Probing the hadron mass spectrum in two-color dense QCD with the linear sigma model

Daiki Suenaga (RIKEN Nishina Center)

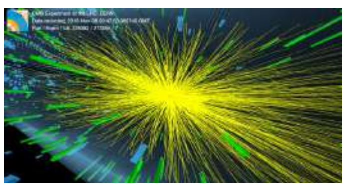
1. Introduction

- **QCD phase diagram**: experimental aspect

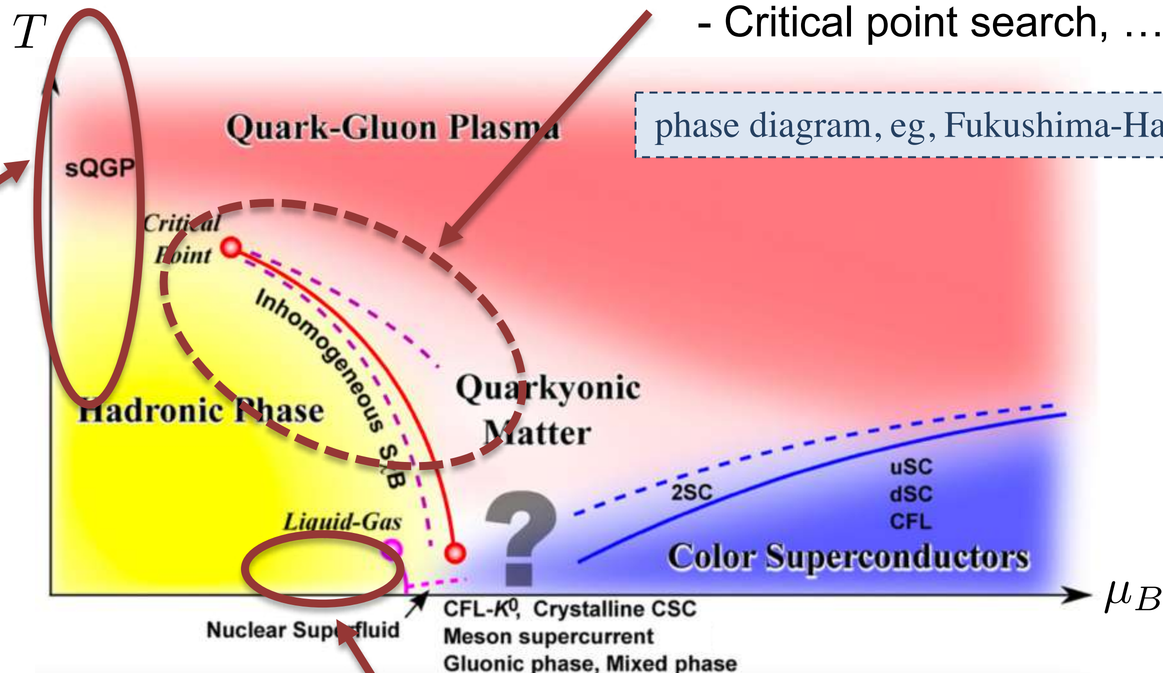
Beam energy scan (at RHIC)
- Critical point search, …

phase diagram, eg, Fukushima-Hatsuda (2017)

Heavy-ion collision
at RHIC, LHC, etc.



- quark-gluon plasma
- chiral restoration
...



mesic nuclei (atom),
E16 experiment, etc.
- partially chiral restoration, ...

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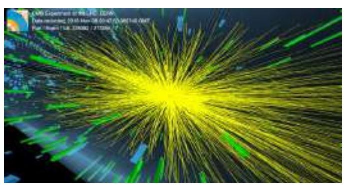

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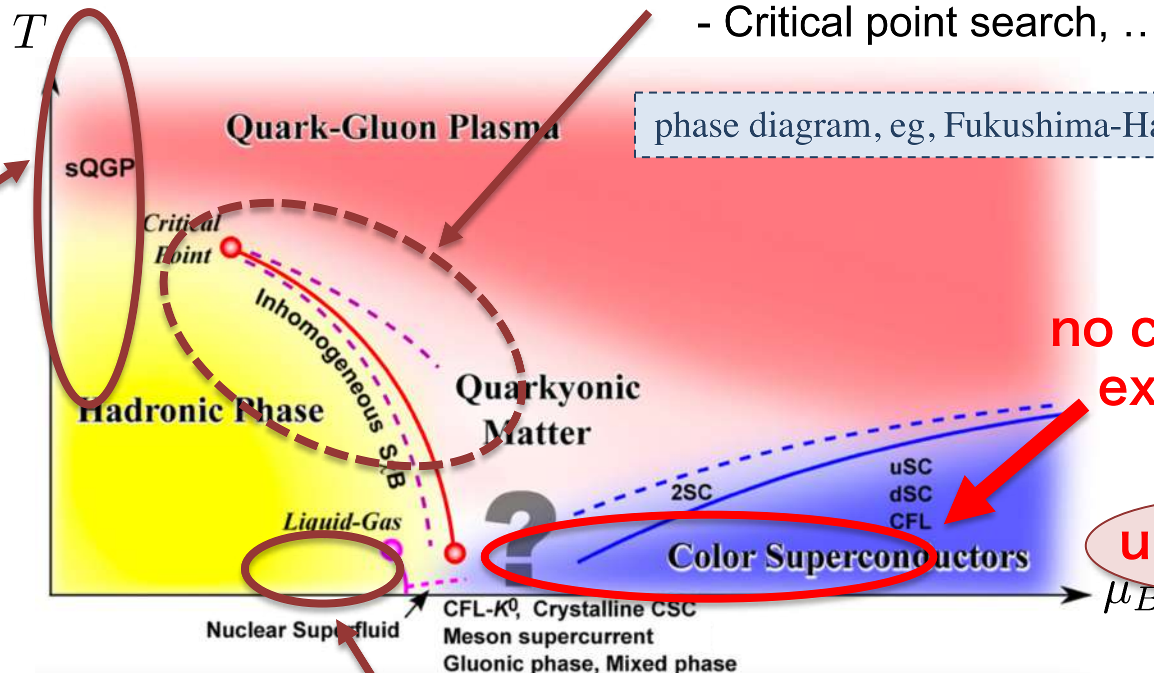


no controllable experiment

unknown

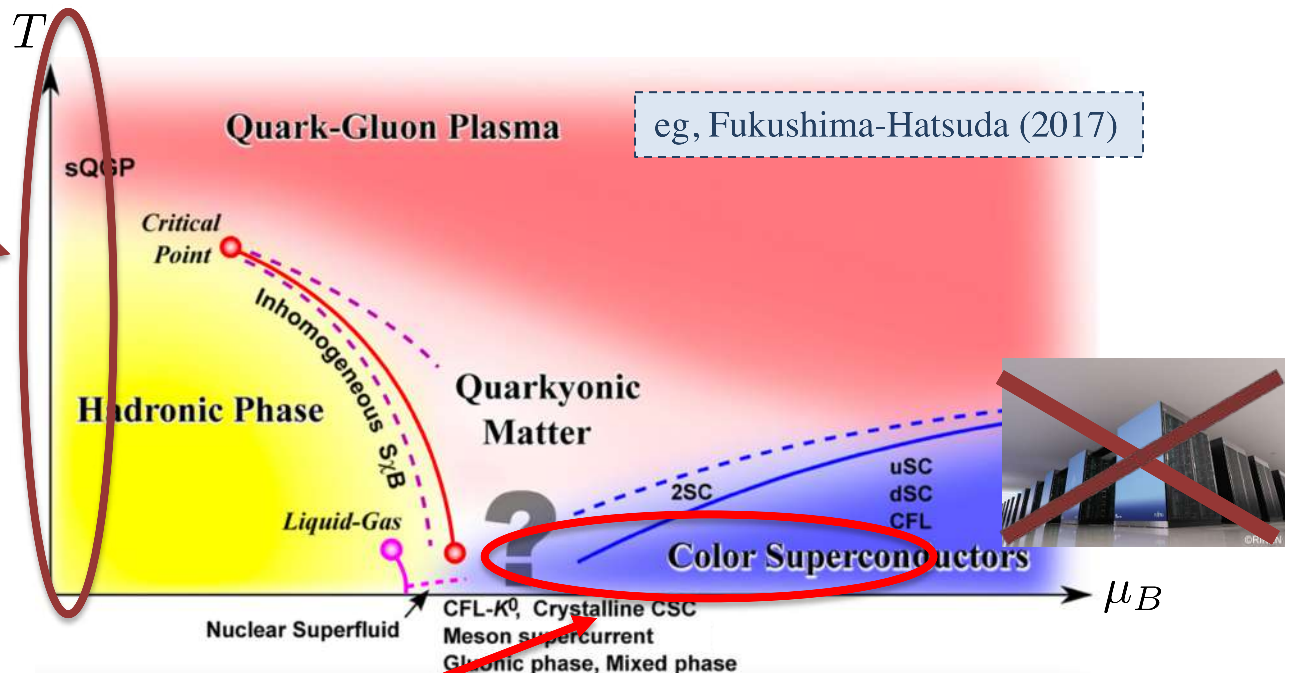
mesic nuclei (atom), E16 experiment, etc.

- partially chiral restoration, ...

1. Introduction

- **QCD phase diagram**: lattice QCD aspect

Lattice QCD



eg, Fukushima-Hatsuda (2017)

- lattice cannot apply due to the *sign problem*

⇨ frontier of QCD

eg, G Aarts (2016) J. Phys.: Conf. Ser. 706 022004

1. Introduction

• Two-color QCD world

three-color QCD (our world)

- Lattice QCD at density is not easy

sign problem

- Baryon is made of three quarks

nucleon

⋮

two-color QCD (imaginary world)

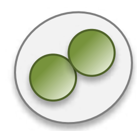
- Lattice QCD at density is possible!

sign problem disappears

↑

pseudo reality of $SU(2)_c$

- Baryon is made of two quarks



diquark baryon

⋮

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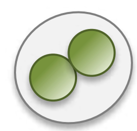


diquark baryon

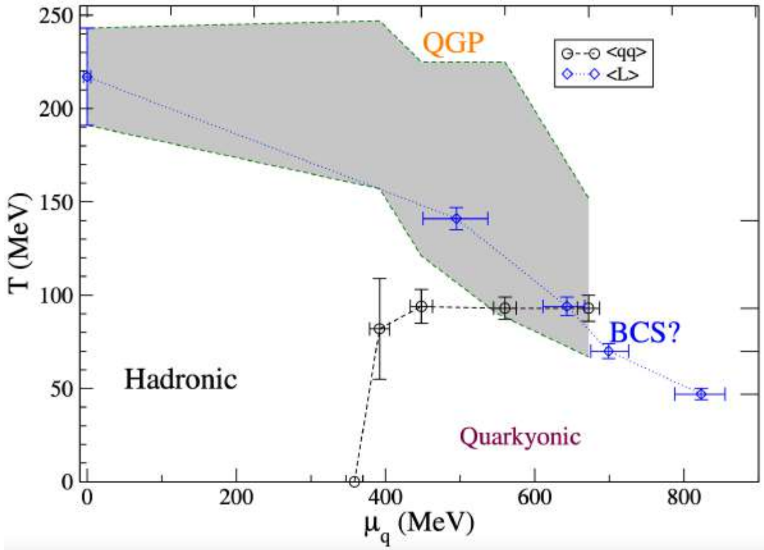
⋮

- Two-color lattice simulations are valuable numerical experiments of dense QCD
⇨ Pursue understanding of dense three-color QCD matter

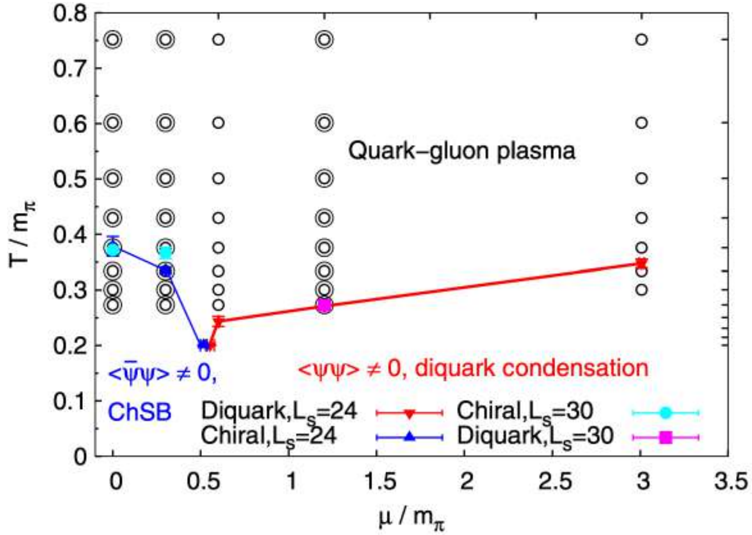
1. Introduction

• Phase diagram in two-color QCD (=QC_2D)

- Examples of simulation results of phase diagram in QC_2D



Boz-Cotter-Fister-Mehta-Skullerud (2013)



Buividovich-Smith-Smekal (2020)

…

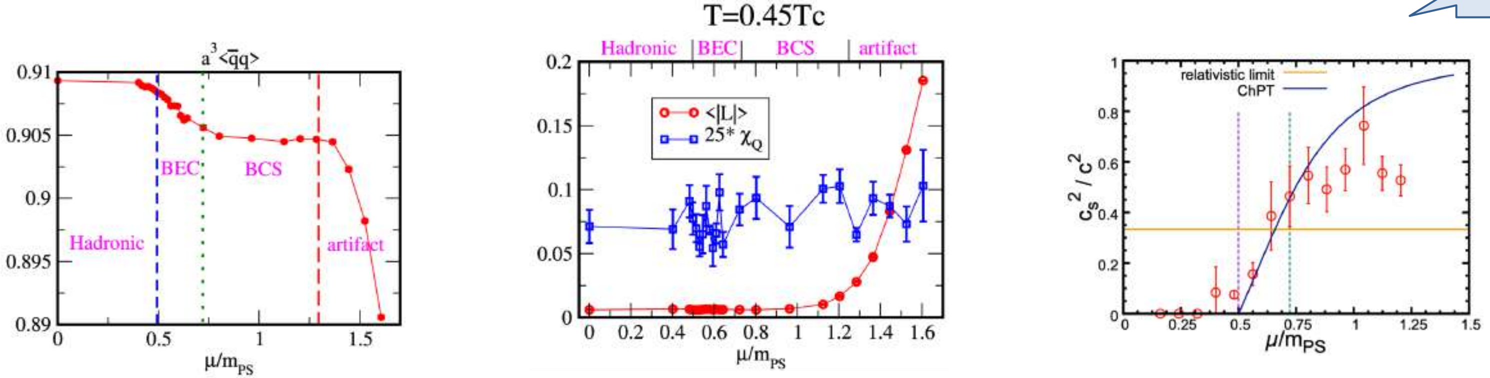
- Currently at least four lattice simulation groups are active
  - Ireland/UK group (Hands, Skullerud, …)
  - Russian group (Bornyakov, ...)
  - UK group (Buividovich, ...)
  - Japanese group (Iida-san, Itou-san, …)

1. Introduction

- **Lattice results**

  - In addition to phase diagram, hadron mass spectrum, gluon propagator, transport coefficient, $\langle\bar{\psi}\psi\rangle, \langle\psi\psi\rangle, \langle L\rangle$, etc. have been simulated

Japanese group



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My approach

– (i) Regard QC_2D lattice simulations as useful "numerical experiments" of dense QCD, and (ii) give interpretation based on effective models

My publications on QC_2D

Gluon propagator: **Suenaga**-Kojo(2019), Kojo-**Suenaga**(2021), CSE effect: **Suenaga**-Kojo(2021),
Sound velocity: Kojo-**Suenaga**(2022), Topological susceptibility: Kawaguchi-**Suenaga**(2023),
Hadron mass: **Suenaga**-Murakami-Itou-Iida (2023), **Suenaga**-Murakami-Itou-Iida (in preparation)

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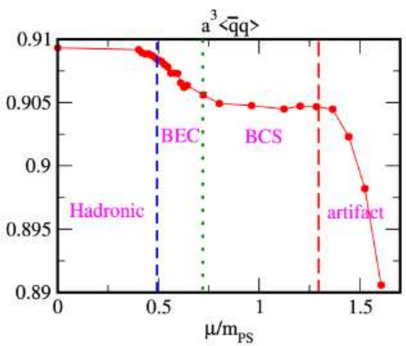

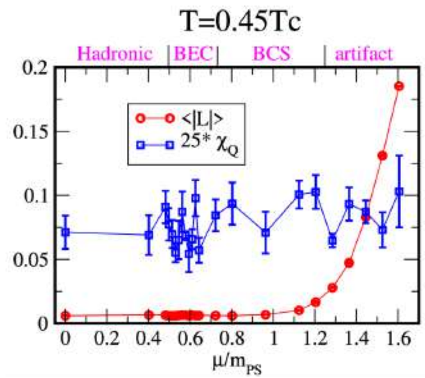

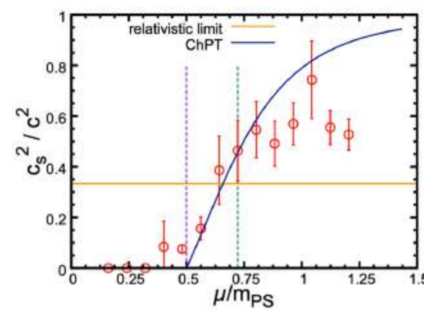

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This talk

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2. LSM construction

• Pauli-Gursey SU(4) symmetry

- Pseudo reality of $SU(2)_c$ allows us to rewrite QC_2D Lagrangian as

$$\mathcal{L}_{\mathrm{QC_2D}} = \bar{\psi} i \partial\!\!\!/ \psi - g_s \bar{\psi} A\!\!\!/^a T_c^a \psi = \Psi^\dagger i\partial_\mu \sigma^\mu \Psi - g_s \Psi^\dagger A_\mu^a T_c^a \sigma^\mu \Psi$$

In two-flavor: $\Psi = (\psi_R, \tilde{\psi}_L)^T = (u_R, d_R, \tilde{u}_L, \tilde{d}_L)^T$ with $\tilde{\psi}_L = \sigma^2 \tau_c^2 \psi_L^*$

Four-dimensional Pauli matrix: $\sigma^\mu = (1, \sigma^i)$

- $\mathcal{L}_{\mathrm{QC_2D}}$ is invariant under $\Psi \to g\Psi$ $[g \in SU(4)]$

⇓

- In QC_2D, $SU(2)_L \times SU(2)_R$ chiral symmetry is extended to SU(4) symmetry

Pauli-Gursey SU(4) symmetry

Pauli (1957), Gursey (1958)

2. LSM construction

• Spin-0 hadron field

- Introduce the following spin- and color-singlet quark bilinear field

$$\Sigma_{ij} \sim \Psi_j^T \sigma^2 \tau_c^2 \Psi_i = \begin{pmatrix} 0 & d_R^T\sigma^2\tau^2 u_R & u_L^\dagger u_R & d_L^\dagger u_R \\ -d_R^T\sigma^2\tau^2 u_R & 0 & u_L^\dagger d_R & d_L^\dagger d_R \\ -u_L^\dagger u_R & -u_L^\dagger d_R & 0 & d_L^\dagger \sigma^2\tau^2 u_L^* \\ -d_L^\dagger u_R & -d_L^\dagger d_R & -d_L^\dagger\sigma^2\tau^2 u_L^* & 0 \end{pmatrix}_{ij}$$

$$\Psi = (\psi_R, \tilde{\psi}_L)^T = (u_R, d_R, \tilde{u}_L, \tilde{d}_L)^T$$

- Assignment of hadron fields

$$B \sim -\frac{i}{\sqrt{2}}\psi^T C\gamma_5 \tau_c^2 \tau_f^2 \psi \quad B' \sim -\frac{1}{\sqrt{2}}\psi^T C \tau_c^2\tau_f^2\psi \quad \sigma \sim \bar{\psi}\psi$$

$$a_0^a \sim \bar{\psi}\tau_f^a\psi \quad \eta \sim \bar{\psi}i\gamma_5\psi \quad \pi^a \sim \bar{\psi}i\gamma_5\tau_f^a\psi$$

Hadron	J^P	Quark number	Isospin
σ	0^+	0	0
a_0	0^+	0	1
η	0^-	0	0
π	0^-	0	1
B $(\bar{B})$	0^+	$+2(-2)$	0
B' $(\bar{B}')$	0^-	$+2(-2)$	0

- 4×4 matrix Σ reads (with normalization of 1/2)

$$\Sigma \to g\Sigma g^T \quad [g \in SU(4)]$$

$$\Sigma = \frac{1}{2}\begin{pmatrix} 0 & -\frac{B'-iB}{2\sqrt{2}} & \frac{\sigma-i\eta+a_0^0-i\pi^0}{4} & \frac{a_0^+-i\pi^+}{2\sqrt{2}} \\ \frac{B'-iB}{2\sqrt{2}} & 0 & \frac{a_0^- -i\pi^-}{2\sqrt{2}} & \frac{\sigma-i\eta-a_0^0+i\pi^0}{4} \\ -\frac{\sigma-i\eta+a_0^0-i\pi^0}{4} & -\frac{a_0^- -i\pi^-}{2\sqrt{2}} & 0 & -\frac{\bar{B}'-i\bar{B}}{2\sqrt{2}} \\ -\frac{a_0^+-i\pi^+}{2\sqrt{2}} & -\frac{\sigma-i\eta-a_0^0+i\pi^0}{4} & \frac{\bar{B}'-i\bar{B}}{2\sqrt{2}} & 0 \end{pmatrix}$$

2. LSM construction

- **Linear sigma model (LSM)**

- (approximately) $SU(4)$-invariant LSM Lagrangian is given by

$$\mathcal{L} = \mathrm{tr}[D_\mu \Sigma^\dagger D^\mu \Sigma] - m_0^2 \mathrm{tr}[\Sigma^\dagger \Sigma] - \lambda_1 \left(\mathrm{tr}[\Sigma^\dagger \Sigma]\right)^2 - \lambda_2 \mathrm{tr}[(\Sigma^\dagger \Sigma)^2] + \underbrace{\mathrm{tr}[H^\dagger \Sigma + \Sigma^\dagger H]}_{\text{explicit breaking}} + \underbrace{c(\det\Sigma + \det\Sigma^\dagger)}_{U(1)_A \text{ anomaly}}$$

$$\begin{cases} D_\mu \Sigma = \partial_\mu \Sigma - i\mu_q \delta_{\mu 0}\{J, \Sigma\} \text{ with } J = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix} & \leftarrow \text{chemical potential effect} \\ H = h_q E \text{ with } E = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix} & \leftarrow \text{current-quark mass effect} \end{cases}$$

- $\bar{\psi}\psi = \frac{1}{2}\left(\Psi^T \sigma^2 \tau_c^2 E^T \Psi + \Psi^\dagger \sigma^2 \tau_c^2 E \Psi^*\right)$ is invariant under $\Psi \to h\Psi$ $(h^T E h = E)$

⇨ "chiral" symmetry breaking pattern reads $SU(4) \to Sp(4)$

- Advantage of LSM
  → we can see mass relation between parity partners

parity partner
$B(\bar{B}) \leftrightarrow B'(\bar{B}')$
$\eta, \pi \leftrightarrow \sigma, a_0$

3. Hadron mass

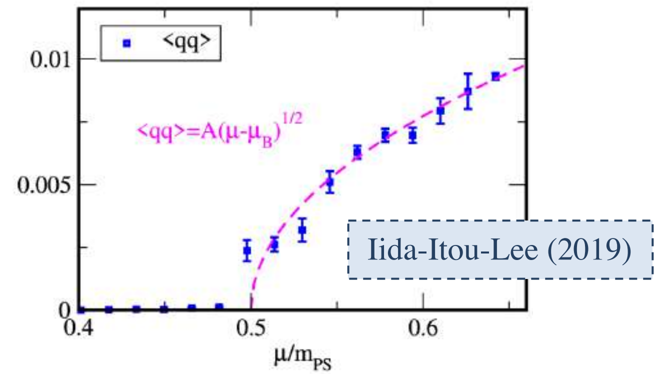
- **Mean field**

- The mean fields are $\sigma_0 \equiv \langle\sigma\rangle$ and $\Delta \equiv \left\langle \frac{B+\bar{B}}{\sqrt{2}} \right\rangle$

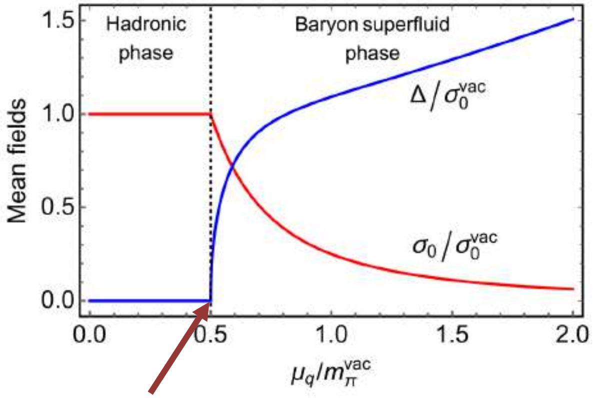
$\sigma_0 \sim \langle\bar{\psi}\psi\rangle$: chiral condensate

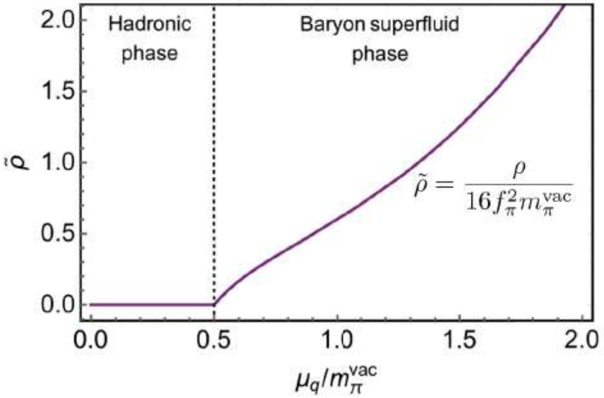
$\Delta \sim -\frac{i}{2}\langle\psi^T C\gamma_5\tau_c^2\tau_f^2\psi\rangle + \mathrm{h.c.}$: diquark condensate

diquark cond. by lattice



- μ_q dependence of σ_0 and Δ , and number density ρ





2nd order phase transition at $\mu_q = m_\pi^{\mathrm{vac}}/2$

Input here

$\sigma_0^{\mathrm{vac}} = 250\,\mathrm{MeV}$

$\lambda_1 = c = 0$ (large Nc)

$m_\pi^{\mathrm{vac}} = 738\,\mathrm{MeV}$

$m_{a_0}^{\mathrm{vac}}/m_\pi^{\mathrm{vac}} = 2.18$

} lattice
Murakami et al

3. Hadron mass

- **Hadron mass**

  - I take these three inputs ⇨

		c	λ_1	λ_2	m_0^2	h_q
large N_c →	Set (I)	0	0	65.6	$-(693\ \mathrm{MeV})^2$	$(364\ \mathrm{MeV})^3$
	Set (II)	0	−7	65.6	$-(206\ \mathrm{MeV})^2$	$(364\ \mathrm{MeV})^3$
w/ anom. →	Set (III)	15	0	58.1	$-(495\ \mathrm{MeV})^2$	$(364\ \mathrm{MeV})^3$

3. Hadron mass

Suenaga-Murakami-Itou-Iida
Phys.Rev.D 107, 054001 (2023)

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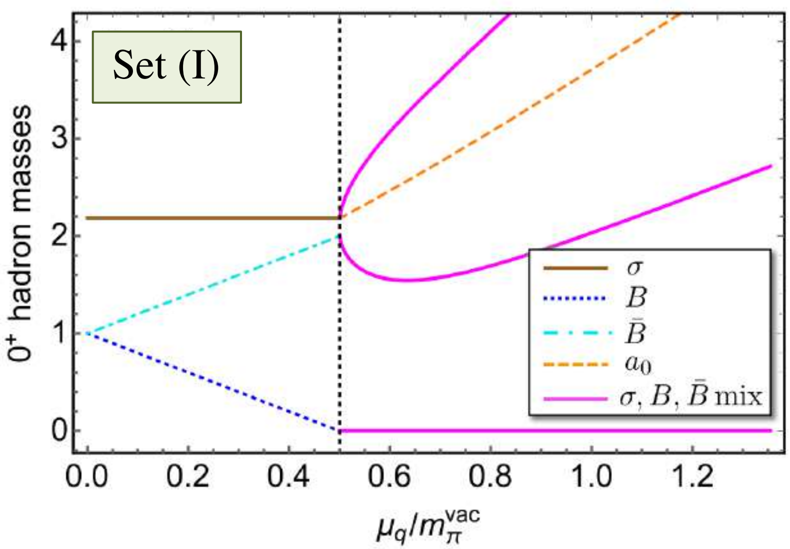

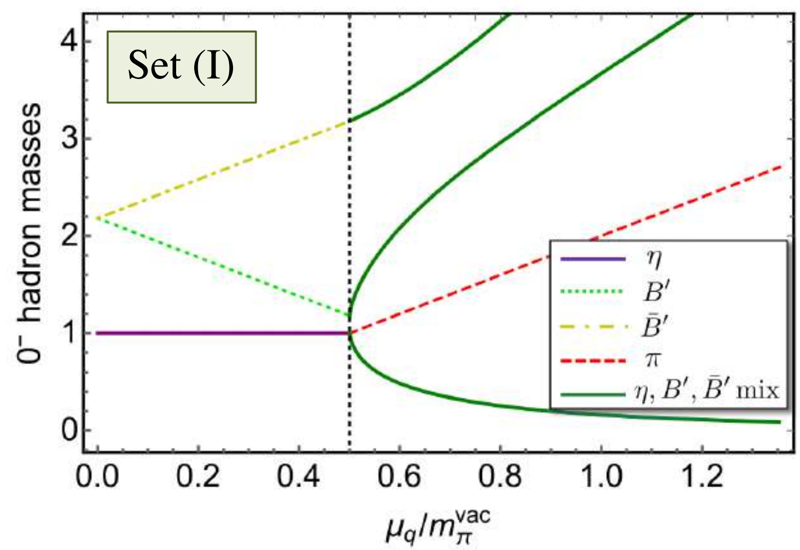

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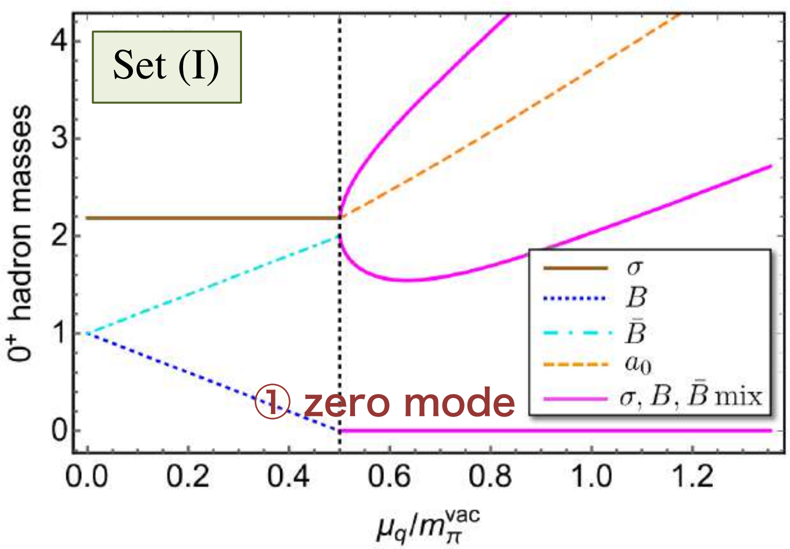

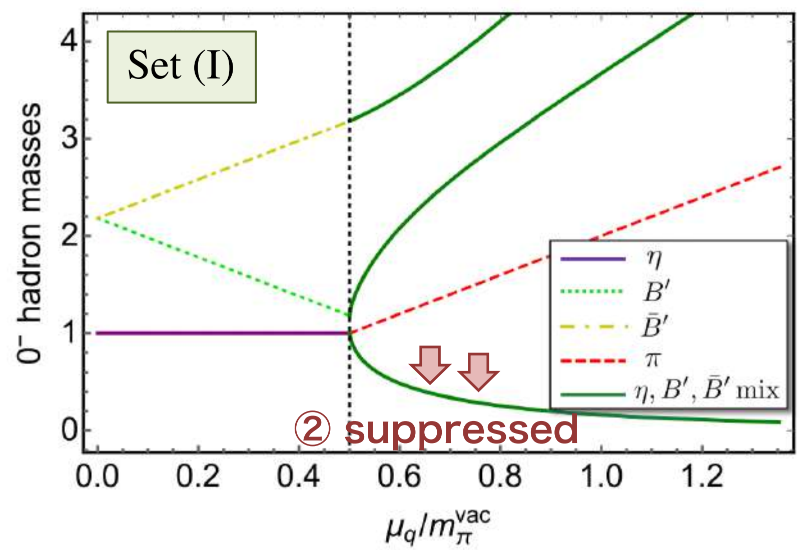

- Baryon number violation in superfluid phase

$$\begin{cases} \sigma \leftrightarrow B \leftrightarrow \bar{B} \ \text{mixing } (0^+) \\ \eta \leftrightarrow B' \leftrightarrow \bar{B}' \ \text{mixing } (0^-) \end{cases}$$

① zero model (NG mode of $U(1)_B$ breaking)
② suppression of " η " mass

3. Hadron mass

Suenaga-Murakami-Itou-Iida
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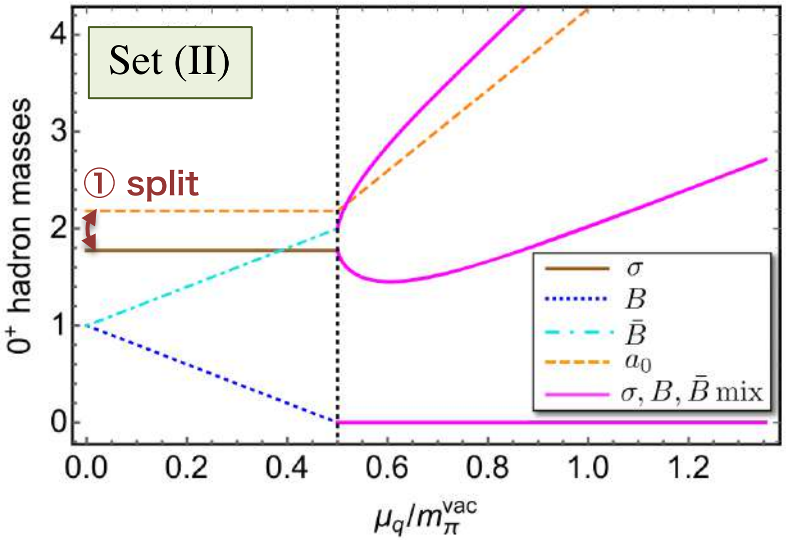

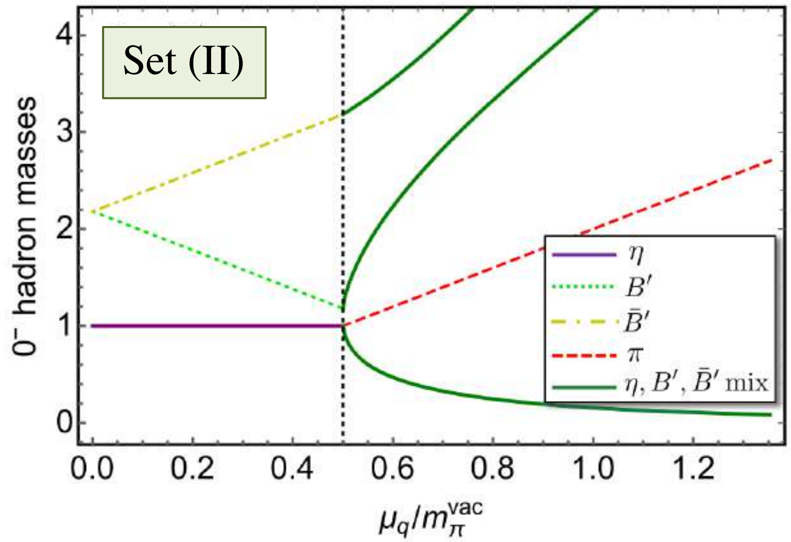

- Baryon number violation in superfluid phase

$\sigma \leftrightarrow B \leftrightarrow \bar{B}$ mixing (0^+)

$\eta \leftrightarrow B' \leftrightarrow \bar{B}'$ mixing (0^-)

① mass splitting of (σ, a_0) induced by λ_1

- **Hadron mass**

- I take these three inputs ⇨

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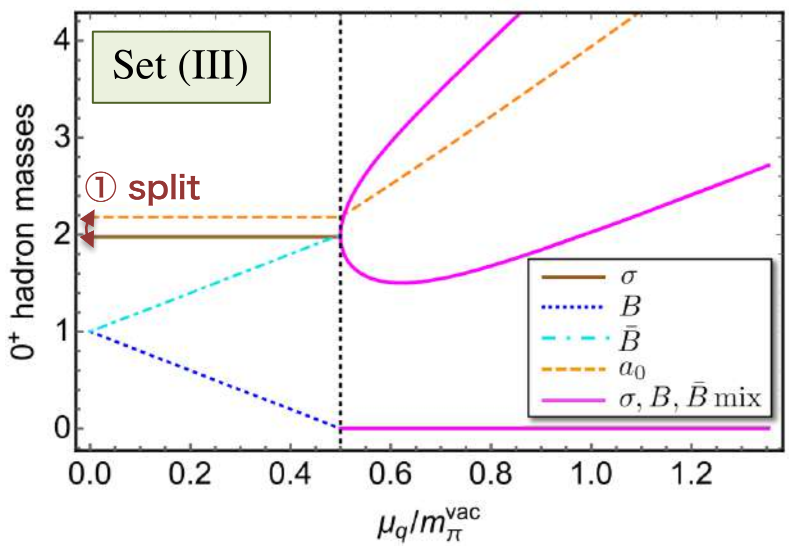
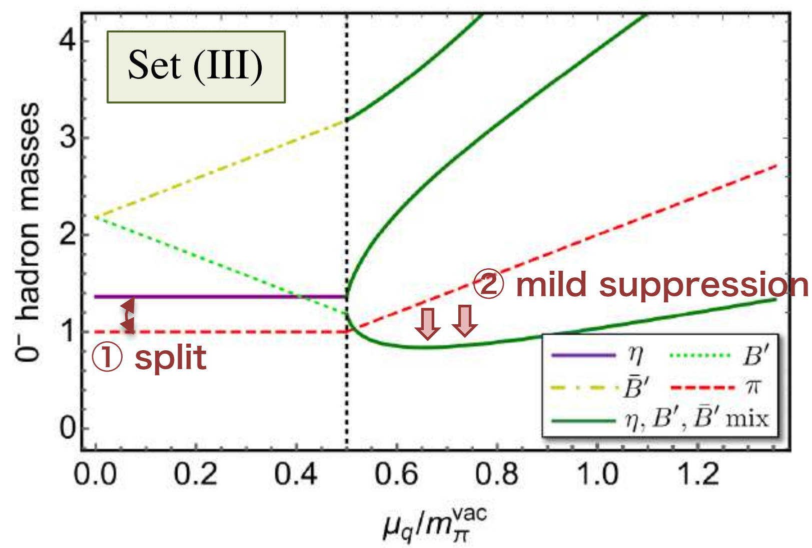




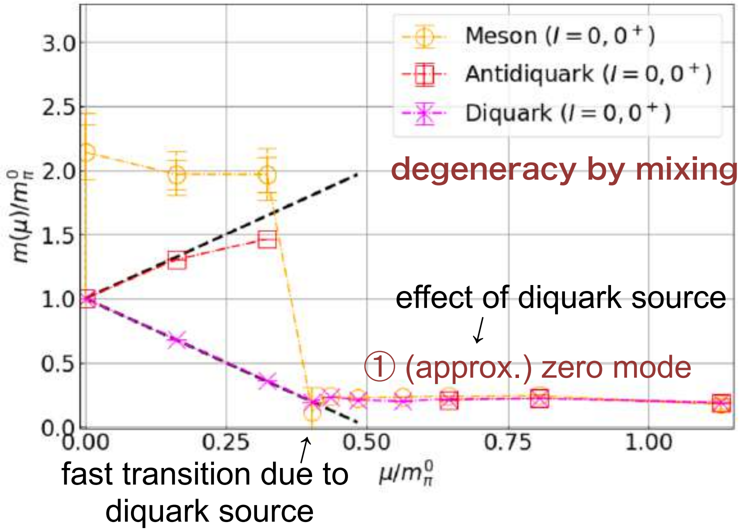
- Baryon number violation in superfluid phase

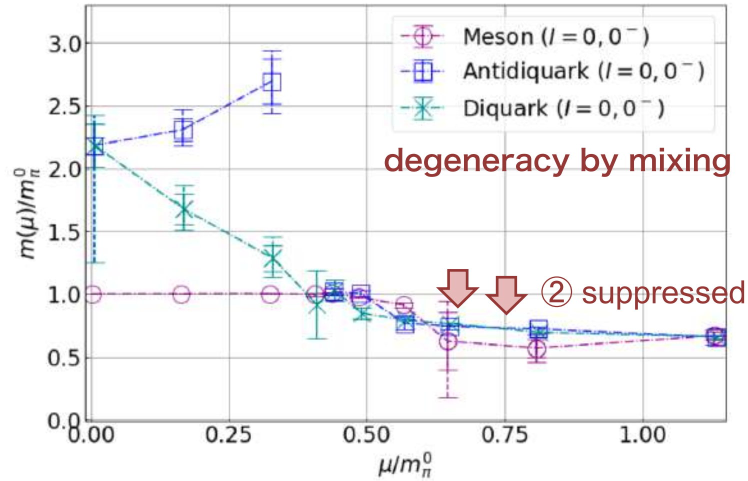
$\sigma \leftrightarrow B \leftrightarrow \bar{B}$ mixing (0^+)
$\eta \leftrightarrow B' \leftrightarrow \bar{B}'$ mixing (0^-)

① mass splitting of (σ, a_0) and of (η, π) induced by anomaly
② Anomaly makes the suppression milder

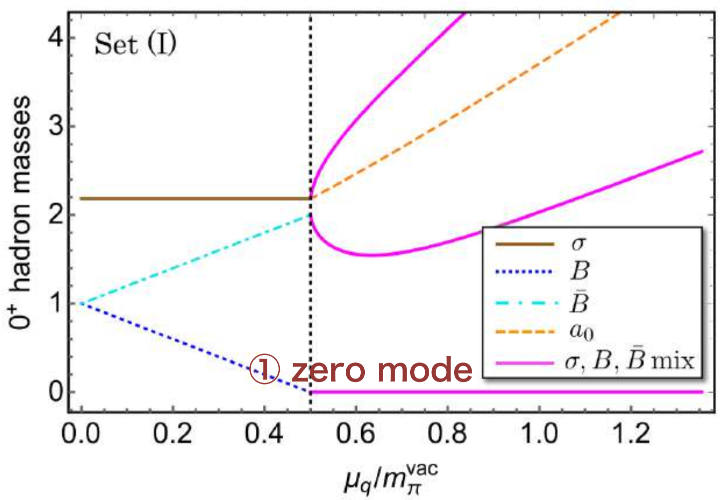
• Comparison with lattice -Set (I)-

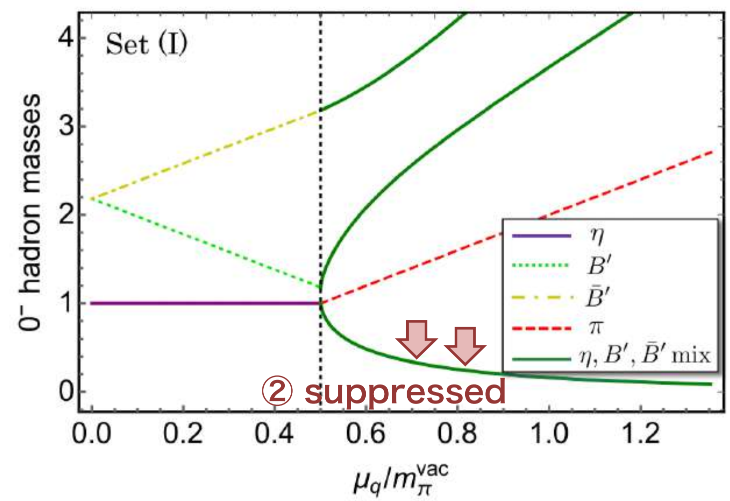
Lattice (Murakami et al)





My model

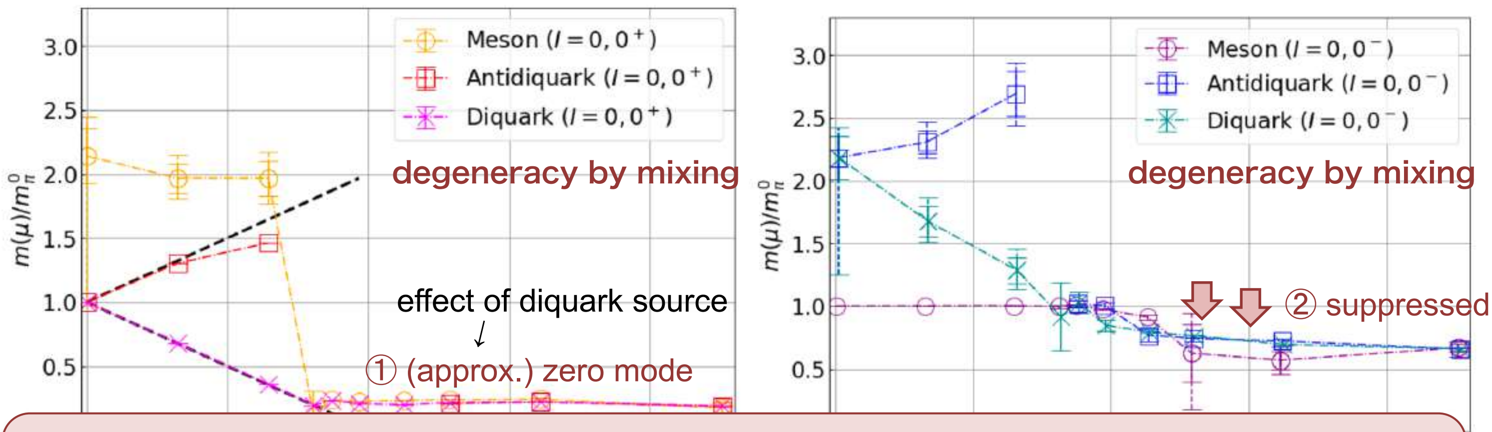




3. Hadron mass

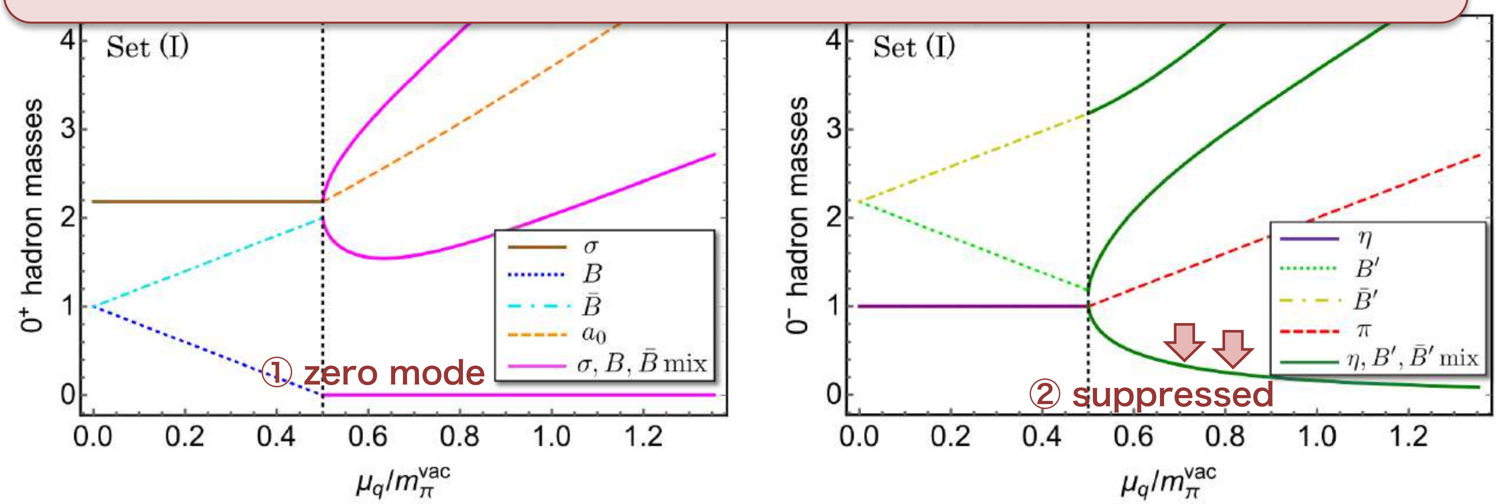
- **Comparison with lattice** -Set (I)-

Lattice (Murakami et al)



Succeeded in qualitative understanding!

My model



3. Hadron mass

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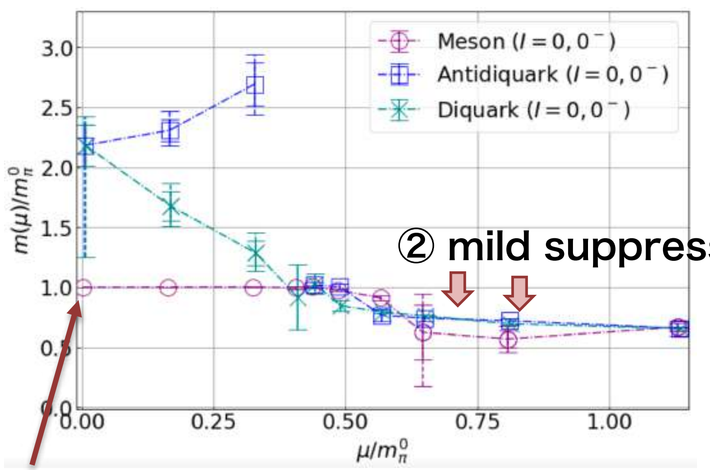
- **Comparison with lattice –focused on anomaly-**

Lattice QCD (Murakami et al)

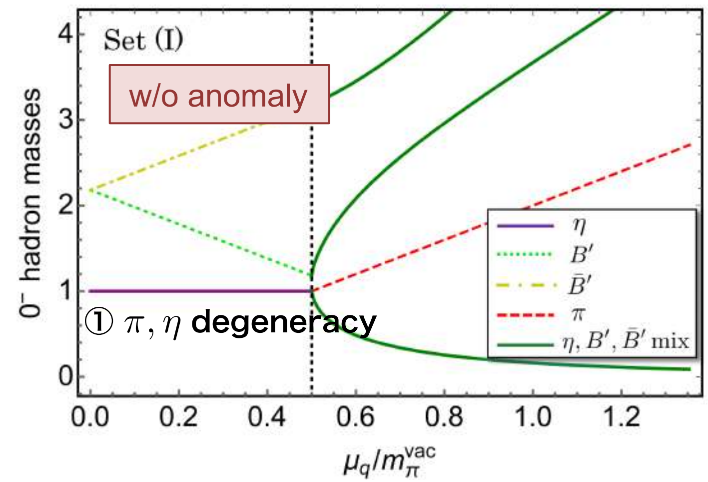


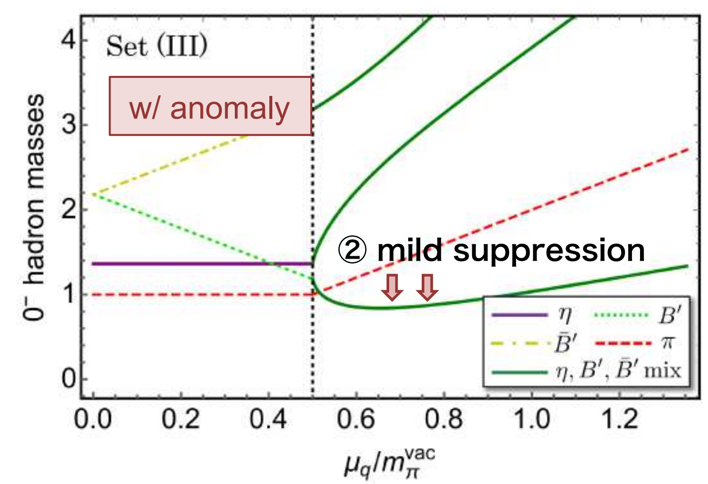
① degeneracy of π and η (← no disc. diagrams)

⇕

does not change largely with disc. diagrams

My model





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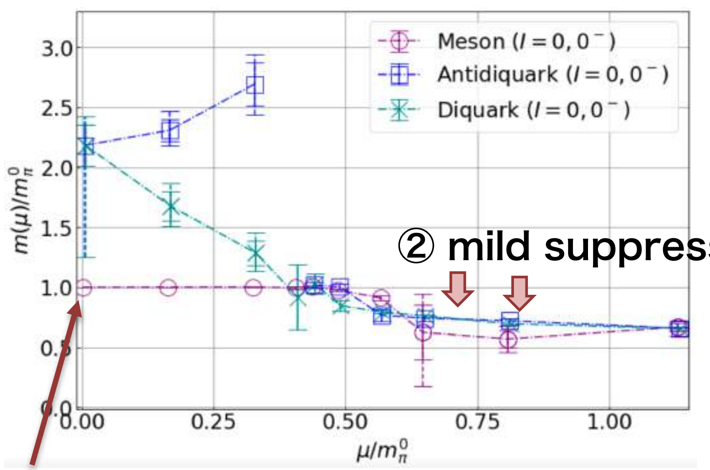
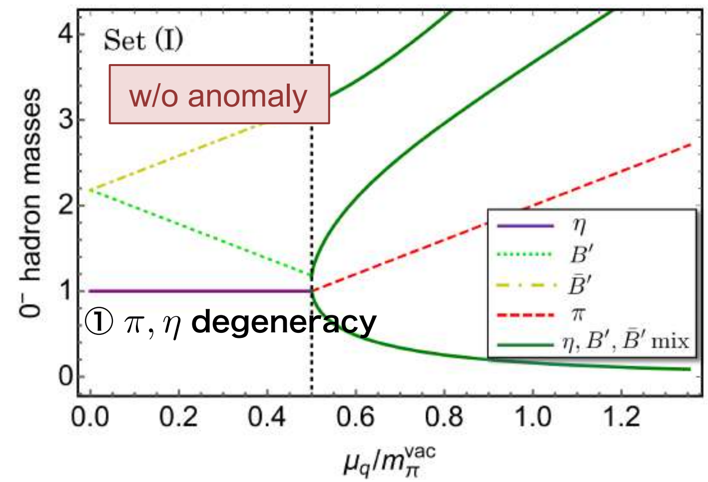


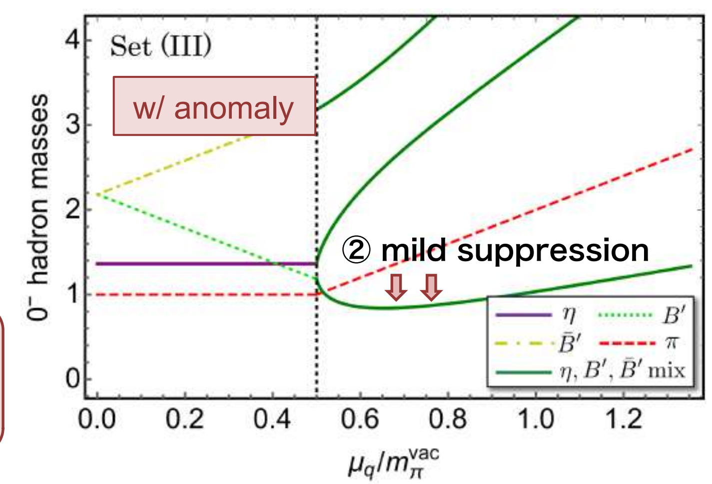
My model



① degeneracy of π and η (← no disc. diagrams)

⇕

does not change largely with disc. diagrams



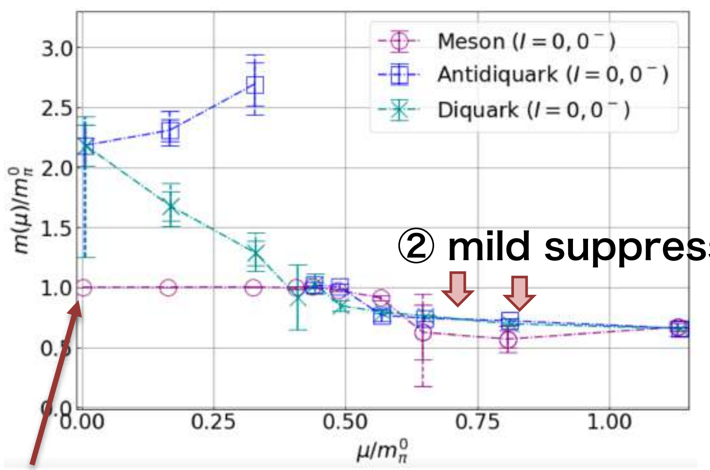
- At zero density anomaly effect is suppressed,
  but at finite density anomaly would be enhanced

3. Hadron mass

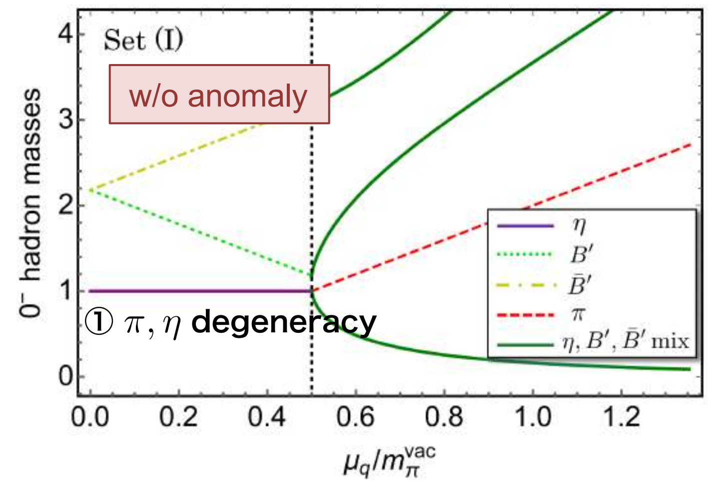
Suenaga-Murakami-Itou-Iida
Phys.Rev.D 107, 054001 (2023)

- **Comparison with lattice –focused on anomaly-**

Lattice QCD (Murakami et al)



My model

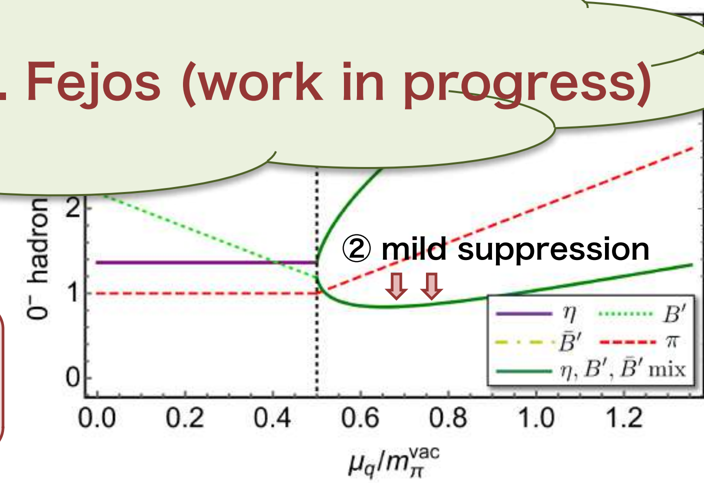


① degeneracy of π and η

⇕

does not change largely with disc. diagrams

FRG analysis with G. Fejos (work in progress)



- At zero density anomaly effect is suppressed,
 but at finite density anomaly would be enhanced

4. LSM with spin-1

Suenaga-Murakami-Itou-Iida, in preparation

• LSM with spin-1 hadrons

- Introduce the following 4×4 matrix representing spin-1 hadrons

$$\Phi^{\mu}_{ij} \sim \Psi^{\dagger}_{j}\sigma^{\mu}\Psi_{i} = \frac{1}{2}\begin{pmatrix} \frac{\omega+\rho^0-(f_1+a_1^0)}{\sqrt{2}} & \rho^+ - a_1^+ & \sqrt{2}B_S^{I=+1} & B_S^{I=0}-B_{AS} \\ \rho^- - a_1^- & \frac{\omega-\rho^0-(f_1-a_1^0)}{\sqrt{2}} & B_S^{I=0}+B_{AS} & \sqrt{2}B_S^{I=-1} \\ \sqrt{2}\bar{B}_S^{I=-1} & \bar{B}_S^{I=0}+\bar{B}_{AS} & -\frac{\omega+\rho^0+f_1+a_1^0}{\sqrt{2}} & -(\rho^- + a_1^-) \\ \bar{B}_S^{I=0}-\bar{B}_{AS} & \sqrt{2}\bar{B}_S^{I=+1} & -(\rho^+ + a_1^+) & -\frac{\omega-\rho^0+f_1-a_1^0}{\sqrt{2}} \end{pmatrix}^{\mu}_{ij}$$

spin-1 mesons

$$\omega^{\mu} \sim \bar{\psi}\gamma^{\mu}\psi\ ,\quad f_1^{\mu} \sim \bar{\psi}\gamma_5\gamma^{\mu}\psi\ ,$$

$$\rho^{0,\mu} \sim \bar{\psi}\tau_f^3\gamma^{\mu}\psi\ ,\quad \rho^{\pm,\mu} \sim \frac{1}{\sqrt{2}}\bar{\psi}\tau_f^{\mp}\gamma^{\mu}\psi\ ,$$

$$a_1^{0,\mu} \sim \bar{\psi}\tau_f^3\gamma_5\gamma^{\mu}\psi\ ,\quad a_1^{\pm,\mu} \sim \frac{1}{\sqrt{2}}\bar{\psi}\tau_f^{\mp}\gamma_5\gamma^{\mu}\psi$$

spin-1 diquarks

$$B_S^{I=0,\mu} \sim -\frac{i}{\sqrt{2}}\psi^T C\gamma^{\mu}\tau_c^2\tau_f^1\psi$$

$$B_S^{I=\pm1,\mu} \sim -\frac{i}{2}\psi^T C\gamma^{\mu}\tau_c^2(\mathbf{1}_f \pm \tau_f^3)\psi\ ,$$

$$B_{AS}^{\mu} \sim -\frac{1}{\sqrt{2}}\psi^T C\gamma_5\gamma^{\mu}\tau_c^2\tau_f^2\psi$$

$$\bar{B}_S^{I=0,\mu} = (B_S^{I=0,\mu})^{\dagger}\ ,\quad \bar{B}_S^{I=\pm1,\mu} = (B_S^{I=\mp1,\mu})^{\dagger}$$

$$\bar{B}_{AS}^{\mu} = (B_{AS}^{\mu})^{\dagger}\ ,$$

Hadron	J^P	Quark number	Isospin
ω	1^-	0	0
ρ	1^-	0	1
f_1	1^+	0	0
a_1	1^+	0	1
B_S ($\bar{B}_S$)	1^+	+2 (−2)	1
B_{AS} ($\bar{B}_{AS}$)	1^-	+2 (−2)	0

4. LSM with spin-1

Suenaga-Murakami-Itou-Iida, in preparation

- **LSM with spin-1 hadrons**
  - Φ^μ transforms as $\Phi^\mu \to g\Phi^\mu g^\dagger \; [g \in SU(4)]$

cf, eLSM by Frankfurt group
↔ HLS, Harada-Nonaka-Yamaoka(2010)

$$
\begin{aligned}
\mathcal{L}_{\rm eLSM} &= {\rm tr}[D_\mu\Sigma^\dagger D^\mu\Sigma] - m_0^2{\rm tr}[\Sigma^\dagger\Sigma] - \lambda_1\left({\rm tr}[\Sigma^\dagger\Sigma]\right)^2 - \lambda_2{\rm tr}[(\Sigma^\dagger\Sigma)^2] + {\rm tr}[H^\dagger\Sigma + \Sigma^\dagger H] + c({\rm det}\Sigma + {\rm det}\Sigma^\dagger) \\
&- \frac{1}{2}{\rm tr}[\Phi_{\mu\nu}\Phi^{\mu\nu}] + m_1^2{\rm tr}[\Phi_\mu\Phi^\mu] + ig_3{\rm tr}\left[\Phi_{\mu\nu}[\Phi^\mu, \Phi^\nu]\right] + h_1{\rm tr}[\Sigma^\dagger\Sigma]{\rm tr}[\Phi_\mu\Phi^\mu] + h_2{\rm tr}[\Sigma\Sigma^\dagger\Phi_\mu\Phi^\mu] \\
&+ h_3{\rm tr}[\Phi_\mu^T\Sigma^\dagger\Phi^\mu\Sigma] + g_4{\rm tr}[\Phi_\mu\Phi_\nu\Phi^\mu\Phi^\nu] + g_5{\rm tr}[\Phi_\mu\Phi^\mu\Phi_\nu\Phi^\nu] + g_6{\rm tr}[\Phi_\mu\Phi^\mu]{\rm tr}[\Phi_\nu\Phi^\nu] + g_7{\rm tr}[\Phi_\mu\Phi_\nu]{\rm tr}[\Phi^\mu\Phi^\nu]
\end{aligned}
$$

$$
\left\{
\begin{aligned}
\Phi_{\mu\nu} &\equiv D_\mu\Phi_\nu - D_\nu\Phi_\mu \\
D_\mu\Sigma &\equiv \partial_\mu\Sigma - iG_\mu\Sigma - i\Sigma G_\mu^T - ig_1\Phi_\mu\Sigma - ig_2\Sigma\Phi_\mu^T \\
D_\mu\Phi_\nu &\equiv \partial_\mu\Phi_\nu - i[G_\mu, \Phi_\nu]
\end{aligned}
\right.
$$

and $G_\mu \to \mu_q\delta_{\mu 0}J \quad {\rm with} \quad J = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}$

4. LSM with spin-1

Suenaga-Murakami-Itou-Iida, in preparation

• LSM with spin-1 hadrons

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$$\begin{cases}\Phi_{\mu\nu} \equiv D_\mu\Phi_\nu - D_\nu\Phi_\mu \\ D_\mu\Sigma \equiv \partial_\mu\Sigma - iG_\mu\Sigma - i\Sigma G_\mu^T - ig_1\Phi_\mu\Sigma - ig_2\Sigma\Phi_\mu^T \\ D_\mu\Phi_\nu \equiv \partial_\mu\Phi_\nu - i[G_\mu, \Phi_\nu]\end{cases}$$

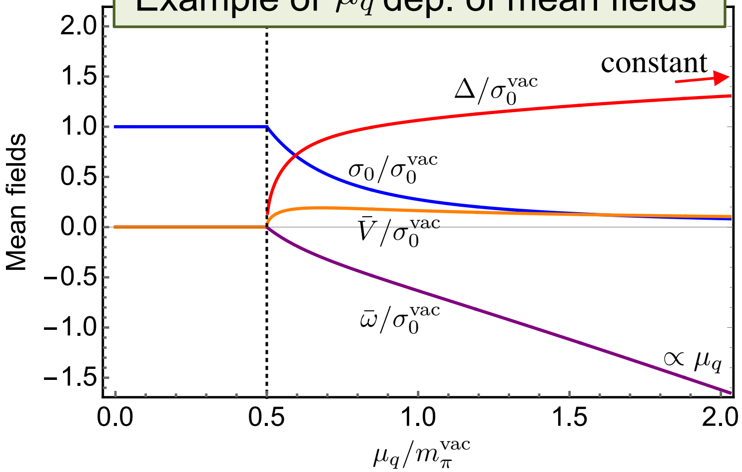
and $G_\mu \to \mu_q\delta_{\mu 0}J$ with $J = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1}\end{pmatrix}$

- There are four possible mean fields

$$\sigma_0 = \langle\sigma\rangle \qquad \bar{\omega} = \langle\omega^{\mu=0}\rangle$$

$$\Delta = \left\langle\frac{B + \bar{B}}{\sqrt{2}}\right\rangle \qquad \bar{V} = \left\langle\frac{\bar{B}_{AS}^{\mu=0} - B_{AS}^{\mu=0}}{\sqrt{2}i}\right\rangle$$

Example of μ_q dep. of mean fields



4. LSM with spin-1

Suenaga-Murakami-Itou-Iida, in preparation

• LSM with spin-1 hadrons

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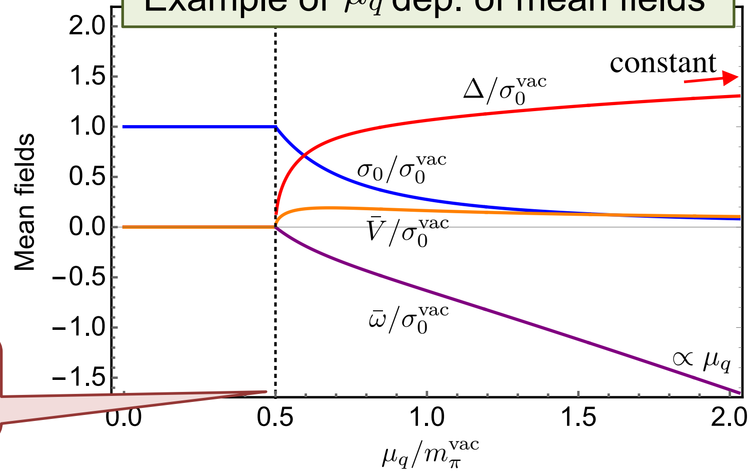
- There are four possible mean fields

$$\sigma_0 = \langle\sigma\rangle \qquad \bar\omega = \langle\omega^{\mu=0}\rangle$$

$$\Delta = \left\langle\frac{B+\bar B}{\sqrt2}\right\rangle \qquad \bar V = \left\langle\frac{\bar B_{AS}^{\mu=0} - B_{AS}^{\mu=0}}{\sqrt2 i}\right\rangle$$

Example of μ_q dep. of mean fields



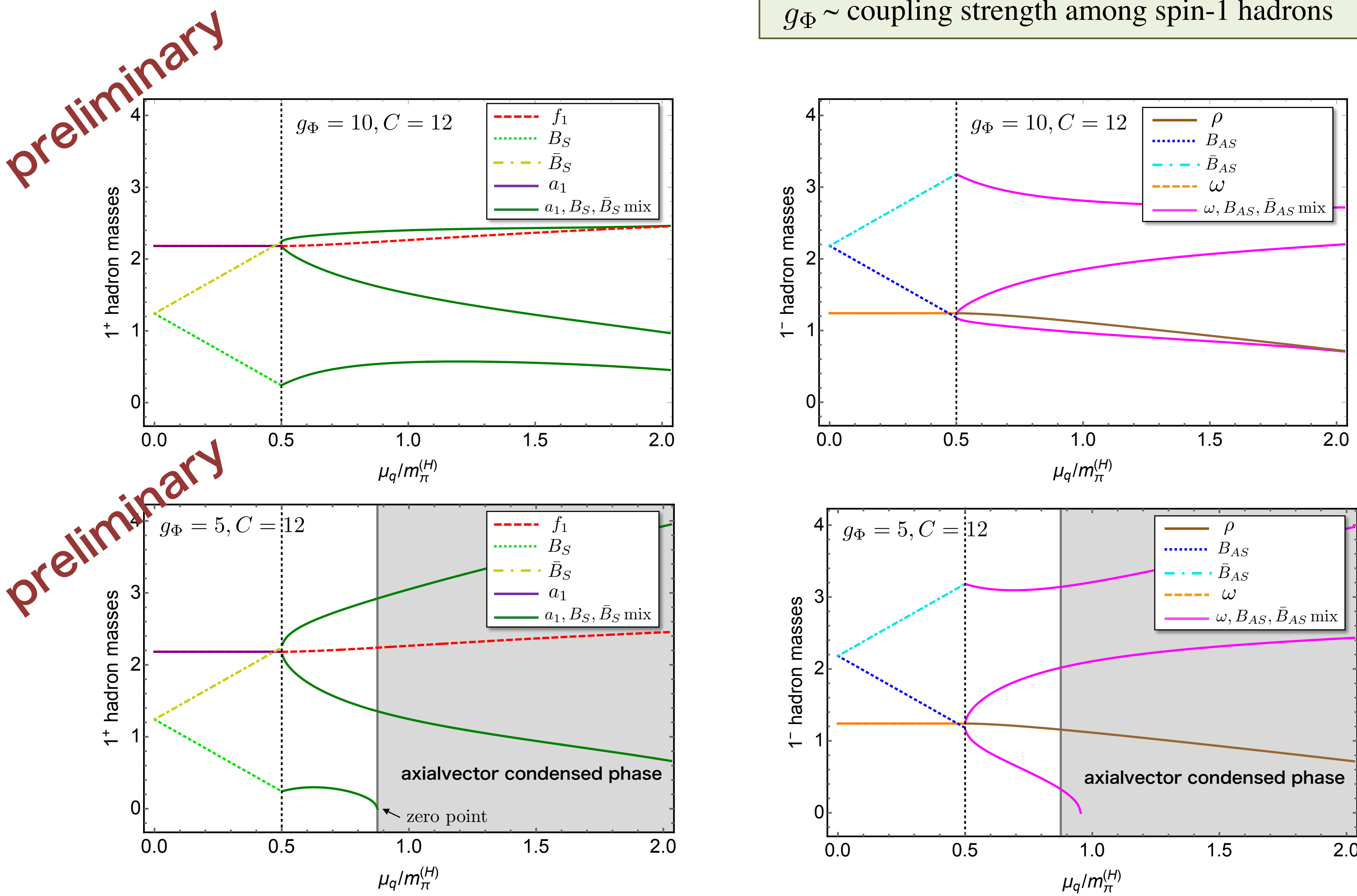
- Without $\bar\omega$ and $\bar V$, critical μ_q does not read $\frac{m_\pi^{\rm vac}}{2}$

4. LSM with spin-1

Suenaga-Murakami-Itou-Iida, in preparation

- **Spin-1 mass spectrum**

C ~ mixing strength between spin-0 and spin-1 hadrons
g_Φ ~ coupling strength among spin-1 hadrons



5. Conclusions

- I constructed Linear sigma model (LSM) in QC_2D and studied masses of spin-0 hadrons including parity partners at finite μ_q

⇩ comparison with lattice

Murakami-Suenaga-Iida-Itou, PoS(2022)

- The $U(1)_A$ was found to be possibly enhanced at large μ_q

↔ Need accurate information of f_π and m_η to evaluate the anomaly enhancement more quantitatively

cf, topological susceptibility, Kawaguchi-Suenaga, JHEP(2023)

<u>Ongoing project</u>

- Extension of LSM to study spin-1 mass spectrum at finite μ_q

→ importance of vector-diquark mean field, possible (axial)vector condensate, etc.

Suenaga et al, in preparation

- Examination of anomaly-effect enhancement with FRG

WIP with Fejos (Eotvos U.)