langle \bar{\psi}_2 \gamma_5 \psi_1(t)\ (\bar{\psi}_2 \gamma_5 \psi_1)^\dagger(0) \rangle =$$



  - scalar diquark 2pt function

$$\langle \psi_1 C\gamma_5 \psi_2(t)\ (\psi_1 C\gamma_5 \psi_2)^\dagger(0) \rangle =$$



neglected in this work

Simulation details

- Conf.: Iwasaki gauge action + 2-flavor Wilson quark action in QC2D in finite μ and j (gauge fixing, 400 confs.)

- $\beta = 0.8$, 32^4 lattices, $m_\pi/m_\rho \approx 0.81$ at $\mu = 0$, $T \approx 0.19T_c$

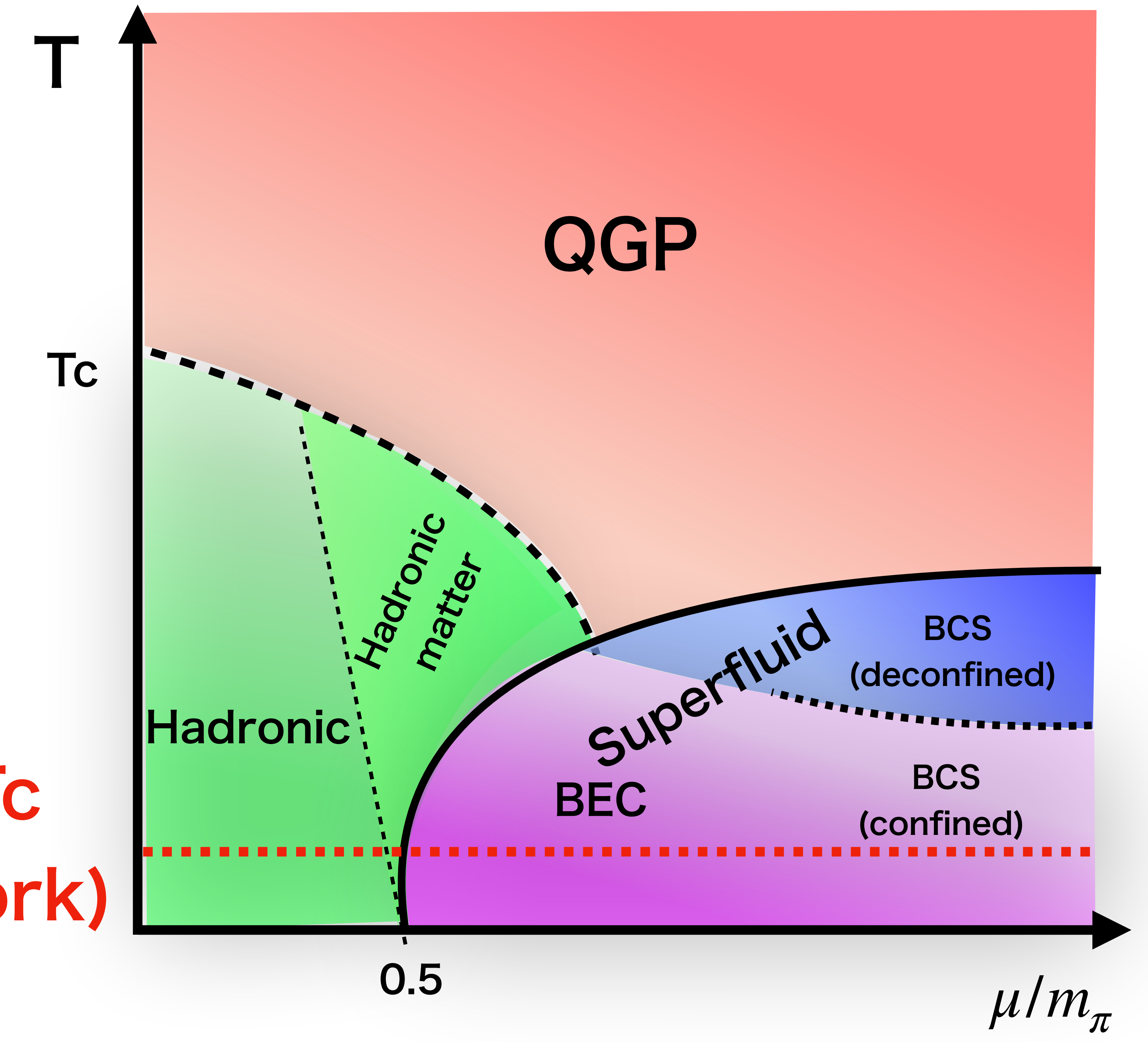
- target:
  - π, ρ meson (meson with $J^P = 1^-$ and isospin $I = 1$)
  - (pseudo) scalar ($J^P = 0^\pm$) meson/diquark/antidiquark with isospin $I = 0$

- at this moment,
  - use $j \neq 0$

    (not so small value in superfluid phase)
  - neglect disconnected diagrams

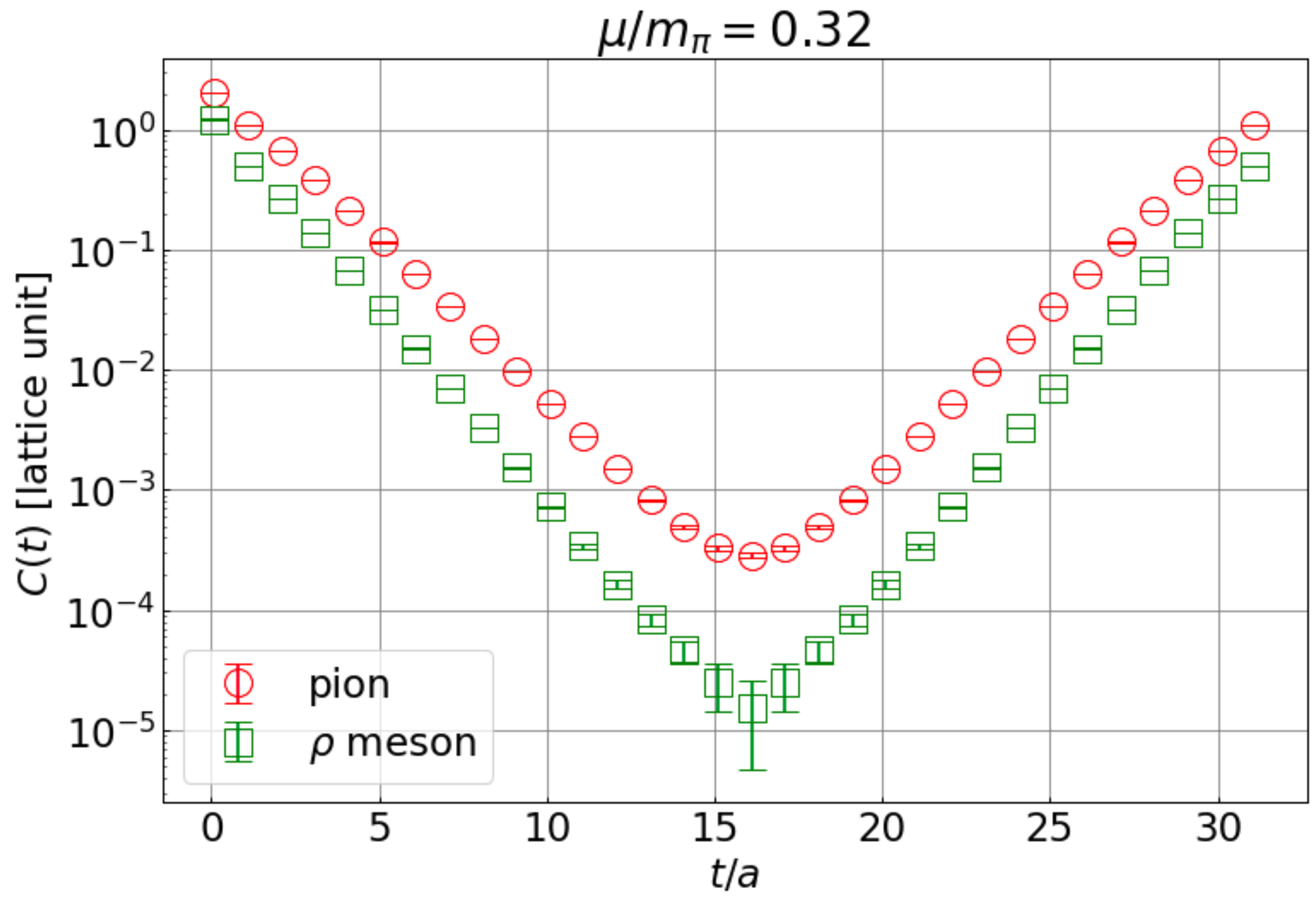
(K. Iida, E. Itou and T. -G. Lee, PTEP **2021**, no.1, 013B05 (2021))

0.19Tc
(this work)



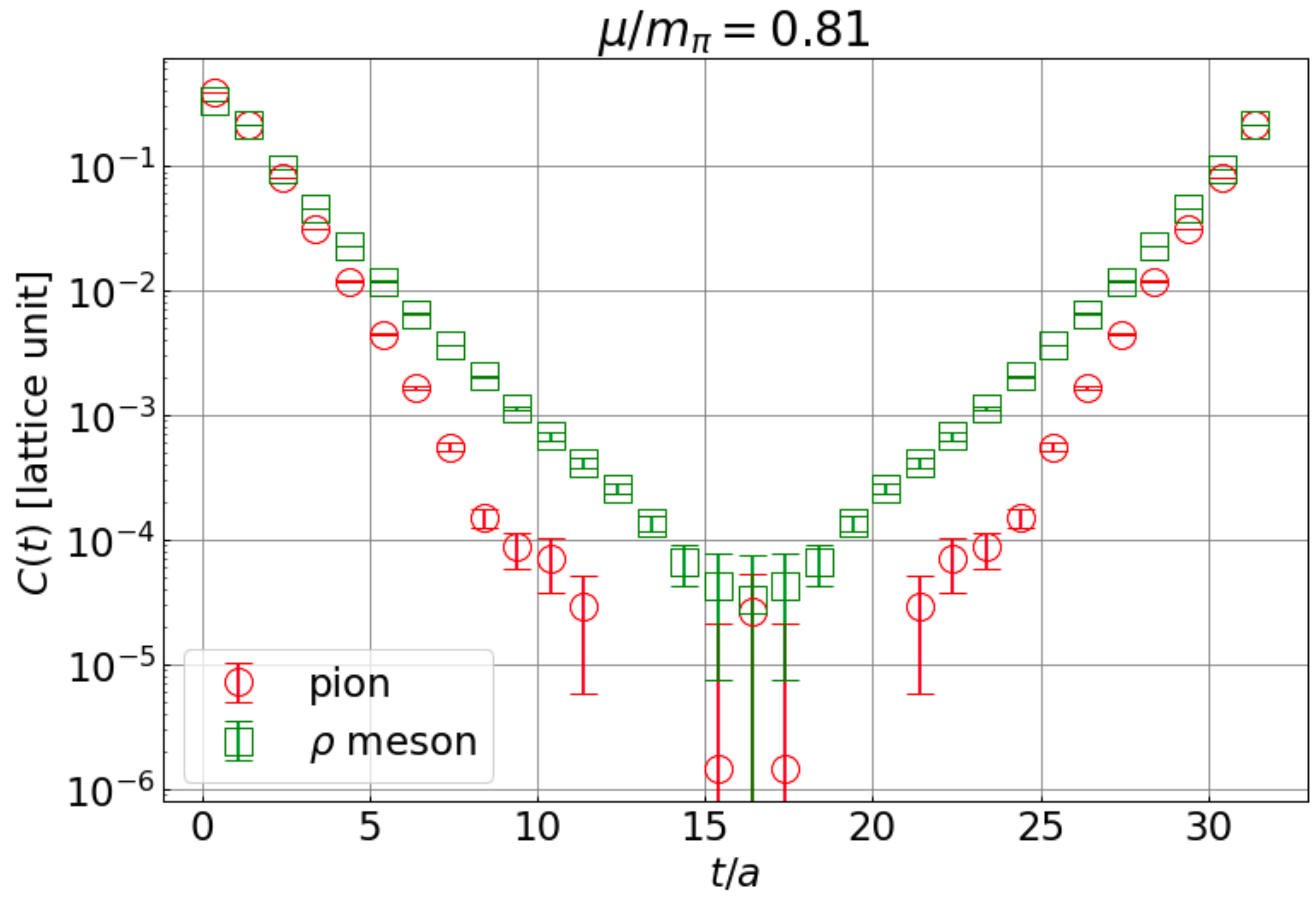
2pt functions of π and ρ meson

in hadronic phase



- $C^\rho(t) < C^\pi(t)$ holds
- both signals are clear

in superfluid phase

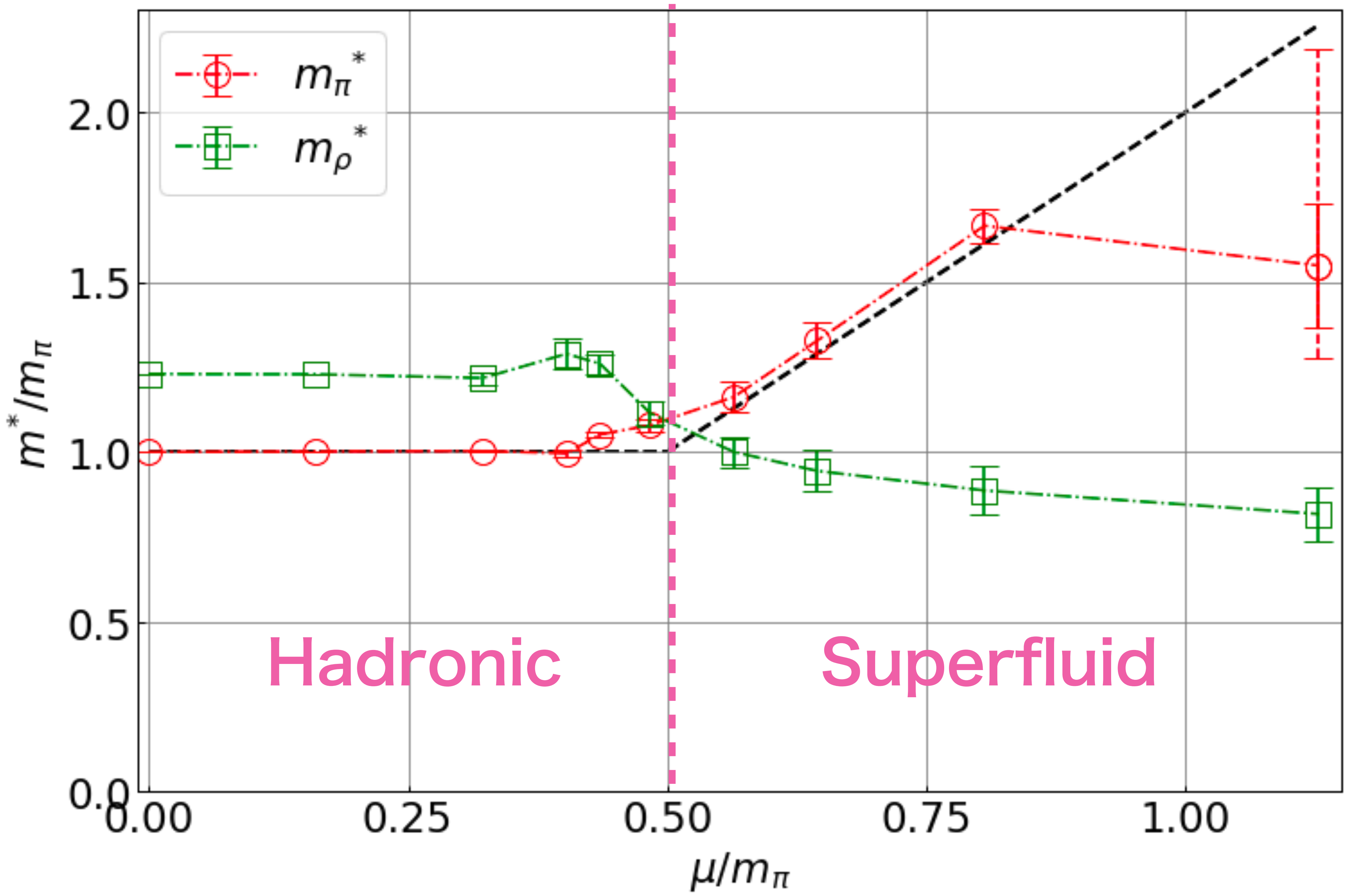


- $C^\rho(t) < C^\pi(t)$ breaks down
- pion becomes noisy

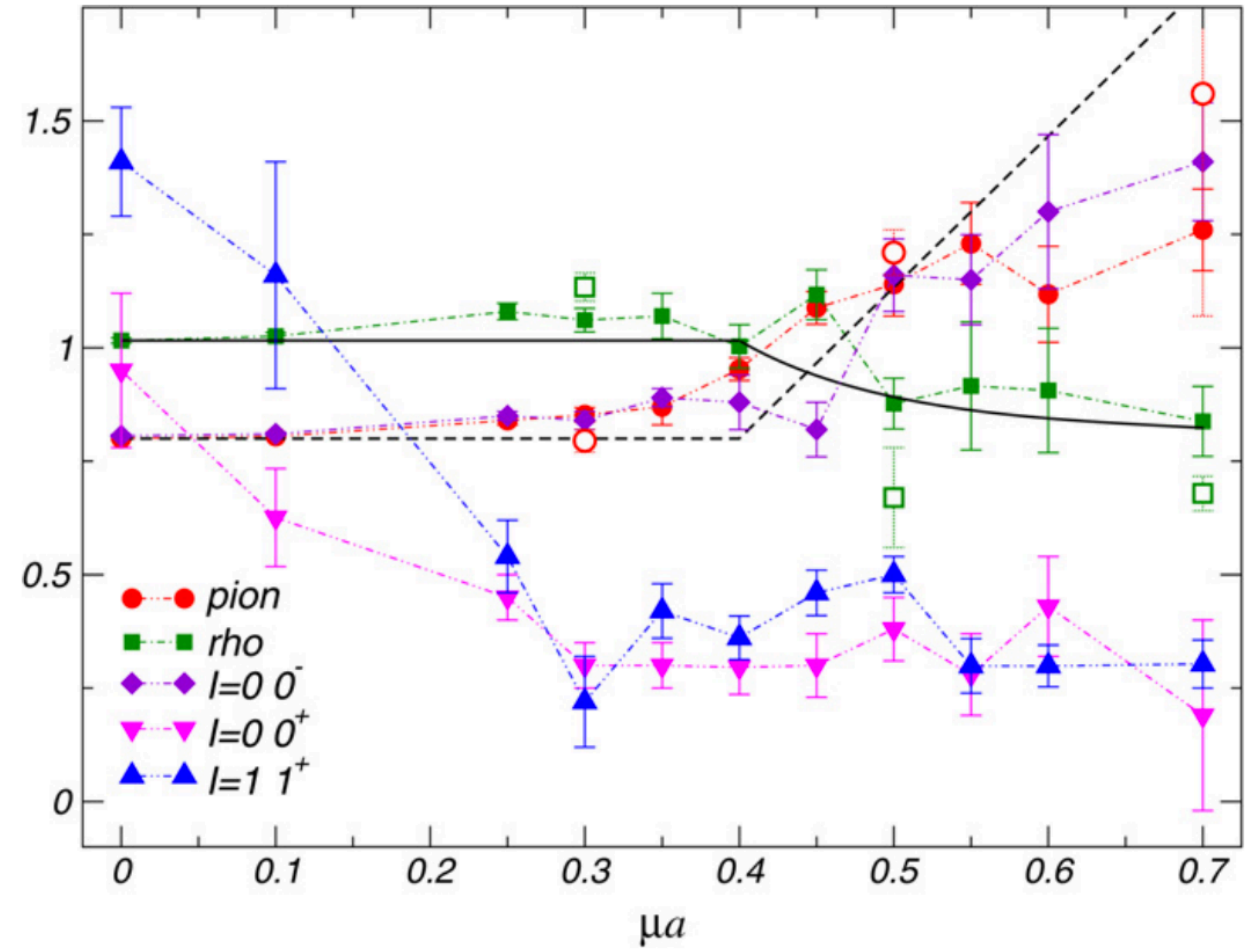
(⬅ $\pi \to \rho\rho$ channel?)

Masses of π and ρ meson

Our results



(S. Hands, P. Sitch and J. I. Skullerud, Phys. Lett. B 662 (2008), 405-412)



- - - : m_π^* from χPT in 2-color QCD [Kogut et al., 2000]

- hadronic phase: no μ dependence

consistent with $E = \sqrt{\mathbf{p}^2 + m^2} - \mu n$ with $\mathbf{p} = 0$, $n = 0$

- **ordering of π and ρ flips at** $\mu \approx m_\pi/2$

[MK, Itou, Iida, submitting to journal]

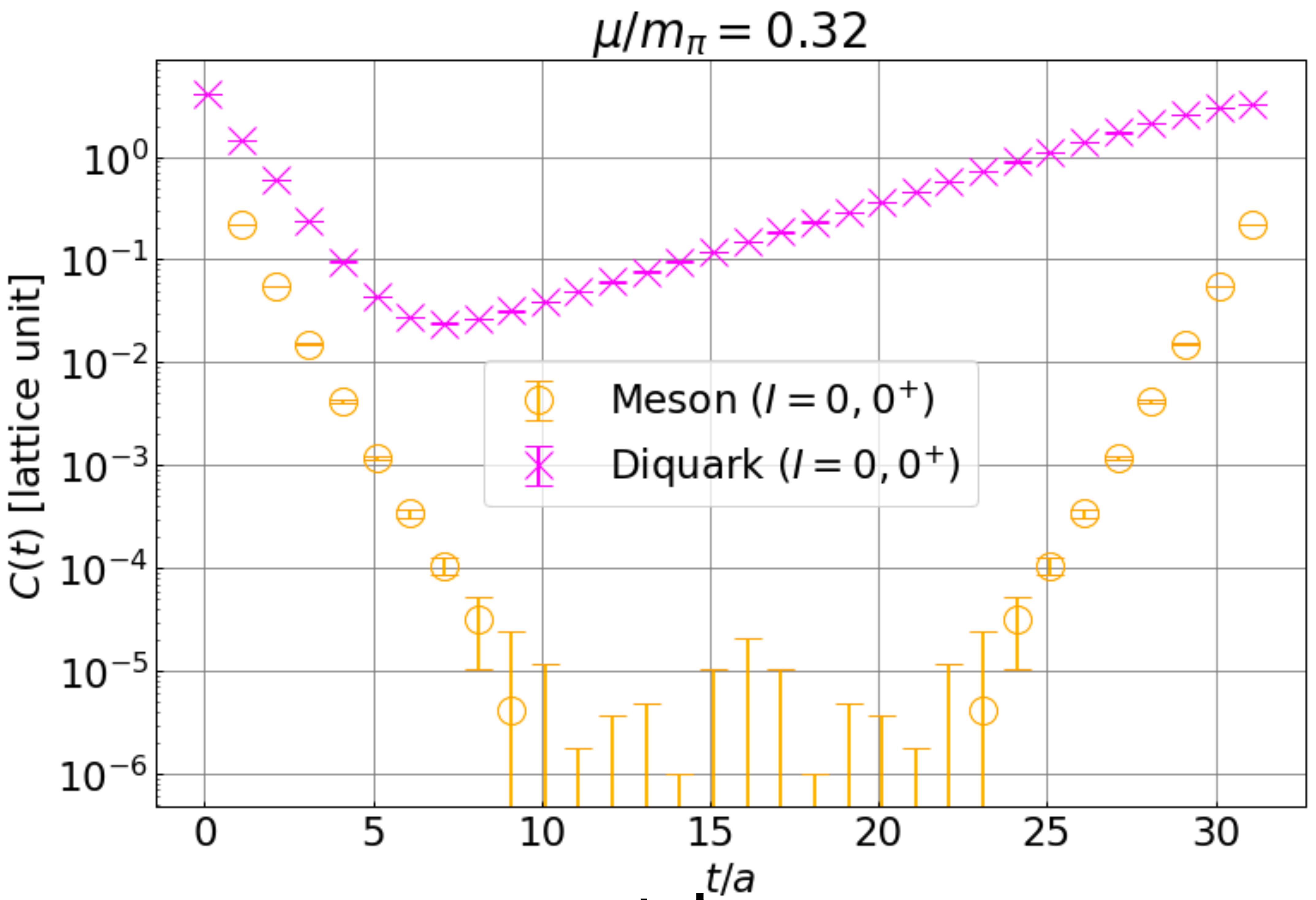
- consistent with the results in the previous lattice studies and studies by effective models

[Muroya et al., 2003; Hands et al., 2008; Wilhelm et al., 2019]

[Hatsuda, Lee,1992; Kogut et al., 2000]

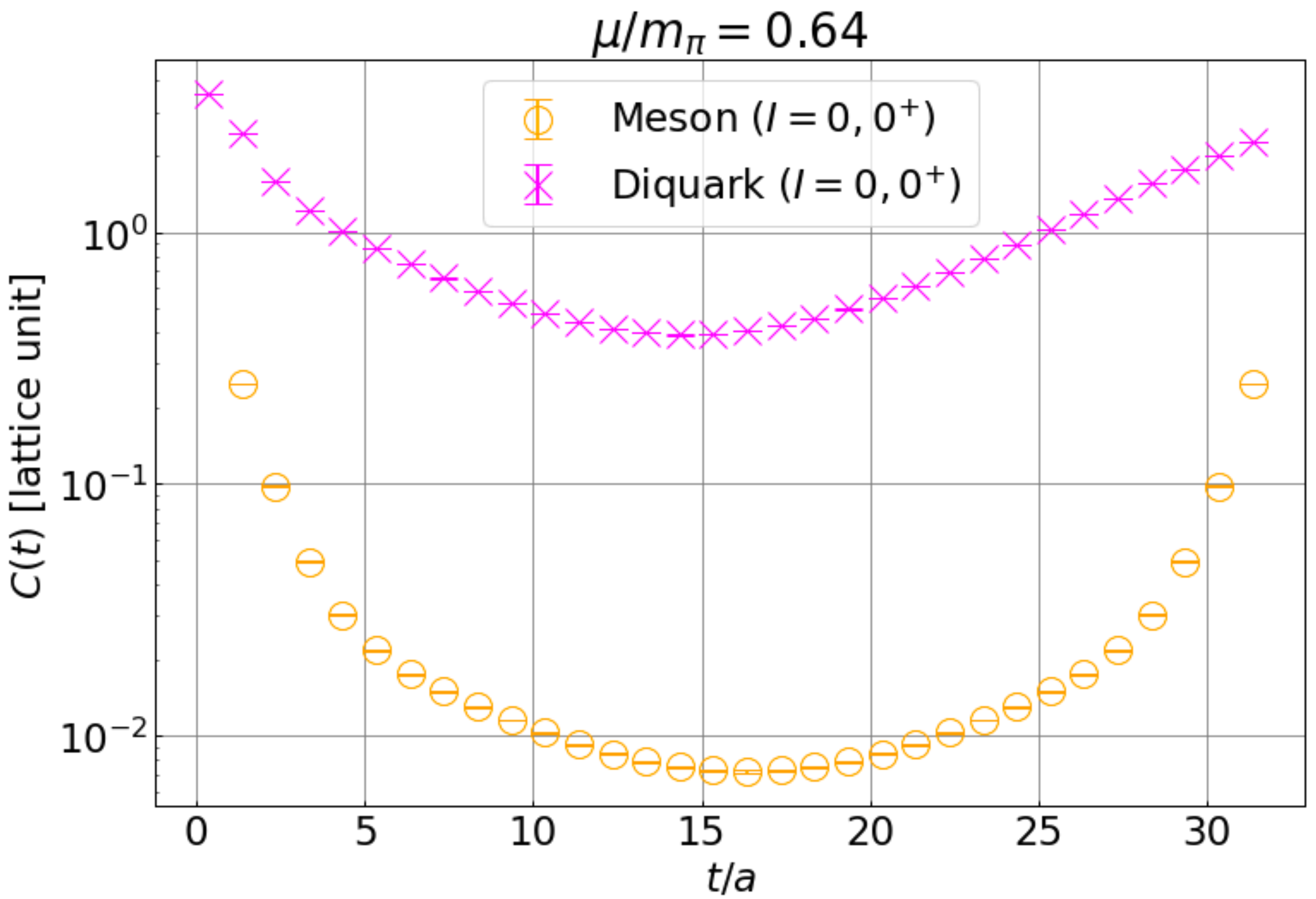
2pt function in scalar channel

in hadronic phase



- meson: symmetric
- diquark: **asymmetric**
- we define
  - diquark mass: fit from $C(T-t)$
  - antidiquark mass: fit from $C(t)$

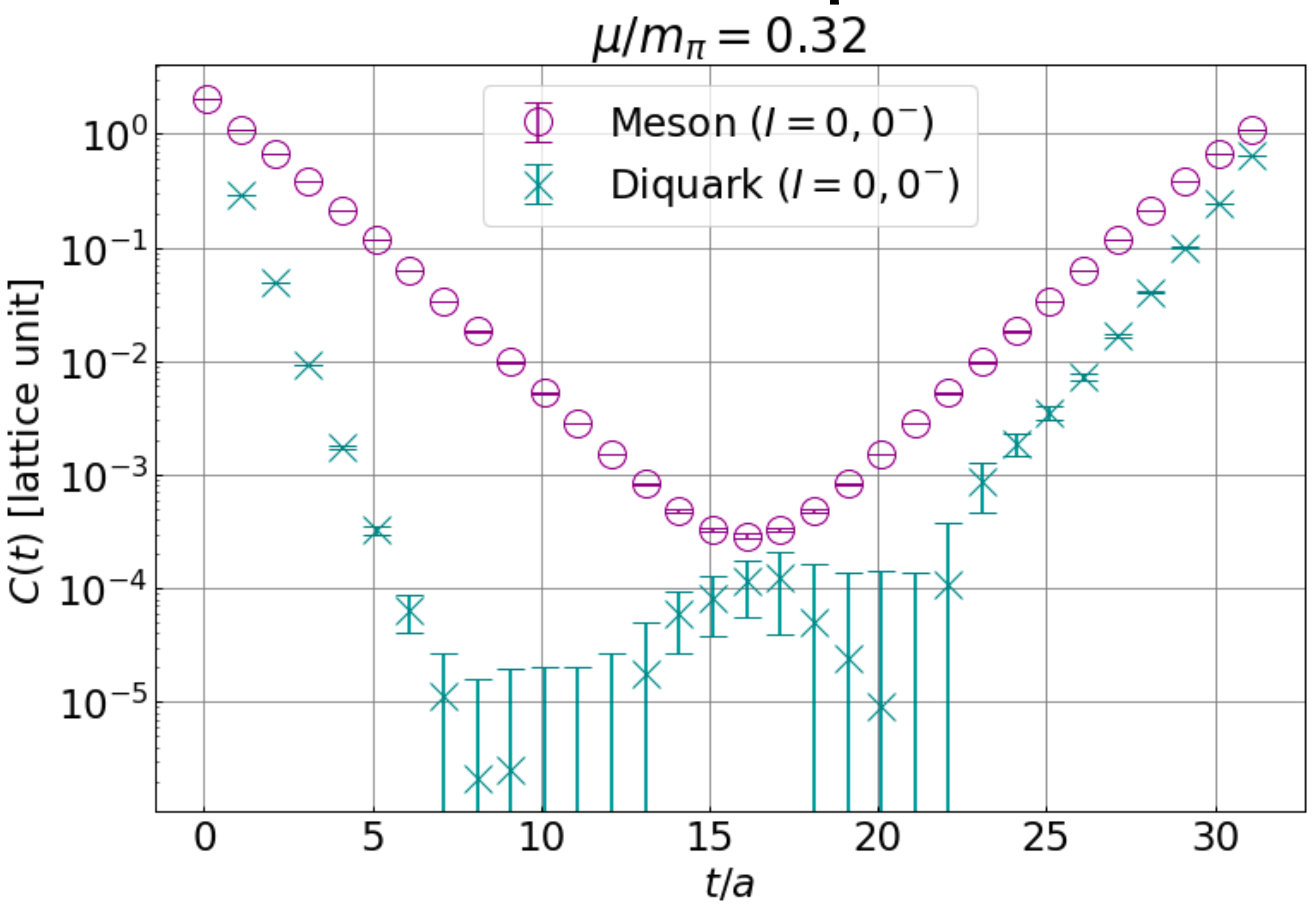
in superfluid phase



- meson: signal gets clear
- diquark: **getting symmetric**

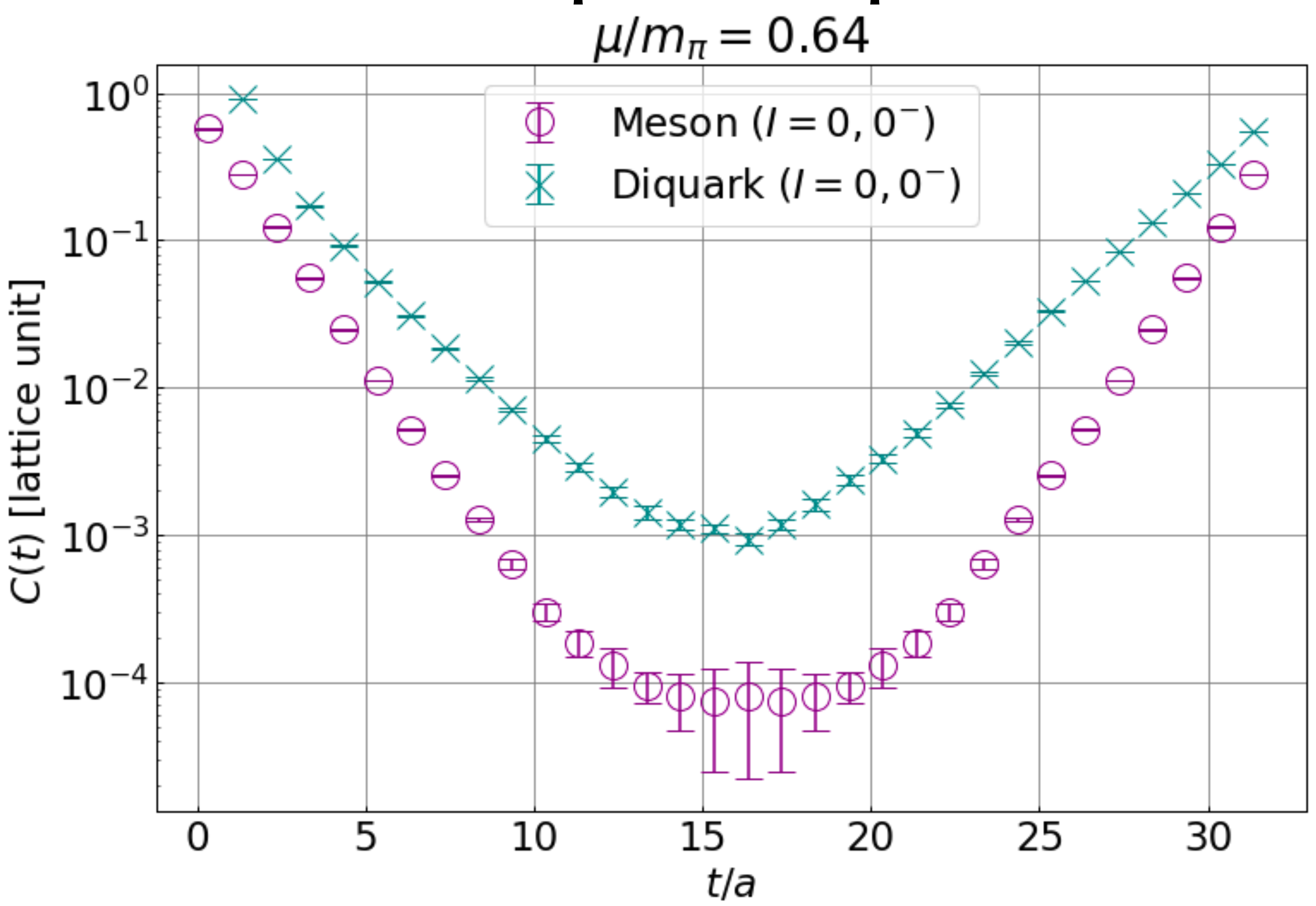
2pt function in pseudo-scalar channel

in hadronic phase



- meson: symmetric
- diquark: asymmetric
- we define
  - diquark mass: fit from $C(T-t)$
  - antidiquark mass: fit from $C(t)$

in superfluid phase



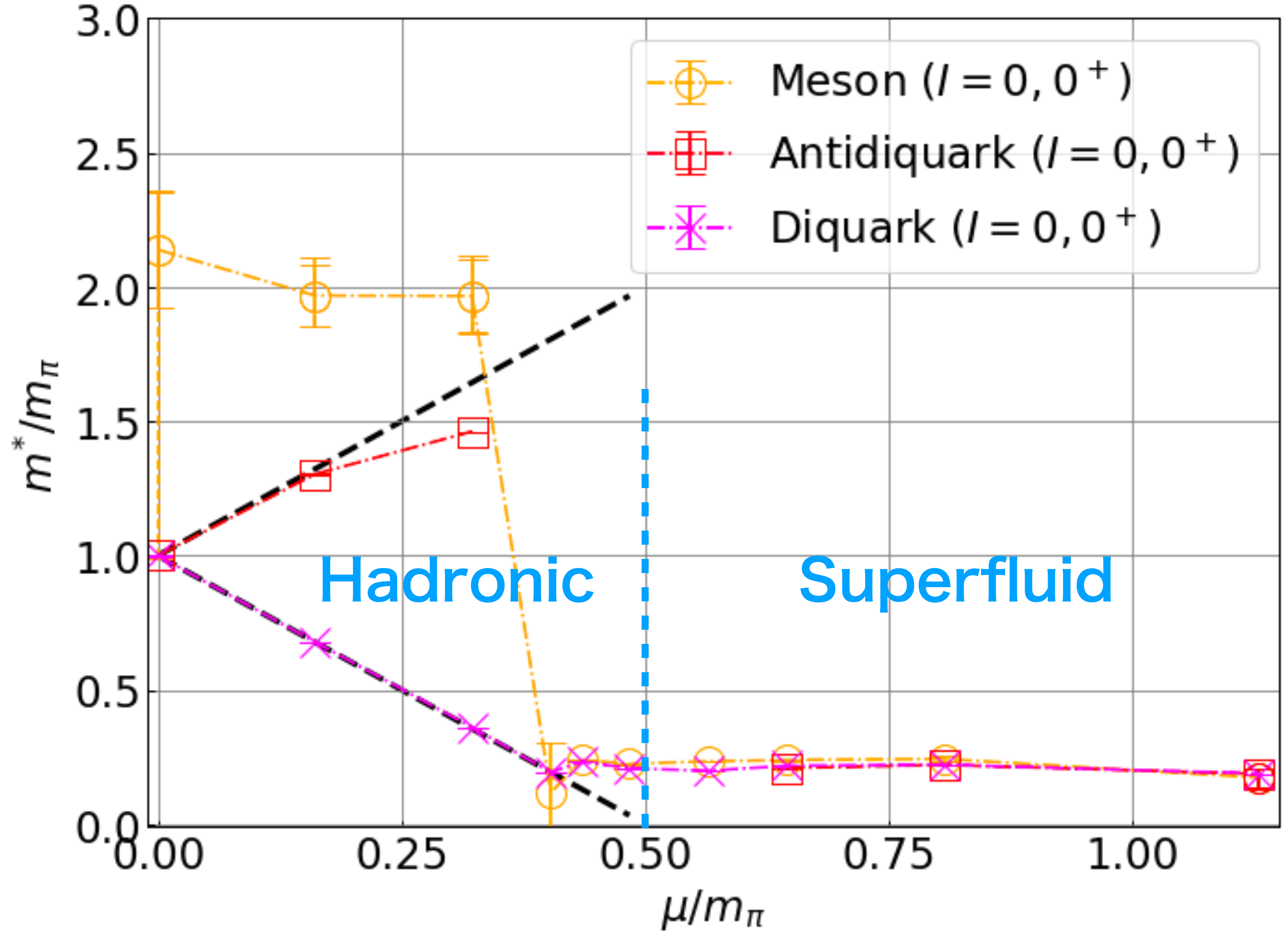
- diquark: getting symmetric
- meson: **different magnitude of slopes in early and late** t

← ground + 1st excited?

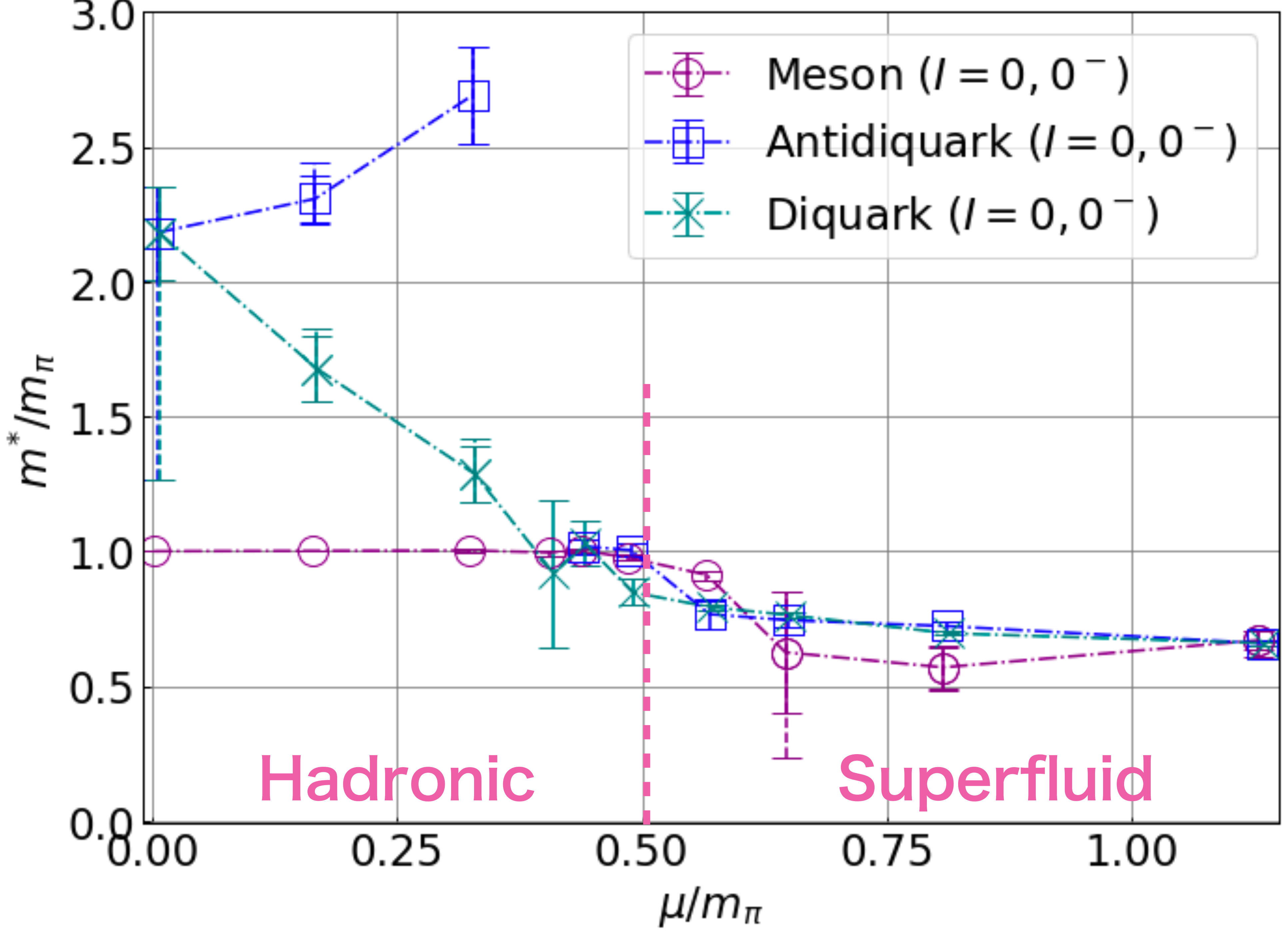
Masses in (pseudo) scalar channels

- - - - - : $m_\pi \pm 2\mu$ (= results from χPT in 2-color QCD)
[Kogut et al., 2000]

scalar



pseudo scalar



- hadronic phase: consistent with $E(\mathbf{p}=0) = m - \mu n$

[MK, Itou, Iida, submitting to journal]

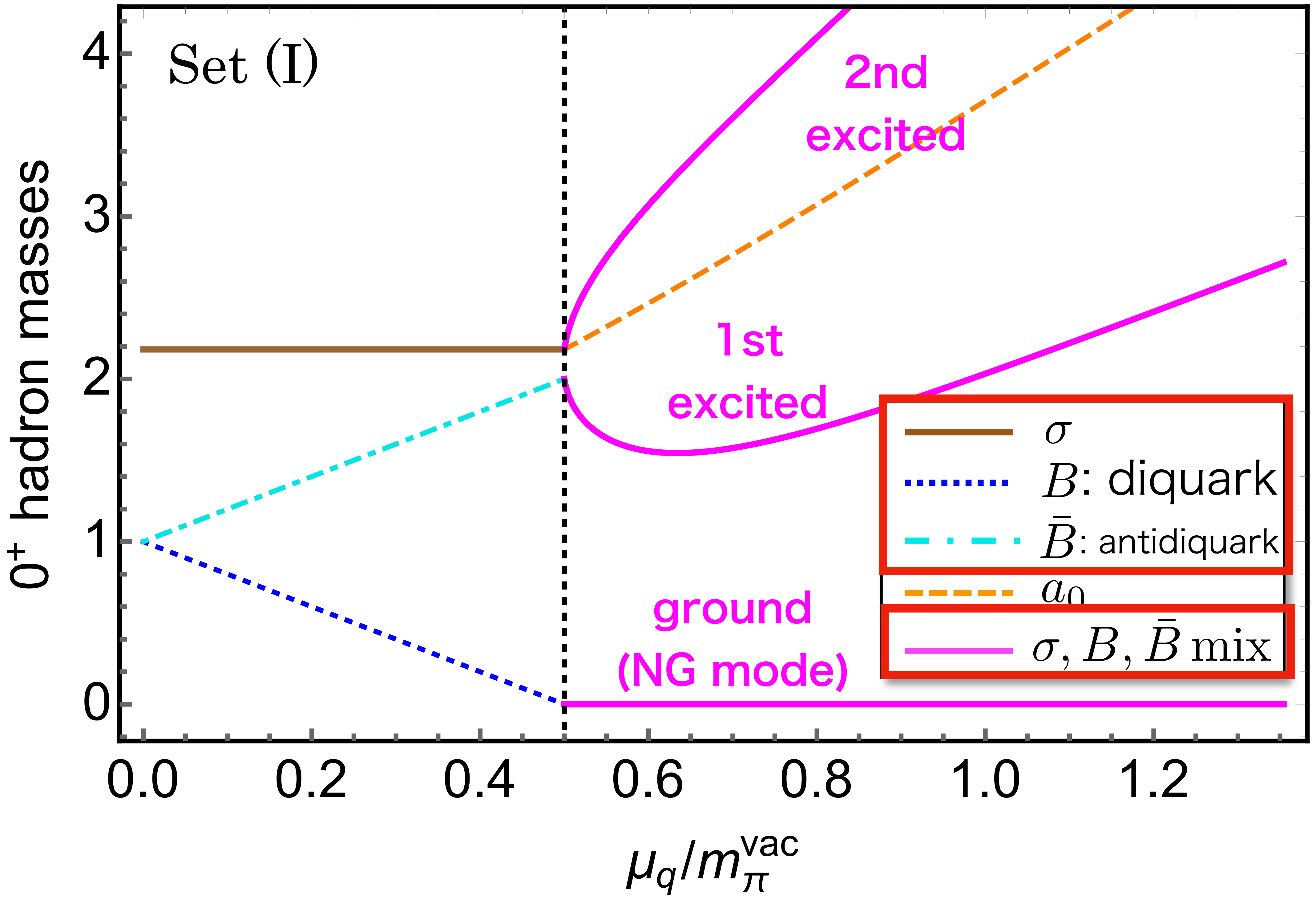
- **very light mode in scalar channel in superfluid phase ← NG mode?**
- in superfluid phase, **all three masses have the same values ← Why?**

Linear sigma model with diquark gap

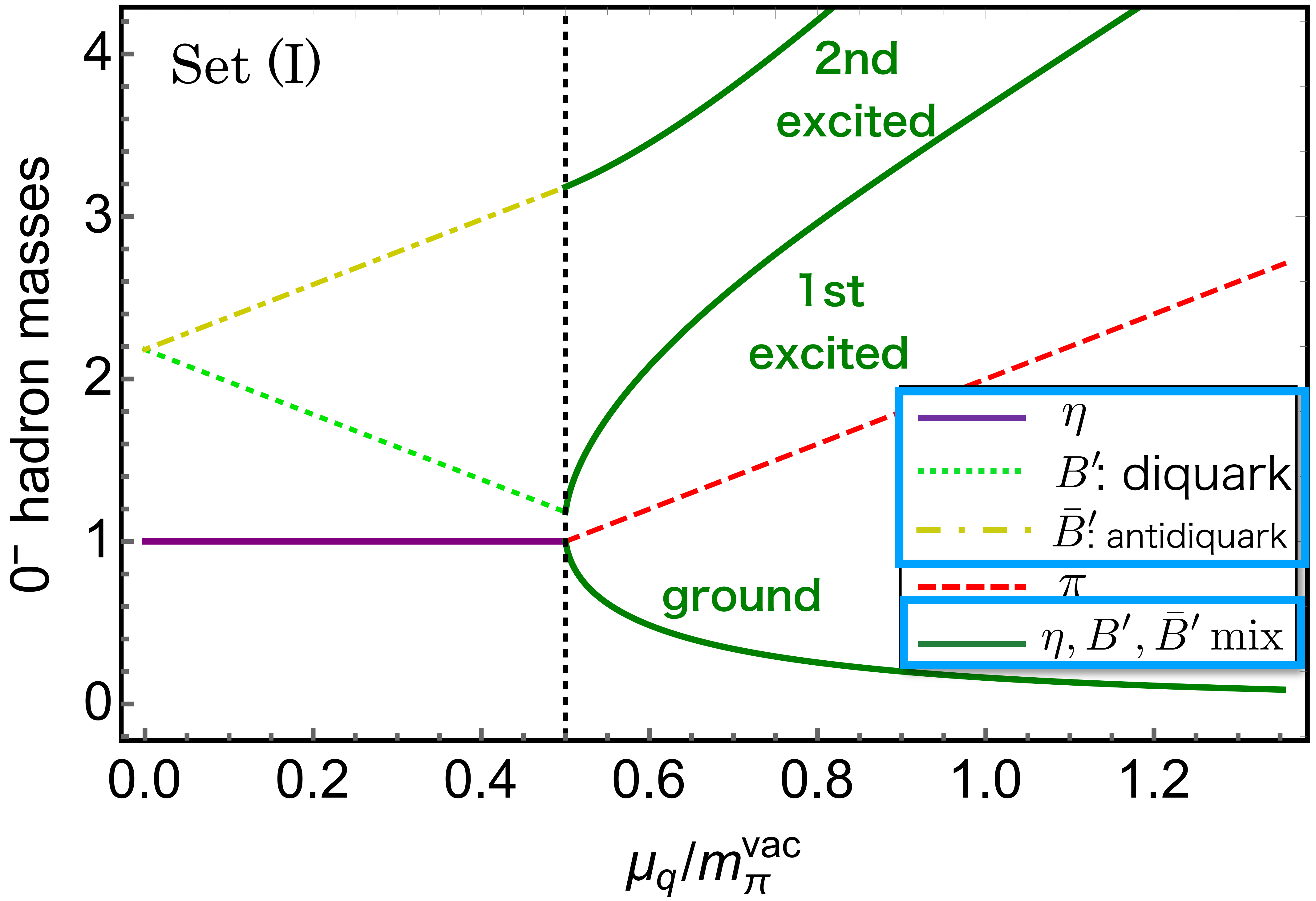
- **Linear sigma model with diquark gap** explains such phenomenon

(Suenaga, KM, Itou, Iida, Phys. Rev. D107, no.5, 054001 (2023) [arXiv:2211.01789 [hep-ph]])

scalar



pseudo scalar



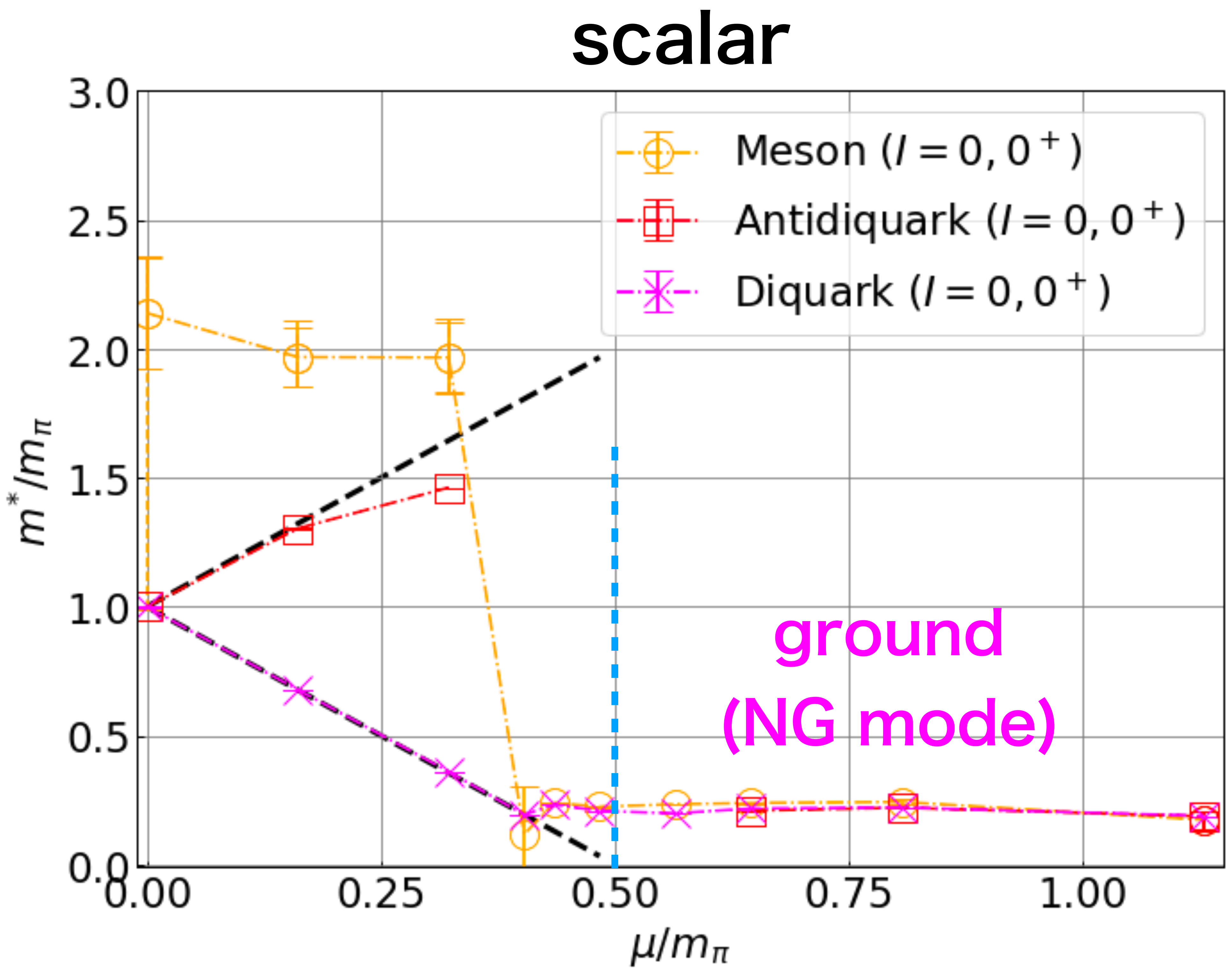
- hadronic phase: $E(\mathbf{p}=0) = m - \mu n$

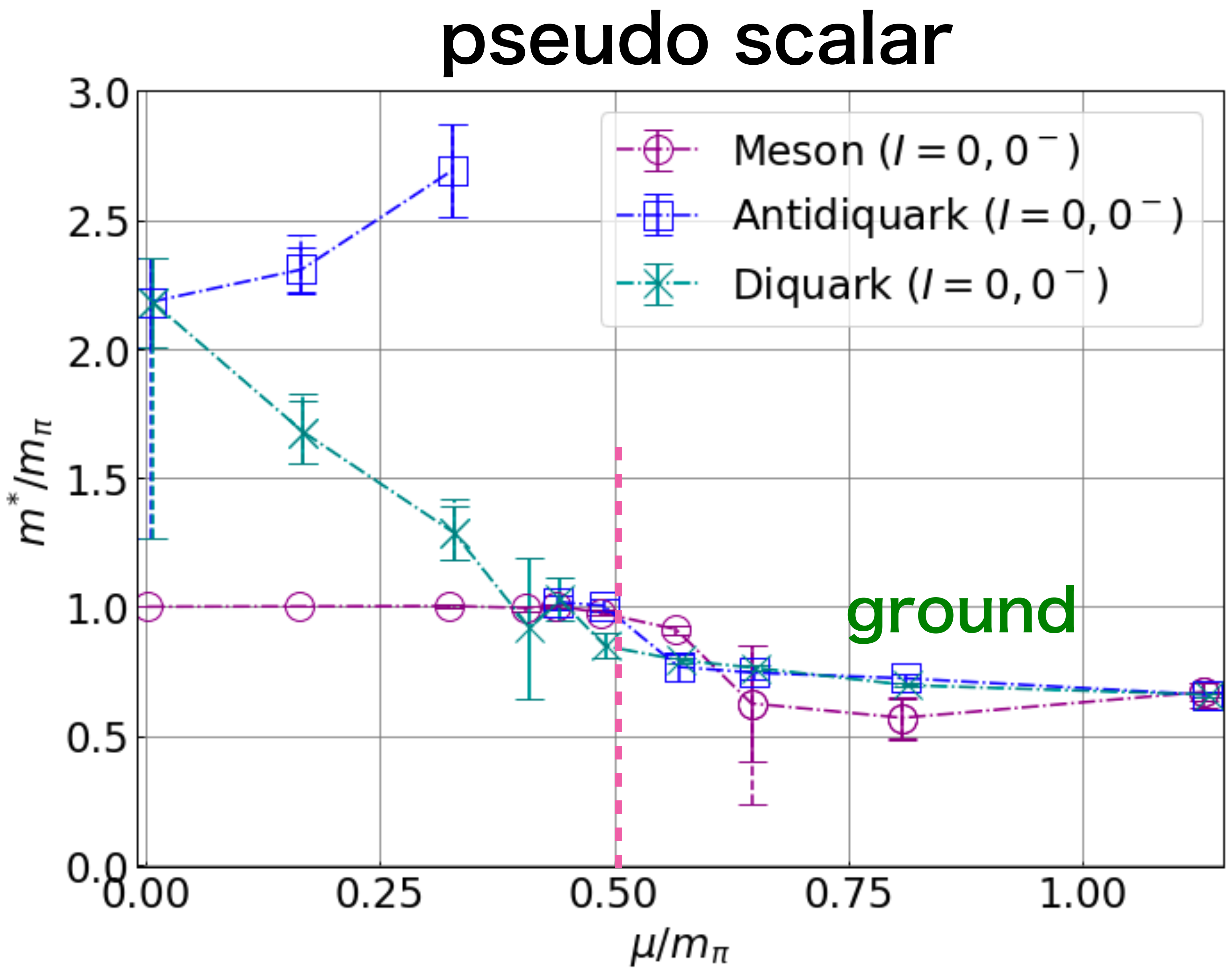
(input of LSM: masses of pion and pseudo-scalar diquark, use approximation $m_\sigma \approx m_{a_0}$)

- superfluid phase: **meson-diquark-antidiquark mixing due to SSB of** $U(1)_B$

Meson-diquark-antidiquark mixing in lattice results

- “hadron mass” in this calculation: fit results of the 2pt function for the **corresponding hadron operator**





- in superfluid phase, **meson, diquark and antidiquark 2pt functions observe the same ground state?**

- still some discrepancies ⟵ $j \neq 0,\ T \neq 0$ effect?

Summary/Future work

- we study hadron spectrum in low temperature and finite density
- order of π and ρ meson spectrum flips at $\mu \approx m_\pi/2$
- signature of meson-diquark-antidiquark mixing in (pseudo) scalar channels in superfluid phase
- Future work:
  - investigate other channels
  - $j \to 0$ extrapolation, include disconnected diagrams
  - estimate mixing angle quantitatively
    ⟵ variational method?

Backups

Quark propagators in detail

- 4 types of quark propagators

- $\Delta(\mu)$: (usual) Dirac operator at chemical potential μ
- $J = \kappa j,\ K = (C\gamma_5)\tau_2$

$(f = 1{,}2)$

- $\psi_f \bar{\psi}_f = S_N = (\Delta^\dagger(\mu)\Delta(\mu) + J^2)^{-1}\Delta^\dagger(\mu)$: normal propagator ($S_N(\mu, j=0) = \Delta^{-1}(\mu)$)

- $\bar{\psi}_f^T \psi_f^T = \bar{S}_N = K\gamma_5(\Delta^\dagger(-\mu)\Delta(-\mu) + J^2)^{-1}\Delta^\dagger(-\mu)K\gamma_5$: back propagator

  ($\bar{S}_N = -S_N^T$)

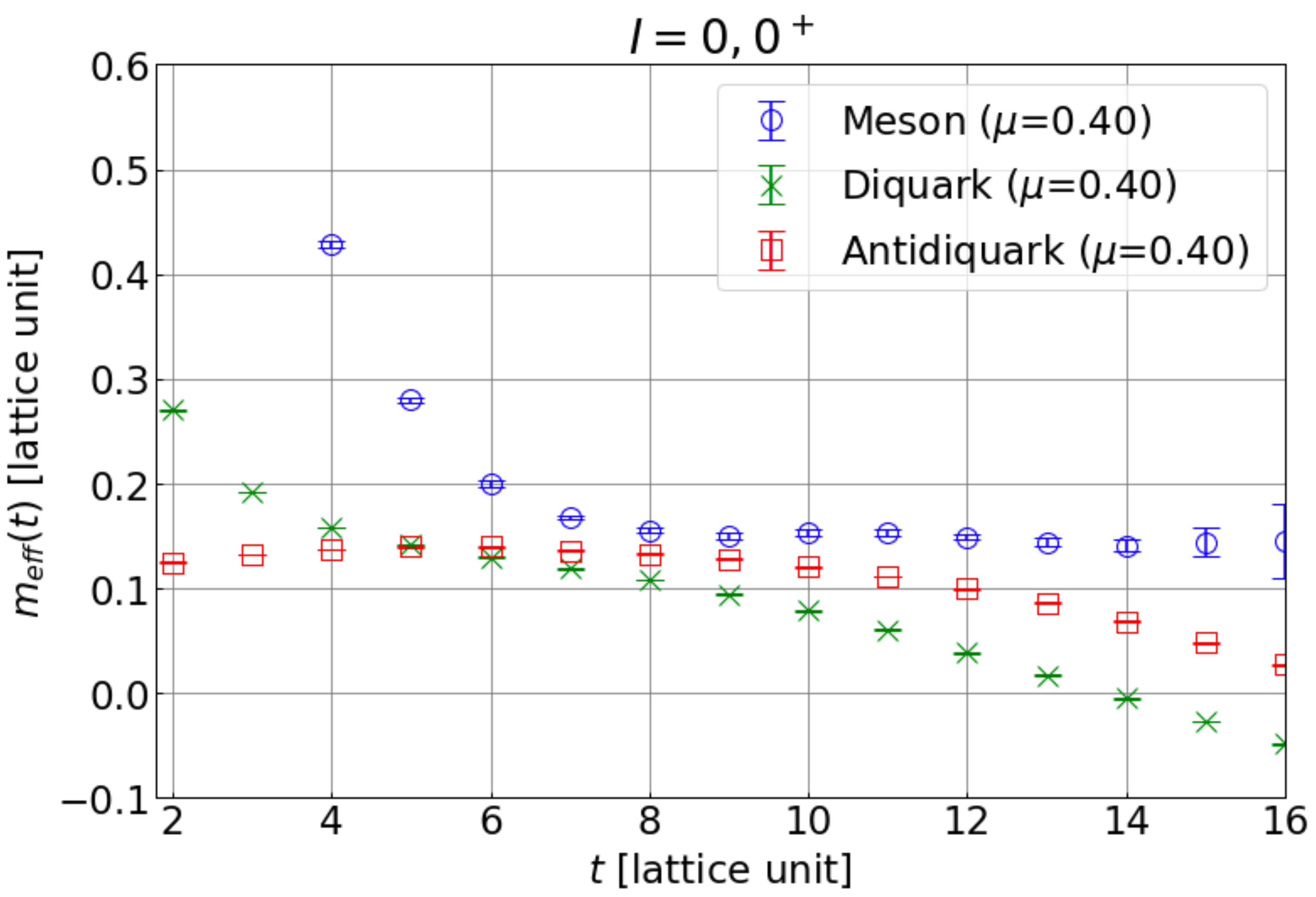
- $\psi_2 \psi_1^T = S_A = J(\Delta^\dagger(\mu)\Delta(\mu) + J^2)^{-1}K$: anomalous propagator

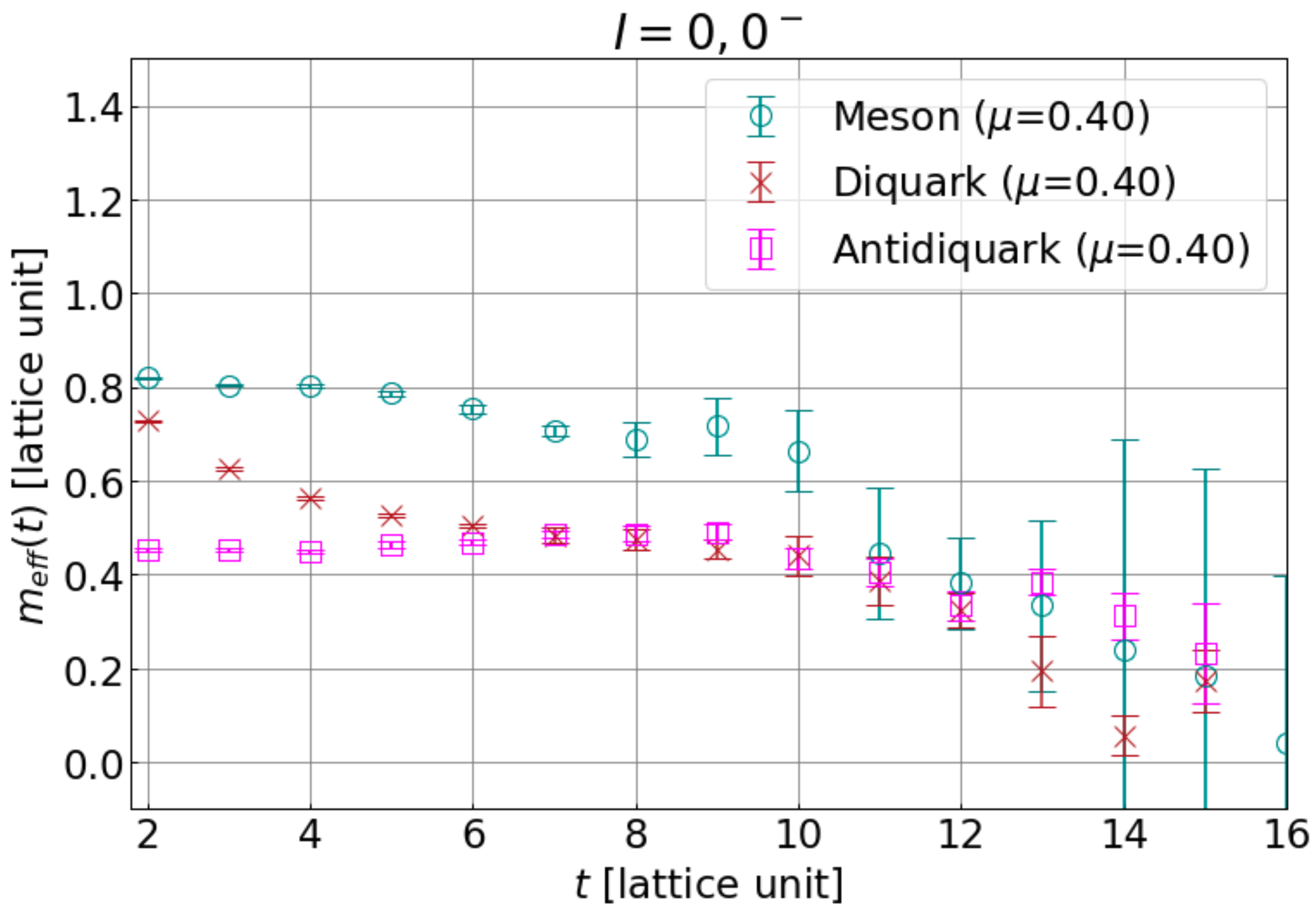
  (quark -> antiquark)

- $\bar{\psi}_2^T \bar{\psi}_1 = \bar{S}_A = JK\gamma_5(\Delta^\dagger(-\mu)\Delta(-\mu) + J^2)^{-1}\gamma_5$: anomalous propagator

  (antiquark -> quark)

Effective mass in $I = 0,\ J^P = 0^{\pm}$ channels in superfluid phase





- position of plateaux of all three hadrons looks the same

  ← **meson-diquark-antidiquark mixing?**

- 0^- meson: decrease slowly ← contribution from excited state?

LSM with diquark gap

(Suenaga, KM, Itou, Iida, Phys. Rev. D107, no.5, 054001 (2023) [arXiv:2211.01789 [hep-ph]])

- QC2D ($N_f = 2$)
  - quark field

$$\Psi_i = \begin{pmatrix} \psi_R \\ \sigma^2\tau^2\psi_L^* \end{pmatrix}$$

$$\Psi \to g\Psi \qquad g \in SU(4)$$

  - meson, diquark op

$$\Phi_{ij} \sim \Psi_j^T \sigma^2 \tau^2 \Psi_i$$

$$\Phi \to g\Phi g^T$$

- LSM($N_f = 2$)
  - meson, diquark field

$$\Sigma = \frac{1}{2}\begin{pmatrix} 0 & -B' + iB & \frac{\sigma - i\eta + a^0 - i\pi^0}{\sqrt{2}} & a^+ - i\pi^+ \\ B' - iB & 0 & a^- - i\pi^- & \frac{\sigma - i\eta - a^0 + i\pi^0}{\sqrt{2}} \\ -\frac{\sigma - i\eta + a^0 - i\pi^0}{\sqrt{2}} & -a^- + i\pi^- & 0 & -\bar{B}' + i\bar{B} \\ -a^+ + i\pi^+ & -\frac{\sigma - i\eta - a^0 + i\pi^0}{\sqrt{2}} & \bar{B}' - i\bar{B} & 0 \end{pmatrix}$$

$$\Sigma \to g\Sigma g^T$$

  - chiral and diquark gap

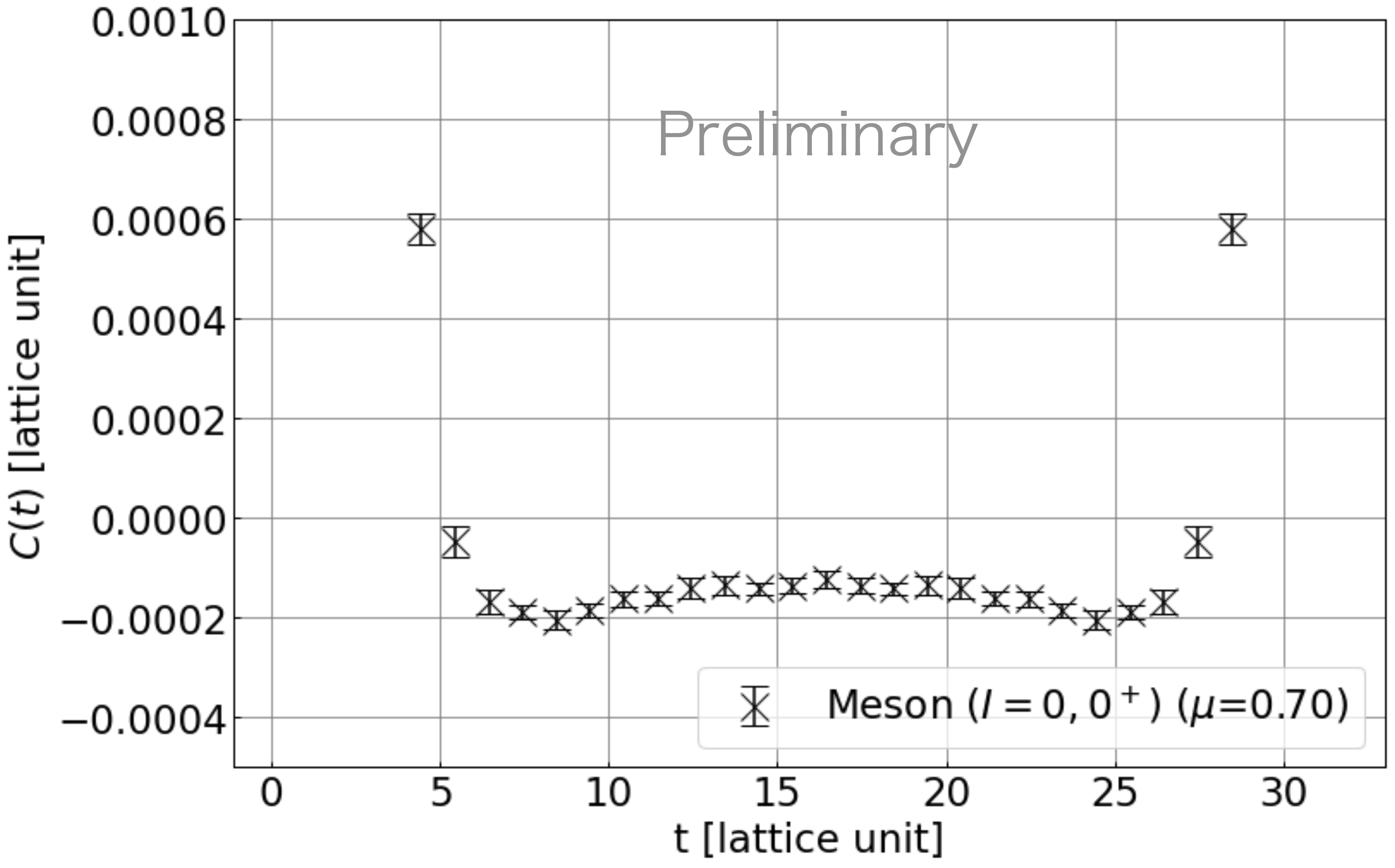
$$\sigma \to \sigma + \sigma_0$$

$$\frac{B + \bar{B}}{\sqrt{2}} \to \Delta + \frac{B + \bar{B}}{\sqrt{2}}$$

Contribution from excited states

- for some channels **in large** μ, we see an explicit sign of contribution from excited states

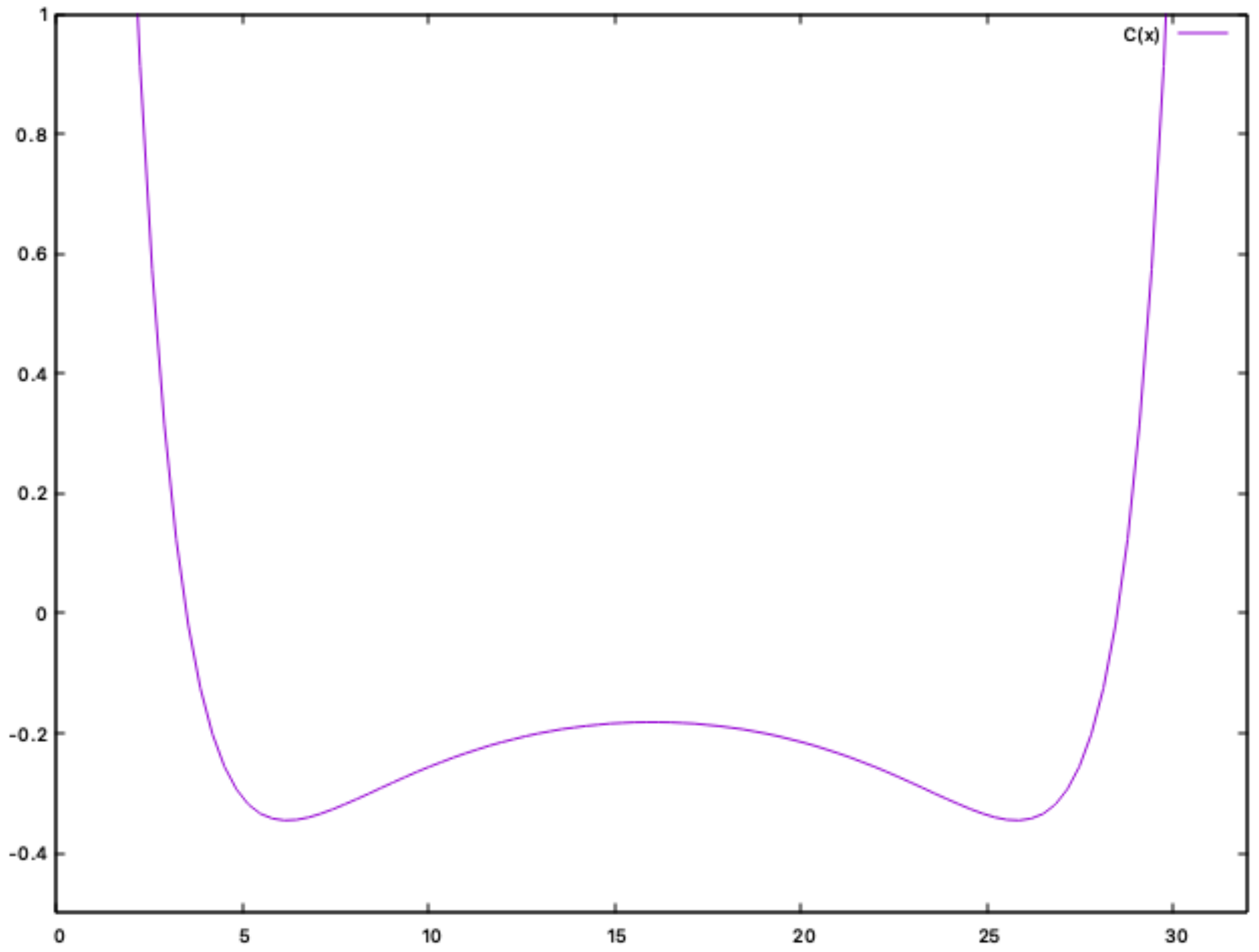
2pt function of meson $(I = 0,\, 0^+,\, a\mu = 0.7)$



plot of

$$f(t) = A_1(e^{-m_1 t} + e^{-m_1(32-t)}) + A_2(e^{-m_2 t} + e^{-m_2(32-t)})$$

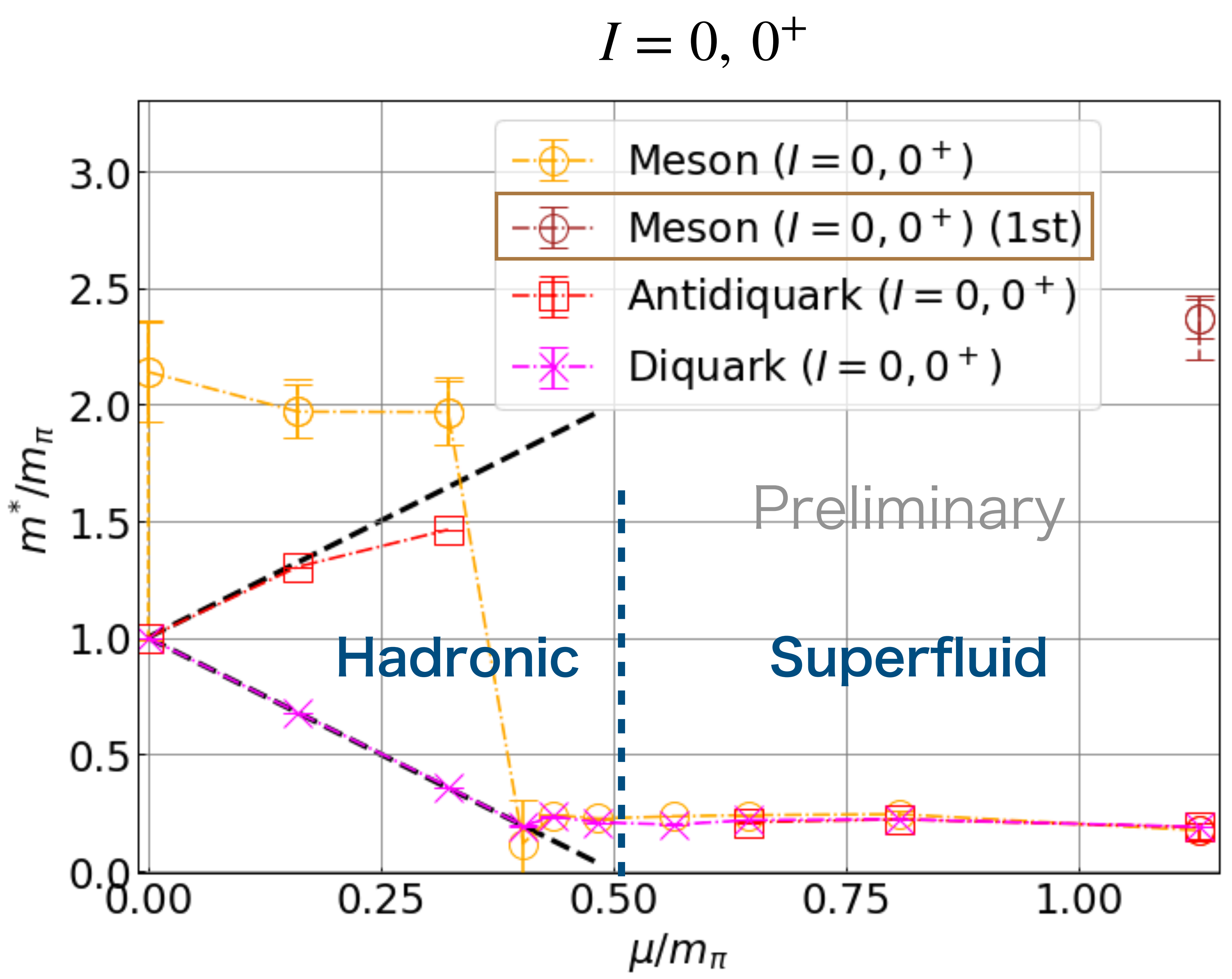
with $A_1 = -1.0,\ m_1 = 0.15,\ A_2 = 10.0,\ m_2 = 0.8$

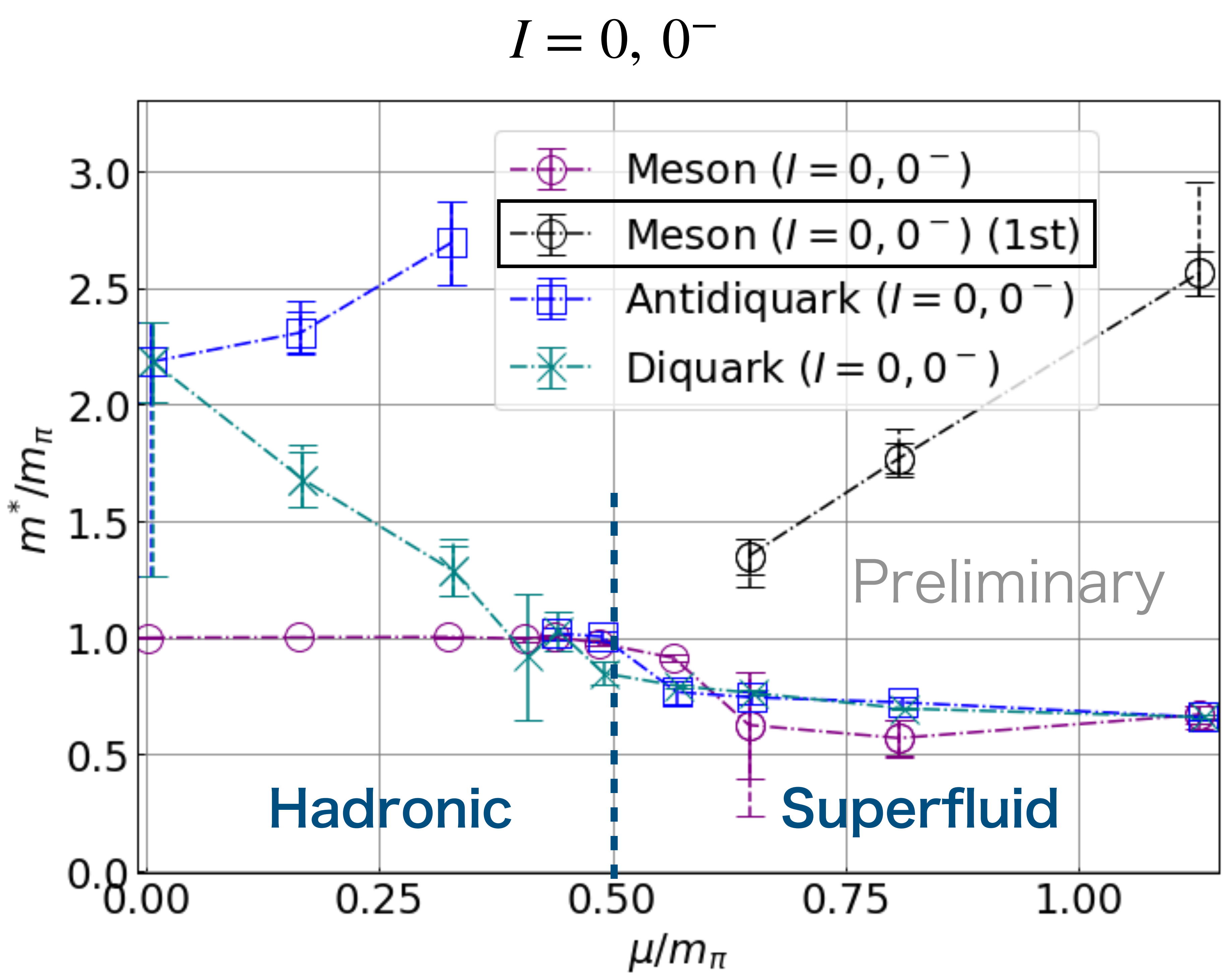


- imply the comparable overlap with excited states

- we use **double cosh function** in fitting for **meson in** $I = 0,\, 0^+$ **at** $a\mu = 0.7$ **and** $I = 0,\, 0^-$ **at** $a\mu = 0.4 - 0.7$

Results of excited states



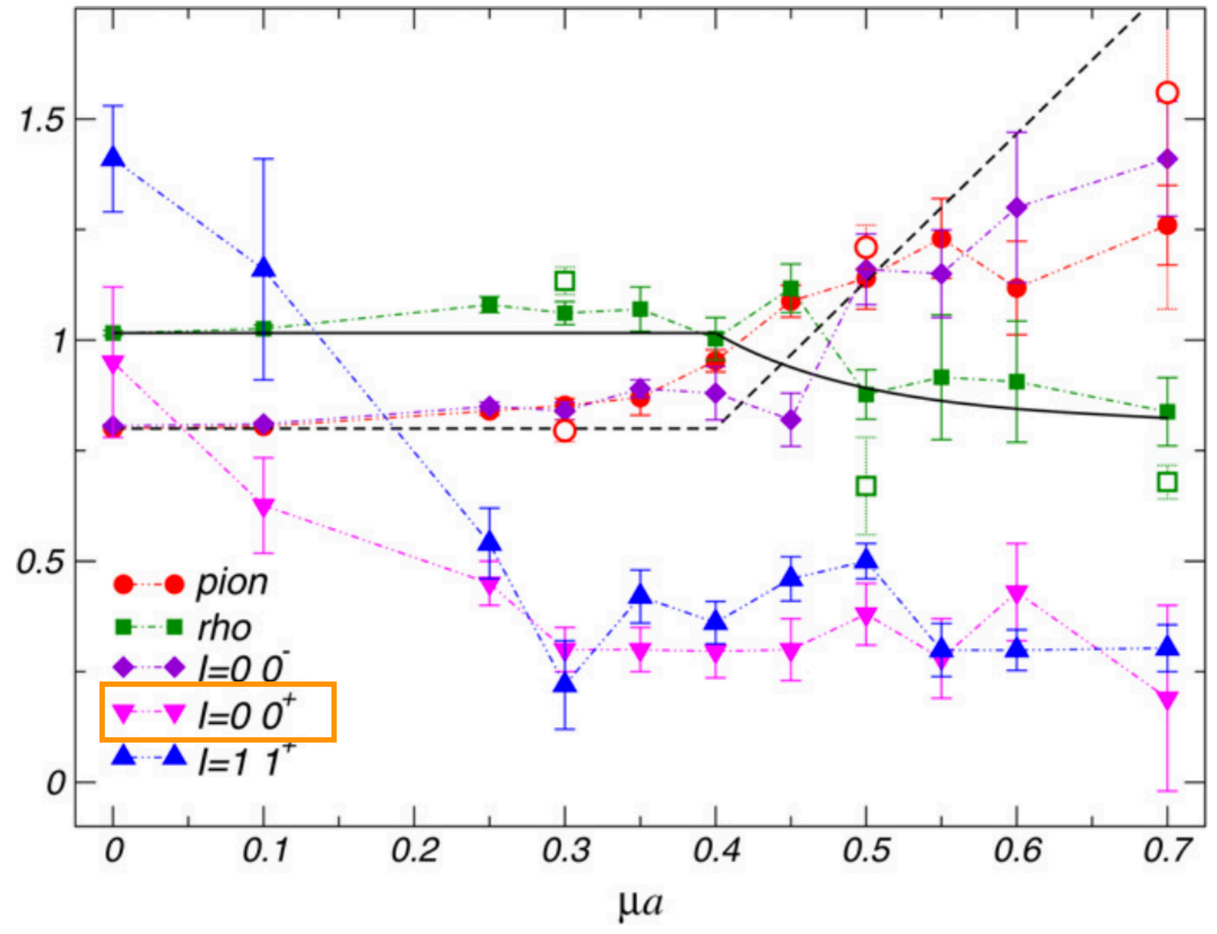


- massive excited states
- level repulsion in 0^{-} channel?

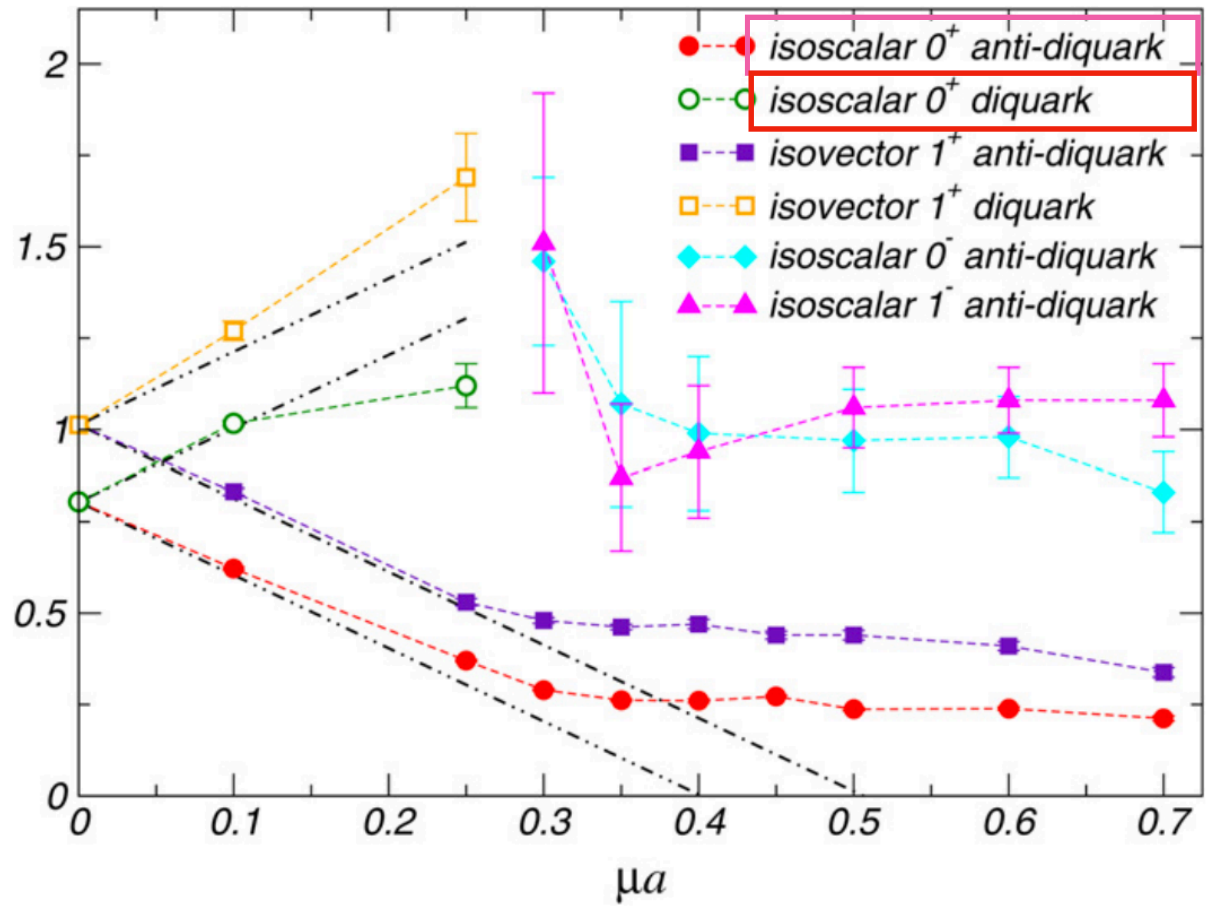
Consistency with the previous studies for $I=0,\ 0^+$ channel

(S. Hands, P. Sitch and J. I. Skullerud, Phys. Lett. B 662 (2008), 405-412)

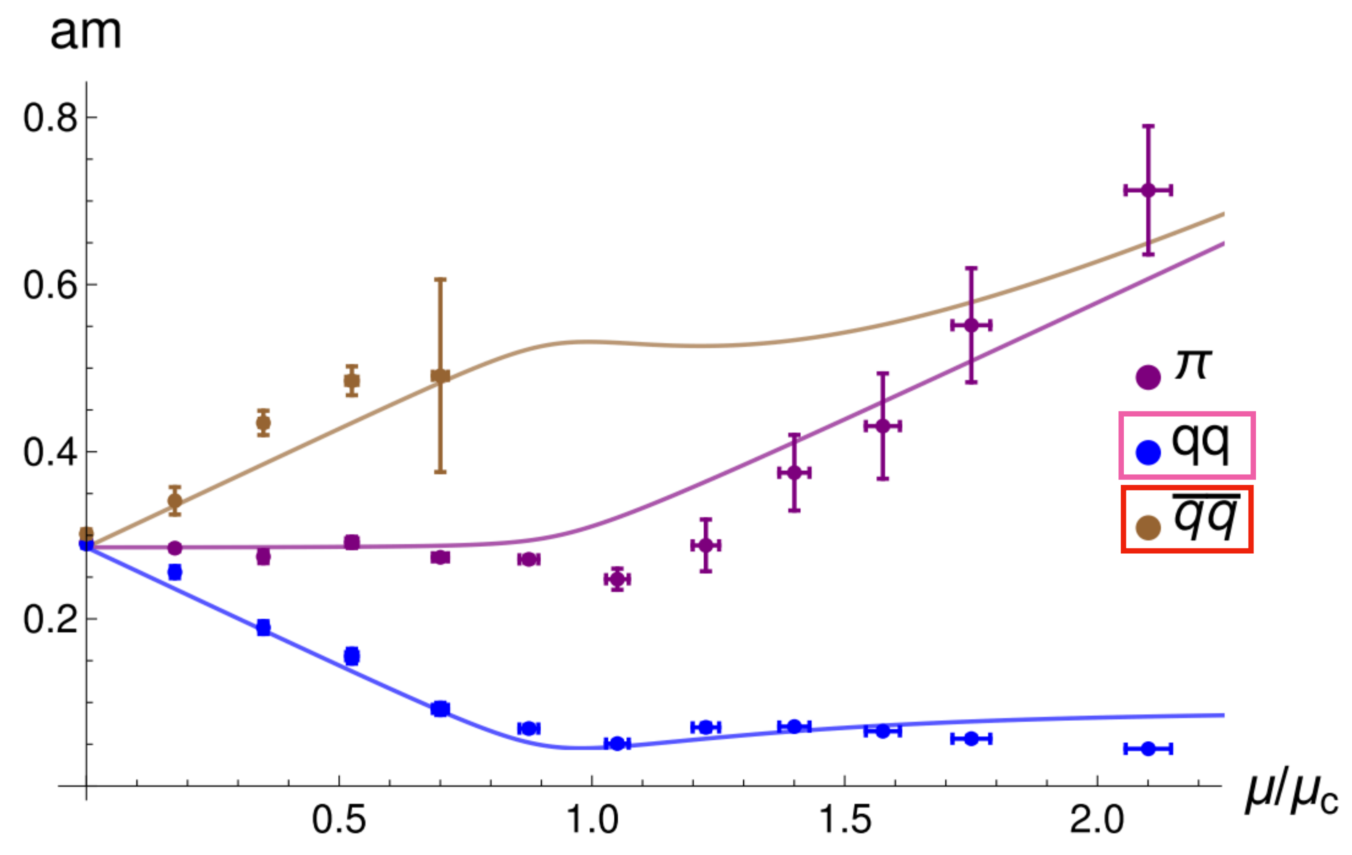
Meson



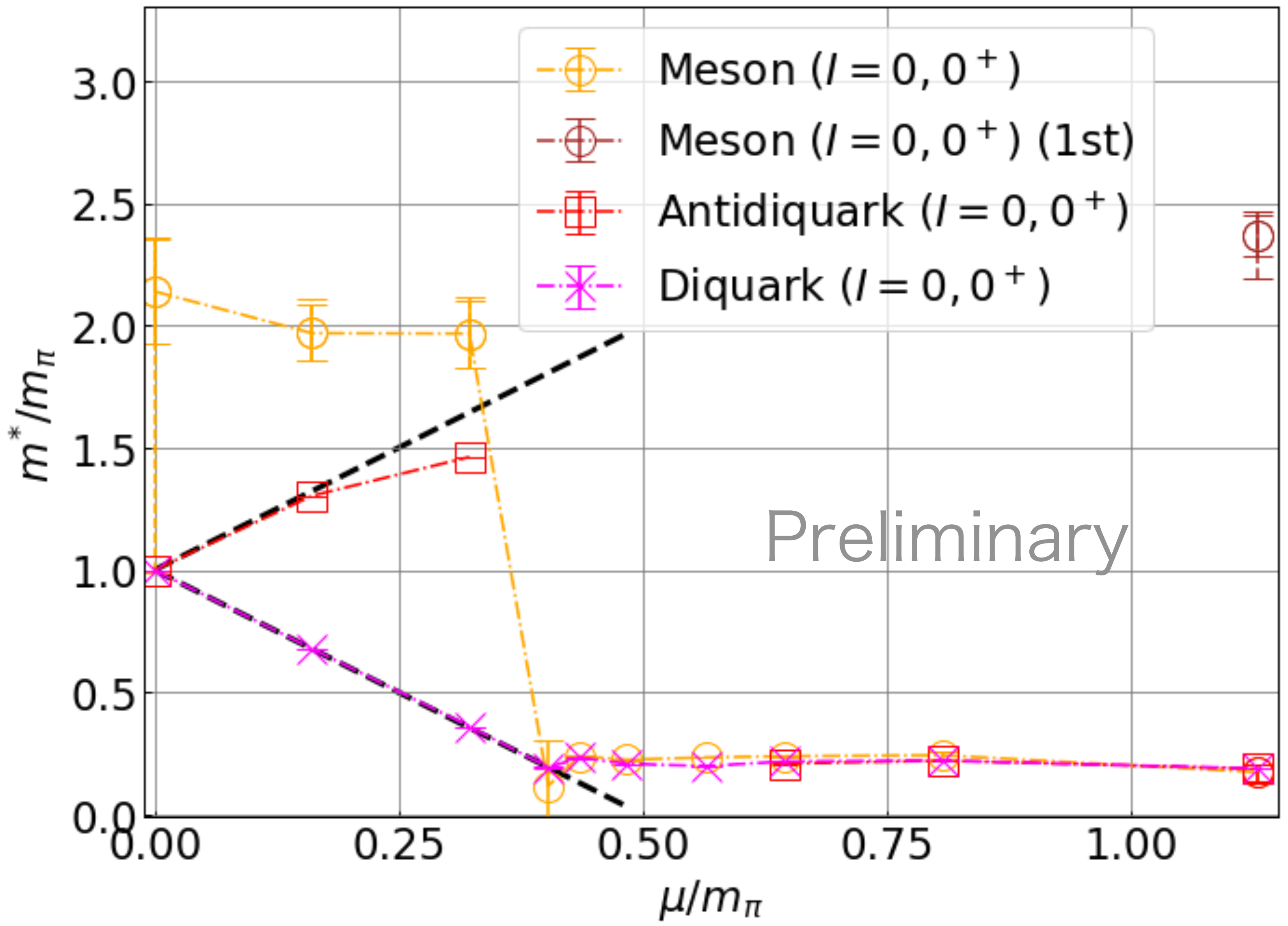
Diquark/Antidiquark



(J. Wilhelm et al., Phys. Rev. D 100 (2019) no.11, 114507)



Our results

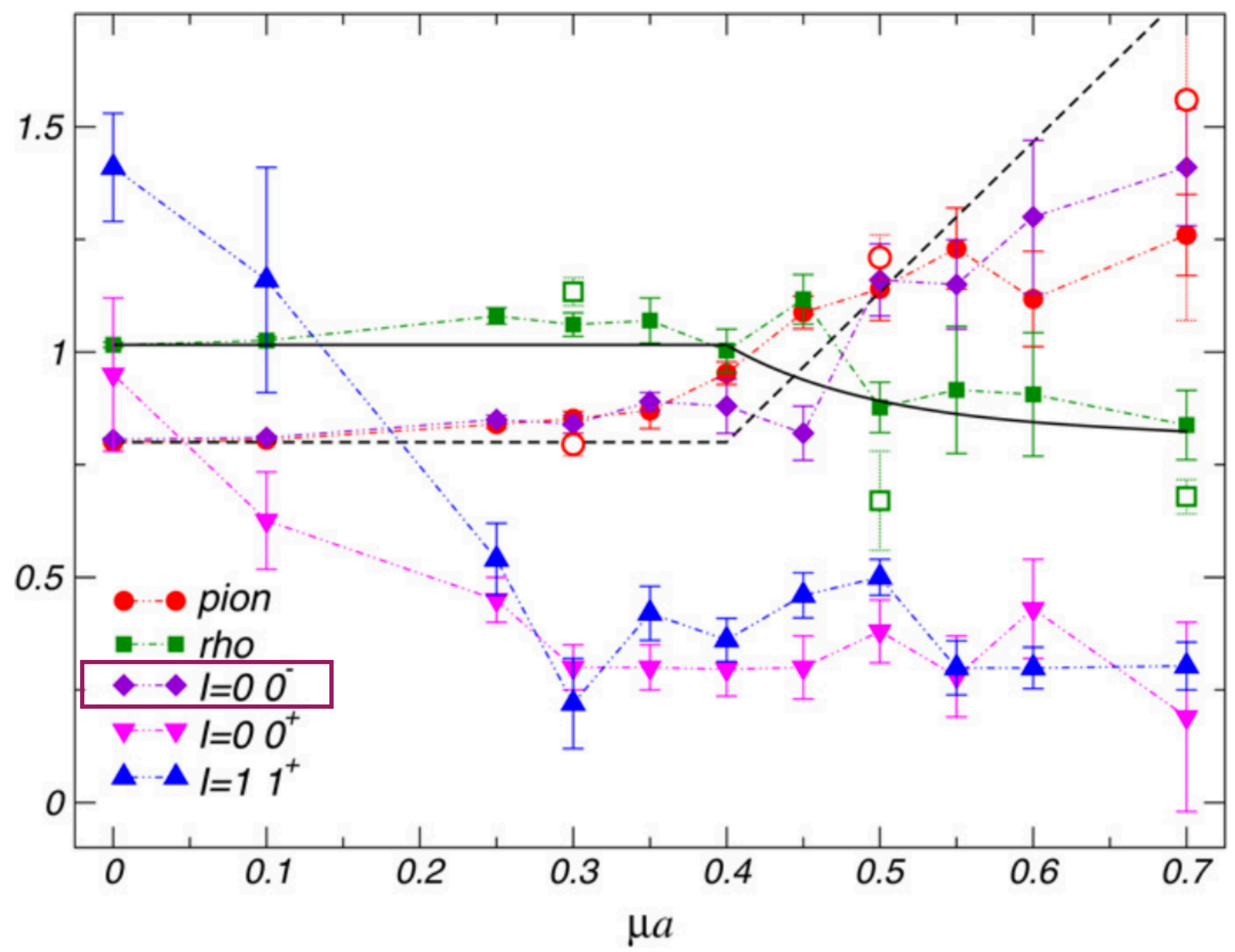


- diquark/antidiquark: consistent
- meson: different behavior in small μ ⟵ finite j effect?

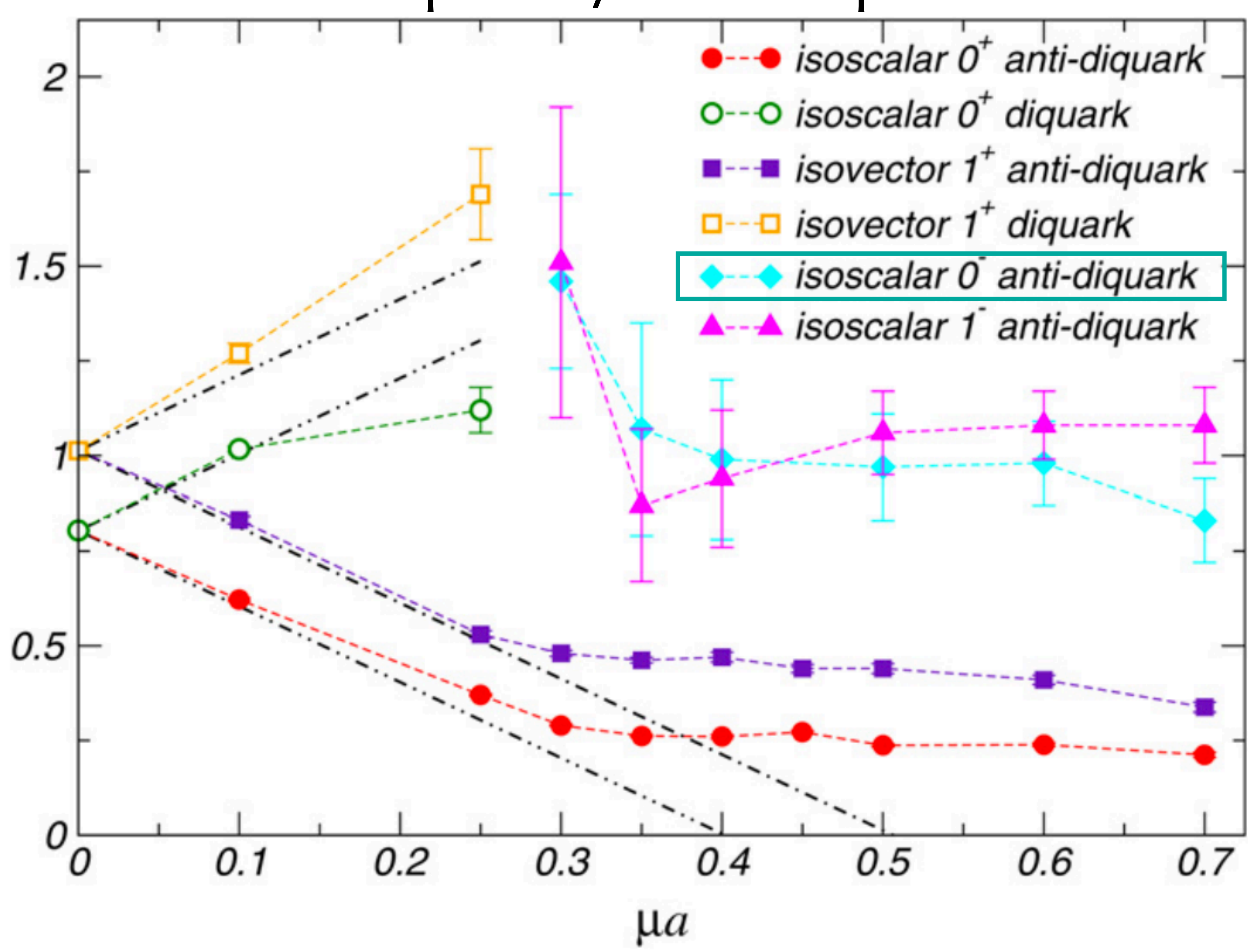
Consistency with the previous studies for $I=0,\ 0^-$ channel

(S. Hands, P. Sitch and J. I. Skullerud, Phys. Lett. B 662 (2008), 405-412)

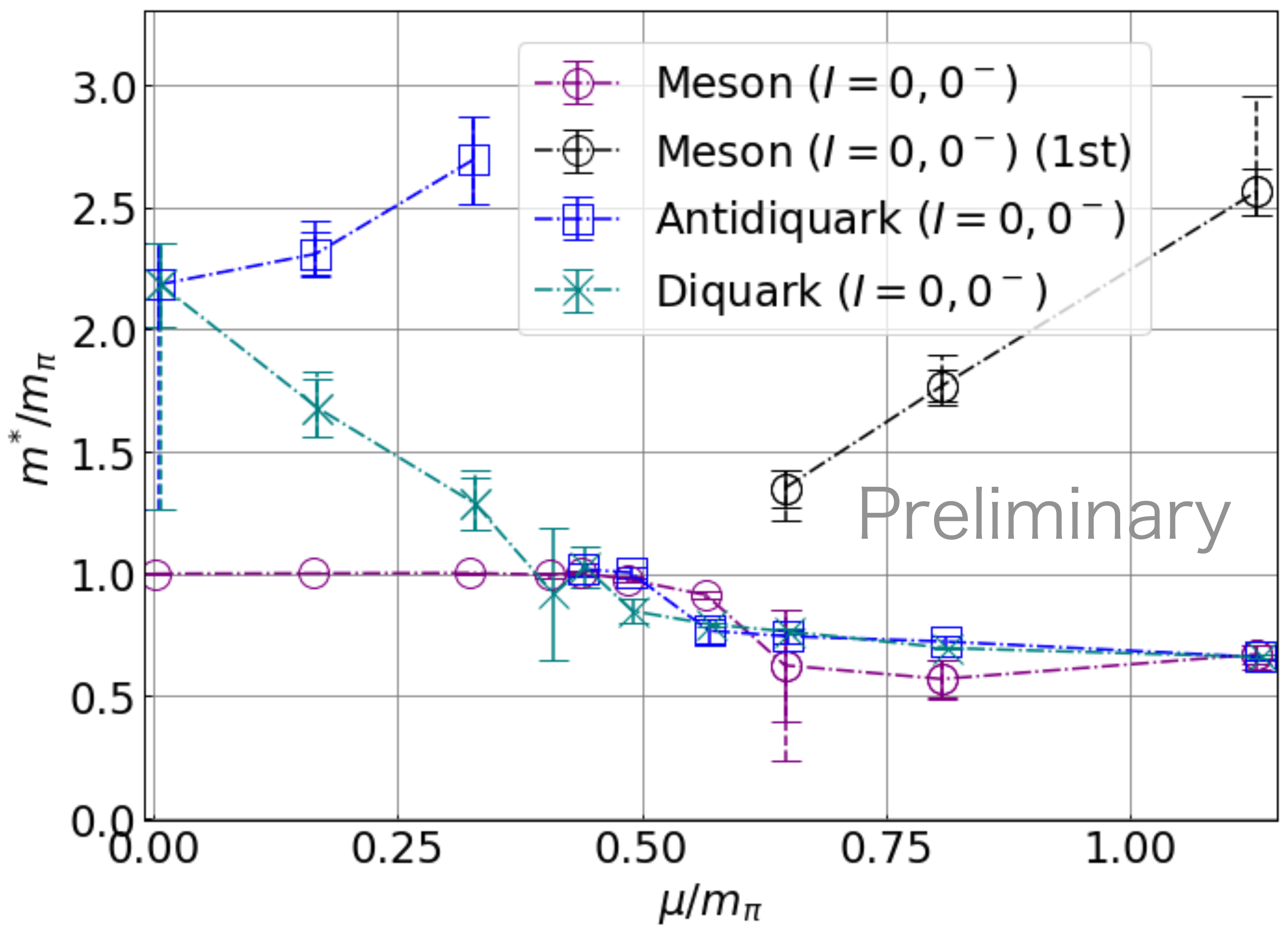
Meson



Diquark/Antidiquark



Our results



- diquark/antidiquark: consistent (in large μ)

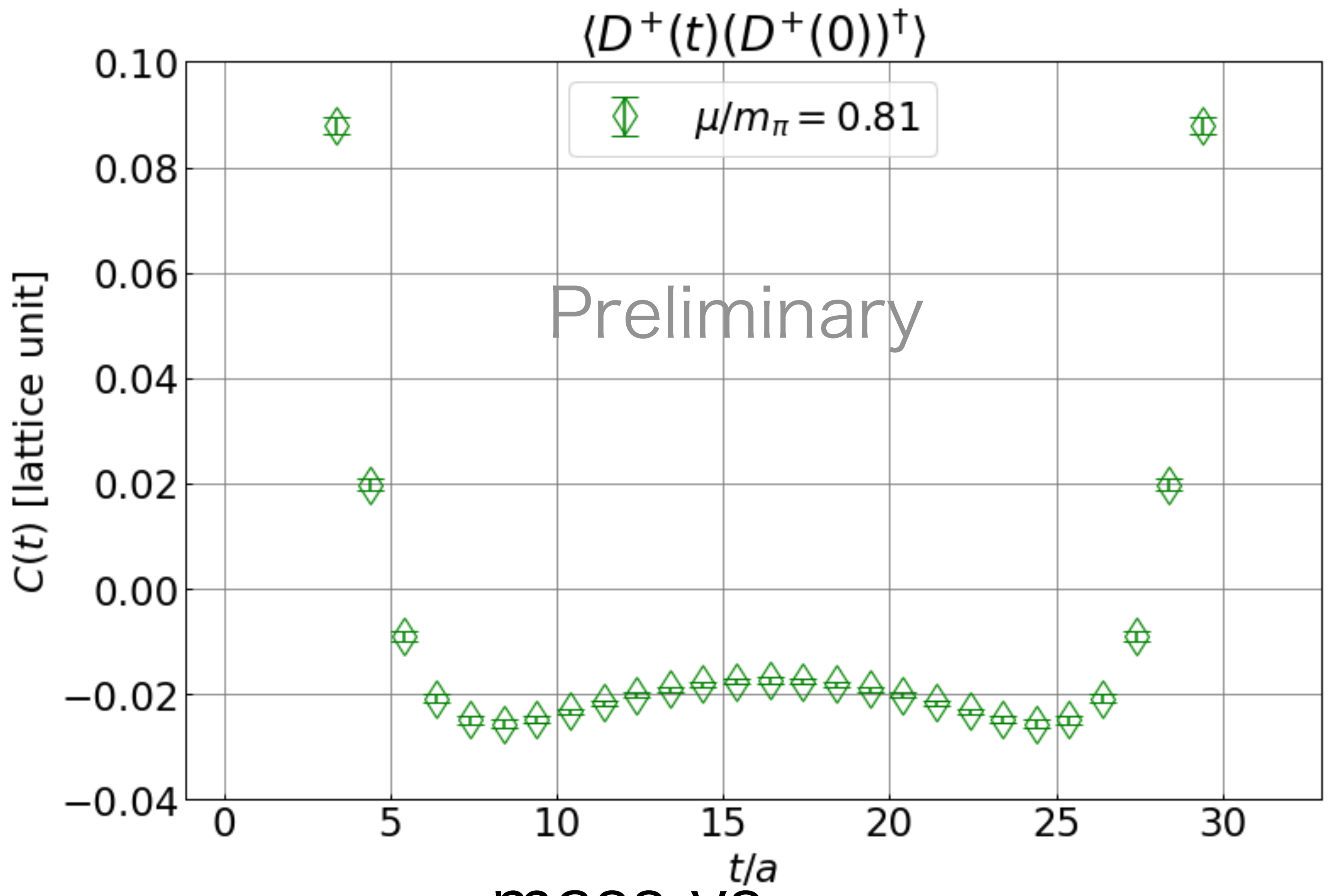
- meson: different in large μ

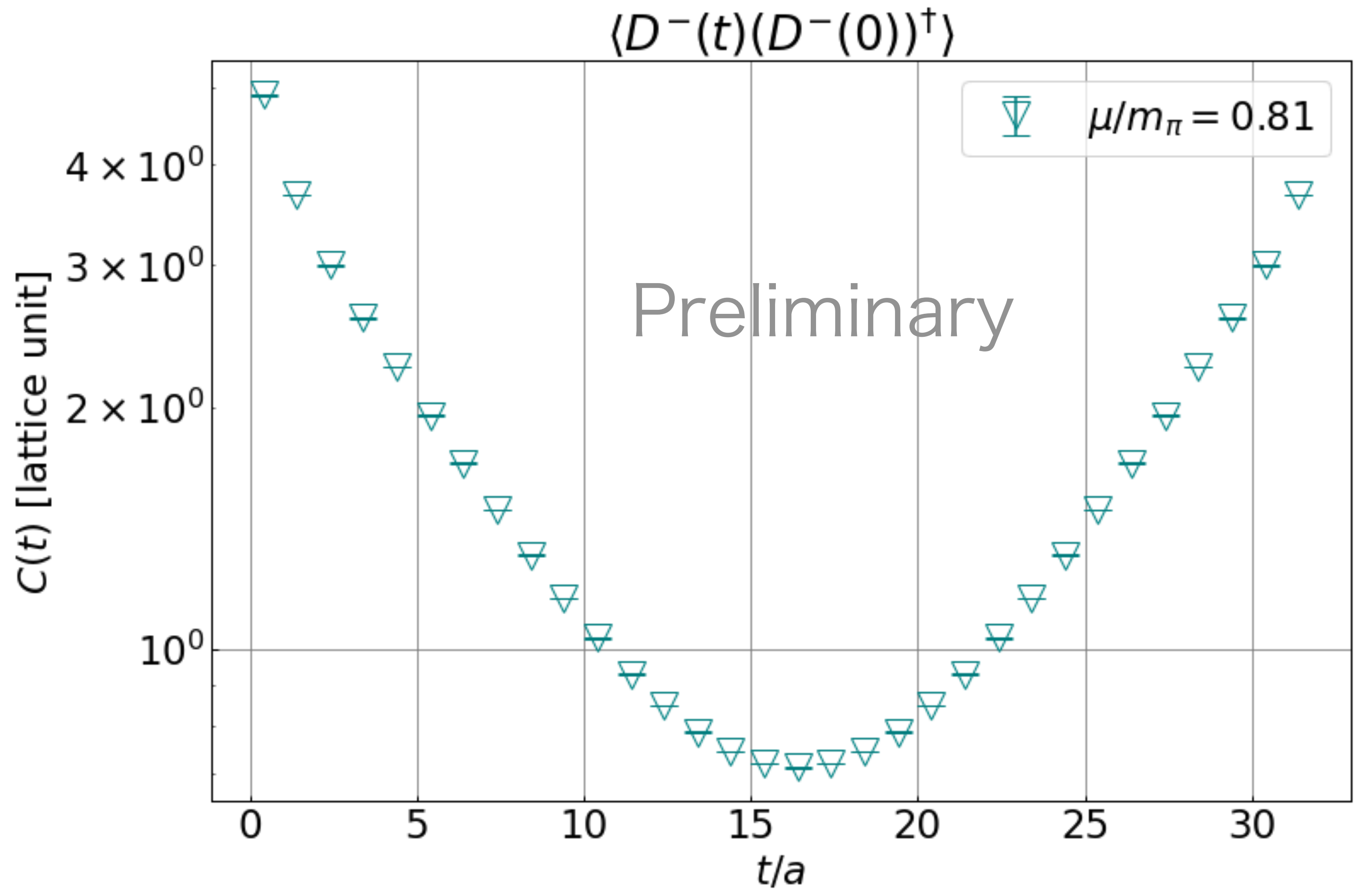
  ← detect excited state?

“Higgs” and “NG” 2pt functions

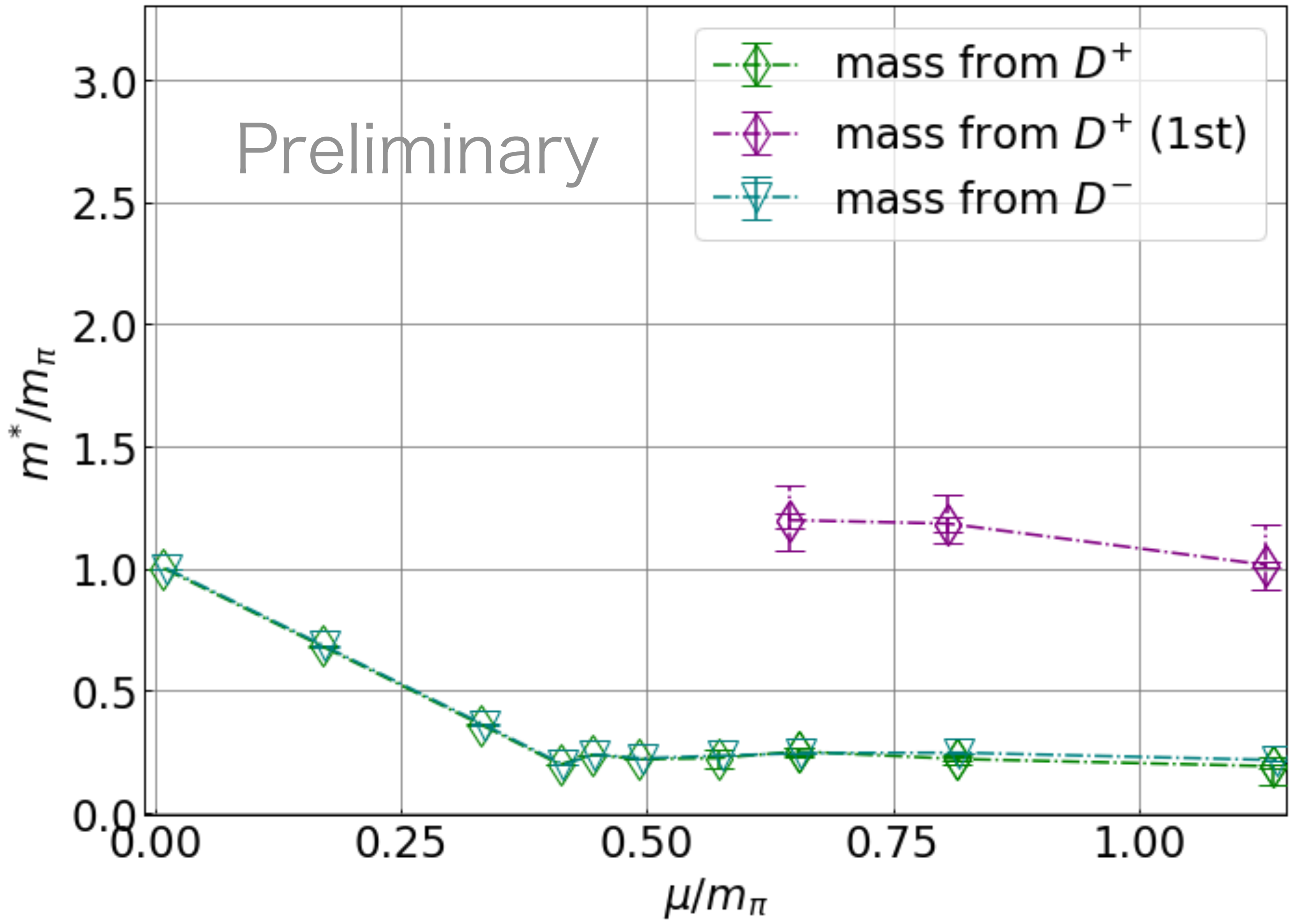
- $D^{\pm} = \bar{\psi}_1 C\gamma_5\tau_2\bar{\psi}_2^T \pm \psi_1^T C\gamma_5\tau_2\psi_2$: would-be Higgs(+)/NG(–) operators

[S. Hands, P. Sitch and J. I. Skullerud, 2008]





mass vs. μ



- D^-: large overlap with light mode

➡ almost NG mode 2pt func.

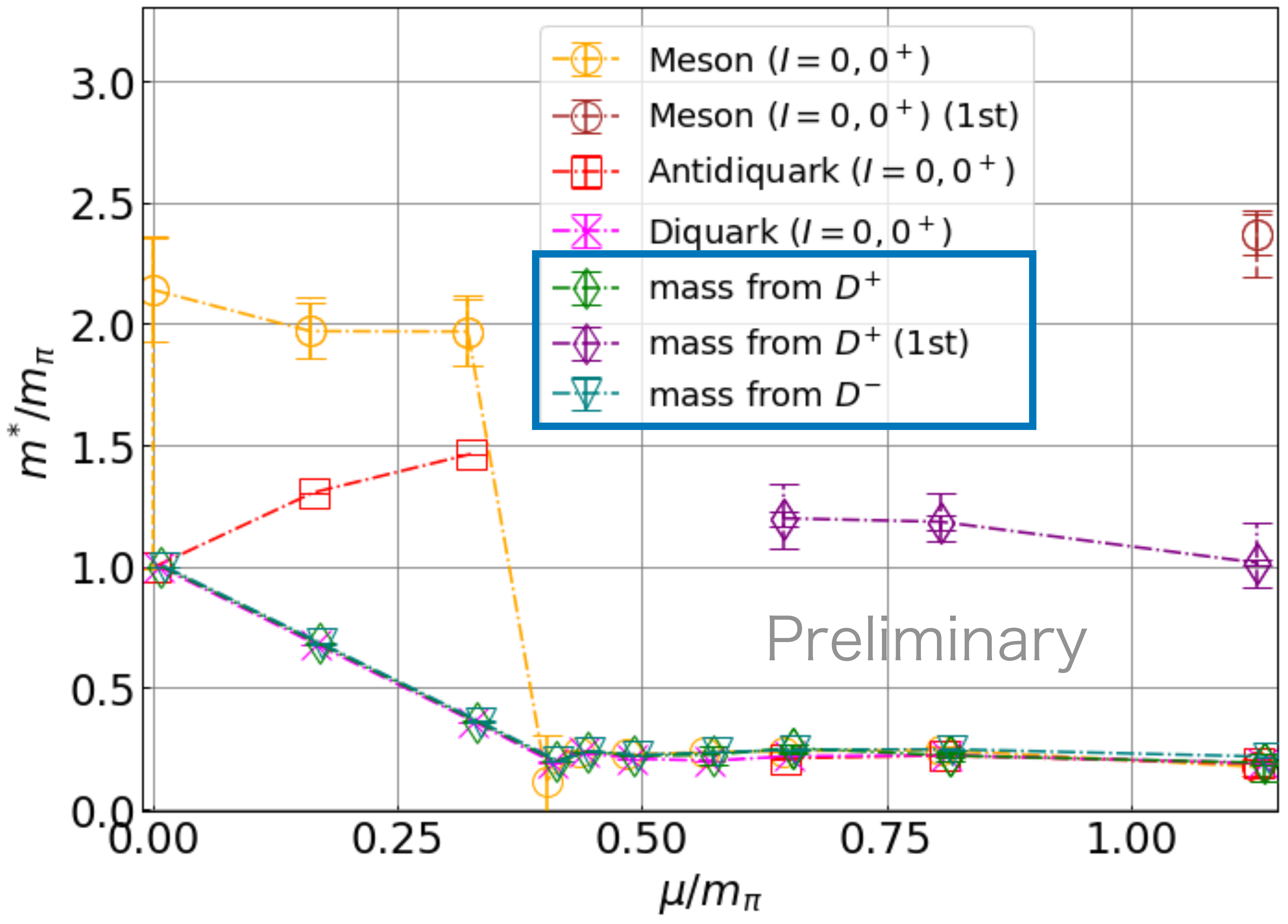
- D^+: heavy + light mode

➡ not pure Higgs

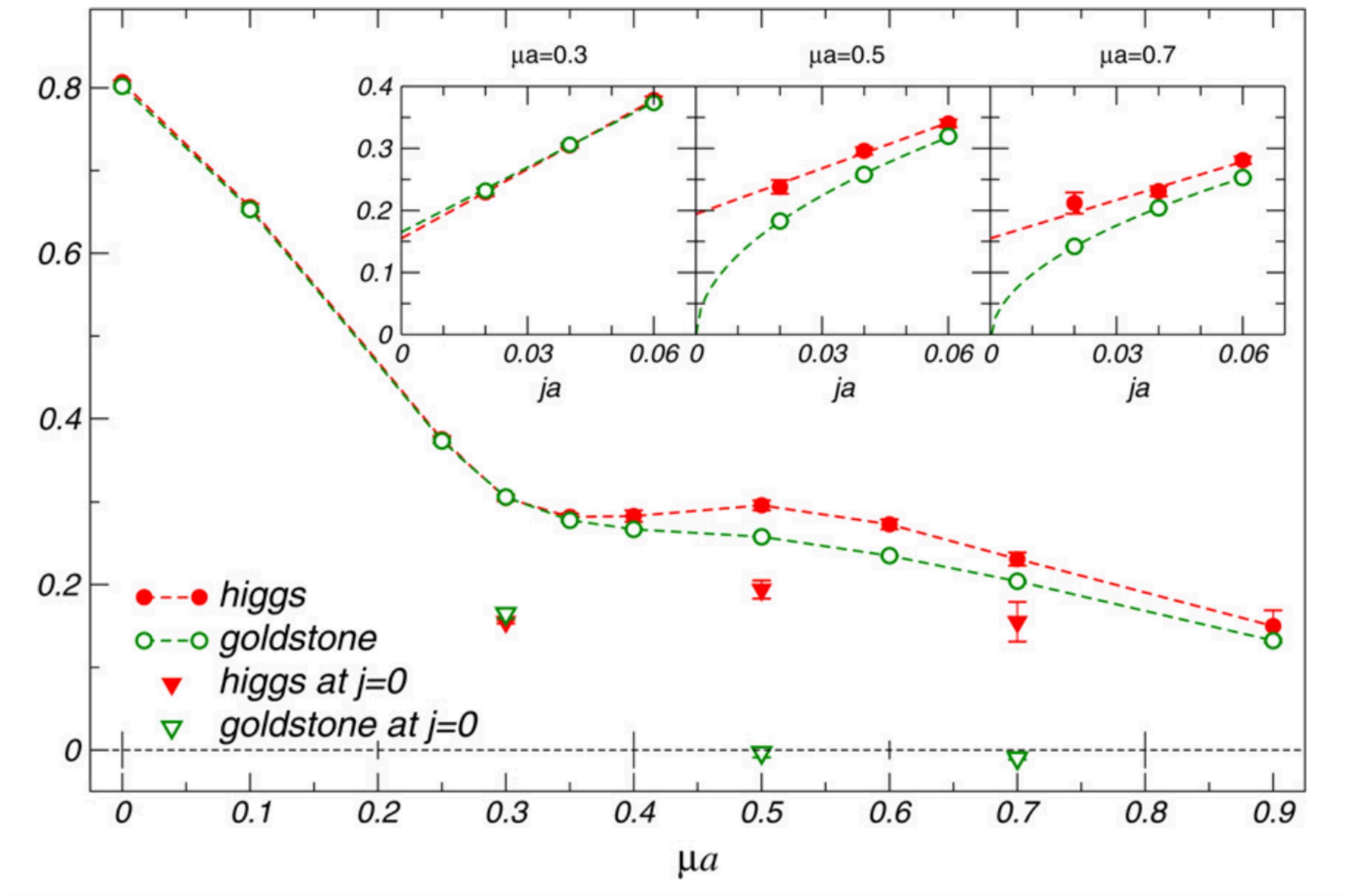
(mixing with meson?)

Comparison of $D^{\pm}$ with other results

Our results



(S. Hands, P. Sitch and J. I. Skullerud,
Phys. Lett. B 662 (2008), 405-412)



- ground state of $D^{\pm}$: same as diquark mass,
  consistent behavior with other lattice results

- excited states of D^{+}: discrepancies among all results
  ← systematics for fitting? lattice artifact?