



# Charge Radii and Neutron Skin Thickness of Unstable Nuclei

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**Low-Energy Electron Scattering for Nucleon and Exotic Nuclei (LEES2024)** October 28 - November 1, 2024





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**Myeong-Hwan Mun, Seonghyun Kim, Myung-Ki Cheoun, W.Y. So, Soonchul Choi, Phys. Lett.B 847, 138298 (2023)**

3. OES and Kink of Hg and Au by DRHBc and other Skyrme DFTs 3-1. OES of Hg and Au 3-2. Kink of Hg and Pb **Myeong-Hwan Mun, E. Ha, Y. B. Choi, Myung-Ki Cheoun, PRC 110, 024310 (2024)** 

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4. CM corrections and Magnetic Moments **Y. Tanimura and M. K. Cheoun, 5. Summary** LEES2024, Oct.28-Nov. 1 2024 **PRC 109, 054323 (2024)**

#### **1. Odd Even Shape staggering** Charge Radii of Hg isotopes in ISOLDE



Fig. 1 | An overview of the in-source resonance ionization spectroscopy study of radiogenic mercury isotopes. a, Mercury isotopes are produced by proton-induced nuclear reactions in a molten lead target. The vapour effuses into the anode volume of the VADLIS ion source<sup>23</sup>, where the atoms are ionized using a three-step resonance photo-ionization scheme (inset). The ions are extracted as a mono-energetic beam at 30 keV. The isotope of interest is selected using the general-purpose mass separator (GPS) and directed towards the most appropriate of the three detection systems shown (see Methods). b, By scanning the laser wavelength of the spectroscopic transition (in this case the first step), the isotope shift (IS) and hyperfine structure (HFS) are examined. The isotope shifts are used to calculate the changes in mean-square charge radii  $\delta(r^2)$  with respect to  $A = N + Z = 198$  along the isotopic chain. c, The results of this study appear as filled red circles (ground states, gs) or open red circles (isomeric states, is) 177-189 Hg are new measurement wherease is - 185 Hg were re-measured and the data points overlap with those of the literature values (blue circles), as can also be seen by the close agreement between values in Table 1. The error bars correspond to the standard deviation of measurements. Additional scaling uncertainties (not shown) arise from the indeterminacy of the F factor (7%; see ref. <sup>19</sup>) and the specific mass shift, M<sub>SMS</sub>. These are provided in Table 1. The additional continuous black line illustrates the previously measure bauke ballas is poetical trend of the lead nuclei<sup>6</sup>. Sketched insets, representative of the shapes of <sup>190</sup>Hg and <sup>181</sup>Hg with a deformation parameter  $(\beta_2)^{1/2} = 0.174$  and 0.313 respectively, are provided.

#### **1. Odd Even Shape staggering** Charge Radii of Bi and Pb isotopes

#### PHYSICAL REVIEW LETTERS 127, 192501 (2021)

#### Large Shape Staggering in Neutron-Deficient Bi Isotopes

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> $^{185}$ Hgs  $-0.6$  $-0.8$  $\delta \langle r^2\rangle_{_{N,N_0}}$  (fm²)  $-1.0$  $-1.2$  $-1.4$  $-1.6$ 98 100 102 104 106 108 110 112 114 116 118 120 N





FIG. 2. Changes in the mean-square charge radii for Bi (downward triangles, present work; upward triangles, Refs. [27,55,56]), Pb (leftward triangles, Refs. [5,57,58]), and Hg (squares, Refs. [10–12]) isotopes. Full and hollow symboles label the gs's and isomers, respectively. Data for each chain are shifted along the Y axis to improve visibility. Red dashed line<br>connects the data for the even-N  $9/2^-$  Bi gs's to demonstrate<br>connects the data for the even-N  $9/2^-$  Bi gs's to demonstrate deviation from the Pb trend.

FIG. 3. HFB PESs obtained by blocking the first  $9/2^-$  qp in odd-A bismuth isotopes (solid lines),  $10<sup>-</sup>$  in even-A ones (dashed lines), and  $1^+$  in <sup>188</sup>Bi (squares).  $E_0$  is the minimal energy of the corresponding PES. For each PES, at least one of the minima has a magnetic moment compatible with experimental data. For 9/2<sup>-1</sup> and 10<sup>-</sup> states it is a minimum at  $\beta \approx -0.07$ , whereas for 1<sup>+</sup> state PHYSICAL REVIEW LETTERS 131, 202501 (2023)

**Editors' Suggestion** 

#### Deformation versus Sphericity in the Ground States of the Lightest Gold Isotopes

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FIG. 2. Comparison between experimental  $\delta(r^2)_{4,197}$  values ( $\bullet$ ) for gold isotopes with HFB calculations without  $(\blacksquare)$  and with  $(\blacktriangle)$  CM included. The filled symbols connected by lines indicate ground states, whilst the hollow symbols represent the isomers in <sup>178,187</sup>Au and the high-spin state in  $176$  Au. The  $11/2$ <sup>-</sup> isomers have been excluded for clarity. The vertical, blue, dashed line represents the  $N = 104$  mid-LEES2024, Oct.28-Nov. **shell.** 5

#### **1. Kink structure** Kink structure in Pb and Hg isotopes

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FIG. 3. Panels (a) and (b) show  $\delta \langle r^2 \rangle^{A,A'}$  of lead and mercury isotopes relative to <sup>208</sup>Pb and <sup>206</sup>Hg ( $N = 126$ ), respectively. Experimental mercury data: this Letter and Ref. [16]. Experimental lead data: Ref. [18]. The statistical uncertainties are smaller than the data points. RHB(DD-ME2) results: this Letter. NR-HFB(M3Y-P6a) results: this Letter (mercury) and Ref. [14] (lead). NR-HFB(UNEDF1) results: Ref. [35].

energies of experimental states in model calculations [47,48]. However, it is neglected in the present study since its impact on charge radii is still an open theoretical question.

The results of the RHB and NR-HFB calculations are presented in Fig. 3, together with the experimental results for the lead and mercury chains. In both cases, the kink at  $N = 126$  is visibly better reproduced in the RHB (DD-ME2) calculations. To facilitate a quantitative comparison of the experimental and theoretical results, two indicators are employed. OES is quantified considering the isotope's nearest neighbors via the commonly used three-point indicator

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- ➢ Indeed, charge radii reflect various nuclear structure phenomena such as halo structures, shape staggering and shape coexistence, pairing correlations, neutron skins and the occurrence of nuclear magic numbers.
- $\triangleright$  The remarkably abrupt changes in charge radii are observed naturally across the neutron-closure shells, namely the kinks at  $N = 20$ , 28, 50, 82, 126  $[1\text{-}4]$ .



[3] Farooq-Smith, G. J. et al. Phys. Rev. C 96, 044324 (2017). [4] Kreim, K. et al. Phys. Lett. B 731, 97 – 102 (2014). [1] Gorges, C. et al. Phys. Rev. Lett. 122, 192502 (2019). [2] Anselment, M. et al. Nucl. Phys. A  $451, 471$   $-480, (1986)$ .

### Relative differences by two-point and three-point of mean square radii and binding energies of light nuclei around  $N = 28$  and 32



Nature Physics volume 17, pages439–443 (2021)

A Brief summary

Thanks to the recent development of the laser spectroscopy, MR ToP and RI accelerator facilities (and electron scattering) in the world

- 1. OES in Hg and Au isotopes turns out to be significant compared to any other nuclei , for example, Pb isotopes. How to understand it ?
- 2. Kink structures appear above any magic shell in light and heavy nuclear isotopes. How to interpret this behavior ?

3. We need a more refined nuclear model, which needs to explain main properties of the whole nuclei in the nuclear chart by including the relativistic description, deformation, continuum, driplines etc. and also proper description of odd nuclei.





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## Theoretical framework for DRHBc

 $\mathcal{L}$ 

### The 7th workshop on nuclear mass table **With DRHBc theory**

Date: July 1 to 4, 2024

Place: Gangneung Green City Experience Center, Gangneung, Korea

#### **Scientific Contents**

- . Overview of the DRHBc calculations for odd-Z nuclei
- DRHBc calculations results for odd-Z nuclei
- Progress Reports from PIs for different regions
- Interesting topics of DRHBc mass table
- · Future working plan toward a complete DRHBc mass table

#### **Schedule Overview**

- July 1, 2024 (Mon)
- Arrival & Registration / Domestic meeting
- July 2 (Tue)
- Morning Session / Afternoon Session / Banquet
- July 3 (Wed)
- Morning Session / Excursion
- July 4  $(\bar{T}hu)$ - Morning Session / Discussions

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 $\triangleright$  We start from the following point-coupling Lagrangian density<sup>1</sup>

$$
= \bar{\psi}(i\gamma_{\mu}\partial^{\mu} - M)\psi - \frac{1}{2}\alpha_{S}(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_{V}(\bar{\psi}\gamma_{\mu}\psi)(\bar{\psi}\gamma^{\mu}\psi) - \frac{1}{2}\alpha_{TV}(\bar{\psi}\vec{\tau}\gamma_{\mu}\psi)(\bar{\psi}\vec{\tau}\gamma^{\mu}\psi) - \frac{1}{2}\alpha_{TS}(\bar{\psi}\vec{\tau}\psi)(\bar{\psi}\vec{\tau}\psi) - \frac{1}{3}\beta_{S}(\bar{\psi}\psi)^{3} - \frac{1}{4}\gamma_{S}(\bar{\psi}\psi)^{4} - \frac{1}{4}\gamma_{V}[(\bar{\psi}\gamma_{\mu}\psi)(\bar{\psi}\gamma^{\mu}\psi)]^{2} - \frac{1}{2}\delta_{S}\partial_{\nu}(\bar{\psi}\psi)\partial^{\nu}(\bar{\psi}\psi) - \frac{1}{2}\delta_{V}\partial_{\nu}(\bar{\psi}\gamma_{\mu}\psi)\partial^{\nu}(\bar{\psi}\gamma^{\mu}\psi) - \frac{1}{2}\delta_{TV}\partial_{\nu}(\bar{\psi}\vec{\tau}\gamma_{\mu}\psi)\partial^{\nu}(\bar{\psi}\vec{\tau}\gamma^{\mu}\psi) - \frac{1}{2}\delta_{TS}\partial_{\nu}(\bar{\psi}\vec{\tau}\psi)\partial^{\nu}(\bar{\psi}\vec{\tau}\psi)
$$
  
 - \frac{1}{2}\delta\_{TS}\partial\_{\nu}(\bar{\psi}\vec{\tau}\psi)\partial^{\nu}(\bar{\psi}\vec{\tau}\psi)   
 - \frac{1}{4}F^{\mu\nu}F\_{\mu\nu} - e\bar{\psi}\gamma^{\mu}\frac{1-\tau\_{3}}{2}A\_{\mu}\psi,

[1] J. Meng (Editor). Relativistic Density Functional for Nuclear Structure (2016) LEES2024, Oct.28-Nov. 1 2024

#### **2. Theoretical Approaches**

#### DRHBc (Relativistic MF approach with point coupling)

$$
\mathcal{L} = \bar{\psi}(i\gamma_{\mu}\partial_{\mu} - M)\psi - \frac{1}{2}\alpha_{s}(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_{V}(\bar{\psi}\gamma_{\mu}\psi)(\bar{\psi}\gamma^{\mu}\psi) - \frac{1}{2}\alpha_{TV}(\bar{\psi}\vec{\tau}\gamma_{\mu}\psi)(\bar{\psi}\vec{\tau}\gamma^{\mu}\psi) -\frac{1}{2}\alpha_{TS}(\bar{\psi}\vec{\tau}\psi)(\bar{\psi}\vec{\tau}\psi) - \frac{1}{3}\beta_{S}(\bar{\psi}\psi)^{3} - \frac{1}{4}\gamma_{S}(\bar{\psi}\psi)^{4} - \frac{1}{4}\gamma_{V}[(\bar{\psi}\gamma_{\mu}\psi)(\bar{\psi}\gamma^{\mu}\psi)]^{2} -\frac{1}{2}\delta_{S}\partial_{\nu}(\bar{\psi}\psi)\partial^{\nu}(\bar{\psi}\psi) - \frac{1}{2}\delta_{V}\partial_{\nu}(\bar{\psi}\gamma_{\mu}\psi)\partial^{\nu}(\bar{\psi}\gamma^{\mu}\psi) - \frac{1}{2}\delta_{TV}\partial_{\nu}(\bar{\psi}\vec{\tau}\gamma_{\mu}\psi)\partial^{\nu}(\bar{\psi}\vec{\tau}\gamma^{\mu}\psi) -\frac{1}{2}\delta_{TS}\partial_{\nu}(\bar{\psi}\vec{\tau}\psi)\partial^{\nu}(\bar{\psi}\vec{\tau}\psi) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - e\bar{\psi}\gamma^{\mu}\frac{1-\tau_{3}}{2}A_{\mu}\psi,
$$
(1)

where M is the nucleon mass, e is the proton charge, and  $A_u$  and  $F_{uu}$  are the fourvector potential and field tensor of the electromagnetic field, respectively. The coupling constant  $\alpha_i$  for four-fermion terms is specified by superscripts  $(i = S, V)$ and  $T$ ) which stand for the scalar, vector, and isovector channels, respectively. The higher-order terms are specified by  $\beta_i$  and  $\gamma_i$ , while  $\delta_i$  refers to gradient terms. The isovector-scalar channels,  $\alpha_{TS}$  and  $\delta_{TS}$ , are neglected in the DRHBc approach [22]. The EDF is derived in terms of nucleon scalar density,  $\rho_s(\mathbf{r}) = \sum_{i=1}^A \bar{\psi}_i(\mathbf{r}) \psi_i(\mathbf{r})$ , and IS and IV currents  $j_{\mu}(\mathbf{r}) = \sum_{i=1}^{A} \bar{\psi}_i(\mathbf{r}) \gamma_{\mu} \psi_i(\mathbf{r})$ , and  $\vec{j}_{\mu}(\mathbf{r}) = \sum_{i=1}^{A} \bar{\psi}_i(\mathbf{r}) \vec{\tau} \gamma_{\mu} \psi_i(\mathbf{r})$  as follows  $[17, 41]$ 

$$
E_{RMF}[\psi, \bar{\psi}, A_{\mu}] = \sum_{i=1}^{A} \int d^{3} \mathbf{r} \psi_{i}^{+} (\boldsymbol{\alpha} \cdot \mathbf{p} + \beta M) \psi_{i} - \frac{1}{2} (\nabla \cdot \mathbf{A})^{2} + \frac{1}{2} e \int d^{3} \mathbf{r} j_{p}^{\mu} A_{\mu} + \int d^{3} \mathbf{r} \left[ \frac{1}{2} (\alpha_{S} \rho_{s}^{2} + \alpha_{V} j^{\mu} j_{\mu} + \alpha_{TV} j^{\mu} \cdot j_{\mu}) + \frac{1}{3} \beta_{S} \rho_{s}^{3} + \frac{1}{4} \gamma_{S} \rho_{s}^{4} + \frac{1}{4} \gamma_{V} (j^{\mu} j_{\mu})^{2} + \frac{1}{2} \delta_{S} \rho_{s} \Box \rho_{s} + \frac{1}{2} \delta_{V} j^{\mu} \Box j_{\mu} + \frac{1}{2} \delta_{TV} \vec{j}^{\mu} \cdot \Box j_{\mu}^{2} \right].
$$
 (2)

The present calculations are carried out by solving the HB equation with the density functional PC-PK1 [47], which reads [41]

$$
\int d^3\mathbf{r}' \begin{pmatrix} h_D(\mathbf{r}, \mathbf{r}') - \lambda_{\tau} & \Delta(\mathbf{r}, \mathbf{r}') \\ -\Delta^*(\mathbf{r}, \mathbf{r}') & -h_D^*(\mathbf{r}, \mathbf{r}') + \lambda_{\tau} \vec{j}^{\mu} \cdot \vec{j}_{\mu} \end{pmatrix} \begin{pmatrix} U_k(\mathbf{r}) \\ V_k(\mathbf{r}) \end{pmatrix} = E_k \begin{pmatrix} U_k(\mathbf{r}) \\ V_k(\mathbf{r}) \end{pmatrix}, \quad (3)
$$

where  $\lambda_{\tau}$  is the Fermi energy  $(\tau = n/p)$  for neutrons or protons.  $E_k$  and  $(U_k, V_k)^T$  are the quasiparticle energy and quasiparticle wave function. In coordinate space, the Dirac Hamiltonian  $h_D$  is given by

$$
h_D(\mathbf{r}, \mathbf{r}') = [\boldsymbol{\alpha} \cdot \boldsymbol{p} + V(\boldsymbol{r}) + \beta(M + S(\boldsymbol{r}))]\delta(\boldsymbol{r}, \boldsymbol{r}'), \tag{4}
$$

where M is the nucleon mass, and  $V(r)$  and  $S(r)$  are the vector and scalar potentials, respectively, given by

$$
S(\mathbf{r}) = \alpha_S \rho_S + \beta_S \rho_S^2 + \gamma_S \rho_S^3 + \delta_S \Delta \rho_S ,
$$
  
\n
$$
V(\mathbf{r}) = \alpha_V \rho_V + \gamma_V \rho_V^3 + \delta_V \Delta \rho_V + eA^0 + \alpha_{TV} \tau_3 \rho_3 + \delta_{TV} \tau_3 \Delta \rho_3 ,
$$
\n(5)

with the following densities represented in terms of the quasiparticle wave functions as follows

$$
\rho_S(\bm{r}) = \sum_{k>0} \bar{V}_k(\bm{r}) V_k(\bm{r}) \ , \ \rho_V(\bm{r}) = \sum_{k>0} \bar{V}_k(\bm{r}) \gamma_0 V_k(\bm{r}) \ , \ \rho_3(\bm{r}) = \sum_{k>0} \bar{V}_k(\bm{r}) \gamma_0 \tau_3 V_k(\bm{r}). \tag{6}
$$

The paring potential  $\Delta(\mathbf{r}, \mathbf{r}')$  is given by the following gap equation in terms of the pairing tensor  $\kappa(r,r')$  [30]

$$
\Delta(\mathbf{r}, \mathbf{r}') = V^{pp}(\mathbf{r}, \mathbf{r}') \kappa(\mathbf{r}, \mathbf{r}') , \ \kappa(\mathbf{r}, \mathbf{r}') (= \langle |a_k a_{k'}| \rangle) = \sum_{k>0} V_k^*(\mathbf{r}) U_k^T(\mathbf{r}') , \tag{7}
$$

where we did not write explicity the sum on the spin degree of freedom and the upper and lower components of Dirac spinors. The DRHBc approach uses a density-dependent zero-range force

$$
V^{pp}(\boldsymbol{r}, \boldsymbol{r}') = \frac{V_0}{2} (1 - P_{\sigma}) \delta(\boldsymbol{r} - \boldsymbol{r}') (1 - \frac{\rho(\boldsymbol{r})}{\rho_{\text{sat}}}), \qquad (8)
$$

where  $\rho_{\text{sat}}$  is the nuclear saturation density,  $V_0$  is the pairing strength, and  $(1 - P_{\sigma})/2$  is the projector for the spin-singlet,  $S = 0$  component in the pairing channel. Then the pairing potential and pairing tensor of Eq.  $(7)$  become local quantities [30],

$$
\Delta(\mathbf{r}) = V_0(1 - \rho(\mathbf{r})/\rho_{sat})\kappa(\mathbf{r}), \ \kappa(\mathbf{r}) = \sum_{k>0} V_k^+(\mathbf{r})U_k(\mathbf{r}) \tag{9}
$$

because of the zero-range and spin-singlet properties of the pairing force in Eq. (8). For the non-local and spin-triplet (isoscalar) interactions, one needs a more careful treatment of the pairing potential and pairing tensor [41].

### **2. Theoretical Approaches DRHBC (Relativistic MF approach with point coupling)**



Lianjian Zou<sup>19</sup>, DRHBc Mass Table Collaboration

### **PC-PK1 + DRHBc : 4829 bound even- nuclei with 8 ≤ Z ≤ 120** DRHBc Collaboration,

**At. Data Nucl. Data Tables** 144, 101488 (2022);158, 101661 (2024)

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#### 3. OE Shape Staggering Results | Total Binding Energy in Hg, Au and Pb isotopes







FIG. 1: (Color online) PECs (a)((b)), BE/A (c)((d)) Au (Pb) isotopes determined by DRHBc calculations without (red circles) and with (blue triangles) rotational correction  $E_{\text{rot}}$ . They are compared with available experimental data taken from Ref. [101]. The numbers in parentheses stand for average rms deviation in MeV to the data [101]. The relative difference between the experimental TBE data and the DRHBc calculations are enlarged in the panel (e) and (f), respectively, for Au and Ph isotopes

3. OE Shape Staggering Results Potential Energy Curve Evolution with Defor. for Hg



FIG. 2: (Color online) Detailed TBE curves in terms of the deformation parameter  $\beta_2$  for <sup>179–189</sup>Hg isotopes. The blue points in oblate region for  $^{181,183,185}$ Hg disclose local minima of the TBEs, that is, they are located a bit higher than 1 MeV compared to those in prolate deformation.

80<sub>Ha</sub>

 $0.6$ 

 $183$ Hg

 $186$ Hg

 $-0.2$ 

 $0.0$ 

 $\beta_2$ 

 $0.4$ 

 $\sum_{\infty}$ -1411

1412

 $-1413$ 

 $-1415$ 

1416

**AAC** 

 $\Sigma$ -1439

 $\boldsymbol{\Sigma}_{-1440}$ 

 $-1441$ 

 $-144$ 

144

 $144!$ 

 $-1447$ 

 $\sum_{-146}$ 

 $-1465$ 

146

 $-146$ 

 $-1470$ 

-147

 $-1472$ ا1473 $0.0$ 

 $\beta_2$ 

 $\beta_2$ 

 $0.0$ 

 $0.2$ 

 $\beta_2$ 

FIG. 1: (Color online) Deformation parameters  $\beta_2$  of Hg isotopes (A = 179 - 210) by DRHBc mass model (a). Total binding energies (TBEs) of some Hg isotopes  $(A = 179 - 189)$  in the prolate region (b). All of the Hg isotopes in this mass region show a possibility of the shape coexistence whose energy differences are within 1 MeV maximally although exact calculations provide prolate deformation. Blue points of <sup>181,183,185</sup>Hg indicate oblate deformations in a local minimum. Detailed  $\mathbf{r}$   $\mathbf{r}$   $\mathbf{r}$   $\mathbf{r}$ 

### **3. OE Shape Staggering Results** Potential Energy Curve Evolution with Defor. for Au



**for Au**

FIG. 4: (Color online) Detailed **PECs** in terms of the deformation parameter  $\beta_2$  for <sup>176-179</sup>Au and  $187,188$  Au isotopes. All of the isotopes demonstrate a possibility of the shape coexistence coming from about 1 MeV energy difference between prolate and oblate minima. But, the violet point in oblate region in the panel (c) for  $178Au$  is located a bit higher than 1 MeV compared to the  $19$ prolate deformation.



### **#. Shape Coexistence of Pb isotopes** 186Pb and Rotational Band and Pairing

In Fig. 3, we provide the results for some specific nuclei,  $^{184,186}Pb$ , which discloses the possibility of the shape coexistence [52]. Similar evolution pattern to the results in the spherical and oblate cases in Fig. 1 and 2 are also found for the mean field energy, pairing (b) in Fig. 3 shows the larger pairing energy in the prolate region (see their absolute vale of the red solid curves) than that in the oblate region, which means the lower moment of inertia by the pairing energy, and consequently the larger energy gap in the rotational **banding structures is expected.** This is quite consistent with the experimental data [6].

FIG. 3: (Color online) Same as Fig. 1, but for possible candidates of shape coexistence nuclei,  $184,186$ Pb isotopes



 $^{220}$ Ph isotope  $^{220}$ Ph isotope  $^{220}$ 



Fig. 3 Partial level scheme of <sup>186</sup>Pb presenting transitions relevant to the present work. The widths of the arrows are proportional to the total

### **3. OE Shape Staggering Results** Nuclear Shape Transition of **Hg isotopes**



 $\Rightarrow$  Can explain small charge radii !!

But still, we cannot explain the OES data of 181,183 and 185 of Hg.

#### **However, remember the oblate minima of 181,183,185 of Hg are 2nd local minima. We have to take prolate deformation.**

Then the OES (blue stars) can be explained clearly.

FIG. 4: (Color online) Charge radii (a) and relative changes of mean square charge radii  $\delta \langle r^2 \rangle^{A,198}$  (b) of Hg isotopes with N = 97-128. Results denoted in a blue color are obtained by prolate shapes calculated by the DRHBc mass model. Red colors in the region are calculated by oblate deformation considering the shape coexistence presented in Figs.1 and 2. Blue stars for  $^{181,183,185}$ Hg with N = 101,103 and 105 denote the results by the prolate deformation, which means that we do not take the shape coexistence for these three nuclei. The OES clearly appear along with the deformation, oblate(o)-prolate(p)-o-p-o-p-o-o-o deformation, for  $N = 100$  to  $N = 108$ . Black boxes (left) for charge radii stand for the experimental data from ADND data [55] and black stars (right) for relative changes of mean square charge radii are from Ref. [4]. 21 LEES2024, Oct.28-Nov. 1 2024



FIG. 3. (Color online) Relative changes of mean square charge radii  $\delta \langle r^2 \rangle^{A,198}$  of Hg isotopes. Black boxes stand for the data [13]. Results of  $N \ge 110$  ( $N \le 109$ ) (blue color) are obtained by OD  $(PD)$  shapes. Red colors are calculated by  $OD$  considering the SC taken from Fig.1. Blue stars for  $^{181,183,185}$ Hg denote the results by the PD obtained w/o SC. The upper contours illustrate the neutron (left half) and proton (right half) prolate shapes of <sup>181,183,185</sup>He by their density distributions. The lower contours illustrate oblate shapes of  $^{182,184}$ Hg.

Myeong-Hwan Mun, Seonghyun Kim, Myung-Ki Cheoun, W.Y. So, Soonchul Choi et al., Phys. Lett.B 847 (2023) 138298

### **3. OE Shape Staggering Results** Potential Energy Curve Evolution with Defor. for **Au**

**Nuclear shape transition of Au and no transition in Pb by DRHBc**



FIG. 7: (Color online) The relative changes of mean square charge radii  $\delta \lt r^2 > A^{.197}$  $(\delta \langle r^2 \rangle^{A,208})$  (a) ((c)) and charge radii (c) ((d)) for Au isotopes with  $N = 96 \sim 121$ (for Pb isotopes with  $N = 100 \sim 132$ ). For Au, the results denoted in blue triangles are obtained by prolate shapes calculated by the DRHBc theory and red circles in the region are calculated by oblate deformation considering the shape coexistence presented in Fig. 3 and Fig. 4. Considering the shape coexistence, the odd-even shape staggering of charge radii and the abrupt change of charge radii are perspicuously visible for  $178Au$  isotope and provides a reasonable description. For Pb (100  $\leq N \leq 117$ ), the red data result from the spherical shape coexistence. Black boxes for relative changes of mean square charge radii of Au and Pb are taken from experimental data  $[7, 35, 110-112, 114]$ . The numbers in parentheses stand for average  $\frac{1}{2}$  rms deviation to the experimental data.

### **3. OE Shape Staggering Results** Nuclear Shape Transition of Hg isotopes by other DFT

#### Expt. data, Hg isotopes in ground state:  $5.50$  $-\blacksquare$  I. Angeli, K. P. Marinova (2013)  $-$  - B. A. Marsh et al. (2018) 5.48 B. Day Goodacre et al. (2021) Mass number A Mass number A 179 180 181 182 183 184 185 186 5.46 187 188 179 180 181 182 183 184 185 186 187 188  $0.2$ 5.44  $(a)$ 7.90  $(b)$  $0.0$ 5.42 ng energy per nucleon [MeV] 7.88 5.40  $-0.2$ 7.86 5.38  $8 < r^2 > A.198$  [fm<sup>2</sup>] FaNDF0 calculation:  $-0.4$  $-\blacksquare$  Hg isotopes in ground state 5.36 -4 weakly prolate isomer 7.84 - strongly prolate isomer  $-0.6$ 5.34 178 182 186 190 194 198 202  $206 \tA$  $-0.8$ 7.80  $-$  EXP.(ANDNT)  $\rightarrow$  DRHBc (0.0160)  $-$  EXP.(ADNDT)  $-1.0$ Fig. 6. Charge radii of mercury isotopes. 7.78 DRHBc w/F  $-$  DRHBc w/SC (0.1997)  $\frac{1}{60}$  7.76  $\rightarrow$  SHF SIII (0.2632)  $\rightarrow$  SHF SIII (0.0311)  $\overline{\text{y}}$  SHF SLy4 (0.3983)  $-1.2$  $-\bullet$  SHF\_SLy4 (0.0254) TABLE I. Deformations of Hg isotopes used in Fig. 3 and 4 by DRHBc and vari  $\leftarrow$  SHF SKI3 (0.4400)  $-$  SHF\_SKI3 (0.0621) parameter sets in Skyrme DFT. 99 100 101 102 103 104 105 106 107 108 109 99 100 101 102 103 104 105 106 107 108 109 **Neutron number N Neutron number N**

#### **Nuclear shape transition of Hg isotopes by Skyrme and Fayans DFTs**

FIG. 4: (Color online) Comparison of DRHBc and Skyrme models for the BE/A (a) and OES (b). The  $BE/A$  (a) corresponds to the Fig. 2 (b) and the OES (b) is the same as the Fig. 3 (a). Relative changes of mean square charge radii  $\delta \langle r^2 \rangle^{A,198}$  of Hg isotopes. Black boxes stand for the data [13].

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 $5.52 \,$ 

### **3. OE Shape Staggering Results** Shape Transition and Occupation Probabilities (Hg)

Red lines for **n1h9/2** shell **decrease** in odd nucleus with the prolate deformation. But **n1i13/2** shell increase with prolate deformation  $(\sqrt{2})/$ Milip by adding a neutrom to e-e  $Hg$ .

 $\Rightarrow$  like a seasaw from even to odd nuclei !!

For proton, **p1h9/2** shell **increase**, but **p3s1/2** decrease !!





FIG. 4: (Color online) Evolution of OPs of the neutron (a) and proton (b) SPSs for the Hg isotopes. In PD region, the OPs of  $\nu 1h_{9/2}$ ,  $\nu 2f_{7/2}$ , and  $\pi 3s_{1/2}$  states decrease while those of  $\nu 1i_{13/2}$  and  $\pi 1h_{9/2}$  states increase. This behaviour can be understood by the shell evolution for  $\beta_2$  with Nilsson quantum numbers  $\Omega(N, n_z, \Lambda)$  in the panel (c).

#### **3. OE Shape Staggering Results** Shape Transition of **Au isotopes**

OP of Proton (neutron) h9/2 increases (decreases) , which increases the charge radii !

Similarly to the OES in Hg case



FIG. 8: (Color online) Evolution of occupation probabilities of the neutron  $\nu 1i_{13/2}$  (a),  $\nu 2f_{7/2}$  (b) and  $\nu 1h_{9/2}$  (c) SPSs and proton  $\pi 3s_{1/2}$  (d) and  $\pi 1h_{9/2}$  (e) SPSs for the Au isotopes. In prolate deformation region, the occupation probabilities of  $\nu 1h_{9/2}$ ,  $\nu 2f_{7/2}$ , and  $\pi 3s_{1/2}$  states decrease while those of  $\nu 1i_{13/2}$  and  $\pi 1h_{9/2}$  states increase compared to the spherical one. The shaded regions indicate the ground state considering the shape coexistence.

### **3. OE Shape Staggering Results** Why Nuclear Shape Transition in **Hg isotopes**



If we add one neutron to e-e nucleus 180Hg, the OC of ni13/2 increases and **n1h9/2 decreases** in 181Hg. The increase of charge radii of 181Hg is due to **the increase of p1h9/2** shell OC. The OES perhaps come from the tensor interaction ??

### **3. Kink Structure Results** Kink structure around  $N = 126$  shell for Hg isotopes

OC of 1i11/2 and 2g9/2 of neutrons increase above N=126 shell. Increase above  $N=126$  shell.<br>But OC of protons  $h9/2$  do not change.

The Kink is to be understood by the swelling of neutrons in the symmetric core, not by the swelling of protons !!





FIG. 5. (Color online) Kink structure in the vicinity of  $^{208}Hg_{126}$  by the relative radius difference (a) and  $R_n$  with  $R_{ch}$  (b) w.r.t. <sup>206</sup>Hg. The dashed lines are just from the continuation of the data before N = 126 case to show the kink structure. Panel (c) show the OPs of  $\nu 2g_{9/2}$  and  $\nu 1i_{11/2}$ state near the magic shell. The larger increase of  $\nu 1i_{11/2}$  than  $\nu 2g_{9/2}$  state for neutron is evident. OPs of proton states (d) are rarely changed.

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#### **3. Kink Structure Results How to understand the Kink**



If we add one neutron to e-e nucleus 208Pb, the OC of ni11/2 and n2g9/2 increase 209Pb, but no change of proton OC g9/2 and i11/2.

The increase of charge radii of 209Pb is due to **the increase of neutron side and attract the proton radius**.

LEES2024, Oct.28-Nov. 1 2024 The Kink perhaps come from the tensor interaction or symmetry energy ??

50  $196Pb$  $g(r) = 4\pi r^2 H_{\text{sym}}$  $40<sup>1</sup>$ g(r) [MeV fm<sup>-1</sup>] SLy4 30 20 – – – HF  $-$ HFB 10  $-10$  $\overline{2}$ 10 6 8  $12$  $r$  [fm] 90  $\sqrt[144]{\text{Sn}}$ 80  $g(r) = 4\pi r^2 H_{sym}$ 70 g(r) [MeV fm<sup>-1</sup>] 60 SLy4 50 – – – HF 40  $-HFB$ 30 20 10  $-10$  $\overline{2}$ 10 6 8  $12$ 0  $\boldsymbol{\Lambda}$  $r$  [fm]

FIG. 4. (Color online) Symmetry energy density distribution  $g(r) = 4\pi r^2 \mathcal{H}_{sym}$  in <sup>196</sup>Pb (upper panel) and <sup>144</sup>Sn (lower panel) obtained by the HF(dashed line) and HFB (solid line) calculation with SLy4 functional. 30 Phys. Rev. C 96, 024311 (2017)





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3. OES and Kink of Hg and Au by DRHBc and other Skyrme DFTs 3-1. OES of Hg and Au 3-2. Kink of Hg and Pb **Myeong-Hwan Mun, E. Ha, Y. B. Choi, Myung-Ki Cheoun, PRC 110, 024310 (2024) Seonghyun Kim, Myeong-Hwan Mun, Myung-Ki Cheoun** 

**and Eunja Ha, PRC 105, 034340 (2022)**

4. CM corrections and Magnetic Moments **Y. Tanimura and M. K. Cheoun, PRC 109, 054323 (2024)**

### **5. Summary** LEES2024, Oct.28-Nov. 1 2024

#### Effects of center-of-mass correction and nucleon anomalous magnetic moments on nuclear charge radii

Yusuke Tanimura  $\mathbb{D}^{1,2}$  and Myung-Ki Cheoun  $\mathbb{D}^1$ 

#### **II. MODEL**

#### A. Relativistic Hartree-Bogoliubov model

We employ an RHB model with DD-ME2 parameter set [50] for the *ph* channel and Gogny D1S interaction [51,52] for the *pp* channel. A remark on DD-ME2 is in order: the parameter fit to charge radii was made by  $r_{ch} =$  $\sqrt{\langle r^2 \rangle_p + (0.8 \text{ fm})^2}$ , where  $\langle r^2 \rangle_p$  is the mean-squared (MS) radius of point-proton density distribution, and  $(0.8 \text{ fm})^2$  is a correction for the charge radius of the proton itself, with BCS calculations instead of Hartree-Bogoliubov. The CM correction and anomalous magnetic coupling described in the following subsections were not considered. See Refs. [50,53– 59] for details of the RHB model and the DD-ME2 parameter set. We impose the spherical symmetry and solve the RHB equations in the radial coordinate space.

#### Contribution of nucleon anomalous magnetic moment to charge density

EM current density operator

$$
\hat{j}_{em}^{\mu}(p',p) = \bar{\psi}(p') \left[ F_1(Q^2) \gamma^{\mu} + F_2(Q^2) \frac{i\hbar}{2mc} Q_{\nu} \sigma^{\mu\nu} \right] \psi(p)
$$
\n
$$
F_1(0) = q
$$

Mean-field approximation

$$
\rho_{\rm ch} \equiv \langle j^0_{\rm em} \rangle = \sum_\alpha v_\alpha^2 q_\alpha \psi_\alpha^\dagger \psi_\alpha + \sum_\alpha v_\alpha^2 \kappa_\alpha \frac{\hbar}{2mc} \boldsymbol{\nabla} \cdot (\bar{\psi}_\alpha i \boldsymbol{\alpha} \psi_\alpha) = \rho_p + \rho_\kappa
$$

Mean-square charge radius

$$
\langle r^2 \rangle_{\rm ch} = \frac{1}{Z} \int d^3r \ r^2 [\rho_p(\mathbf{r}) + \rho_\kappa(\mathbf{r})] + \langle r_p^2 \rangle + \frac{N}{Z} \langle r_n^2 \rangle
$$

$$
= \langle r^2 \rangle_p + \langle r^2 \rangle_\kappa + \langle r_p^2 \rangle + \frac{N}{Z} \langle r_n^2 \rangle
$$

 $Q = p'$ - $p$  (電磁場の運動量)

 $q = 0, \kappa = -1.913$  for n

 $\psi_{\alpha}$ : single-particle wfn  $v_{\alpha}^2$ : occupation probability

Finite size of nucleon  $\langle r^2 P \rangle$  = 0.8409(4)<sup>2</sup> fm<sup>2</sup>  $\langle r^2 \rangle$  = -0.1155(17) fm<sup>2</sup>

https://pdglive.lbl.gov/

### **4. More Corrections CM** corrections and AMM effects on Chare Radius

**Neutron** squared radius

$$
N \langle r^2 \rangle_{n,corr} = \left\langle \sum_{i \in n} (r_i - R_G)^2 \right\rangle
$$
  
=  $\left\langle \sum_{i \in n} r_i^2 \right\rangle - \frac{1}{A} \left[ \left\langle \left( \sum_{i \in n} r_i \right)^2 \right\rangle - \left\langle \left( \sum_{i \in p} r_i \right)^2 \right\rangle \right] - \frac{Z}{A^2} \left\langle \left( \sum_{i=1}^A r_i \right)^2 \right\rangle$   
=  $N \left[ \langle r^2 \rangle_n + \Delta_n^{\text{(CM1)}} + \Delta_n^{\text{(CM2)}} \right]$ 

1- and 2-body parts of the CM correction:

$$
\Delta_n^{\text{(CM1)}} = -\frac{2}{A} \left\langle r^2 \right\rangle_n + \frac{1}{A} \left\langle r^2 \right\rangle_m \qquad \text{TM1 & TM2, Sugahara and Toki, NPA579, 557 (1994). \text{PK1 & PKDD, Long et al., PRC69, 034319 (2004). \text{PK1 & PKDD, Long et al., PRC69, 034319 (2004). \n
$$
\Delta_n^{\text{(CM2)}} = +\frac{1}{NA} \left[ \sum_{\alpha\beta\in n} \left( v_\alpha^2 v_\beta^2 - u_\alpha v_\alpha u_\beta v_\beta \right) \left| \langle \alpha | \mathbf{r} | \beta \rangle \right|^2 - \sum_{\alpha\beta\in p} \left( v_\alpha^2 v_\beta^2 - u_\alpha v_\alpha u_\beta v_\beta \right) \left| \langle \alpha | \mathbf{r} | \beta \rangle \right|^2 \right]
$$
\n
$$
+ \frac{Z}{N} \Delta_m^{\text{(CM2)}}
$$
$$

Note that 
$$
A\Delta_m^{\text{(CMi)}} = N\Delta_n^{\text{(CMi)}} + Z\Delta^{(\text{CMi})}
$$
.

#### **4. More Corrections CM** corrections and AMM effects on Chare Radius

#### YUSUKE TANIMURA AND MYUNG-KI CHEOUN

#### PHYSICAL REVIEW C 109, 054323 (2024)



FIG. 1. Contributions of each correction term to the MS charge radius for (a) He, (b) C, (c) O, (d) Ca, (e) Ni, (f) Zr, (g) Sn, and (h) Pb isotopes. Magenta triangles, the anomalous magnetic contribution  $\langle r^2 \rangle_{\kappa}$ ; sky-blue and purple squares, the one- and two-body CM corrections,  $\Delta_p^{\text{(CM1)}}$  and  $\Delta_p^{\text{(CM2)}}$ , respectively; black dots, the total correction  $\langle r^2 \rangle_{\kappa} + \Delta_p^{\text{(CM1)}} + \Delta_p^{\text{(CM2)}}$ . The gray bands show the size of experimental<br>uncertainty  $\Delta \langle r^2 \rangle_{ch} \in [(r_{ch} - \delta r_{ch})^2 - r_{ch}^2 : (r_{ch} + \delta r_{ch$ are taken from Refs. [1,2]. The vertical lines are drawn at  $N = 2$ , 8, 20, 28, 40, 50, 82, and 126.





### **Summary and Conclusion**

- 1. Our findings highlight the significance of SC in understanding **the OES observed in the Hg isotopes**.
- 2. Taking into account OD for most Hg isotopes, except for **181,183,185Hg**, by **the shape coexistence** is crucial for explaining the OES phenomenon.
- 3. But these three (**prolate-shaped**) odd nuclei are excluded from the SC and exhibit larger charge radii.
- 4. It may indicate that the OES can serve as a useful tool for identifying the SC in Hg isotopes, or vice versa.
- 5. **The nuclear shape transition and the evolution of OP by the additional neutron** in the range of 180-186Hg isotopes is essential for understanding the prominent OES observed in the Hg isotopes. The OES of Au has been calculated by DRHBc and reproduce the OES data.
- 6. Furthermore, the DFT models like DRHBc model successfully reproduces **the kink structure** observed around th
- 7. This kink structure arises from **the increase in the OPs of i\_{11/2} and 2 g\_{9/2} states**
- beyond the magic shell, influenced by the multi-particle and multi-hole interactions.
- 8. **This swelling effect of symmetric core** pushes the proton states to the outer nuclear surface by the symmetry energy residing on the symmetric core surface.
- 9. The second order CM correction may cancel the 1<sup>st</sup> order CM correction, so that the SO effect from ANM moment becomes meaningful.
- LEES2024, Oct.28-Nov. 1 2024 10. It would be interesting to further investigate the OES and the kink structures of charge radii and matter radii in Ca, K, and Pb isotopes **in the vicinity of each magic shell** using the DFT models in future studies. 35





### **Remarks and Conclusion**

- 1. Other corrections, such as **CM correction and ANM contribution** are also the remained quantities in the charge radii study.
- 2. **Charge symmetry breaking** is another interesting topic which affects the charge radii and/or the density distribution in nuclei. (Y. Tanimura Talk)
- 3. Matter effects, such as effective masses, **symmetry energy and equation of state**, are to be discussed in detail in the future.
- 4. Along this line, the study of **NST in the experiment (LEES)** could give valuable information for charge radii study of unstable nuclei.
- 5. Laser spectroscopy for the isotope shift from the hyperfine interaction and LEE scattering could complement the charge radii and density distribution (**proton as well as neutron density**) study. (Xiaofei Yang Talk and LEE scattering talks)





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# Thanks for your attention !!



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