Near-threshold hadron scattering using new parametrization of amplitude

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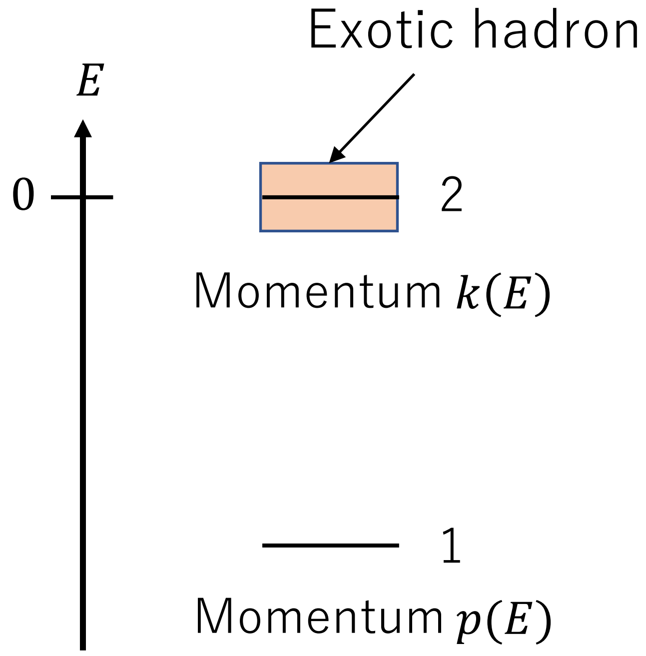
Background

Exotic hadrons ⇨ $T_{cc}, X(3872), f_0(980), a_0, P_c, Z_c$

Internal structure ⇔ Scattering length a

For near-threshold exotic hadrons,
channel couplings are important.



Unstable exotic hadron near the threshold of channel 2

⇨ Flatté amplitude has been used[1].

Scattering length a_F has been determined by the Flatté amplitude[2].

We discuss the behavior of cross section near the threshold in terms of a.

[1] R.Aaij et al. [LHCb], Phys. Rev. D 102, no.9, 092005 (2020)

[2] V. Baru et al. Eur. Phys. J. A, 23, 523-533 (2005)

General form : EFT amplitude

The general form of the scattering amplitude is derived from the optical theorem.

$$f^{-1} = \begin{pmatrix} M_{11}(E) - ip(E) & M_{12}(E) \\ M_{12}(E) & M_{22}(E) - ik(E) \end{pmatrix}$$

One of the general solutions of the above equation derived from EFT.

EFT amplitude[3] up to first order of k.

$$f^{EFT} = \left\{ \frac{1}{a_{12}^2} - \left(\frac{1}{a_{22}} + ik\right)\left(\frac{1}{a_{11}} + ip_0\right) \right\}^{-1} \begin{pmatrix} \left(\frac{1}{a_{22}} + ik\right) & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & \left(\frac{1}{a_{11}} + ip_0\right) \end{pmatrix}$$

The EFT amplitude has three parameters a_{11}, a_{12}, a_{22} near the threshold.

[3]T.D.Cohen et al. , Phys. Lett. B 588 (2004) 57-66

Flatté amplitude

The Flatté amplitude for two channel case

$$f^F = h(E)\begin{pmatrix} g_1^2 & g_1 g_2 \\ g_1 g_2 & g_2^2 \end{pmatrix}$$

The Flatté parameters

g_1, g_2 : Real coupling constants

E_{BW} : Bare energy

The Flatté amplitude satisfies the optical theorem with channel couplings.

The Flatté amplitude has the threshold effect.

$$h(E) = -\frac{1}{2}\frac{1}{E - E_{BW} + i\, g_1^2 p(E)/2 + i g_2^2\, k(E)/2}$$

f_{11}^F, f_{22}^F can be written as the effective range expansion in k.

$$f_{11}^F, f_{22}^F \propto \left(-\frac{1}{a_F} + \frac{1}{2} r_F k^2 - ik + O(k^4)\right)^{-1}$$

a_F : Scattering length

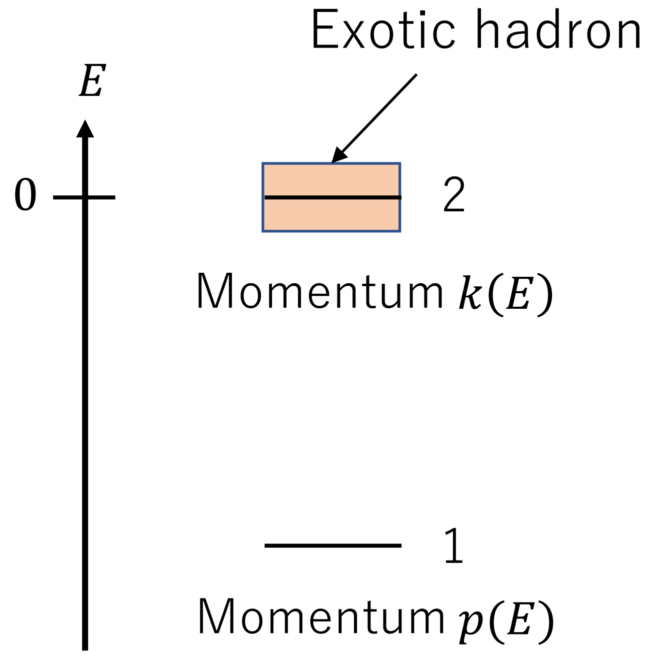
r_F : Effective range

Problem of Flatté amplitude

$1/f_{11}^F$ up to order k^1 can be written only by two parameters R, α[2].

$$f_{11}^F = \frac{g_1^2}{2E_{BW} - ig_1^2 p_0 - ig_2^2 k} = \frac{1/R}{\alpha p_0/R - ip_0/R - ik}$$

$$\alpha = \frac{2E_{BW}}{g_1^2 p_0} \qquad R = \frac{g_2^2}{g_1^2}$$



We find $1/f_{22}^F$ up to k^1 can also be written only by two parameters R, α.

$$f_{22}^F = \frac{g_2^2}{2E_{BW} - ig_1^2 p_0 - ig_2^2 k} = \frac{1}{\alpha p_0/R - ip_0/R - ik}$$

p_0 : channel 1 momentum at $E = 0$

$f^F(g_1^2, g_2^2, E_{BW})$ three parameters ⇒ $f^F(R, \alpha)$ two parameters(near the threshold)

⇒ **Some constraint are imposed to the Flatté amplitude near the threshold.**

[2] V. Baru et al. Eur. Phys. J. A, 23, 523-533 (2005)

Comparison

What is the difference between the EFT and Flatté ?

$\Rightarrow$ **Inverse amplitude**

EFT

$$(f^{EFT})^{-1} = \begin{pmatrix} -\frac{1}{a_{11}} - ip_0 & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & -\frac{1}{a_{22}} - ik \end{pmatrix}$$

Flatté

$$f^F = h(E)\begin{pmatrix} g_1^2 & g_1 g_2 \\ g_1 g_2 & g_2^2 \end{pmatrix}$$

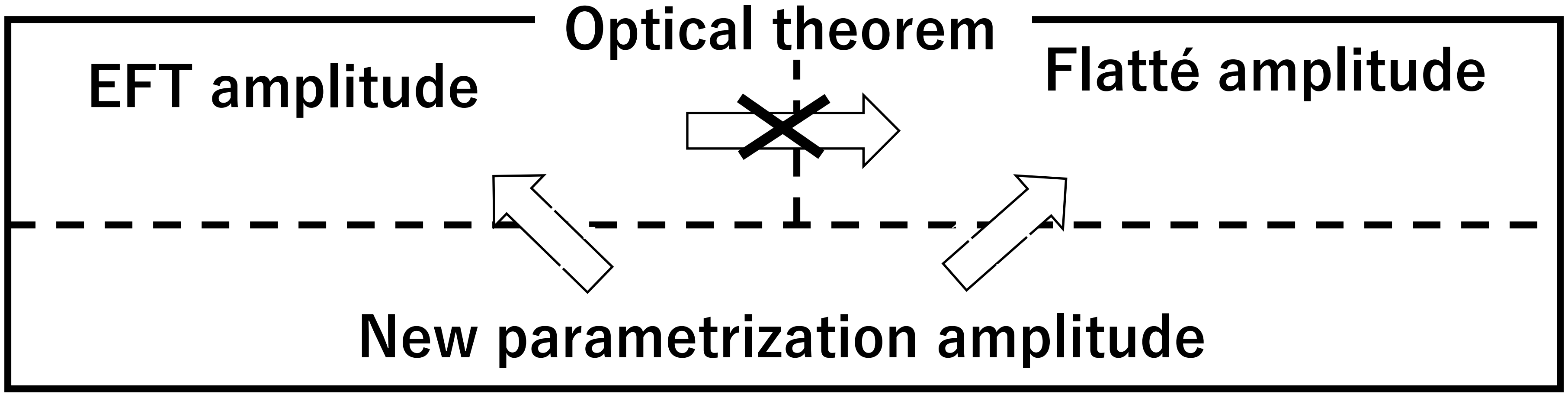
$\Rightarrow$ $(f^F)^{-1} =$ does not exist

unitarity relation

$\boldsymbol{f^{EFT}(a_{11}, a_{12}, a_{22})}$	$\boldsymbol{f^{F}(\alpha, R)}$
・**Inverse matrix exists**	・**Inverse matrix does not exist**

EFT amplitude does not reduce to Flatté amplitude directly

New parametrization amplitude



We construct the new representation including EFT and Flatté.

⇨ **General amplitude $f^{G}(A_{22},\gamma,\epsilon)$**

$$(f^{EFT})^{-1}=\begin{pmatrix}-\dfrac{1}{a_{11}}-ip_0 & \dfrac{1}{a_{12}}\\ \dfrac{1}{a_{12}} & -\dfrac{1}{a_{22}}-ik\end{pmatrix} \quad \Rightarrow \quad (f^{G})^{-1}=\begin{pmatrix}-\dfrac{1}{A_{22}}\dfrac{1}{\gamma}-ip_0 & \dfrac{1}{A_{22}}\dfrac{\sqrt{\epsilon-\gamma}}{\gamma}\\ \dfrac{1}{A_{22}}\dfrac{\sqrt{\epsilon-\gamma}}{\gamma} & -\dfrac{1}{A_{22}}\dfrac{\epsilon}{\gamma}-ik\end{pmatrix}$$

A_{22} : scattering length of channel two in the absence of channel couplings

Property

$$f^{G}(A_{22},\gamma,\epsilon) \xRightarrow{\gamma \neq 0} f^{EFT}(a_{11},a_{12},a_{22})$$

EFT amplitude

$$f^{G}(A_{22},\gamma,\epsilon) \xRightarrow{\gamma = 0} f^{G}(A_{22},0,\epsilon) = \frac{1}{-\frac{1}{A_{22}} - i\epsilon p_0 - ik}\begin{pmatrix} \epsilon & \sqrt{\epsilon} \\ \sqrt{\epsilon} & 1 \end{pmatrix}$$

Flatté form

$$(f^{G})^{-1}(A_{22},\gamma,\epsilon) = \begin{pmatrix} -\frac{1}{A_{22}}\frac{1}{\gamma} - ip_0 & \frac{1}{A_{22}}\frac{\sqrt{\epsilon-\gamma}}{\gamma} \\ \frac{1}{A_{22}}\frac{\sqrt{\epsilon-\gamma}}{\gamma} & -\frac{1}{A_{22}}\frac{\epsilon}{\gamma} - ik \end{pmatrix} \xRightarrow{\gamma = 0} (f^{G})^{-1} = \text{does not exist}$$

Optical theorem

EFT amplitude $\boldsymbol{f^{EFT}(a_{11},a_{12},a_{22})}$ $\not\Rightarrow$ **Flatté amplitude** $\boldsymbol{f^{F}(\alpha,R)}$

$\gamma \neq 0$ $\qquad$ $\gamma = 0$

General amplitude $\boldsymbol{f^{G}(A_{22},\gamma,\epsilon)}$

The scattering length

The scattering length a is obtained from the effective range expansion

$$f_{22}(k) = \frac{1}{-\frac{1}{a} + \frac{r}{2}k^2 + O(k^4) - ik}$$

a : the scattering length

r : the effective range

・Flatté scattering length a_F

$$a_F = \frac{1}{\frac{1}{A_{22}} + i\epsilon p_0}$$

・General scattering length a_G

$$a_G = A_{22}\left(\frac{\frac{1}{A_{22}} + i\gamma p_0}{\frac{1}{A_{22}} + i\epsilon p_0}\right)$$

When a pole is near the threshold, the pole position is related to a

Pole position $k \sim i/a$

Comparison of the cross section

We study the behavior of the scattering cross section near the threshold when the scattering length is stable.

$$\sigma_{ij} = \frac{p_j}{p_i}\int f f^* d\Omega = 4\pi \frac{p_j}{p_i}\left|f_{ij}\right|^2$$

We focus on σ_{11} and σ_{21}

The Flatté amplitude up to first order of k.

$$f_{21}^F = \frac{1/\sqrt{R}}{\alpha p_0/R - i p_0/R - ik} \propto \frac{1}{-\frac{1}{a_F} - ik} \qquad f_{11}^F = \frac{1/R}{\alpha p_0/R - i p_0/R - ik} \propto \frac{1}{-\frac{1}{a_F} - ik}$$

$$\Rightarrow \quad \sigma_{21}^F, \sigma_{11}^F \propto \left|\frac{1}{-\frac{1}{a_F} - ik}\right|^2$$

The Flatté cross sections near the threshold $\boldsymbol{\sigma_{21}^F, \sigma_{11}^F}$ are determined only by $\boldsymbol{a_F}$.

Comparison of the cross section

The General amplitude up to first order of k.

$$f_{21}^{G} = \frac{C_{21}^{G}}{-\frac{1}{A_{22}}\left(\frac{\frac{1}{A_{22}}+i\epsilon p_0}{\frac{1}{A_{22}}+i\gamma p_0}\right)-ik} \propto \frac{1}{-\frac{1}{a_G}-ik}$$

C_{21}^{G}: Real constant

$$f_{11}^{G} = \frac{\frac{\epsilon^2}{\epsilon-\gamma}}{-\frac{1}{A_{22}}\frac{\epsilon}{\epsilon-\gamma}-i\frac{\epsilon^2}{\epsilon-\gamma}p_0-ik}$$

$$\Rightarrow \quad \sigma_{21}^{G} \propto \left|\frac{1}{-\frac{1}{a_G}-ik}\right|^2$$

$\sigma_{21}^{G}(Re(a_G), Im(a_G))$

$$\Rightarrow \quad \sigma_{11}^{G} \propto \left|\frac{1}{-\frac{1}{A_{22}}\frac{\epsilon}{\epsilon-\gamma}-i\frac{\epsilon^2}{\epsilon-\gamma}p_0-ik}\right|^2$$

$\sigma_{11}^{G}(Re(a_G), Im(a_G), \gamma)$

The general cross section $\boldsymbol{\sigma_{11}^{G}}$ is written only by $\boldsymbol{a_G}$.
However, $\boldsymbol{\sigma_{11}^{G}}$ depends on three parameters.

$\Rightarrow$ **When $\boldsymbol{a_G}$ is fixed, $\boldsymbol{\sigma_{21}^{G}}$ is stable, but $\boldsymbol{\sigma_{11}^{G}}$ changes for variation of $\boldsymbol{\gamma}$.**

Cross section

We calculate σ_G varying γ for same value of scattering length:

$$a_G = a_F = +1.0 - i1.0 \text{ [fm]}$$

This a_G makes the sharper peak below the threshold in σ_{21}.

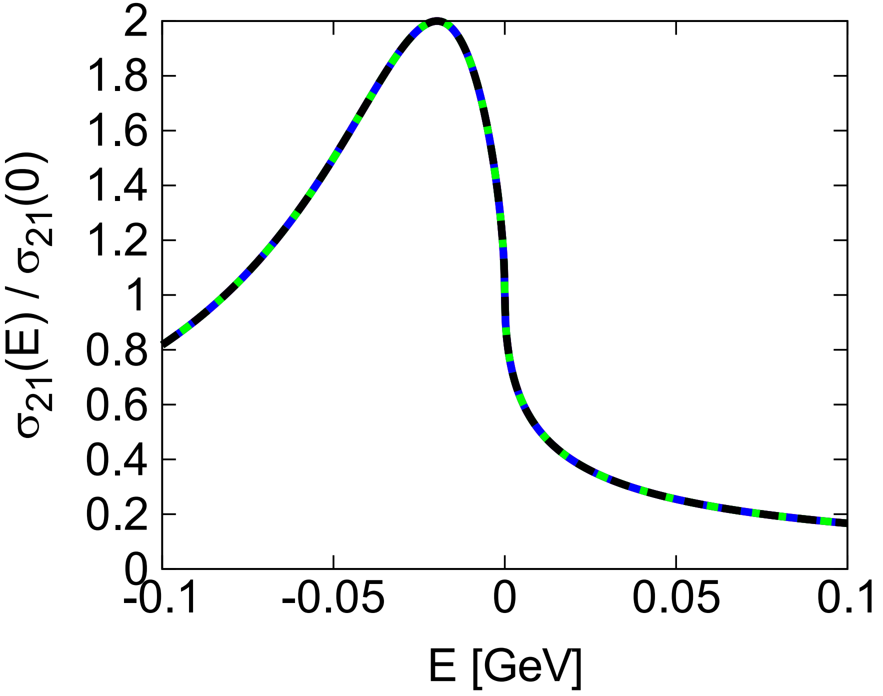
(1) $A_{22} = 3.4\text{[fm]}, \epsilon = 0.3, \gamma = 0.05$

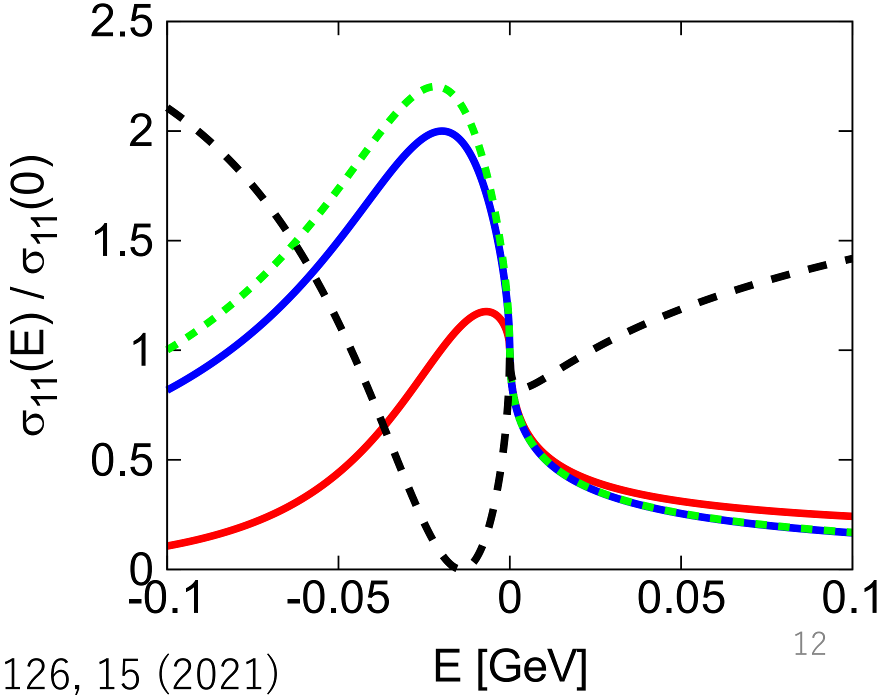
(2) $\gamma = 0.0$ (Flatté)

(3) $A_{22} = 1.9\text{[fm]}, \epsilon = 0.2, \gamma = -0.01$

(4) $A_{22} = 0.27\text{[fm]}, \epsilon = -1.1, \gamma = -10.0$

However, σ^G changes significantly for same a_G. In particular, when $\epsilon < 0$, the dip emerge below the threshold[6].





[6]Dong, Xiang-Kun and Guo, Feng-Kun and Zou, Bing-Song, Phys. Rev. Lett.126, 15 (2021)

Cross section

We calculate σ_G varying γ for same value of scattering length:

$$a_G = a_F = -1.0 - i1.0 \text{ [fm]}$$

This a_G makes the shaper cusp at $E = 0$ for σ_{21}.

(1) $A_{22} = -3.4$[fm], $\epsilon = 0.3$, $\gamma = 0.05$

(2) $\gamma = 0.0$ (Flatté)

(3) $A_{22} = -1.9$ [fm], $\epsilon = 0.2$, $\gamma = -0.01$

(4) $A_{22} = -0.27$[fm], $\epsilon = -1.1$, $\gamma = -10.0$

However, σ^G changes significantly for same a_G. In particular, when $\epsilon < 0$, the dip emerge near the threshold.