UNIVERSITÀ DI TORINO McMULE

LEES2024

NNLO QED with McMULE for lepton-nucleon scattering at ULQ2

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for the McMULE Team

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SENDAI, OCTOBER 31ST, 2024

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- how: McMule might help QED help
- spoiler: QED is not to be underestimated

formalities

- higher-order predictions and comparison with precision experiments
- focus on $2 \to 2$ low-energy QED+ scattering processes
- **input**: matrix elements by us or others (at NNLO + first visits at N3LO)
- **output**: physical cross section for any physical observable at fixed order
- at present an integrator, generator features under testing

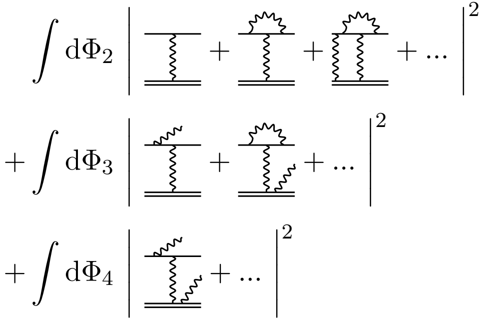
McMule

Monte Carlo for MUons and other LEptons

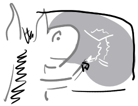
code $\to$ `https://mule-tools.gitlab.io/`

docs $\to$ `https://mcmule.readthedocs.io/`

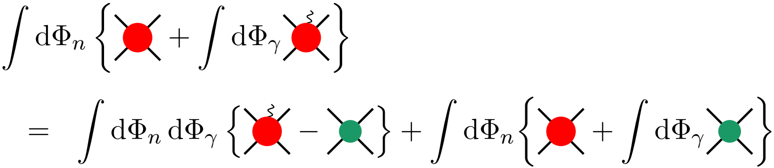
a peek in the ~~stable~~ theory



1. fully-differential PS integration
   → FKS^ℓ
2. numerical instabilities due to pseudo-singularities
   → next-to-soft stabilisation
3. virtual amplitudes with massive particles
   → one-loop: OpenLoops
   → two-loop: massification

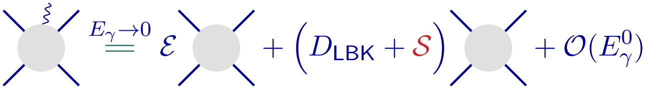


a universal recipe for divergent radiative corrections



- exploits exponentiation of soft singularities [YFS 61]
- works at all orders in QED [Engel, Signer, Ulrich 19]
- singularities are dealt with locally $\to$ stable numerical integration
- no resolution parameters $\to$ theory error: 0

a universal recipe for unstable real-virtual corrections

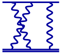


- use OpenLoops [Buccioni, Pozzorini, Zoller 18, Buccioni et al. 19] for the bulk of the phase space
- when unstable, use LBK theorem [LBK 58-61, Engel, Signer, Ulrich 21, 2xEngel 23]
- if $E_\gamma < E_{\rm NTS} \sim 10^{-3}\sqrt{s}/2$ switch to NTS expansion above
- theory error: $\mathcal{O}(10^{-3})\,@\,\rm{NNLO} \sim \mathcal{O}(10^{-6})$

- different carbon footprint: 7 days vs 3 months

a(n almost) universal recipe for two-loop integrals with masses

- use $m_\ell^2 \ll m_p^2 \sim s \sim Q^2 \to$ expand in m_ℓ^2/Q^2



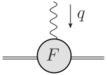
$$\sim A \log^2 \frac{m_\ell^2}{Q^2} + B \log \frac{m_\ell^2}{Q^2} + C + \mathcal{O}\Big(\frac{m_\ell^2}{Q^2}\Big)$$

- can be applied up to three-loop level: $\mathcal{A}(m_\ell) = \mathcal{S}' \times Z \times Z \times \mathcal{A}(0) + \mathcal{O}(m_\ell)$

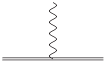
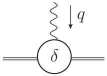
  [Penin 06, Becher, Melnikov 07; Engel, Gnendiger, Signer, Ulrich 18, Ulrich 23]
- use for example full 2-loop 2-fermion amplitude $M \neq 0$, $m = 0$ [Bonciani et al. 21]
- theory error: $\mathcal{O}(10^{-2})\,@\,\mathrm{NNLO} \sim \mathcal{O}(10^{-5})$

results for ULQ2

- simple dipole model for proton form factor $G_E = \frac{G_M}{1+\kappa} = \left(1 + \frac{Q^2}{\Lambda^2}\right)^{-2}$



$$= e\gamma^\mu F_1(G_E, G_M) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_p} F_2(G_E, G_M)$$

$=$  $+$ 

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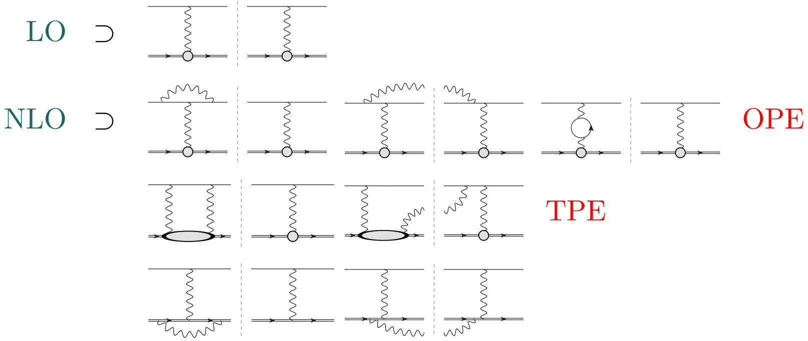

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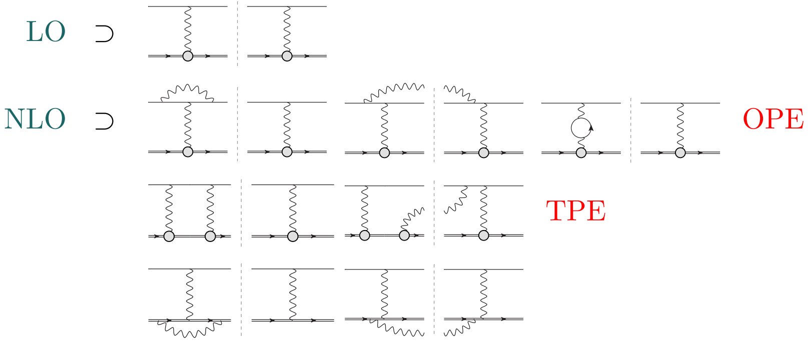

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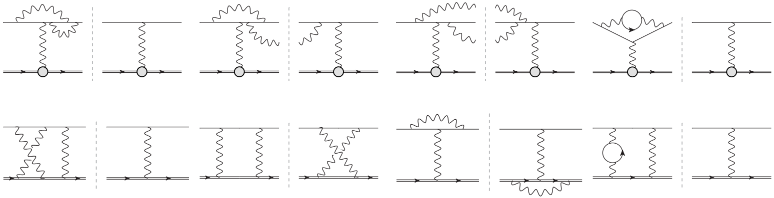
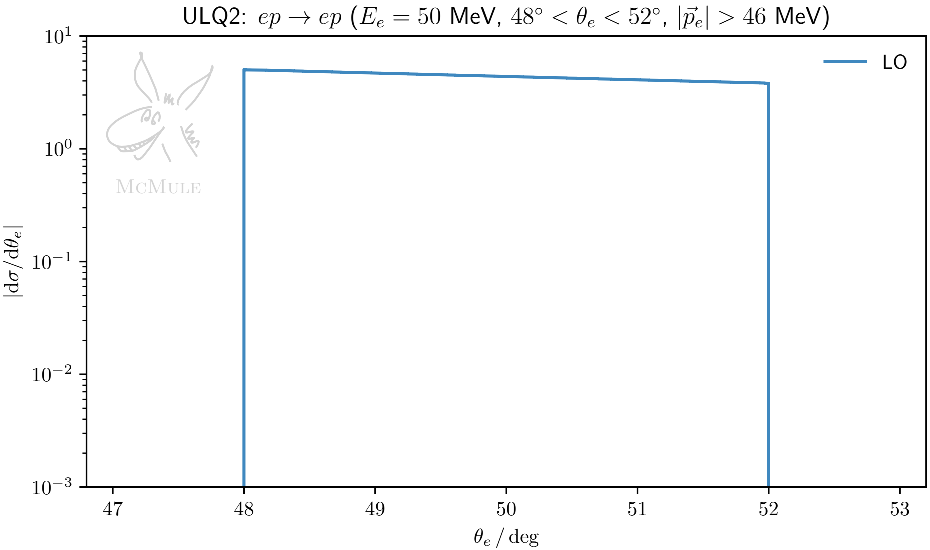
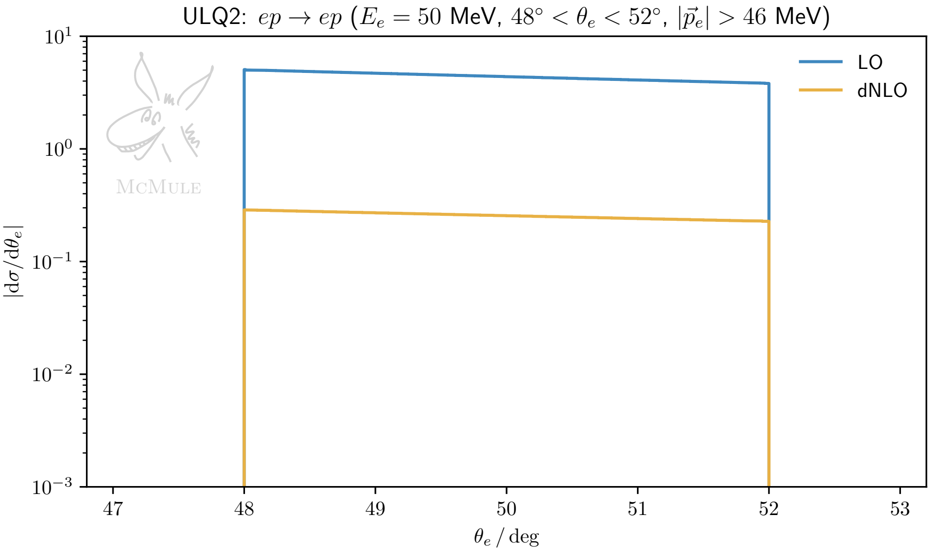
NNLO $\supset$



$+$ many more . . .


ULQ2: $ep \to ep$ ($E_e = 50$ MeV, $48° < \theta_e < 52°$, $|\vec{p}_e| > 46$ MeV)
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
$|d\sigma/d\theta_e|$
LO
McMule
47
48
49
50
51
52
53
θ_e / deg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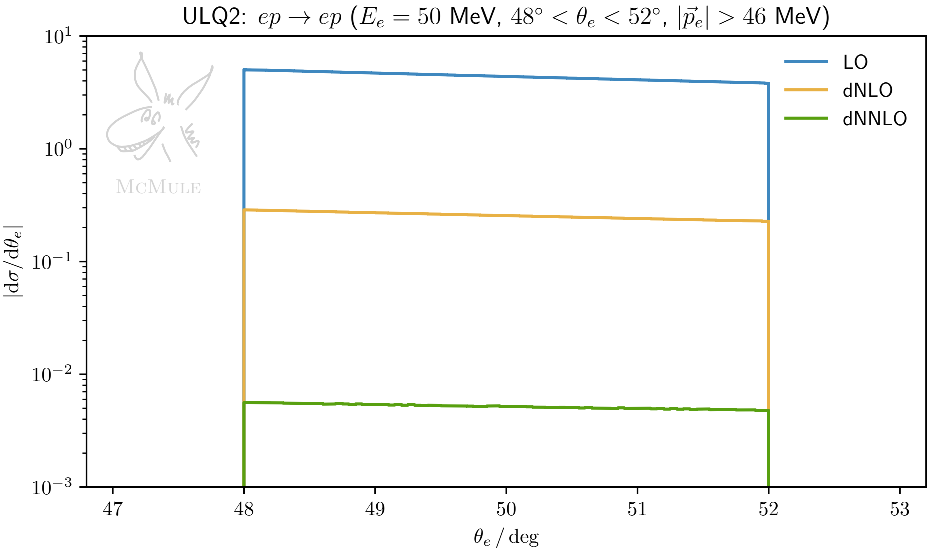
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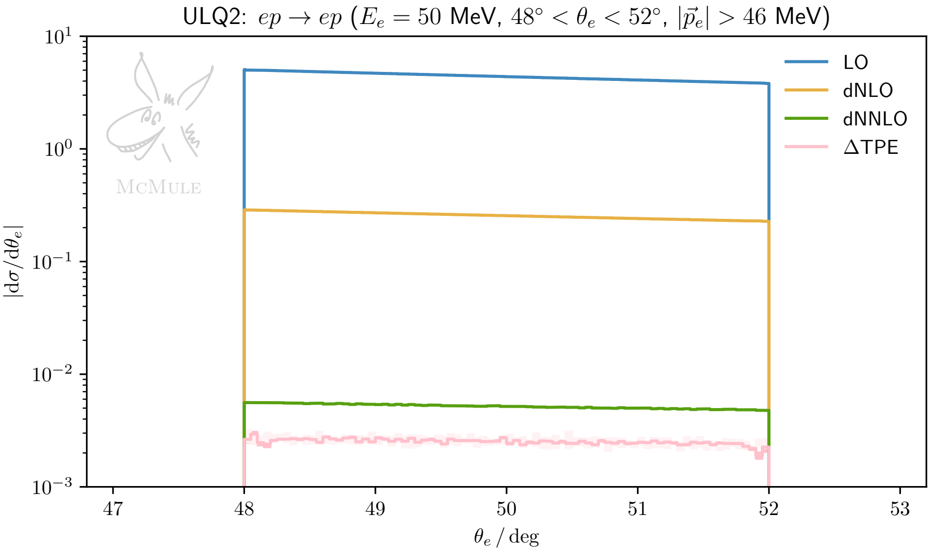
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θ_e / deg
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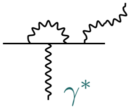
ΔTPE
McMule

what if the proton is a nucleon of any spin?

- McMULE is an OPE professional: 

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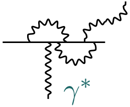
- McMule ~~is~~ will be an OPE ~~professional~~ master: 

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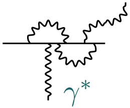
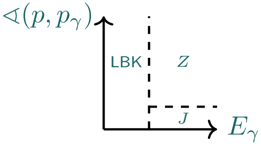


- ◇ VVV: three-loop $ee \to \gamma^*$ with $m_e \neq 0$ is known [Fael, Lange, Schönwald, Steinhauser 22]
- ◇ RVV: two-loop $ee \to \gamma^*\gamma$ with $m_e = 0$ [Badger, Kryś, Moodie, Zoia 23] is implemented



  - soft emissions understood [Engel 23]
  - hard-collinear emission at 2-loop not really
  - calculation with $m_e \neq 0$ extremely challenging ...

- ◇ RRV & RRR: "easy"

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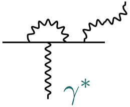
- McMule is an OPE professional: 

- γ^* can be your favourite nucleon:
  spin 0 (^{12}C), spin $\frac{1}{2}$ (^{1}H), and spin 1 (^{2}H) – testing now general case

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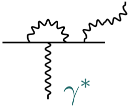

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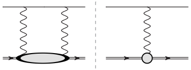
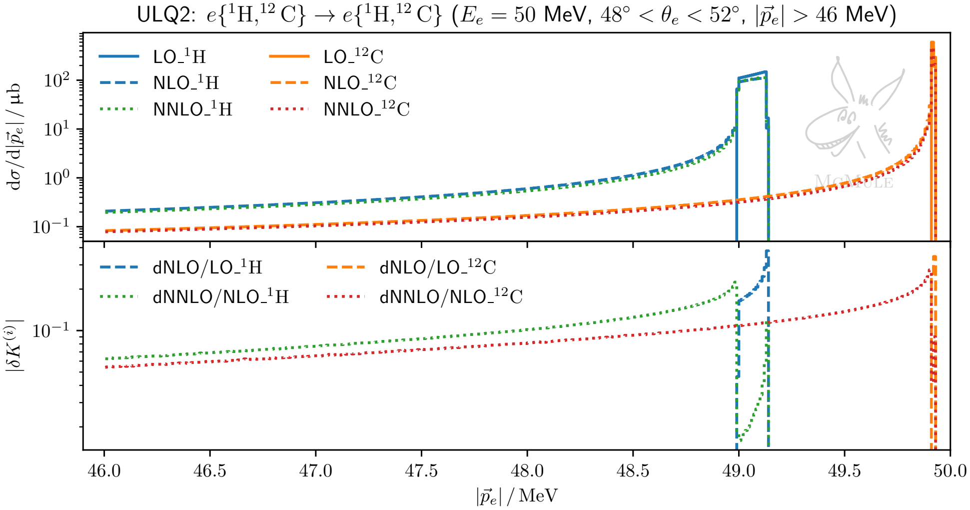
- McMule is learning from pions to become a TPE master

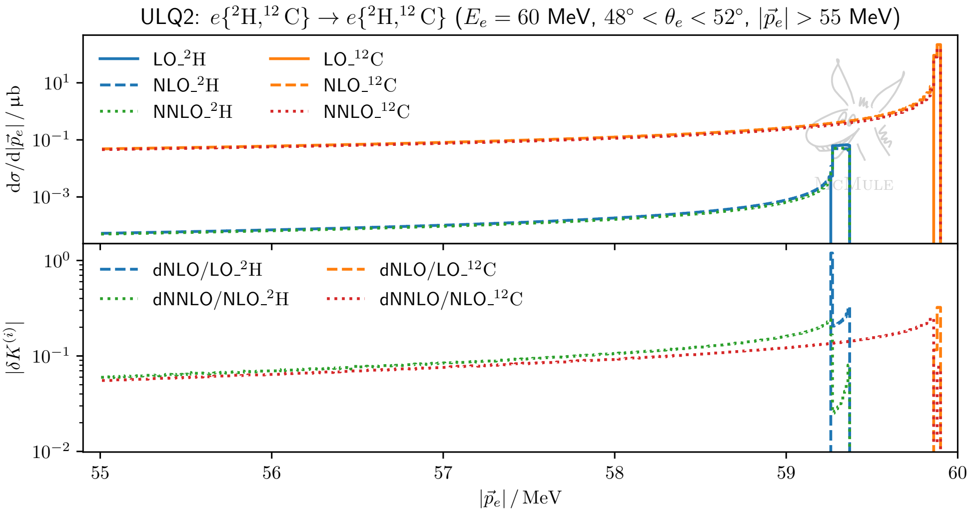
  ↪ dispersive integrals are key!



OPE results for $e\,^1\mathrm{H}$ and $e\,^{12}\mathrm{C}$ scattering



OPE results for $e\,^2\mathrm{H}$ and $e\,^{12}\mathrm{C}$ scattering



- progress in QCD/high-energy seems to be slow to get to QED but
- low-energy precision physics can now be pushed to NNLO+ with McMULE
- assessed importance of NNLO QED corrections wrt (naïve) hadronic effects

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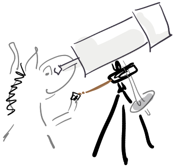
future steps of the mule



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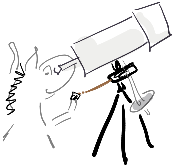

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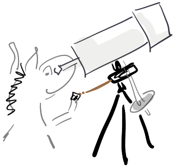
- ◇ integrator → generator
- ◇ N3LO corrections for $\ell\ell \to \gamma^*$



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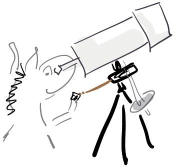
- NNLO corrections for $\ell\ell \rightarrow \ell'\ell'\gamma$



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a mule never stops

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- ◇ integrator $\rightarrow$ generator
- ◇ N3LO corrections for $\ell\ell \to \gamma^*$
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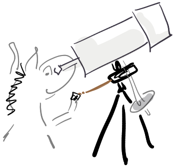
- beyond-fixed-order QED (à la YFS)



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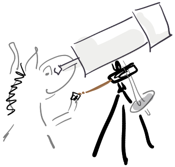
◇ get dirtier with hadrons



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- ◇ beyond-fixed-order QED (à la YFS)
- ◇ get dirtier with hadrons
- ◇ world dominance

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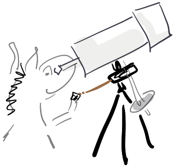

Thank you!

f.l.t.r.: S.Kollatzsch (Zürich & PSI), A.Signer (Zürich & PSI), V.Sharkovska (Zürich & PSI), S.Gündogdu (Zürich & PSI), D. Moreno (PSI), A.Coutinho (IFIC), Y.Ulrich (Liverpool), D. Radic (Zürich & PSI), L.Naterop (Zürich & PSI), M.Rocco (Torino)
not shown: F.Hagelstein (Mainz), N.Schalch (Oxford), T.Engel (Freiburg), A.Gurgone (Pavia), P.Banerjee (Cosenza)

McMule

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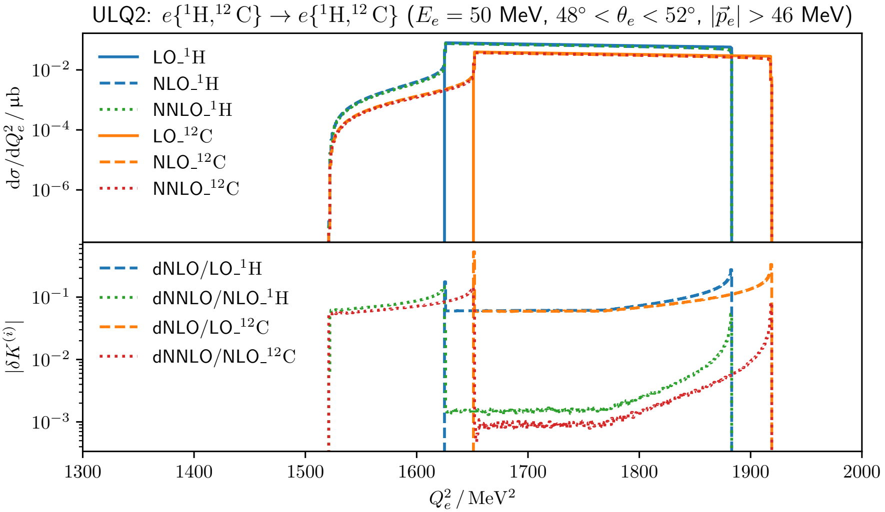
more results for $e\,^1\mathrm{H}$ and $e\,^{12}\mathrm{C}$ scattering



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