# Compositeness of near-threshold s-wave resonances

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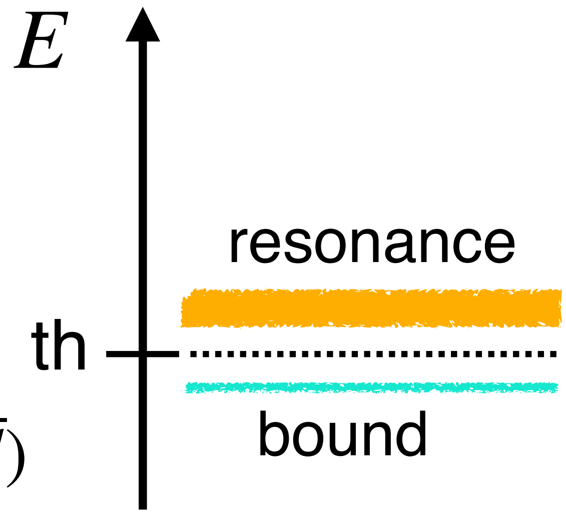
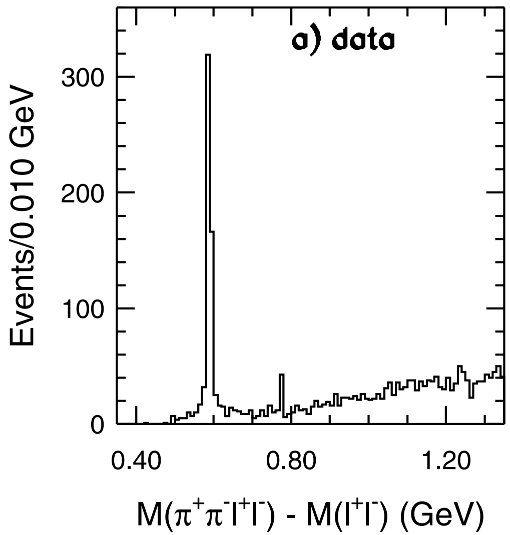
# Background

**Near-threshold** states are interesting!

- exotic hadrons

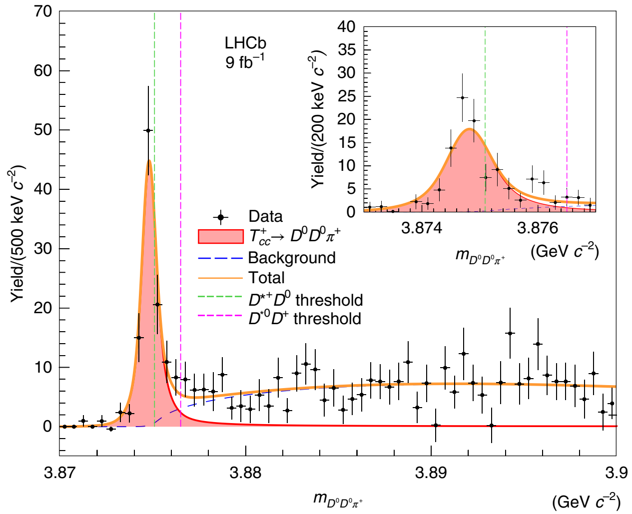



$X(3872) \to \pi^+\pi^- J/\psi$




S. K. Choi *et al*. (Belle), Phys. Rev. Lett. **91**, 262001 (2003).

$T_{cc} \to D^0 D^0 \pi^+ \ (cc\bar{u}\bar{d})$




LHCb Collaboration, Nature Phys. **18** (2022) no.7, 751-754;
LHCb Collaboration, Nat. Commun. **13**, 3351 (2022).

- low-energy universality: shallow **bound** states are molecular dominant

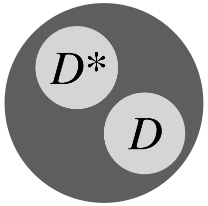
T. Hyodo, Phys. Rev. C **90**, 055208 (2014) ;
C. Hanhart, J. R. Pelaez, and G. Rios, Phys. Lett. B **739**, 375 (2014);
T. Kinugawa, T. Hyodo, 2303.07038 [hep-ph].

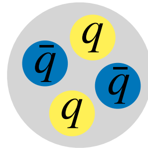
⟶ structure of **resonances** slightly above threshold?

# Compositeness & this work

## ◉ definition

hadron wavefunction




$$|\Psi\rangle = \sqrt{X}\,|\text{molecule}\rangle + \sqrt{Z}\,|\text{non molecule}\rangle$$

**compositeness** **elementarity**

T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013).

$X > 0.5 \Leftrightarrow$ **composite dominant**

$Z > 0.5 \Leftrightarrow$ **elementary dominant**

- **quantitative** analysis of internal structure of bound states

## ◉ this work

internal structure of near-threshold $s$-wave resonances?

1. universality of resonances with effective range expansion
2. interpretation of complex compositeness
3. quantitative examination with compositeness

# Near-th. resonances in ERE



◉ resonance pole written by effective range expansion (ERE)

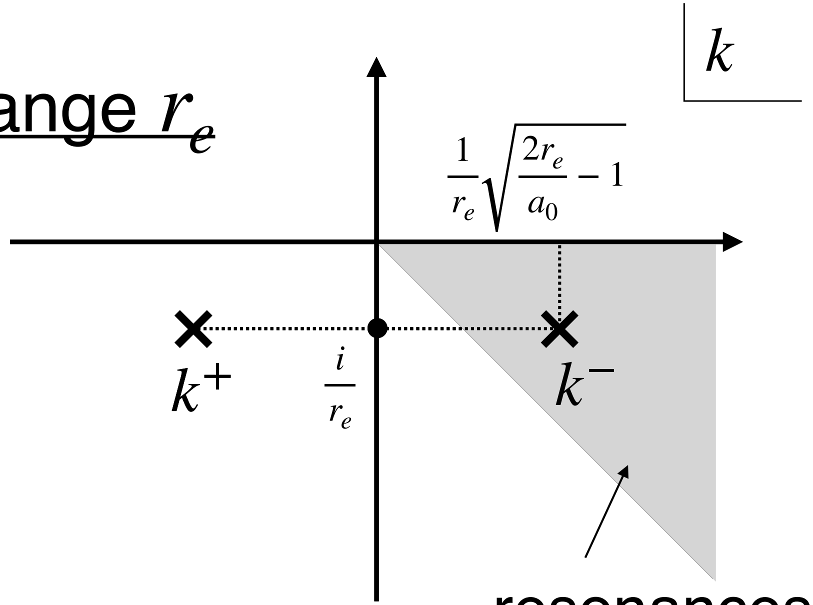
$$f(k)^{-1} = -\frac{1}{a_0} + \frac{r_e}{2}k^2 - ik \longrightarrow k^{\pm} = \frac{i}{r_e} \pm \frac{1}{r_e}\sqrt{\frac{2r_e}{a_0} - 1}$$

T. Hyodo, Phys. Rev. Lett. **111**, 132002 (2013).

- pole position determines $a_0$ and $r_e$

◉ scattering length $a_0$ and effective range $r_e$

- $r_e$ should be negative to obtain resonances
- $i/r_e$ should be small to obtain near-threshold poles (narrow width)




⟶ Effective range should be **large and negative** for near-threshold resonances

# Universality for near-th. resonances



- near-threshold **bound** (and virtual) states

$a_0 \to \infty$ and universality holds in $B \to 0$ limit ○

→ $X \to 1$ (completely composite)

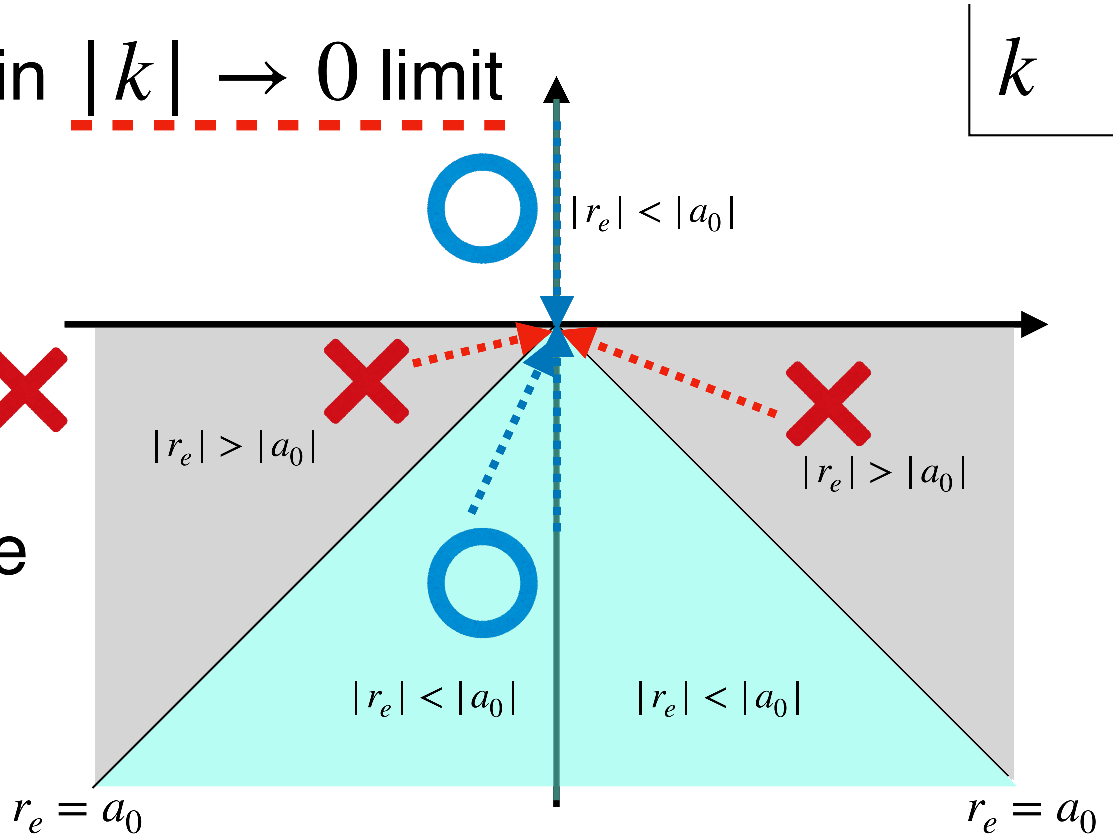
T. Hyodo, Phys. Rev. C **90**, 055208 (2014) ;
T. Kinugawa, T. Hyodo, 2303.07038 [hep-ph].

- near-threshold **resonances**

$a_0 \to \infty$ but also $|r_e| \to \infty$ in $|k| \to 0$ limit

$\because |a_0| \leq |r_e|$

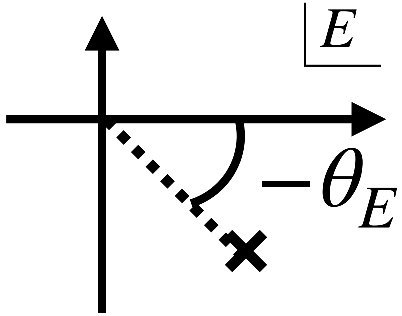
→ universality does not hold ✕




Near-threshold resonances are **not** necessarily composite dominant

# Compositeness in ERE

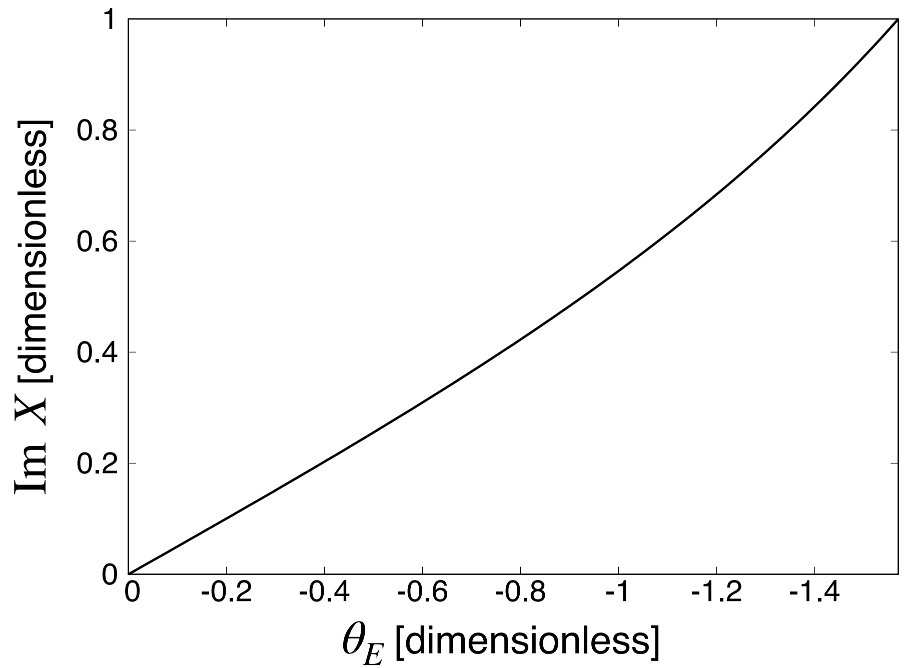
$$X = \sqrt{\frac{1}{1-\frac{2r_e}{a_0}}} = -i\tan\theta_k = -i\tan(\theta_E/2)$$




$$(k = |k|e^{i\theta_k}, E = |E|e^{i\theta_E})$$

→ $X$ in ERE is pure imaginary

Im $X$ as a function of $\theta_E$

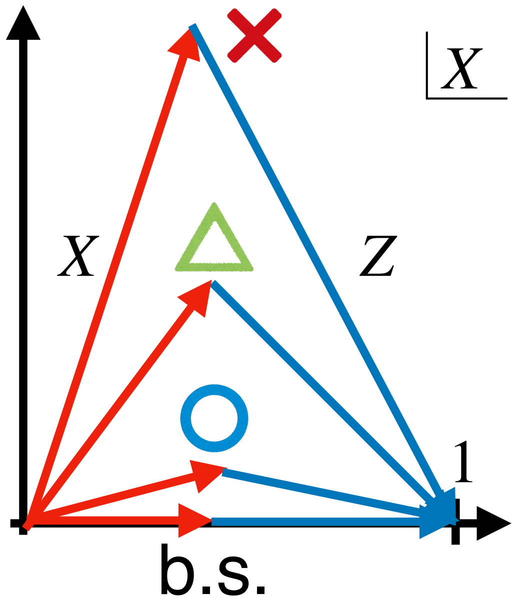



- in general, compositeness $X$ of unstable resonances becomes **complex** by definition

- complex $X$ **cannot** be directly interpreted as a probability

# Complex compositeness






- probabilistic interpretation?

$X \in \mathbb{C}$ and $X + Z = 1$

- proposals of probabilistic interpretation

$X \in \mathbb{C} \longrightarrow \tilde{X} \in \mathbb{R}$

$$\tilde{X}_{\rm KH} = \frac{1 - |Z| + |X|}{2}$$

Y. Kamiya and T. Hyodo, Phys. Rev. C **93**, 035203 (2016).

$$\tilde{X} = \frac{|X|}{|X| + |1 - X|}$$

T. Sekihara, T. Arai, J. Yamagata-Sekihara and S. Yasui, PRC **93**, 035204 (2016).

$0 \leq \tilde{X} \leq 1$ & $\tilde{X} \to X$ in bound state

- If Im $X$ is large, it seems that reasonable interpretation is impossible ✕ △

⟶ We propose new interpretation and quantitatively discuss nature of resonances

# New interpretation

- Berggren's idea T. Berggren, Phys. Lett. B 33, 547 (1970).

transition process which contains resonance

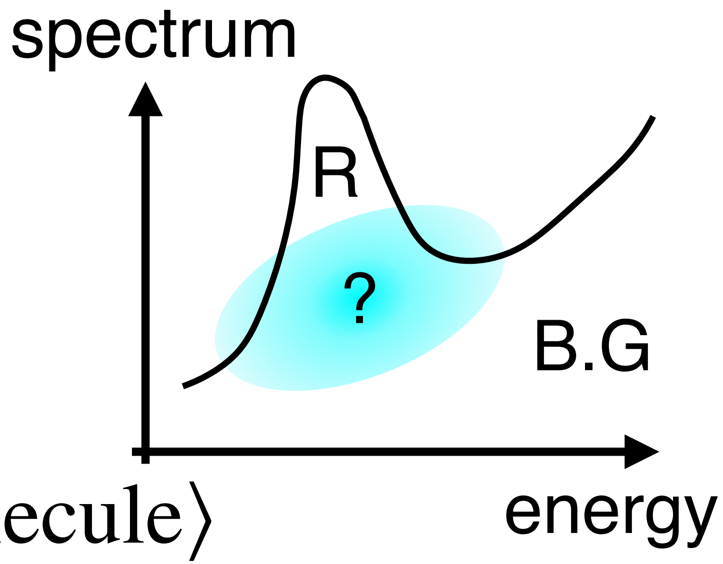
i) practically certain identification as $|\mathrm{resonance}\rangle$

ii) practically certain identification as not $|\mathrm{resonance}\rangle$

iii) uncertain $|\mathrm{resonance}\rangle$ or not

uncertain appears from

- finite lifetime (uncertainty in energy)
- separation from B.G.




- compositeness of resonance

i) $\mathcal{X}$: probability to certainly find $|\mathrm{molecule}\rangle$

ii) $\mathcal{Z}$: probability to certainly find $|\mathrm{not\ morecule}\rangle$

iii) $\mathcal{Y}$: probability of uncertain identification

complex compositeness $X \in \mathbb{C} \longrightarrow \mathcal{X}, \mathcal{Y}, \mathcal{Z}$

# Definition



## ◉ conditions for sensible interpretation

- normalization : $\mathcal{X} + \mathcal{Y} + \mathcal{Z} = 1$ for probabilistic interpretation
- in bound state limit : $\mathcal{X} \to X$, $\mathcal{Z} \to Z$ and $\mathcal{Y} \to 0$

$\mathcal{Y}$ represents property of resonance ◂······▸ distance from b.s.

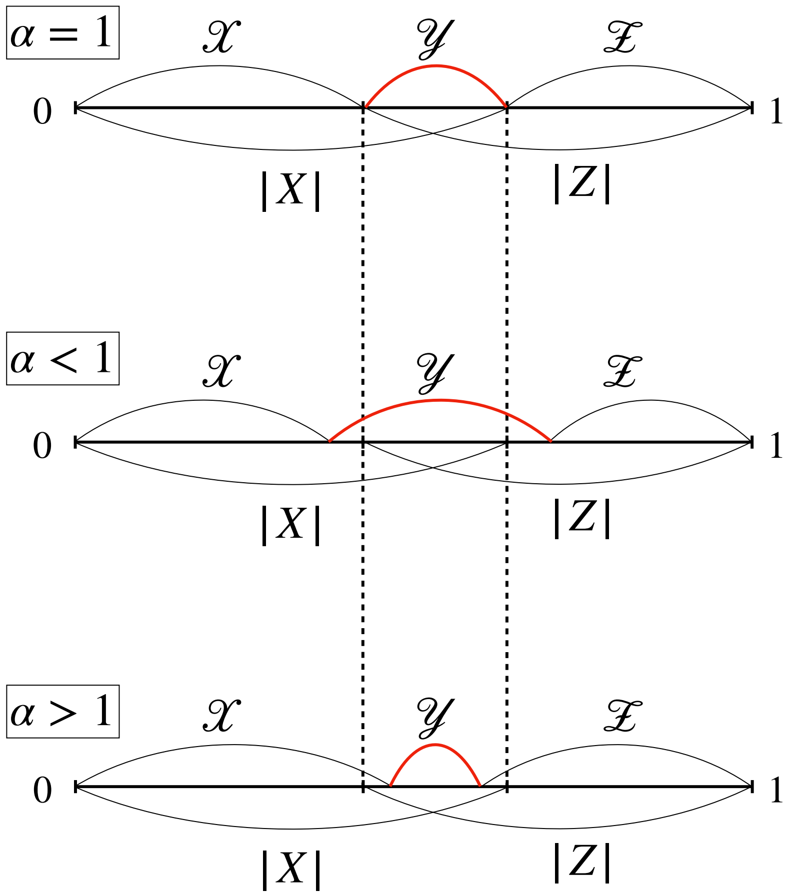
## ◉ new interpretation

$$\mathcal{X} + \alpha\mathcal{Y} = |X|, \quad \mathcal{Z} + \alpha\mathcal{Y} = |Z|$$

$$\mathcal{X} = \frac{(\alpha - 1)|X| - \alpha|Z| + \alpha}{2\alpha - 1}$$

$$\mathcal{Z} = \frac{(\alpha - 1)|Z| - \alpha|X| + \alpha}{2\alpha - 1}$$

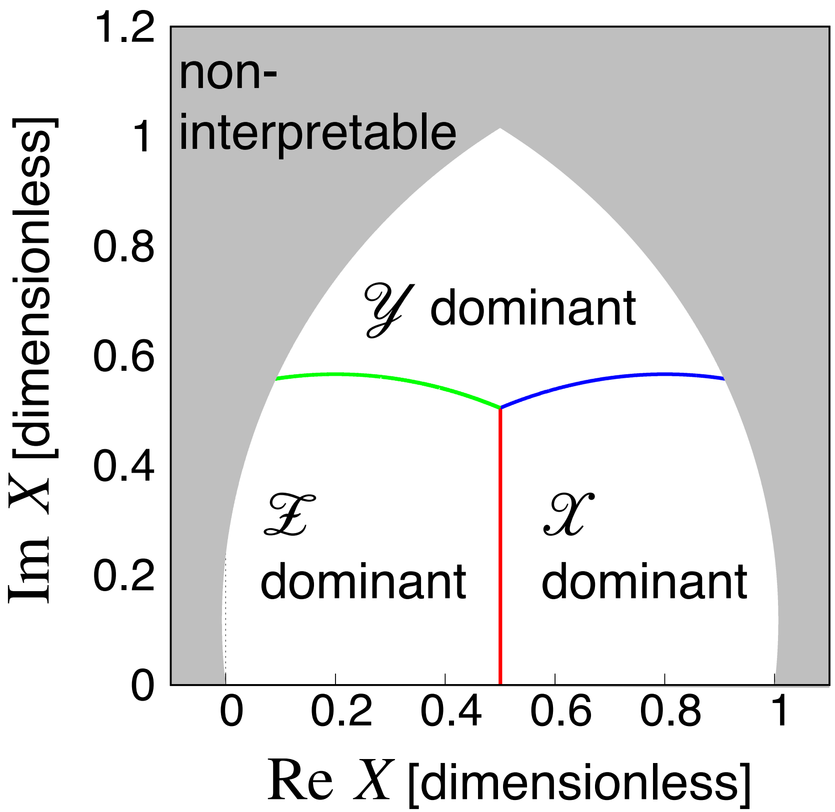
$$\mathcal{Y} = \frac{|X| + |Z| - 1}{2\alpha - 1}$$



# Definition

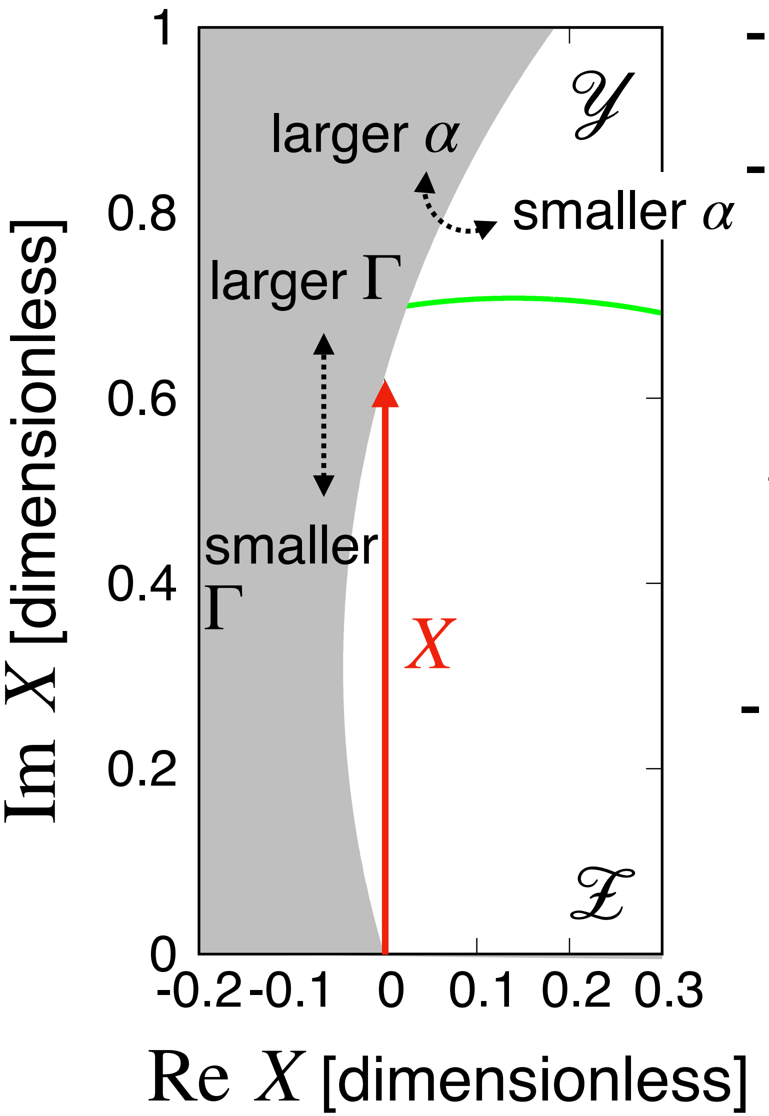
- if $\alpha > 1/2$, $\mathscr{Y}$ is always positive but $\mathscr{X}$, $\mathscr{Z}$ can be negative

| | | |
|---|---|---|
| $\mathcal{X} \geq 0$ and $\mathcal{Z} \geq 0$ | $\mathcal{X} > \mathcal{Y}, \mathcal{Z}$ | composite dominant |
| | $\mathcal{Z} > \mathcal{Y}, \mathcal{X}$ | elementary dominant |
| | $\mathcal{Y} > \mathcal{X}, \mathcal{Z}$ | uncertain |
| $\mathcal{X} < 0$ or $\mathcal{Z} < 0$ | | non-interpretable |




$\mathscr{X}$, $\mathscr{Y}$, $\mathscr{Z}$ dominant regions
and
non-interpretable region

# Choice of $\alpha$ with ERE




- interpretable region depends on $\alpha$
- exclude poles which we cannot regard as physical "state" from probabilistic interpretation

our criterion for "state"

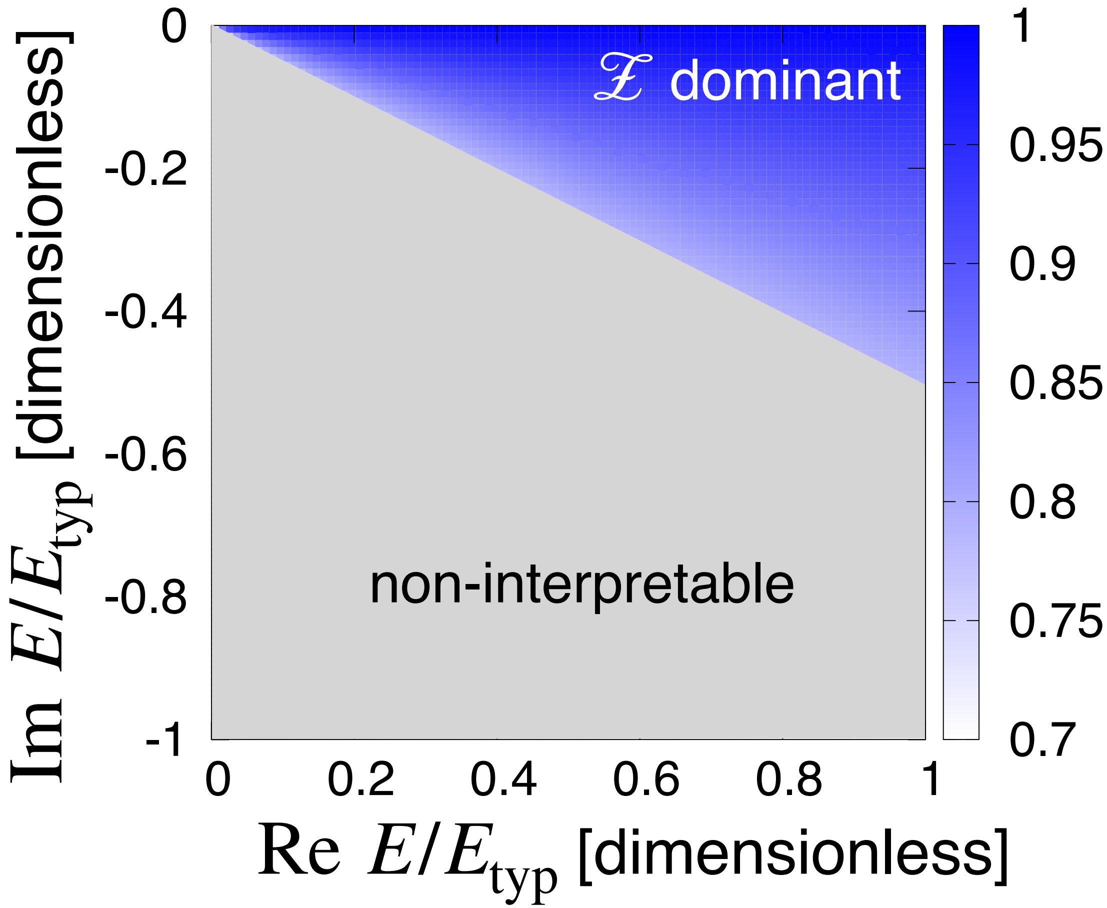
$$\mathrm{Re}\ E \geq \Gamma = -2\mathrm{Im}\ E$$

- use ERE (model independent framework): $X$ is pure imaginary

→ if $\mathrm{Re}\ E < \Gamma$, pole is non-interpretable with $\alpha \sim 1.1318$

# Structure of near-th. resonances






- resonances are **not composite dominant state** ($\mathcal{Z} \gtrsim 0.8$)
- different from near-threshold bound states (composite dominant $X \sim 1$ and $Z \sim 0$)

# Summary

- near-threshold $s$-wave resonances ⟵ ERE
- new interpretation of complex compositeness and elementarity

uncertain and non-interpretable states ⟵ new!

$$\mathscr{X} = \frac{(\alpha - 1)|X| - \alpha|Z| + \alpha}{2\alpha - 1}$$

$$\mathscr{Z} = \frac{(\alpha - 1)|Z| - \alpha|X| + \alpha}{2\alpha - 1}$$

$$\mathscr{Y} = \frac{|X| + |Z| - 1}{2\alpha - 1}$$

| | | |
|---|---|---|
| | $\mathcal{X} > \mathcal{Y}, \mathcal{Z}$ | composite dominant |
| $\mathcal{X} \geq 0$ and $\mathcal{Z} \geq 0$ | $\mathcal{Z} > \mathcal{Y}, \mathcal{X}$ | elementary dominant |
| | $\mathcal{Y} > \mathcal{X}, \mathcal{Z}$ | uncertain |
| $\mathcal{X} < 0$ or $\mathcal{Z} < 0$ | | non-interpretable |

near-threshold resonances are **not composite dominant**

qualitatively different from near-threshold bound states

# Back up

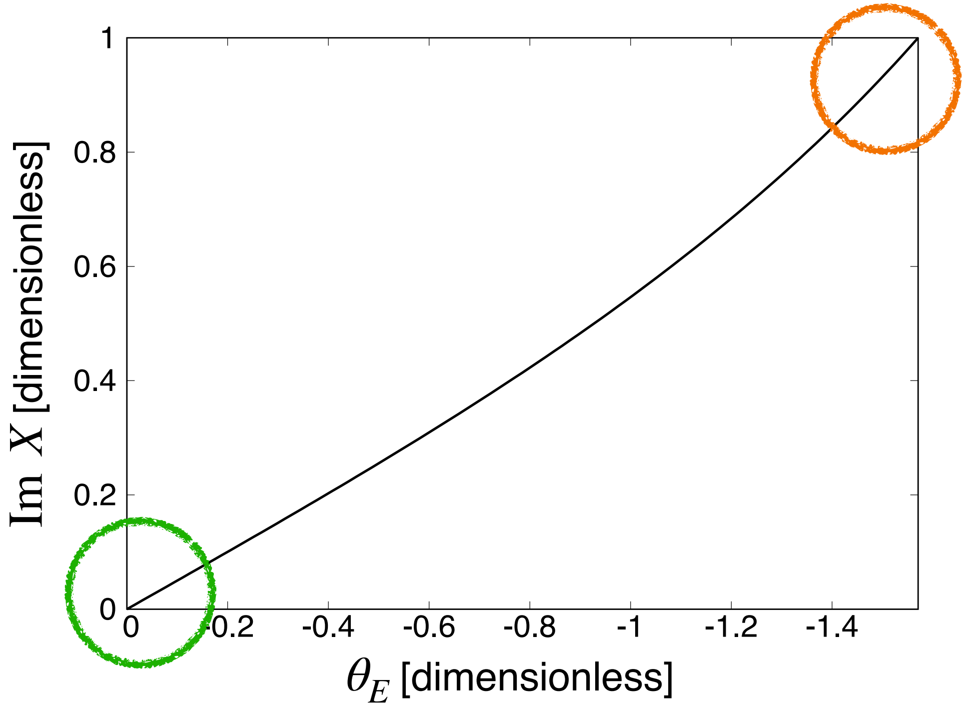
# Compositeness in ERE

$$X = \sqrt{\frac{1}{1 - \frac{2r_e}{a_0}}} = -i \tan \theta_k$$

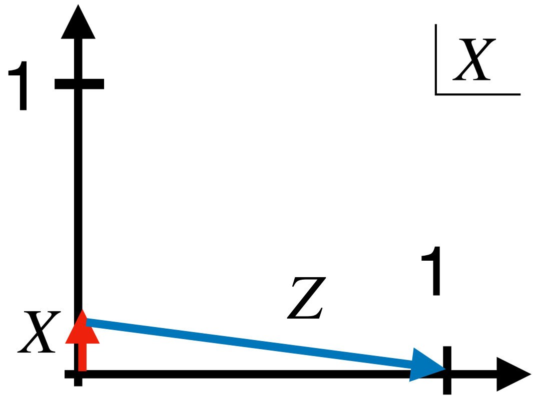
$$(k = |k| e^{i\theta_k})$$

→ $X$ in ERE is pure imaginary

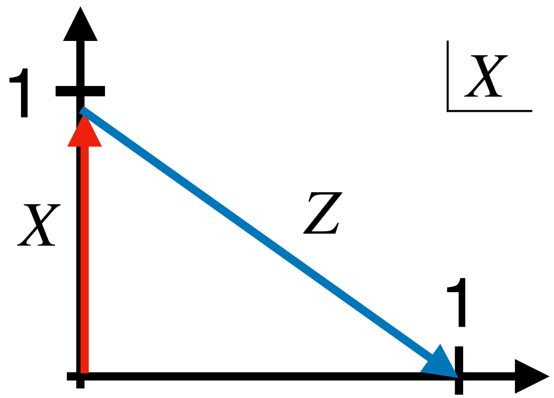
$X$ as a function of $\theta_E$ $(E = |E| e^{i\theta_E})$




small width $(\theta_E \sim 0)$




large width $(\theta_E \sim -\pi/2)$



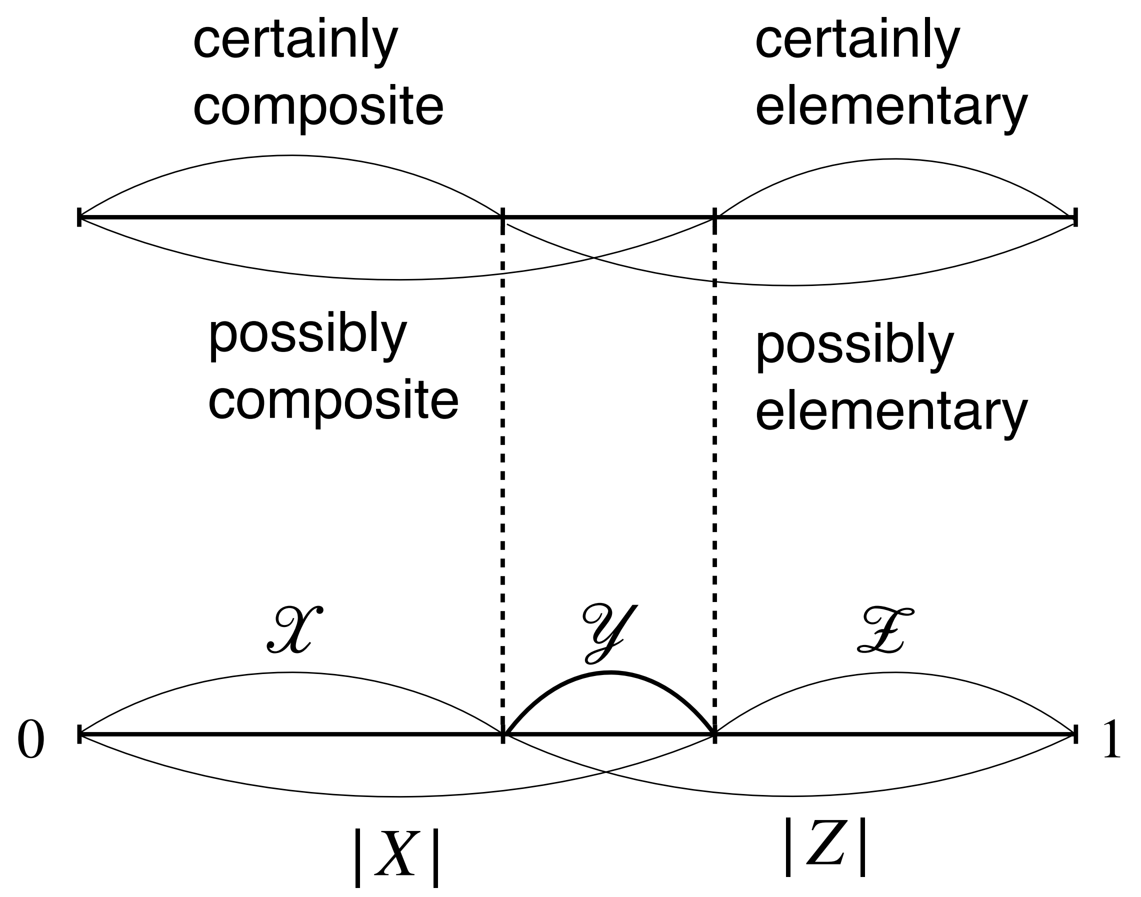
# Definition

◉ new interpretation of complex compositeness & elementarity

⟶ from Berggren's idea T. Berggren, Phys. Lett. B 33, 547 (1970).

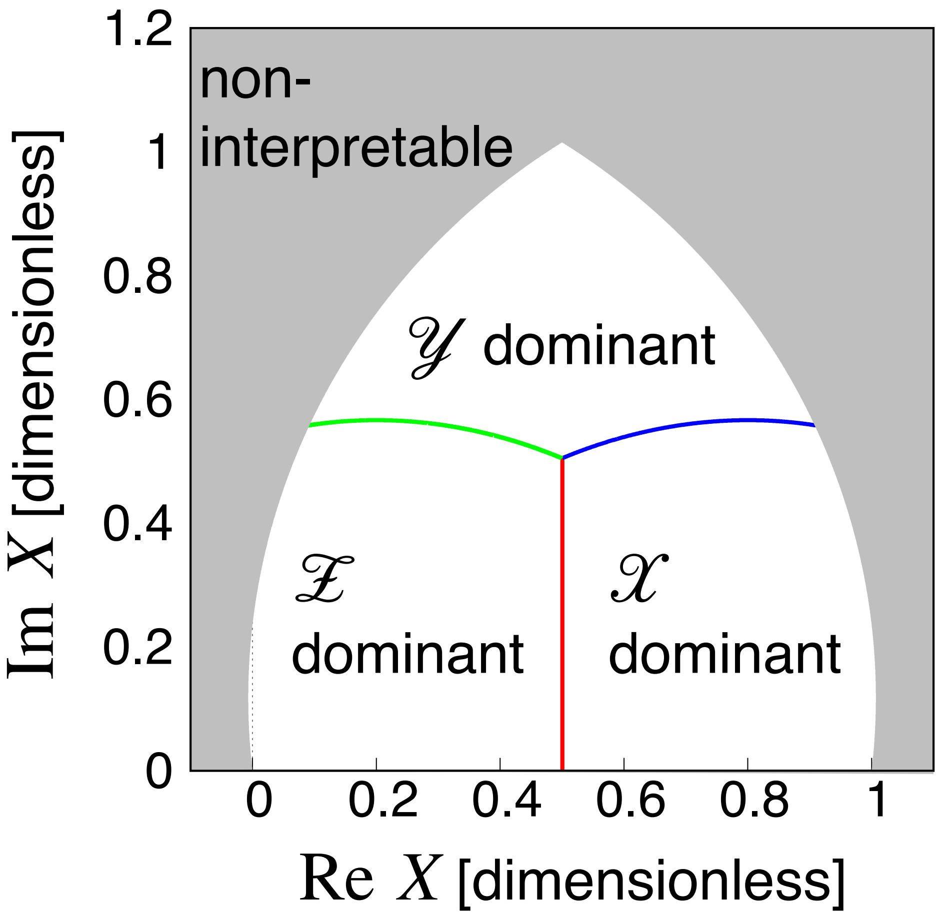
$$\mathscr{X} + \mathscr{Y} = |X| \text{ \& } \mathscr{Z} + \mathscr{Y} = |Z|$$

$$\mathscr{X} = 1 - |Z|$$
$$\mathscr{Z} = 1 - |X|$$
$$\mathscr{Y} = |X| + |Z| - 1$$

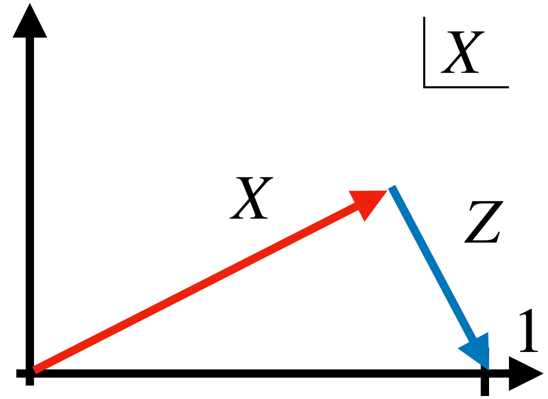


# $\mathscr{X}, \mathscr{Y}, \mathscr{Z}$ dominant region

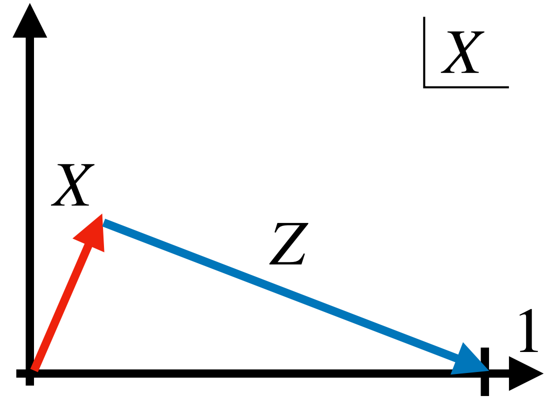
- $\mathscr{X}, \mathscr{Y}, \mathscr{Z}$ dominant region in complex $X$ plane



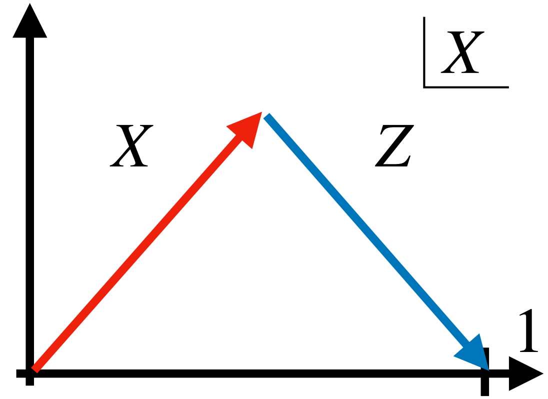

composite dominant




elementary dominant

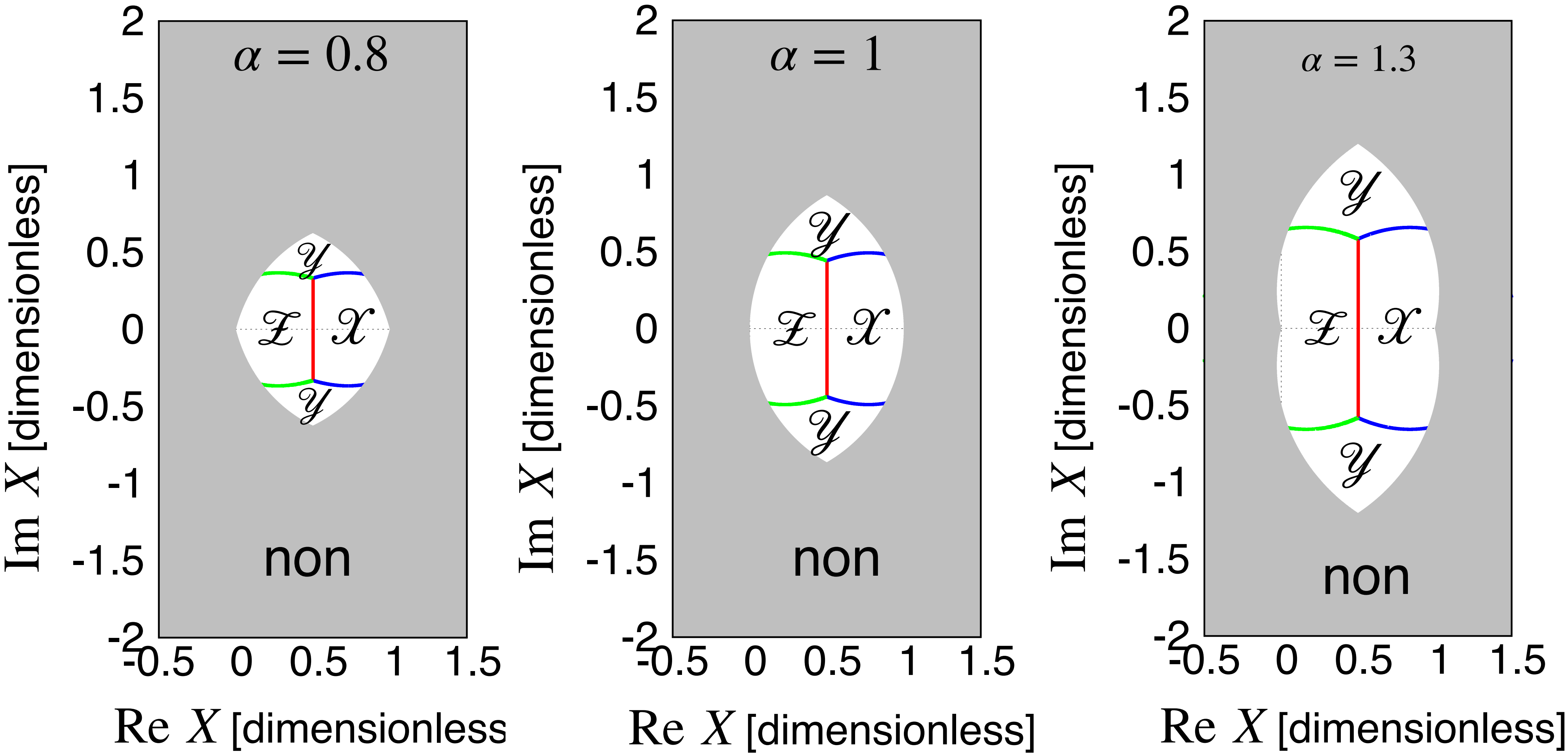



uncertain




We can exclude non-interpretable cases.

# $\mathscr{X}, \mathscr{Y}, \mathscr{Z}$ dominant region




- Interpretable regions become large with increase of $\alpha$

$\alpha \to \infty \longrightarrow \mathscr{X}, \mathscr{Y}, \mathscr{Z}$ reduce to interpretation in previous work

$\mathscr{X} \to \tilde{\mathrm{X}}, \mathscr{Z} \to \tilde{\mathrm{Z}}, \mathscr{Y} \to 0$

Y. Kamiya and T. Hyodo, Phys. Rev. C **93**, 035203 (2016).

# Comparison with previous works

$$\bar{Z} = 1 - \sqrt{\left|\frac{1}{1-2r_e/a_0}\right|}$$

T. Hyodo, Phys. Rev. Lett. **111**, 132002 (2013).

$$\tilde{Z}_{\rm KH} = \frac{1+|Z|-|X|}{2}$$

Y. Kamiya and T. Hyodo, Phys. Rev. C **93**, 035203 (2016).

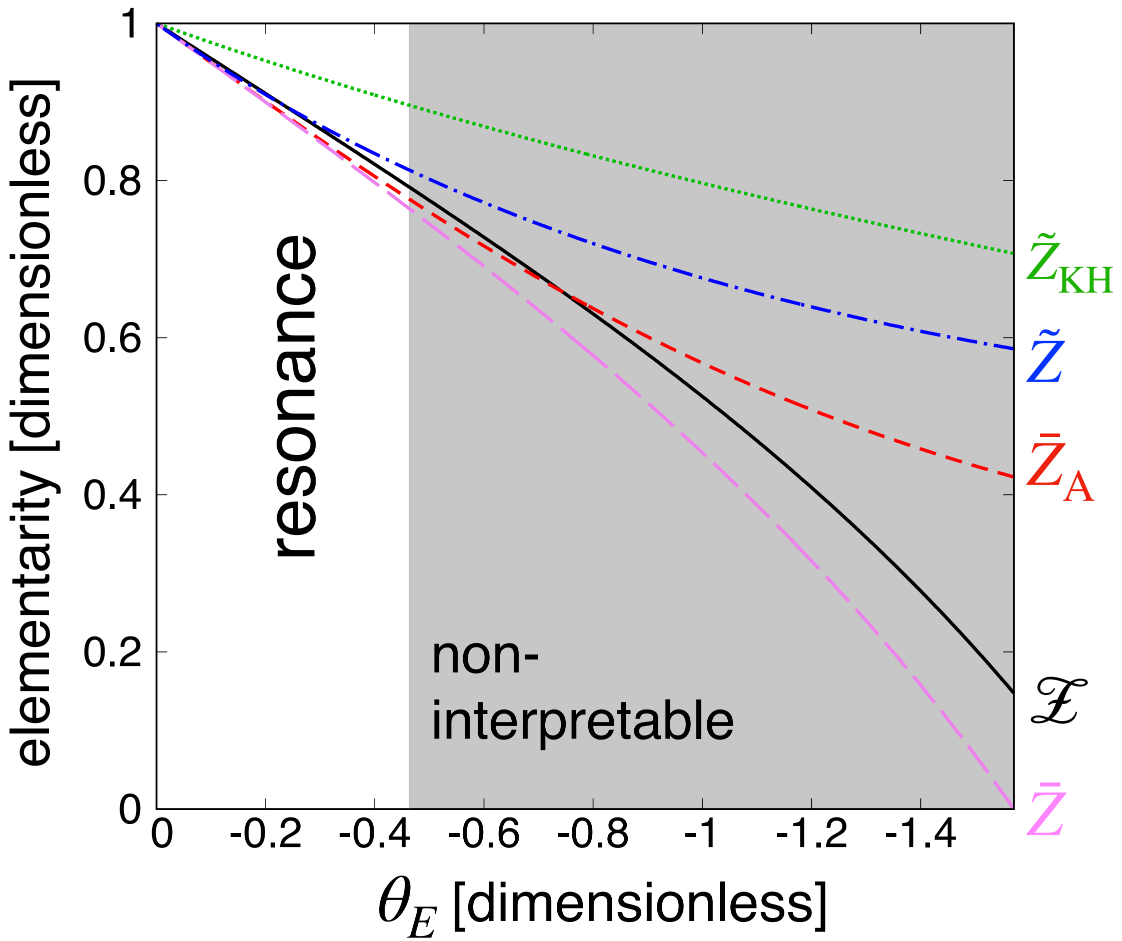
$$\tilde{Z} = \frac{|Z|}{|X|+|Z|}$$

T. Sekihara, T. Arai, J. Yamagata-Sekihara and S. Yasui, PRC **93**, 035204 (2016).

$$\bar{Z}_A = 1 - \sqrt{\frac{1}{1+|2r_e/a_0|}}$$

I. Matuschek, V. Baru, F.-K. Guo, and C. Hanhart, Eur. Phys. J. A **57**, 101 (2021).

interpretations as a function of $\theta_E$




- all interpretations show resonances are elementary dominant