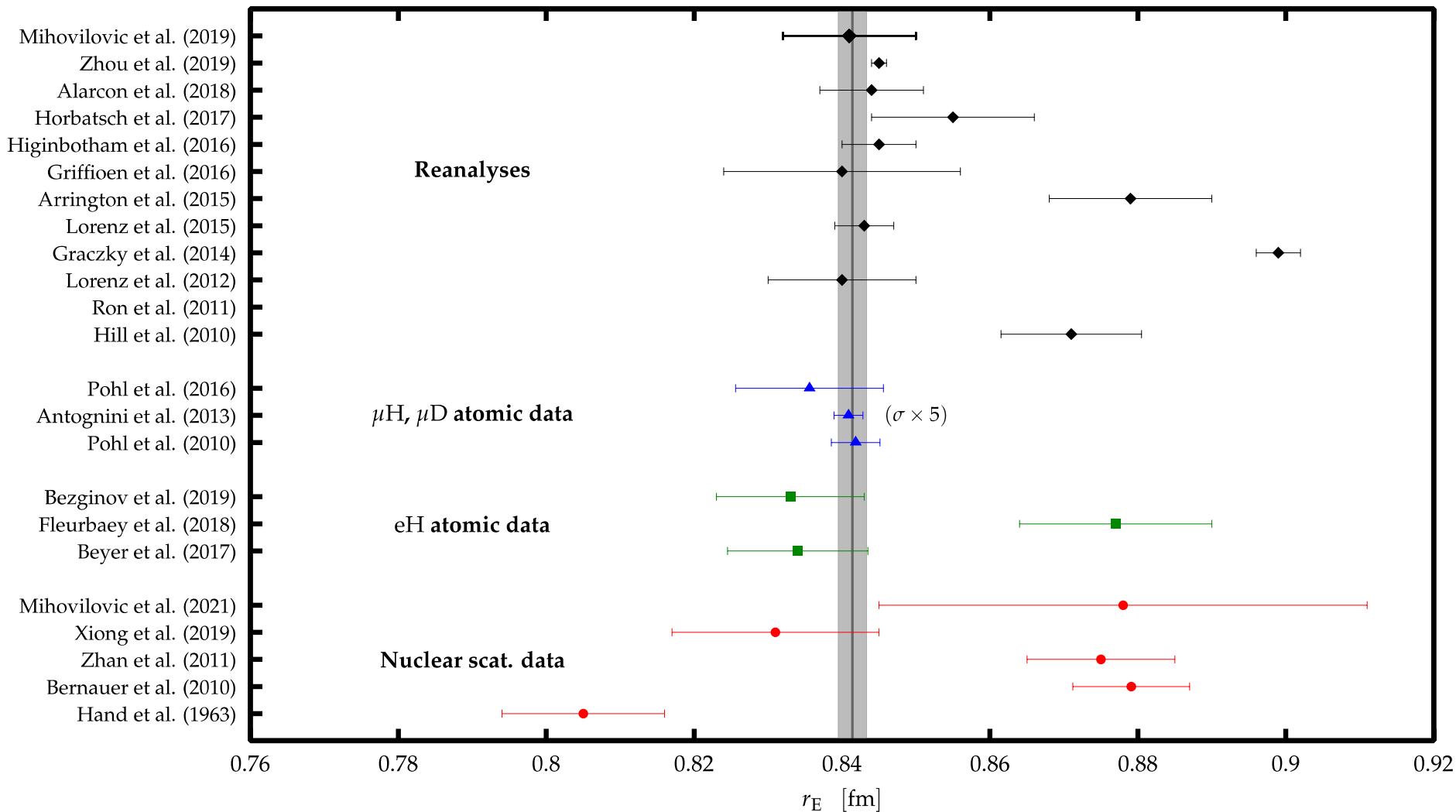


# Proton Charge Radius from Electric Form Factor Measurements at Low $Q^2$

Miha Mihovilovic for A1 Collaboration  
U Ljubljana and JSI

# Motivation - The proton radius problem

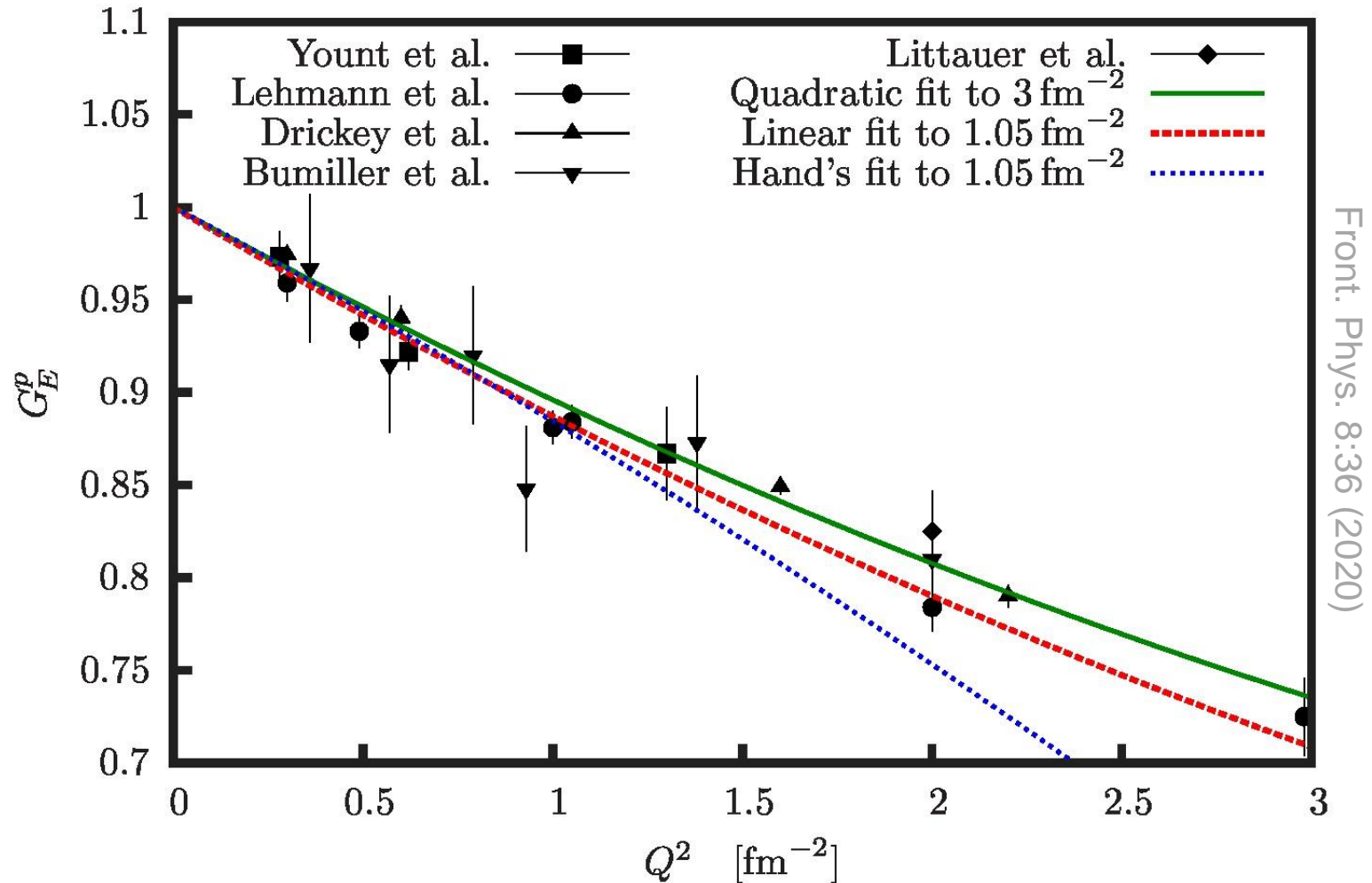
- The “Proton radius puzzle” - a  $6\sigma$  discrepancy in the  $r_p$  measurements.



# Hypotheses for competing $r_p$ values

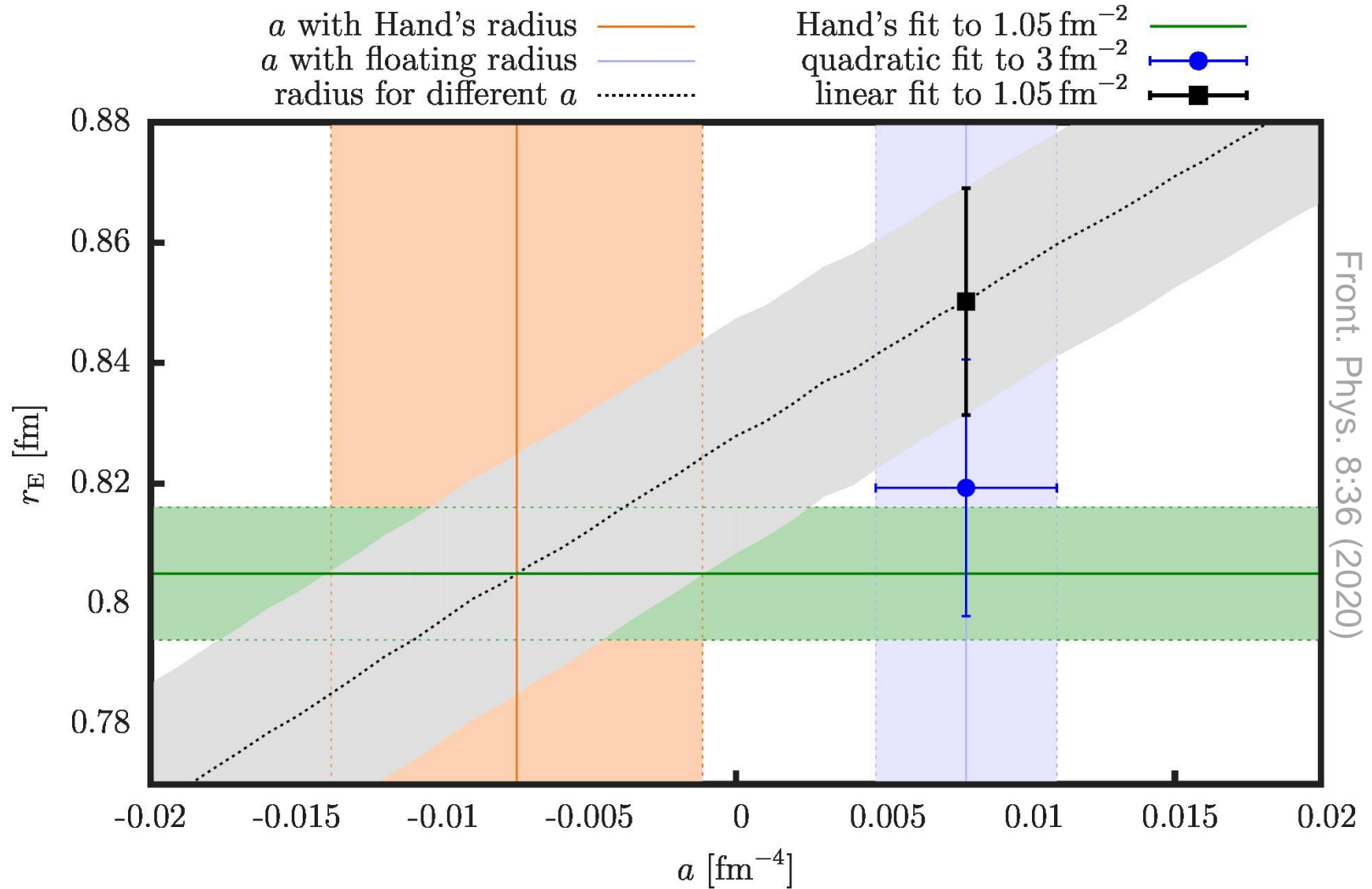
- Inconsistent experimental results:
  - different  $Q^2$  ranges of data.
  - different experimental uncertainties.
  - hidden systematics (luminosity determination).
  - hidden backgrounds (cryogenic depositions).
  
- Differences in the interpretation of experimental results:
  - not knowing the true functional form for  $G_e^p$ .
  - use of different models (model bias) .
  - incomplete models (neglected contributions of higher-order moments).
  - inconsistent use of experimental data (different  $Q^2$  ranges).
  - ignored model-dependent relative normalizations between data.

# Reexamination of first extraction of $r_p$



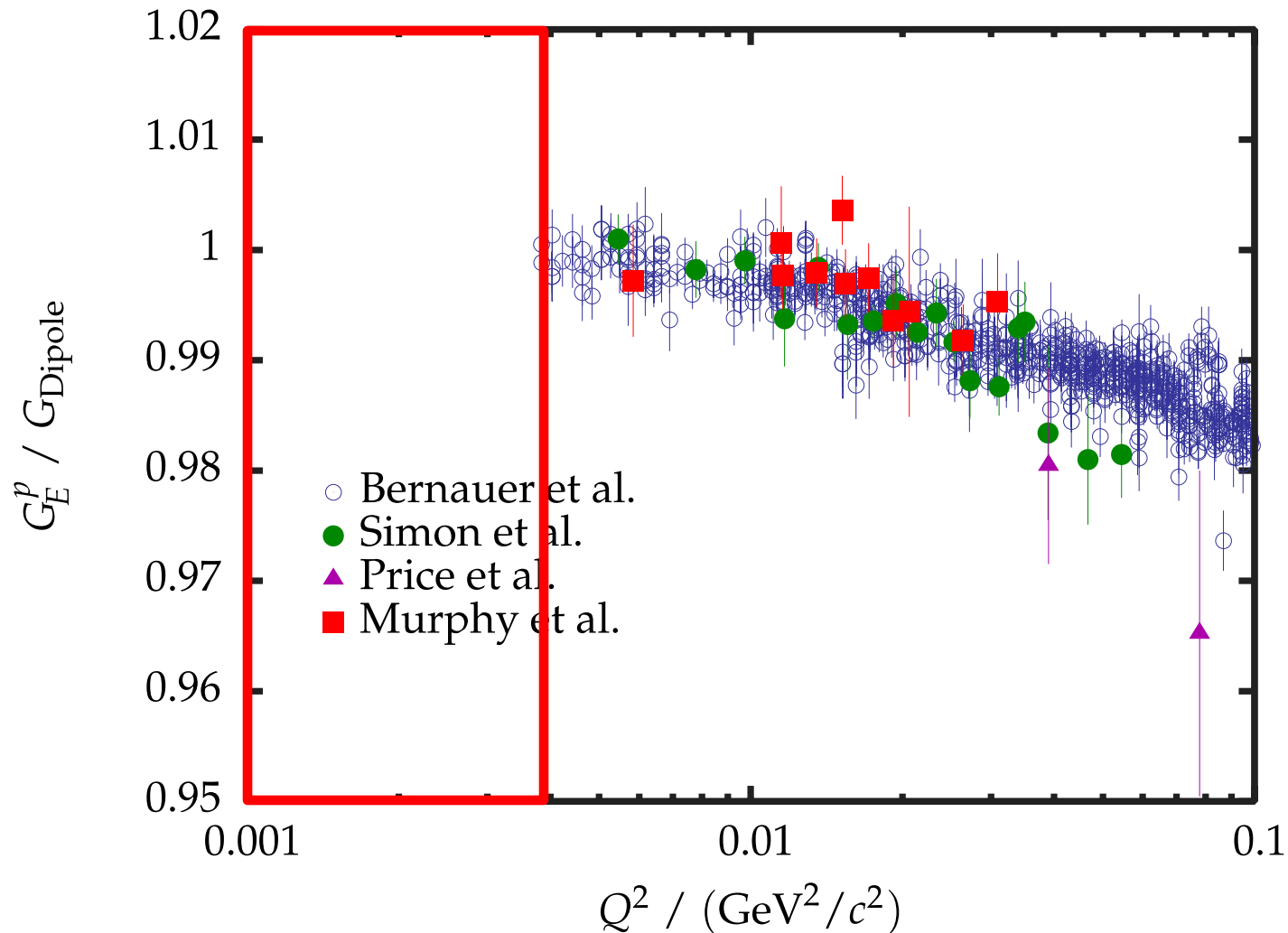
- First determination of proton charge radius done by Hand in 1963.
- Two step fitting technique was applied: quadratic fit up to  $3 \text{ fm}^{-2}$ , linear fit up to  $1 \text{ fm}^{-2}$

# Reexamination of first extraction of $r_p$



- Mistake in an analysis led to a smaller value for the radius.
- Reanalysis of original measurements gives results consistent with CODATA '18.

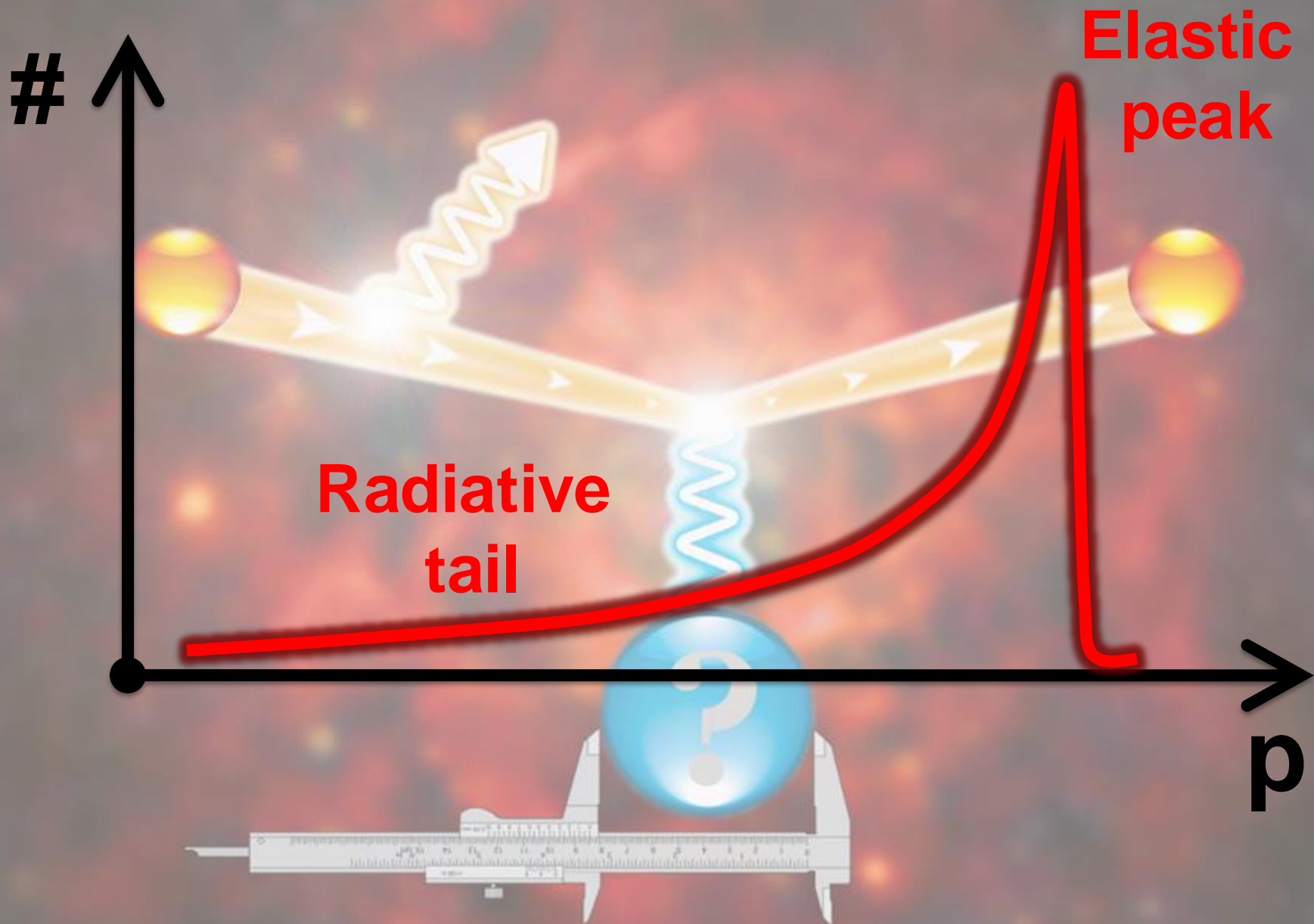
# Proton's charge form-factor



- **In 2013 data available only for  $Q^2 > 0.004 \text{ (GeV/c)}^2$ .**
- More data at even smaller  $Q^2$  needed!



# The idea of ISR Experiment

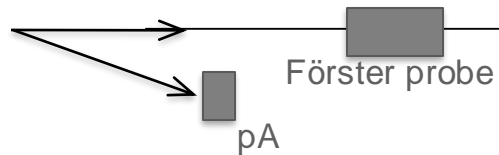


# The ISR experiment

- Full experiment done in August 2013 + additional beam time in 2017.

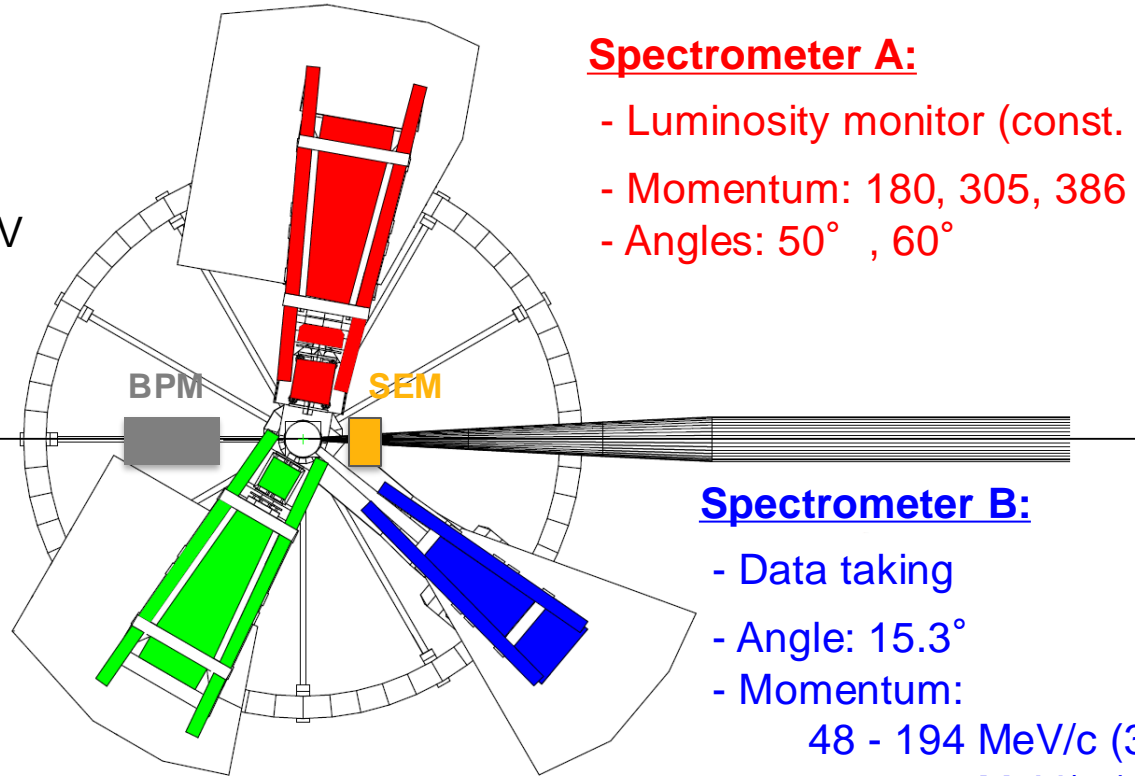
## Electron Beam:

- Energy: 195, 330, 495 MeV
- Current: 10nA – 1 $\mu$ A
- Rastered beam



## Luminosity monitors:

- pA-meter
- Förster probe
- **SEM**



## Spectrometer A:

- Luminosity monitor (const. setting)
- Momentum: 180, 305, 386 MeV/c
- Angles: 50° , 60°

## Spectrometer B:

- Data taking
- Angle: 15.3°
- Momentum:
  - 48 - 194 MeV/c (35 setups)
  - 156 - 326 MeV/c (12 setups)
  - 289 - 486 MeV/c (9 setups)

## Spectrometer C:

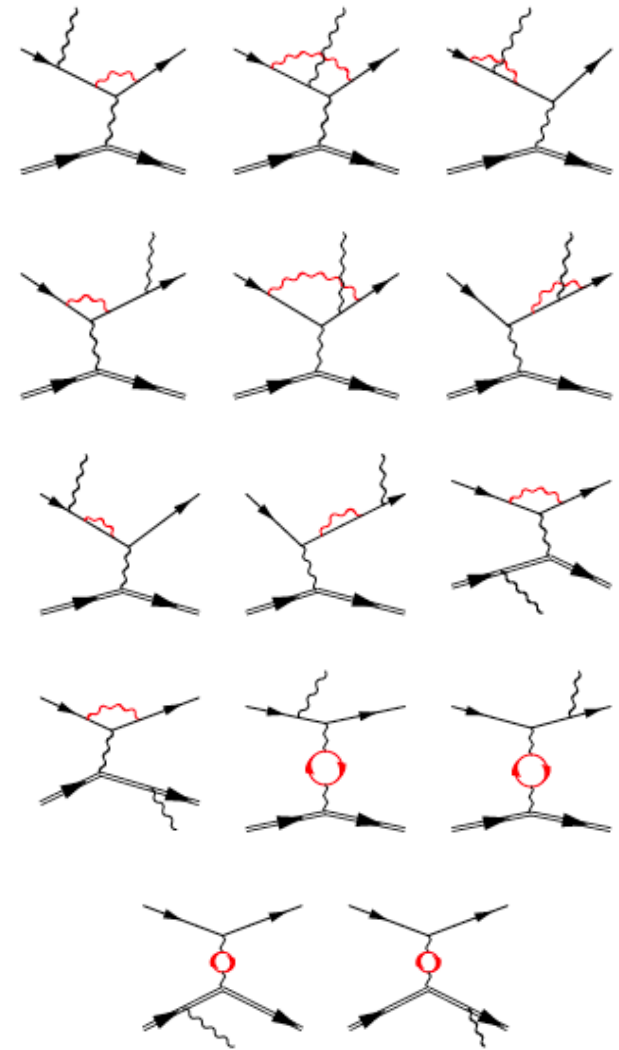
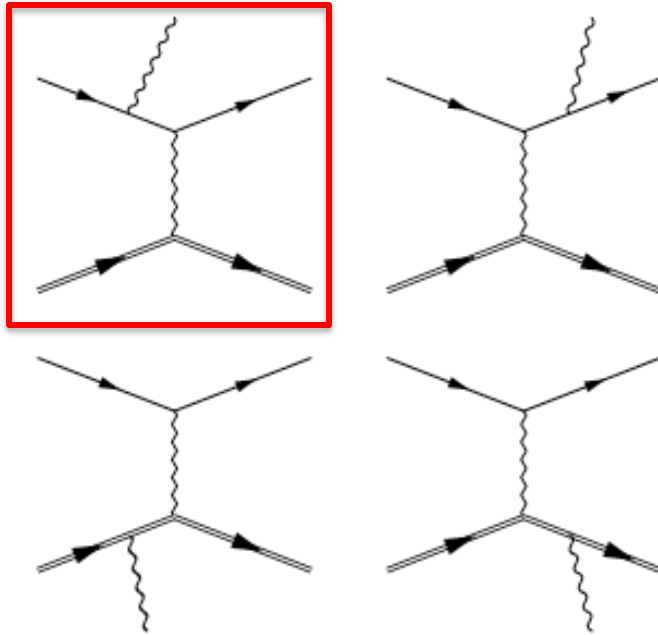
- Not used



# The ISR Simulation

- Based on standard A1 framework.
- Detailed description of apparatus.
- Exact calculation of the leading order diagrams:

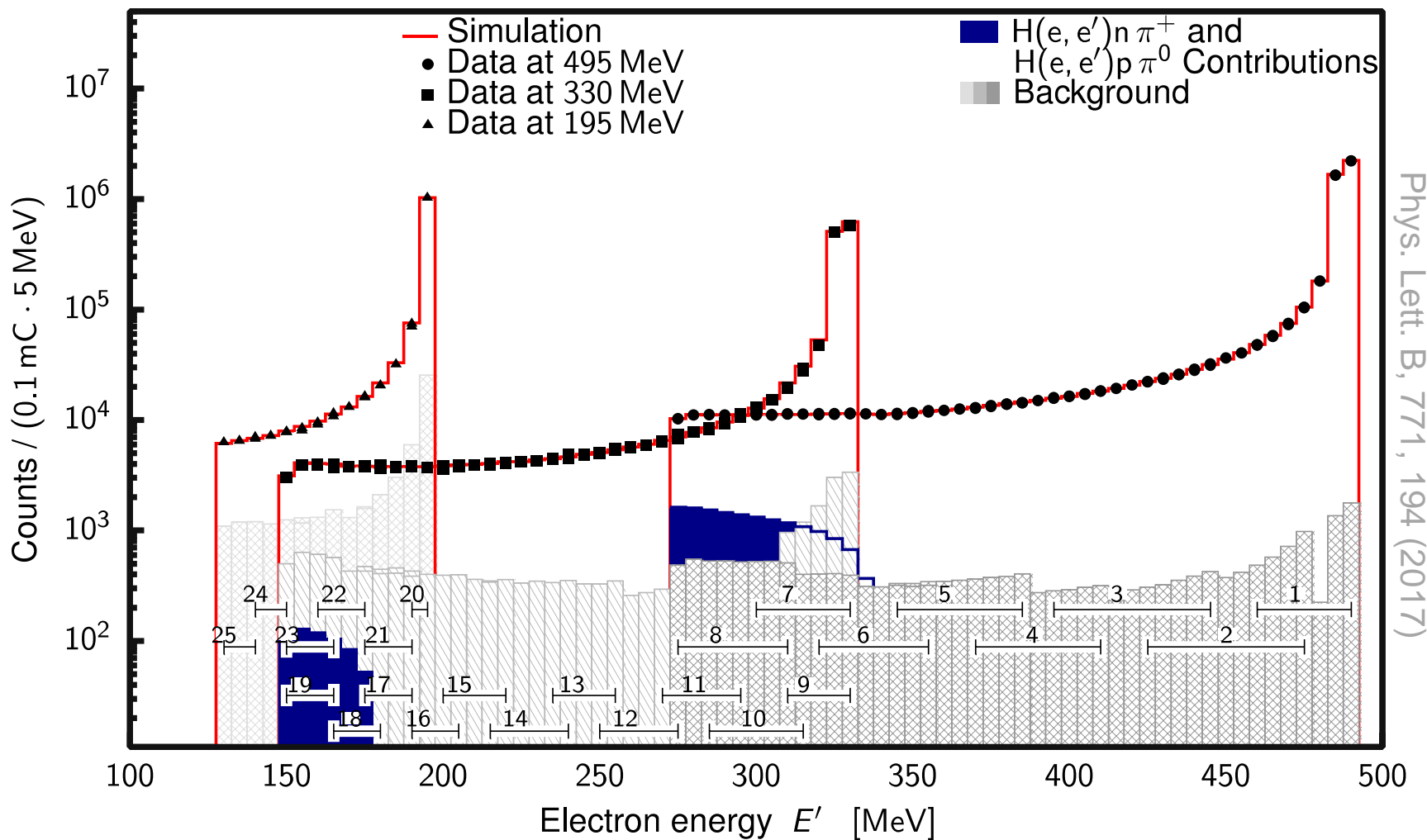
ISR



- The NL-order **virtual and real** corrections included via effective corrections to the cross-section.

# Results

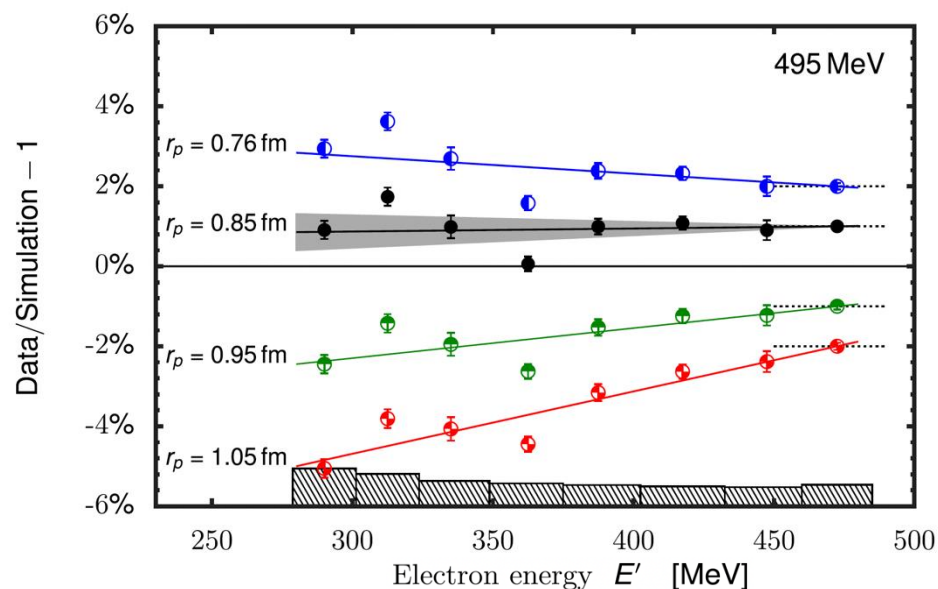
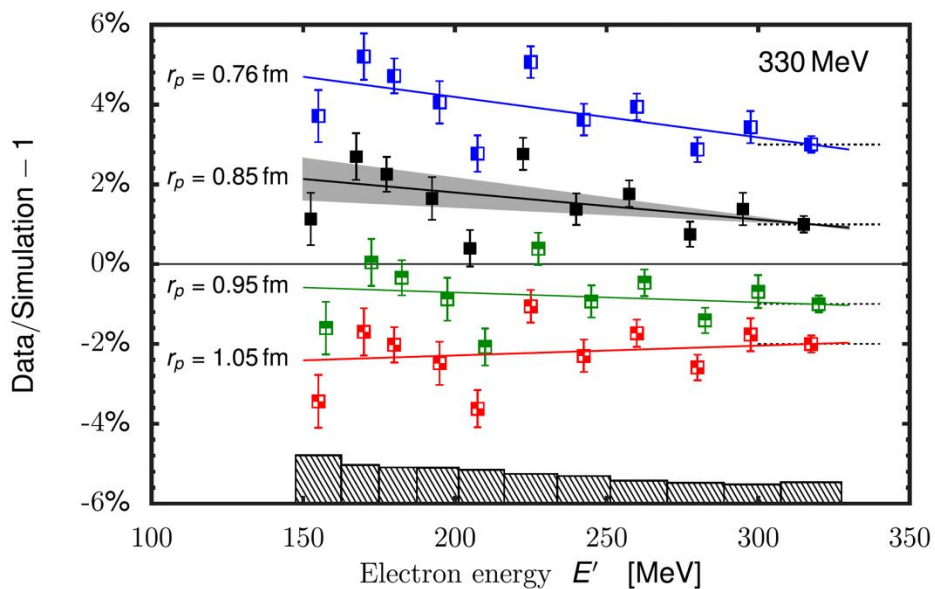
- Existing apparatus limited reach of ISR experiment to  $E' \sim 130$  MeV.



# Analysis of cross-sections

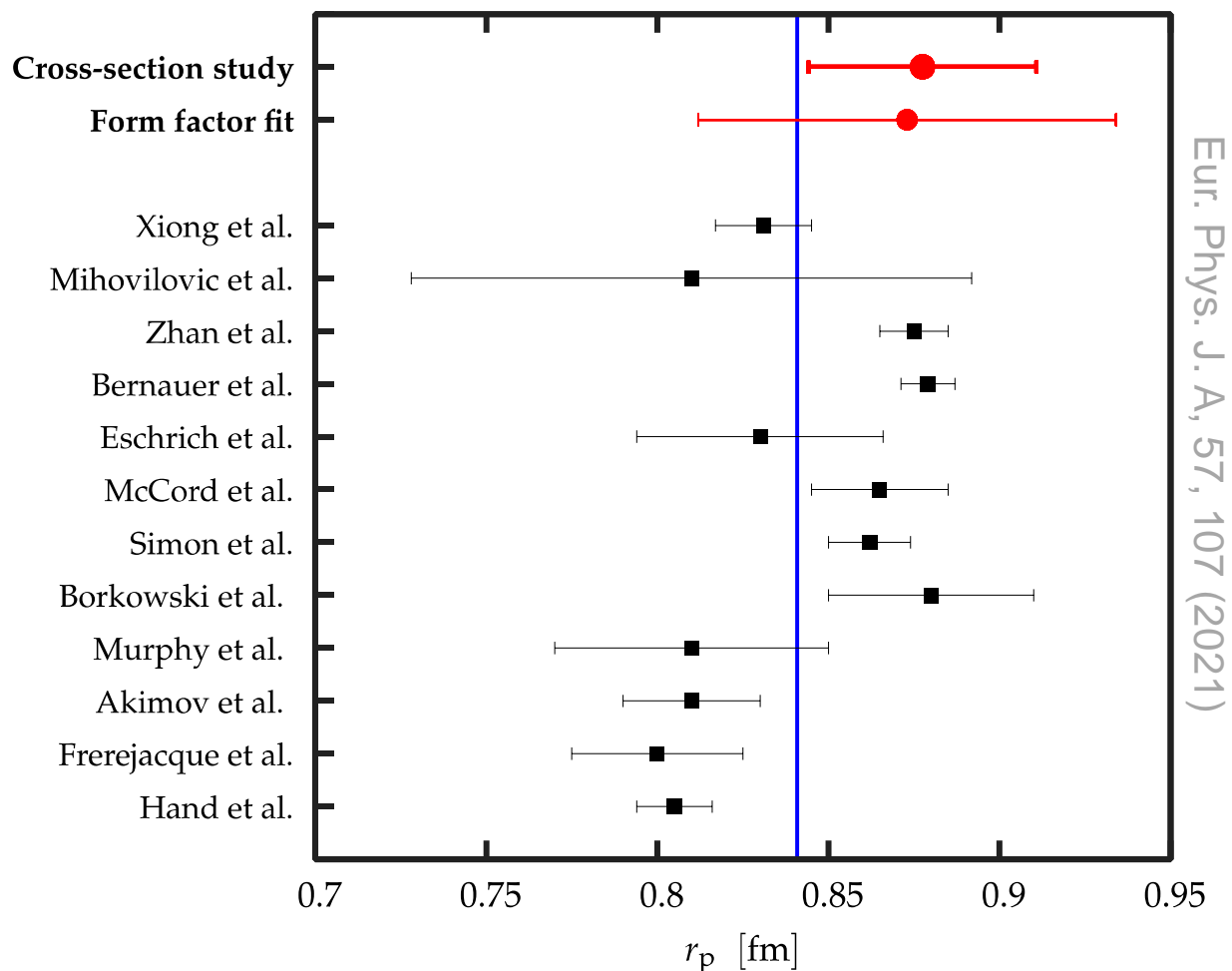
- Determination of the radius directly from the measured cross-sections.
- Small-energy data less sensitive to radius. 195 MeV data excluded.
- Analysis based on a specific form factor model.

$$G_E^p(Q^2) = n \left( 1 - \frac{r_p^2}{6 (\hbar c)^2} Q^2 + \frac{a}{120 (\hbar c)^4} Q^4 - \frac{b}{5040 (\hbar c)^6} Q^6 \right)$$

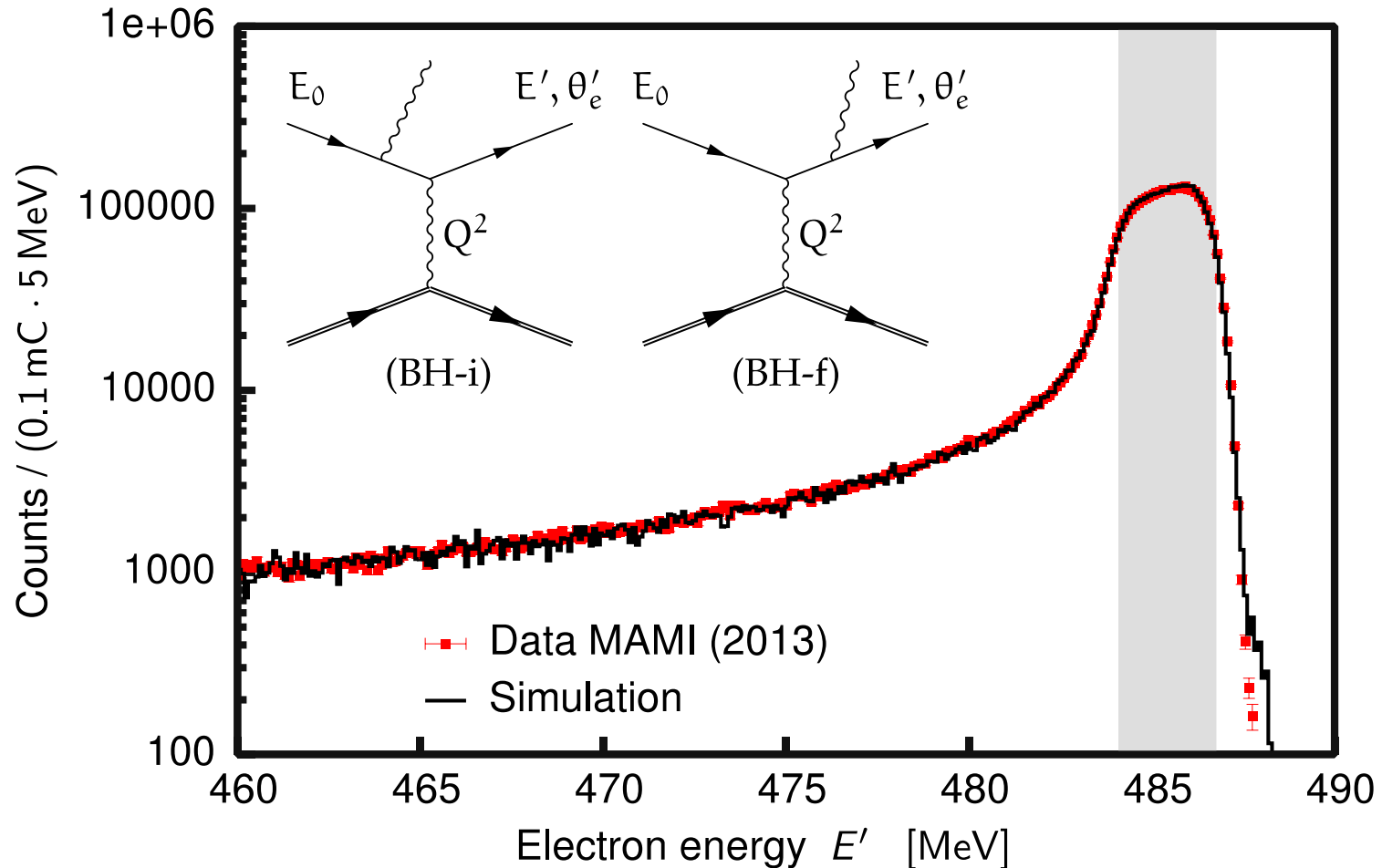


# The result of the ISR experiment

- The values from the direct analysis of cross-sections and fit of extracted form-factor.
- Uncertainty combines statistical and systematic uncertainty.



# Knowing the radiative corrections



- Understanding the radiative corrections to elastic peak at the level of 1% relevant many future experiments also with other targets (*Eur. Phys. J. A*, 59, 225 (2023)).

# Proton radius with Kalman Filtering (KF)

---

- We are interested in proton charge radius, not so much in  $G_E^p$ .
- KF is an alternative approach to determining the proton charge radius.
- We want to estimate  $r_p$  from many available measurements by relying on a dynamical model that dictates the  $Q^2$  dependence of the  $G_E^p$  to get a reliable estimate for the radius that is closest to the real value.
- **The applied form-factor model does not need to be "the correct" model, an approximate model is enough.**
- Kalman filtering is an iterative approach.
- Works with (only) linear problems and assumes normally distributed uncertainties.



# Kalman filtering - Model

- The KF was run with a third-order polynomial model:

$$G_E^p(Q^2) = n \left( 1 - \frac{r_p^2}{6 (\hbar c)^2} Q^2 + \frac{a}{120 (\hbar c)^4} Q^4 - \frac{b}{5040 (\hbar c)^6} Q^6 \right)$$

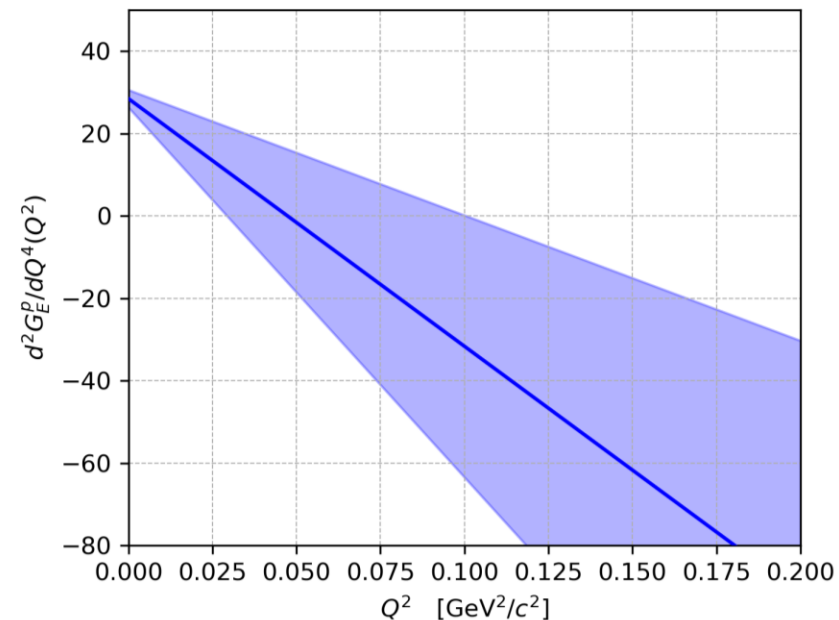
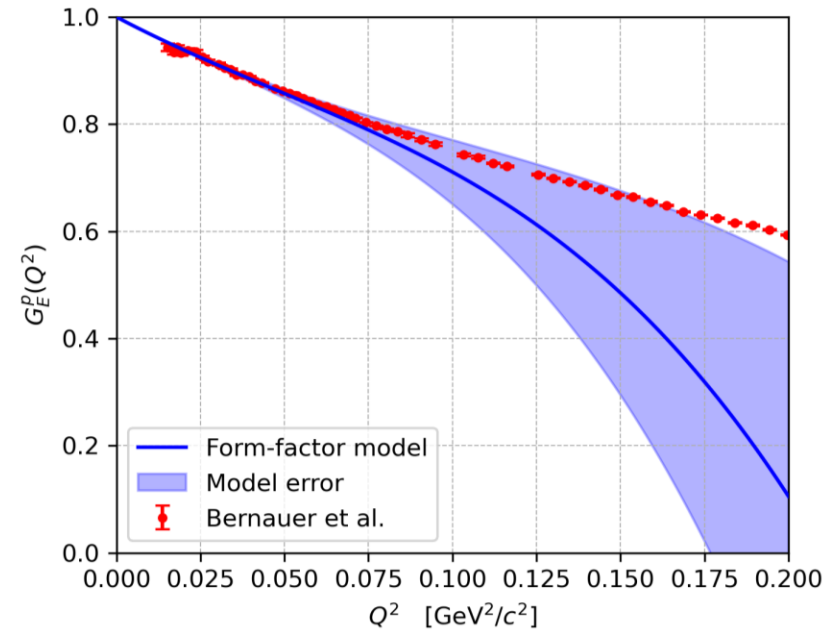
- Higher moments fixed from literature:

$$a = (2.59 \pm 0.194) fm^4$$

$$b = (29.8 \pm 14.71) fm^6$$

- Model fails  $Q^2 \geq 0.1 (GeV/c)^2$ .
- Model enters KF in a form of a second-derivative:

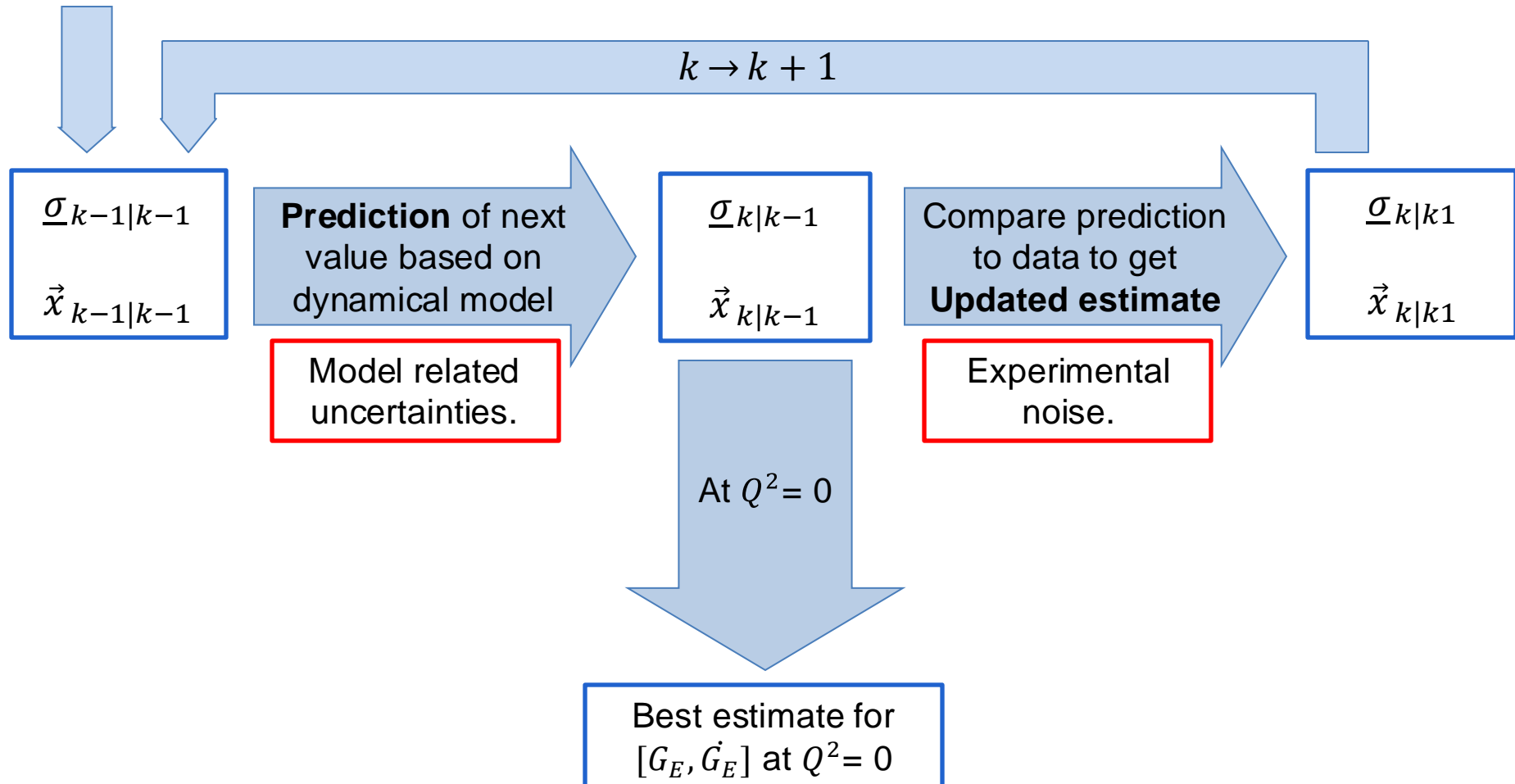
$$\frac{d^2 G_E^p}{d(Q^2)^2} = \frac{a}{60 (\hbar c)^4} - \frac{b}{840 (\hbar c)^6} Q^2$$



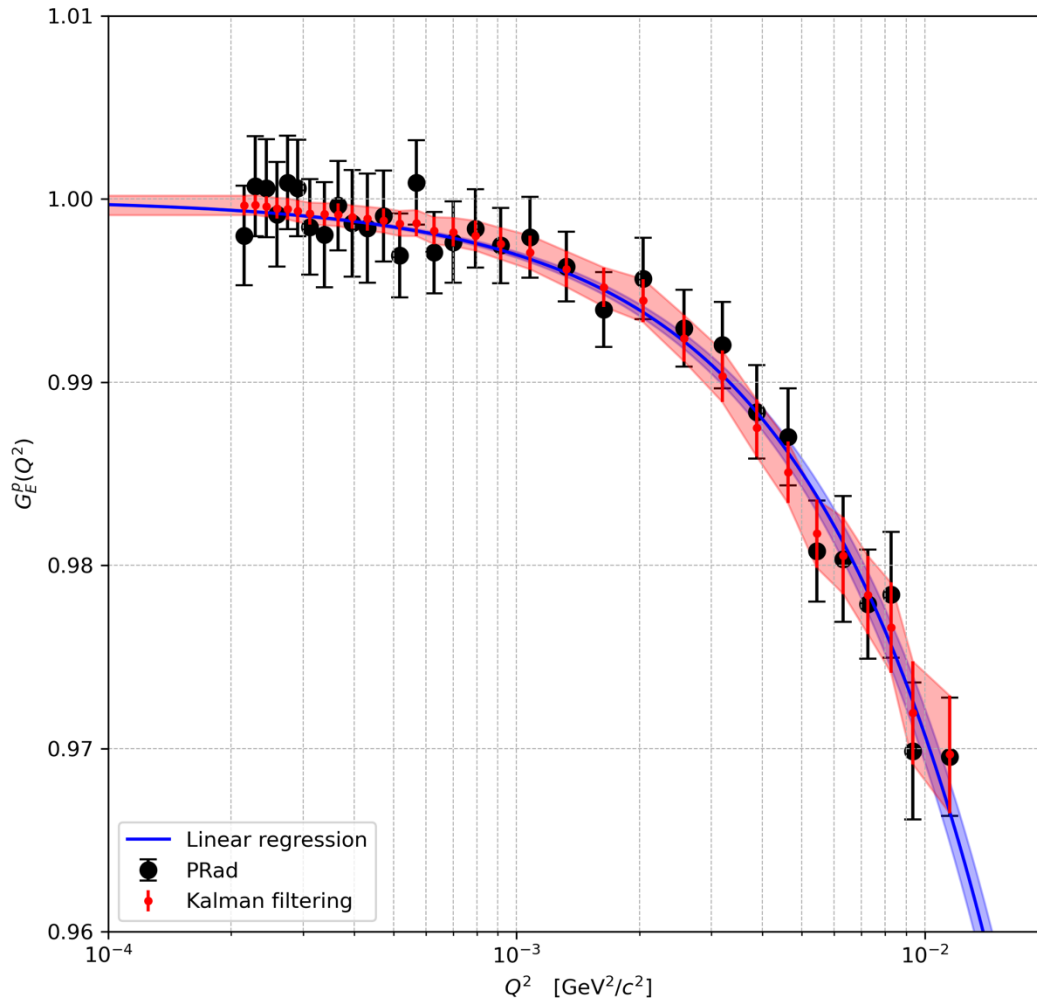
# Proton radius with Kalman filtering #2

Initial estimate of model parameters

$\overrightarrow{x(Q^2)} = [G_E, \dot{G}_E]$  and covariance matrix  $\underline{\sigma}$  at the highest value of  $Q^2$ .

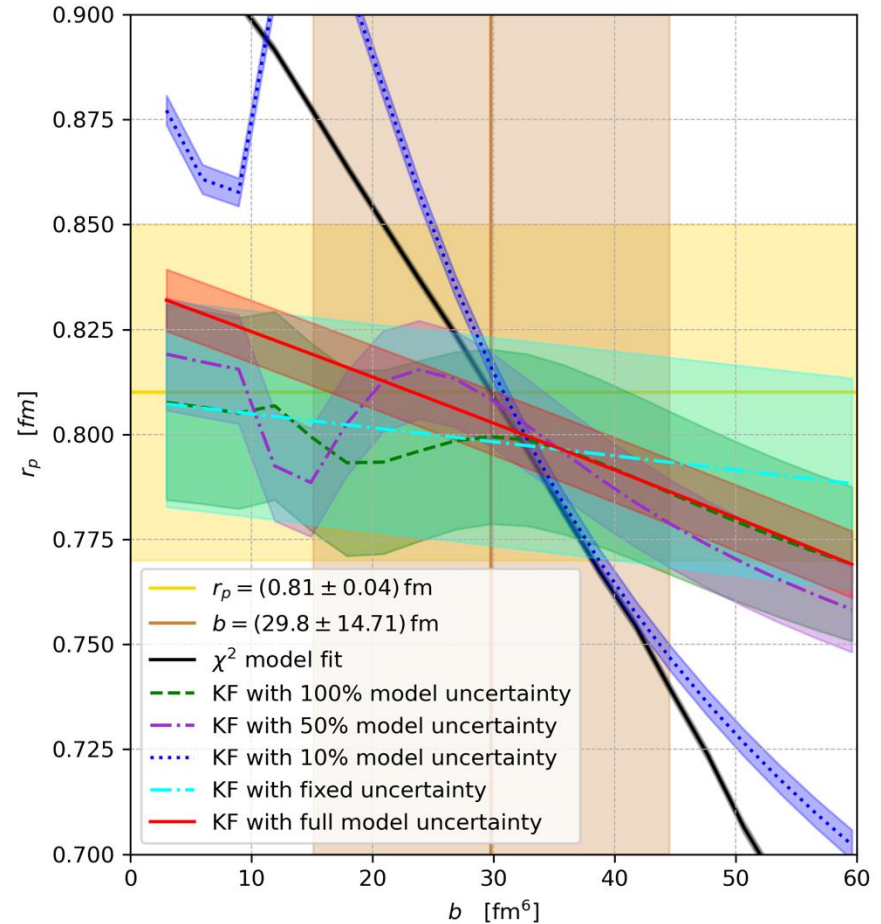
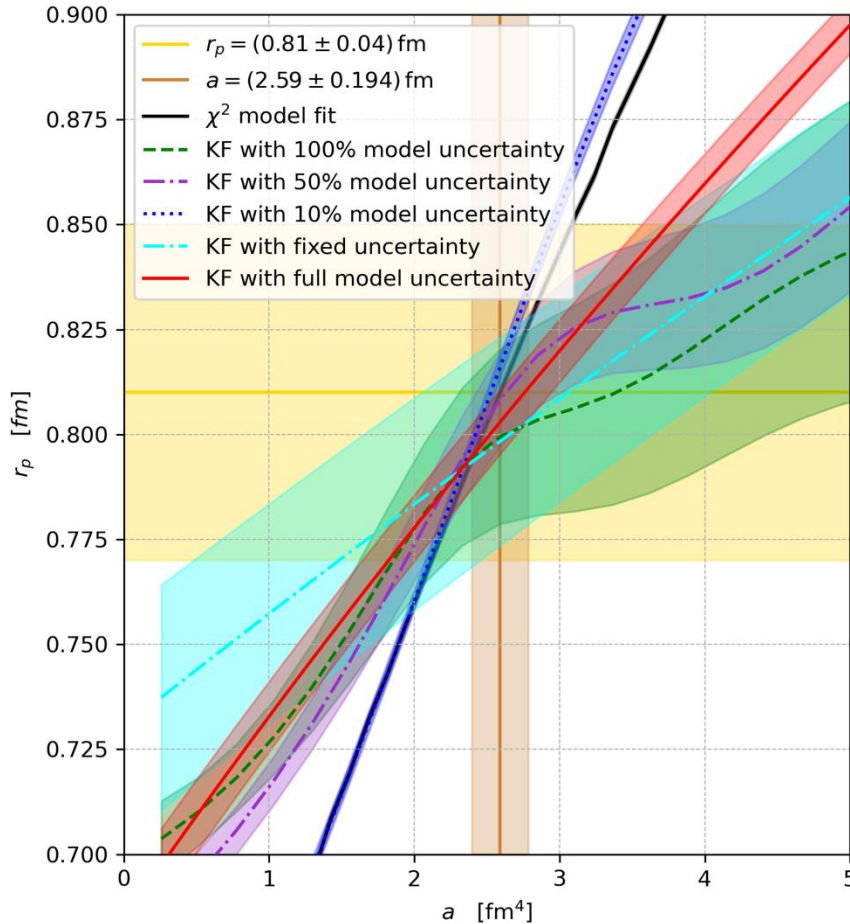


# Kalman filtering - Estimates



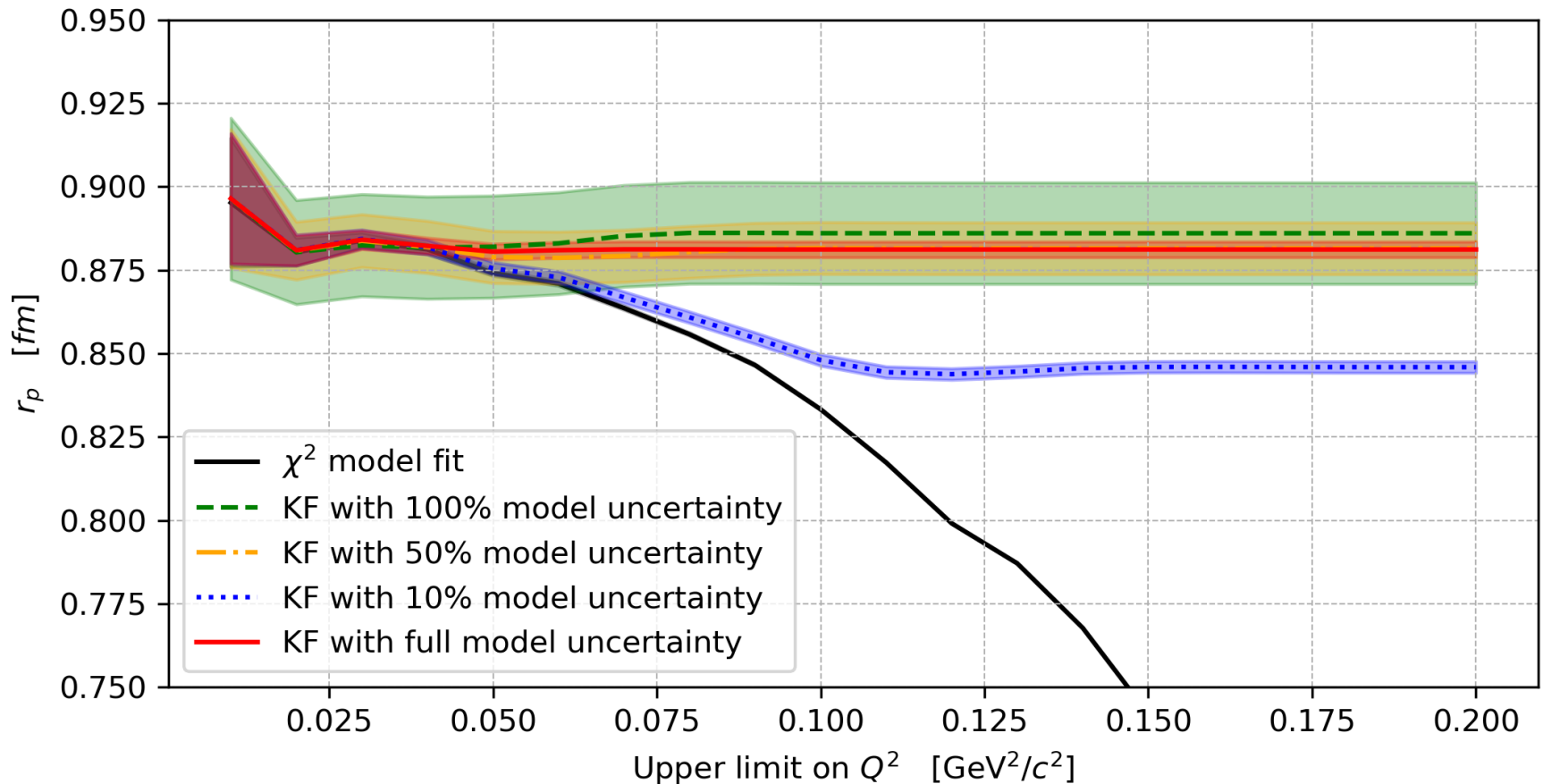
- KF operates as a MSE minimizer.
- In each step KF compares predicted values with the measurements and gives more weight to a more precise value.
- In recursive steps algorithm finds general (smooth) trend through the data to get best estimates of the two open parameters ( $n$ ,  $r_p$ ).

# Kalman filter – Model dependence



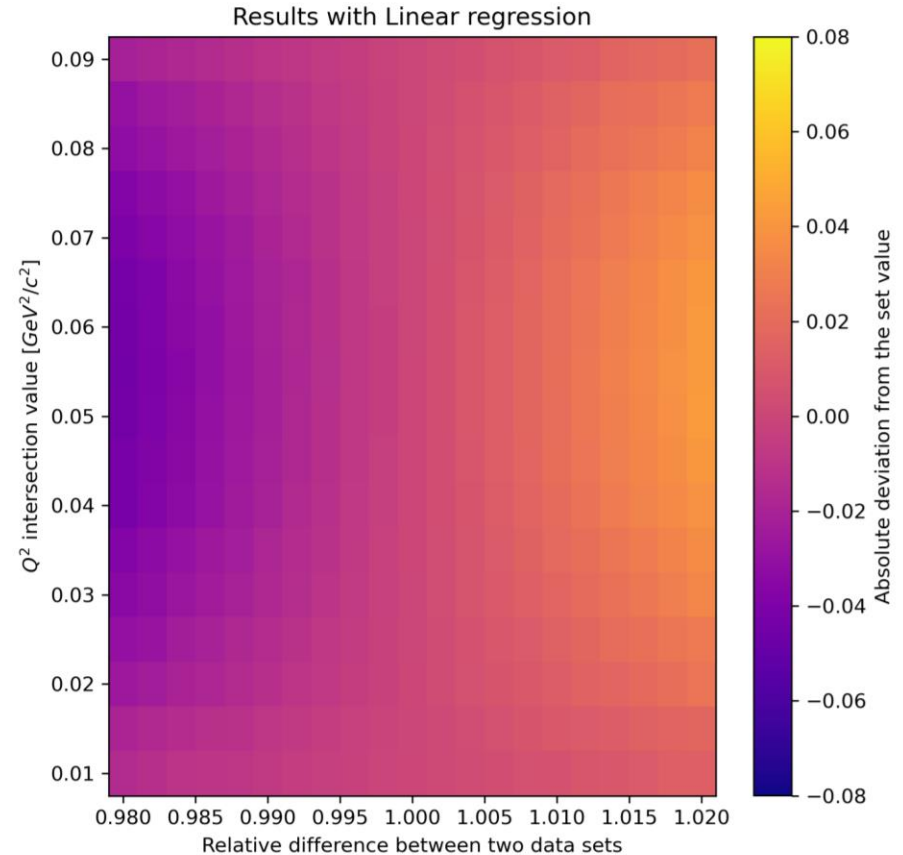
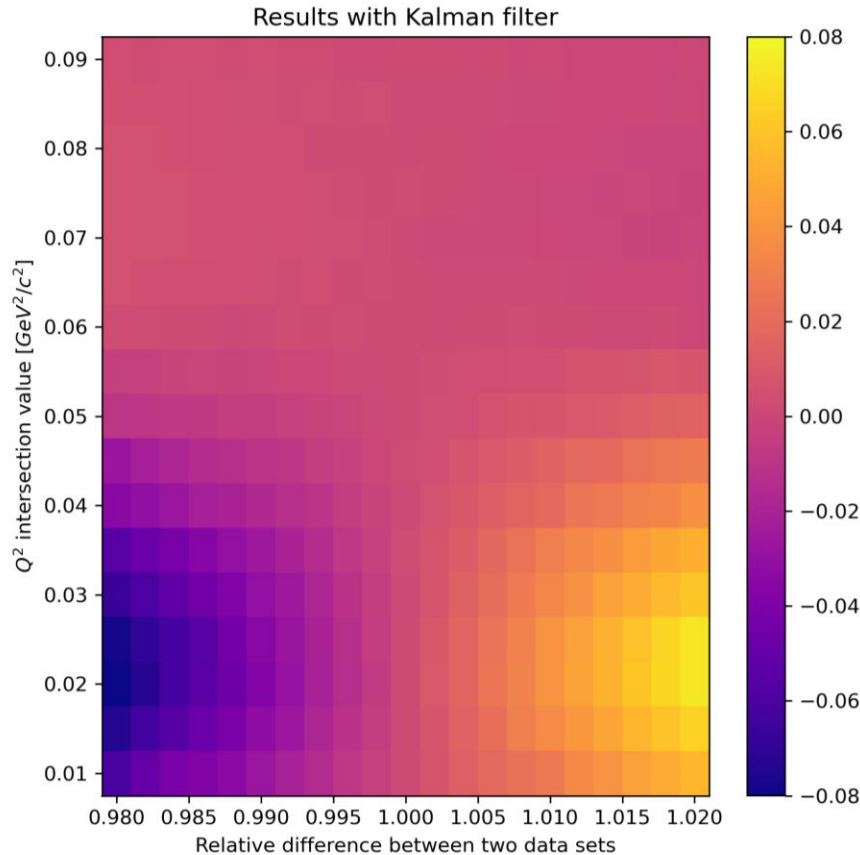
- Model dependence was tested with pseudo-data based on a Polynomial model.
- **Recursive nature of KF reduces the model bias.**
- Smaller model uncertainty brings KF results closer to the results of linear regression.

# Kalman filter – $Q^2$ running



- Results of linear regression strongly depend on the  $Q^2$  range considered in the fit and tend to be biased towards smaller value of  $r_p$ .
- Results of KF avoid bias related to the upper  $Q^2$  cut of available data.**

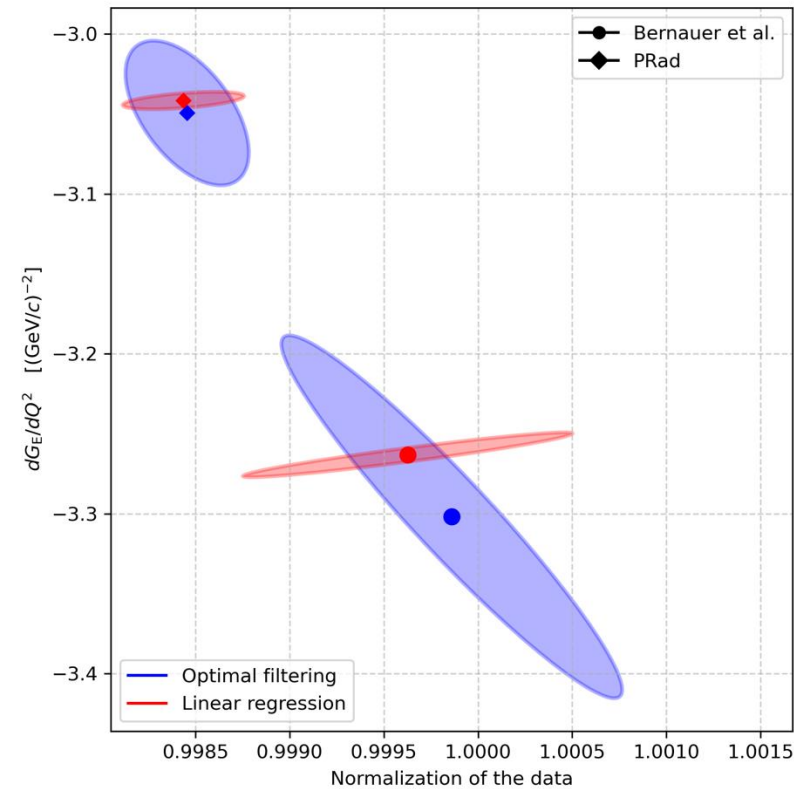
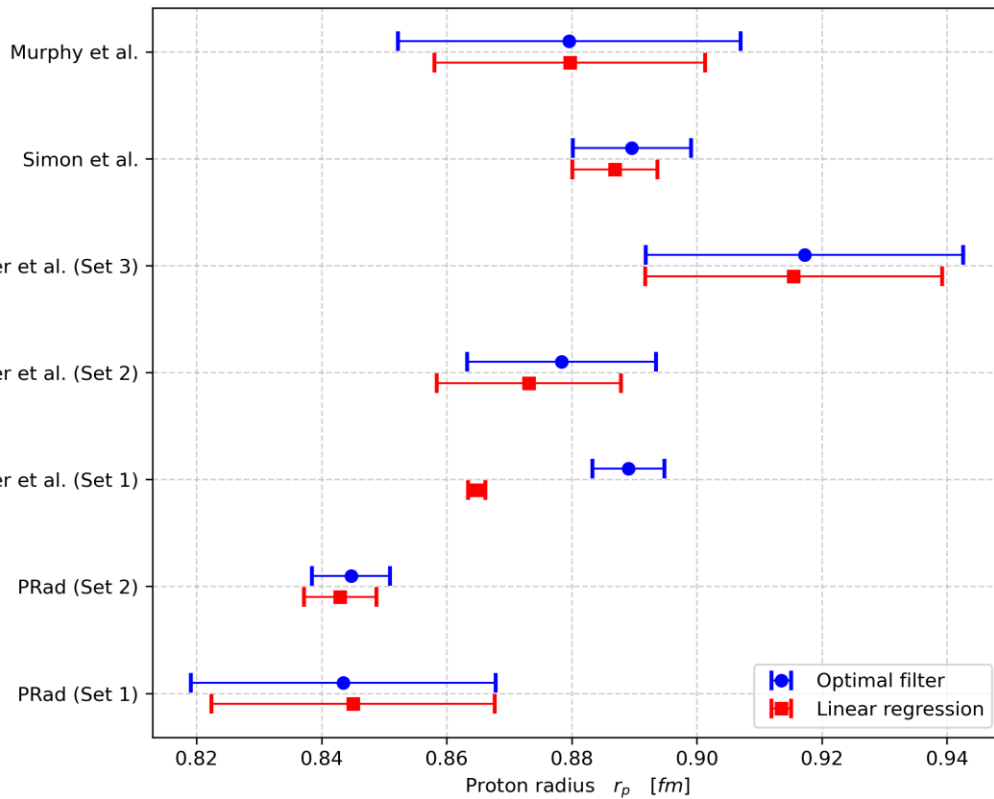
# Kalman filter – Floating Normalizations



- Incorrect consideration of relative normalizations between data can significantly bias results of linear regression.
- **Results of KF avoid this bias if data quality is sufficient for algorithm to correct for the discrepancy between data sets.**



# Kalman filter - Results



- KF of selected data-sets reproduces original results of Simon, Bernauer and PRad.
- **A self-consistent KF analysis indicates discrepancy between data-sets and motivates further experimental verification of existing FF measurements.**
- KF mostly consistent with linear regression but with different parameter correlations.

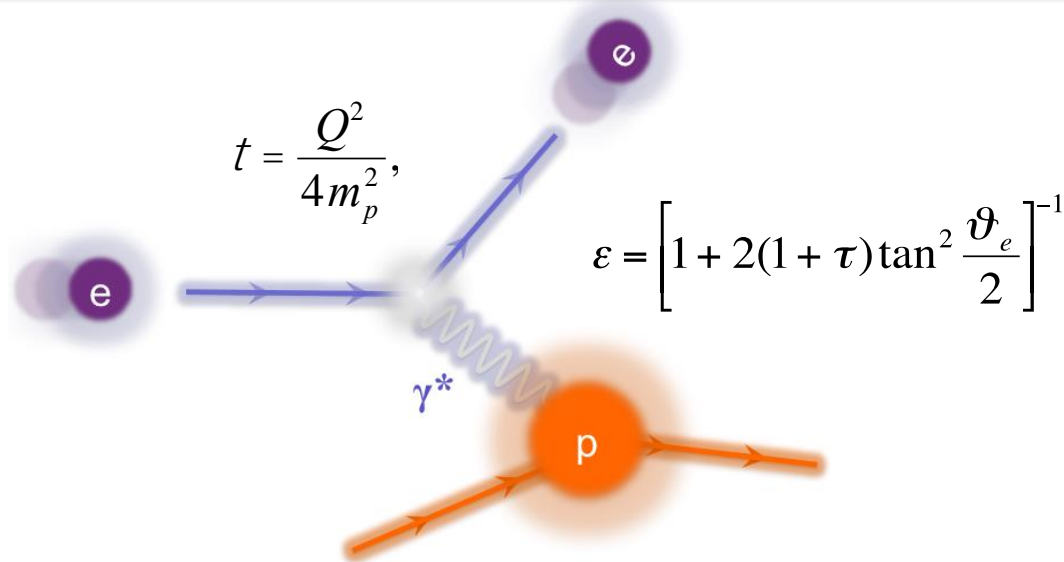
# Summary

---

- Still facing competing values of proton charge radius.
- **Discrepancy due to inconsistent experimental results and ambiguities in the interpretation of available data.**
- The ISR experiment used a new experimental technique for determination of the proton form-factors at very small  $Q^2$ .
- ISR validated radiative corrections far away from elastic settings.
- We need to find consensus on how to fit / interpret the nuclear scattering data.
- **KF as an alternative analysis approach for more robust determination of the proton charge radius.**
- Further measurements are needed – **Magix experiment with Hypersonic gas jet (or plastic) target!**

**Thank you!**

# Radius via Cross-section measurement



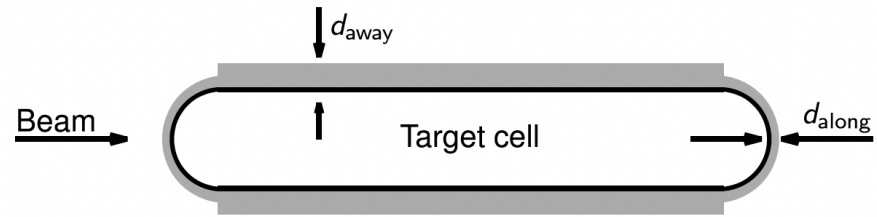
$$\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{Mott} \frac{1}{1 + \tau} \left[ G_E^2(Q^2) + \frac{\tau}{\epsilon} G_M^2(Q^2) \right]$$

- Extraction of FF via Rosenbluth Separation.
- Best estimate for radius:

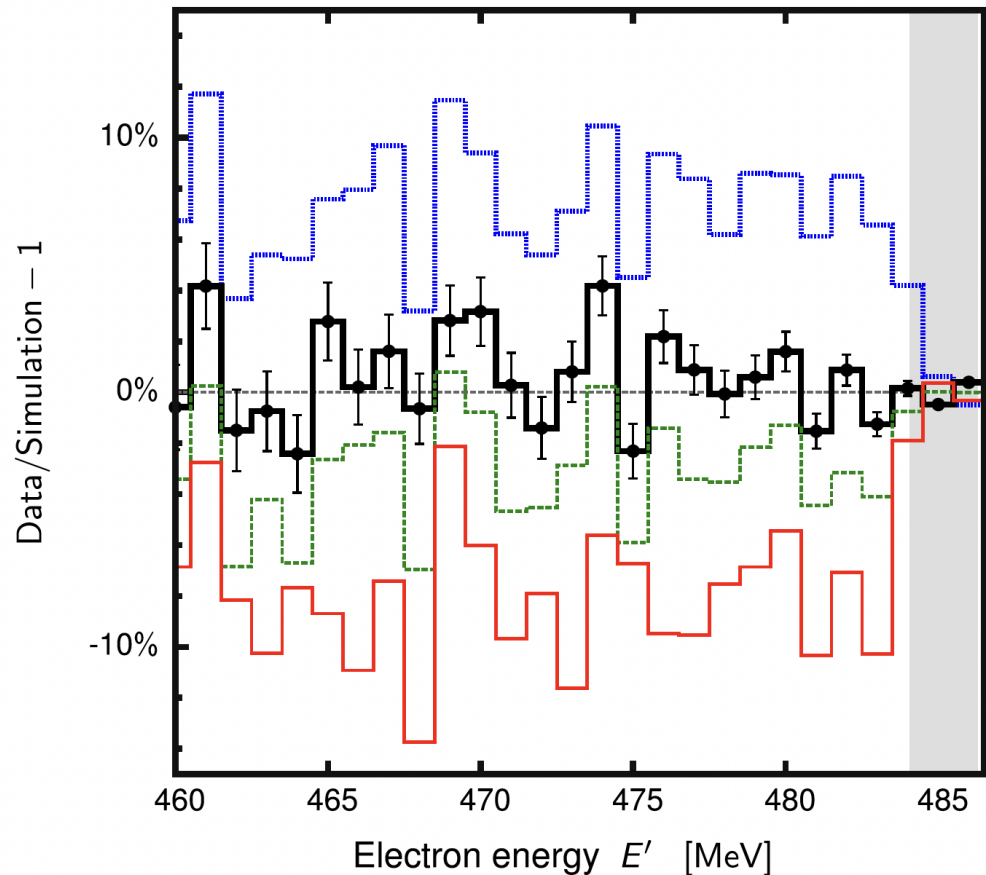
$$r_E^2 = -6\hbar^2 \left. \frac{d}{dQ^2} G_E(Q^2) \right|_{Q^2=0}$$

# Shortcomings of Cryogenic target

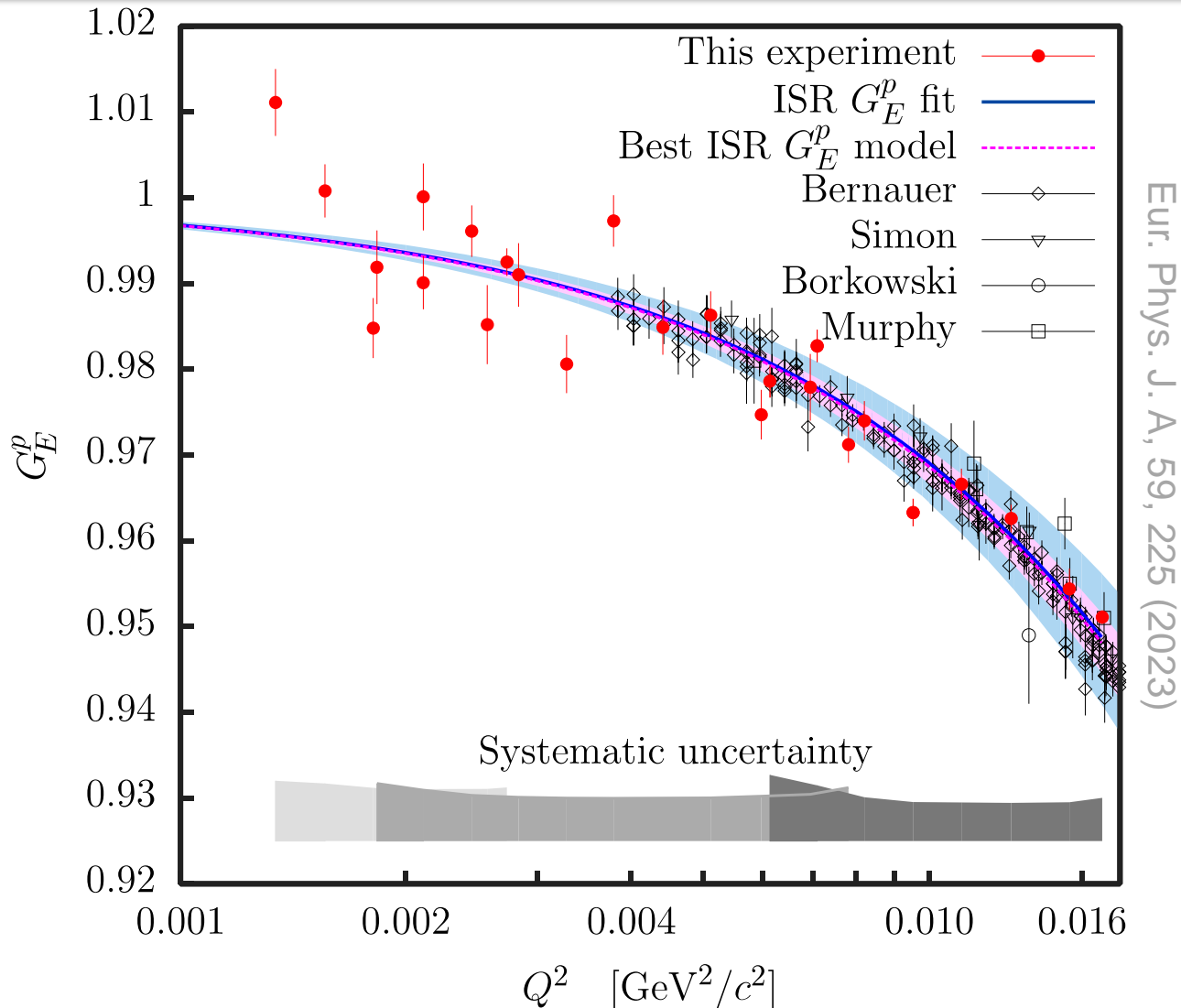
- Employed an extended cryogenic target.
- Backgrounds from target walls and supporting frame.
- Spectra distorted due to cryogenic depositions on the walls.
- Cryogenic layer on the sides much thicker than in the beam direction. **No control over the thickness of the layer.**



Legend for the plot:  
- Blue dotted line:  $d_{away} = d_{along}$   
- Black solid line:  $d_{away} = 200 \cdot d_{along}$   
- Green dashed line:  $d_{away} = 400 \cdot d_{along}$   
- Red solid line:  $d_{away} = 600 \cdot d_{along}$



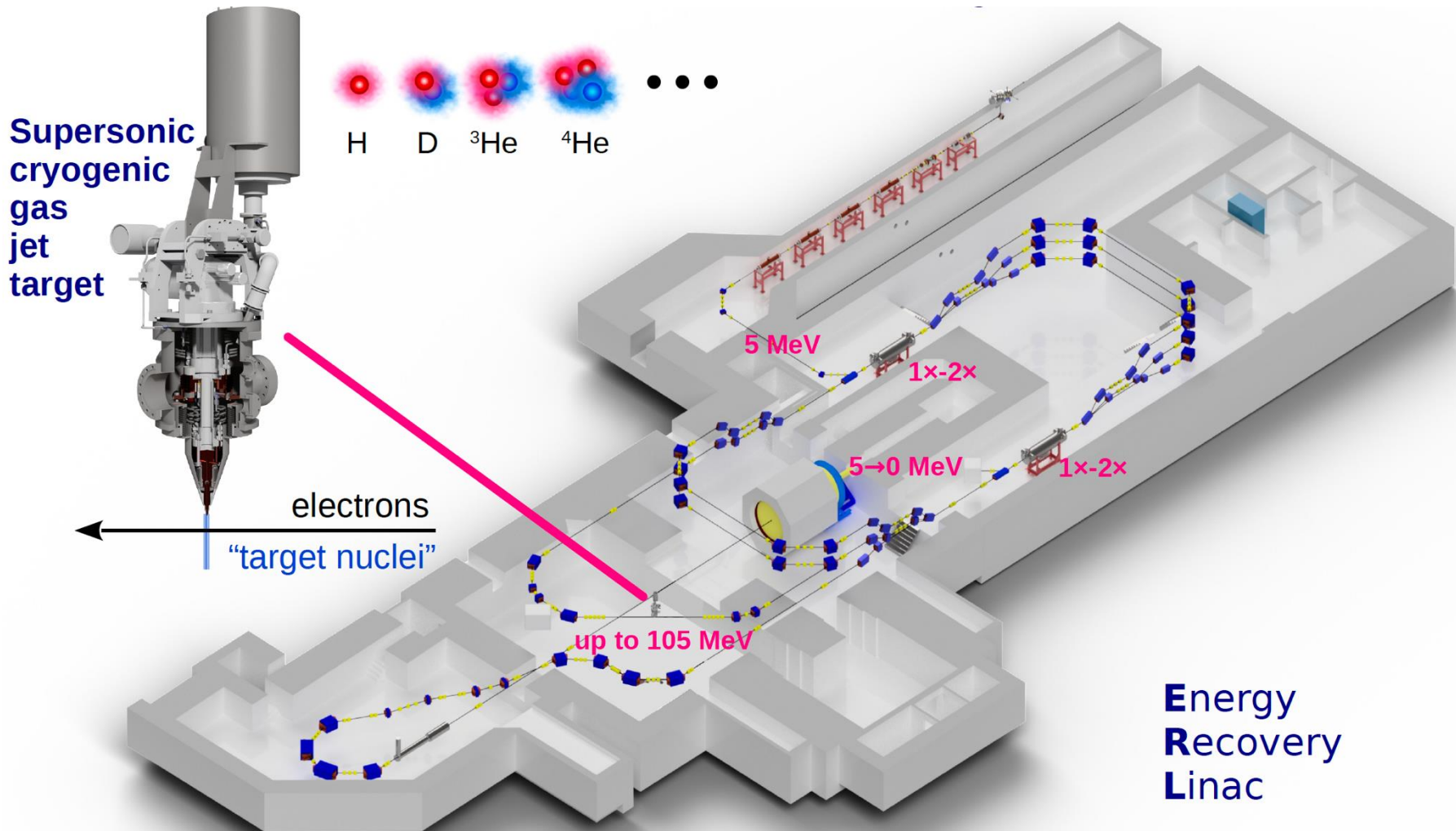
# ISR form-factors



- Form-factors extracted from deviations of the measurements from the Bernauer model, assuming flawless description of radiative corrections.



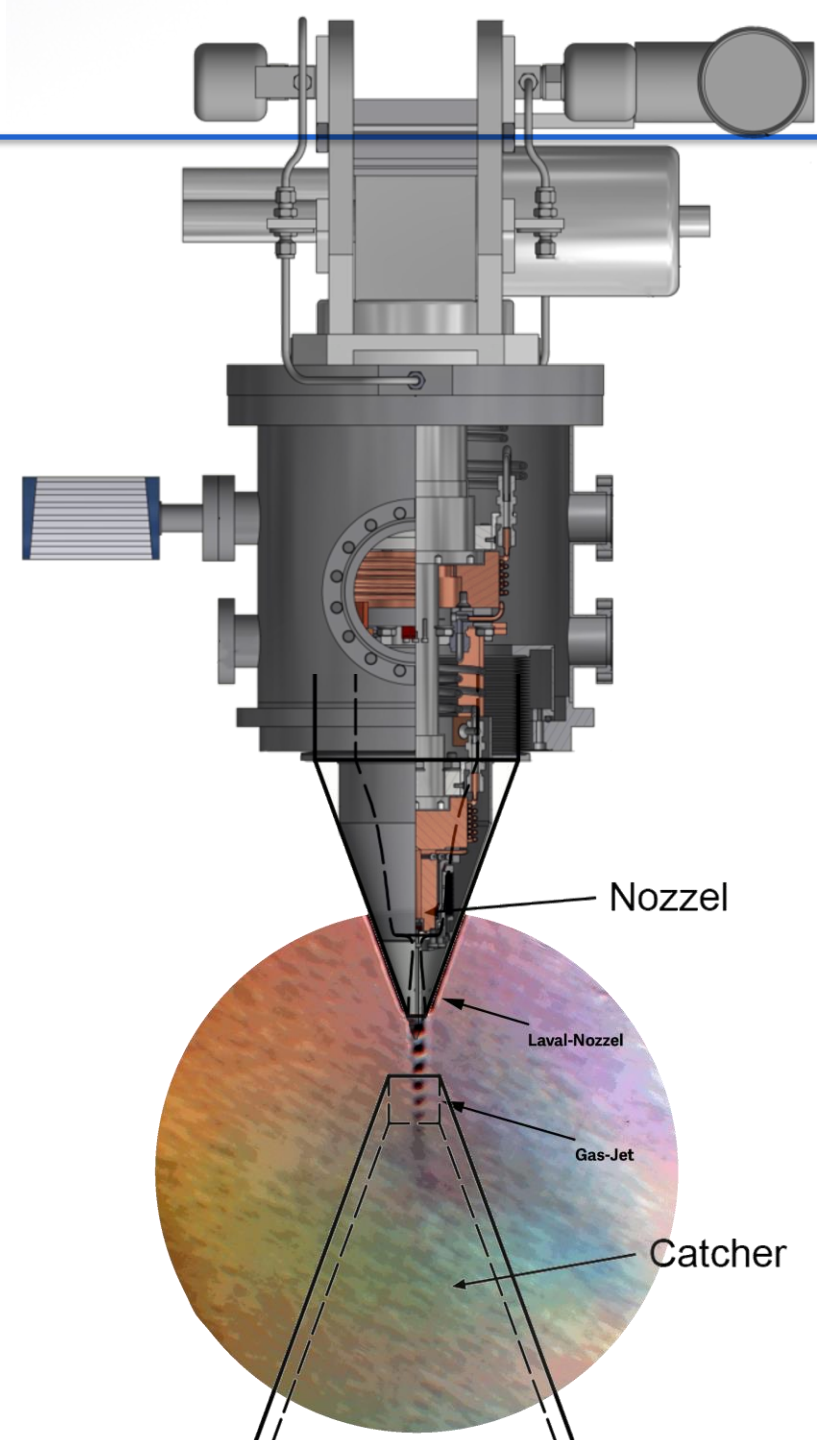
# Magix @ MESA



P2/DM-EB mode: ~155 MeV, 150  $\mu$ A  
MX-EB mode: 20-105 MeV, < 150  $\mu$ A  
ERL mode: 30-105 MeV, 10000  $\mu$ A

# Hypersonic jet target

- Target developed for MAGIX, but could be used also in A1.
- No metal frame near the vertex.
- No target walls.
- Width of the jet 2mm (point-like target)
- Density of  $10^{-4} \text{ g/cm}^3$  at 15 bar.
- Luminosity of  $10^{34}/\text{cm}^2\text{s}$  can be achieved at MAMI.

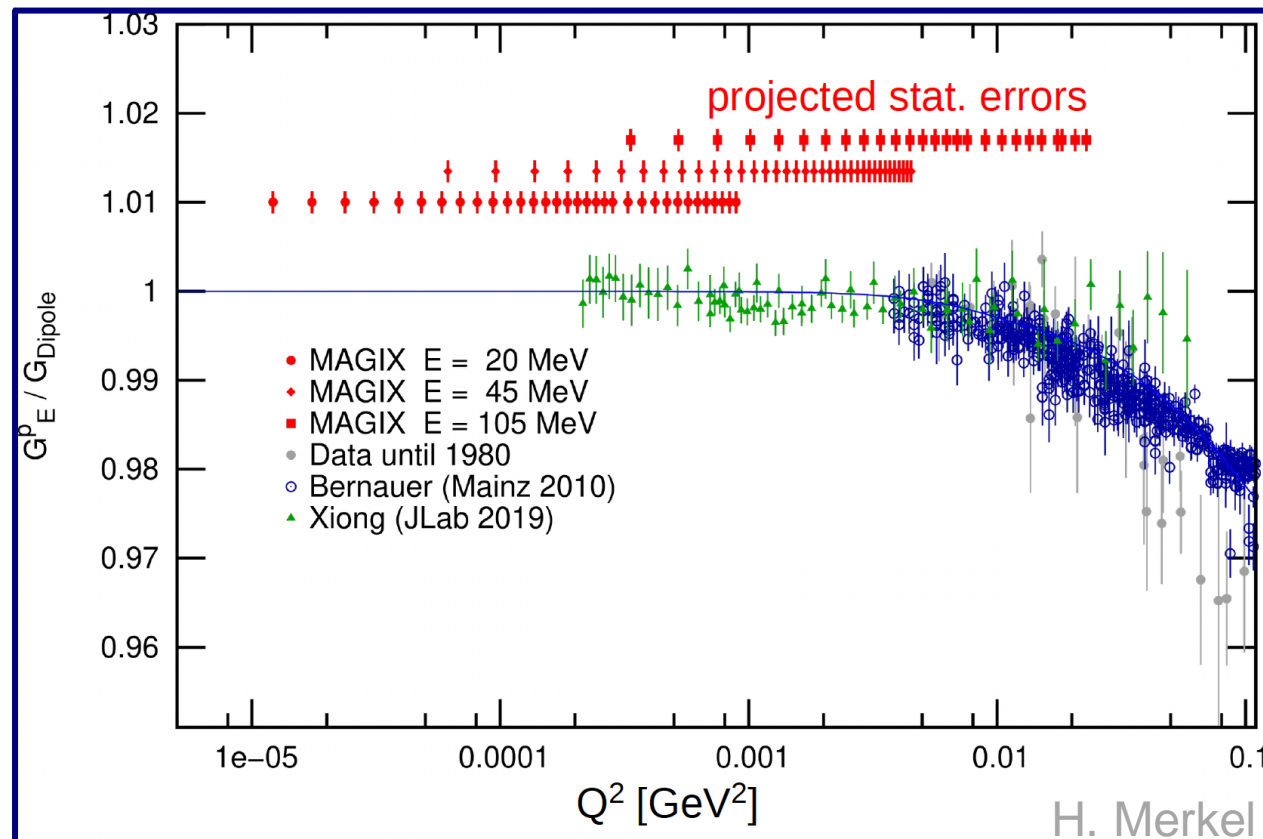


See talk of Yimin Wang

# Radius measurements @ Magix

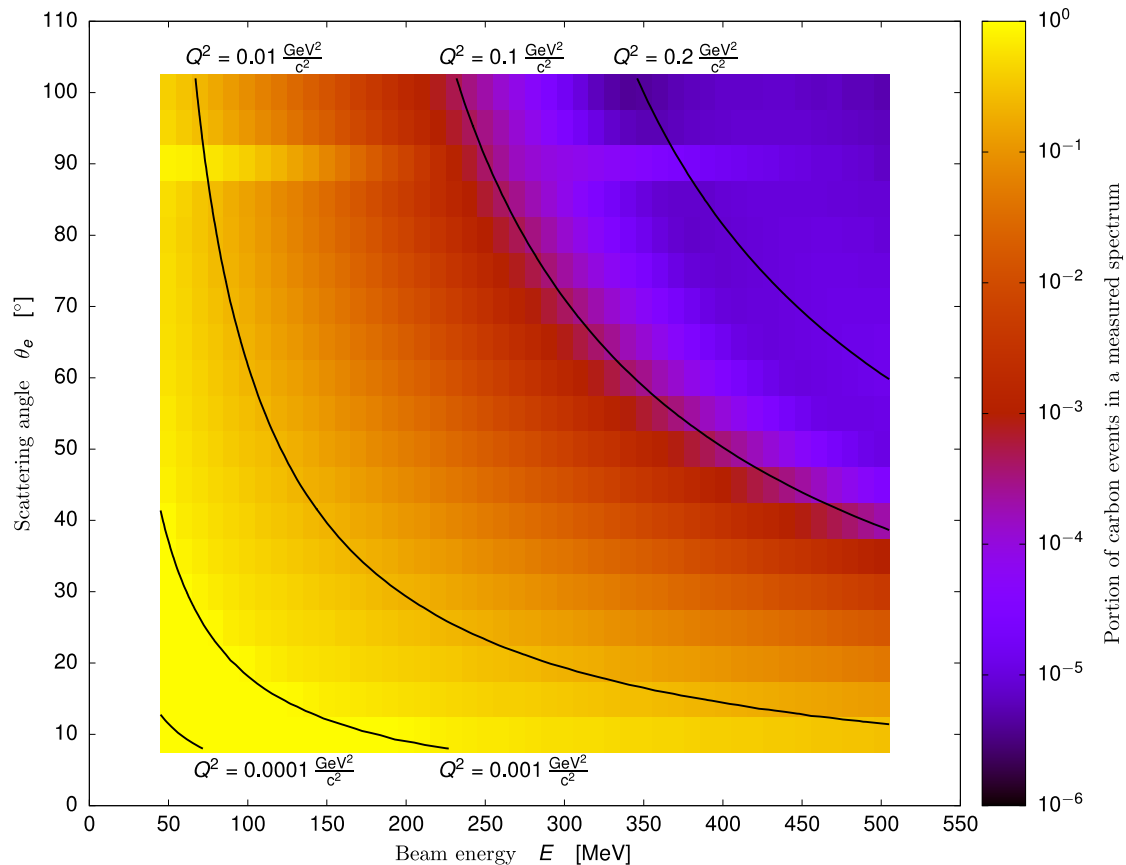
- Persistent discrepancy between different determinations of the proton radius persists demands further measurements.
- New measurement planned also at Magix @ MESA
- Measurement of  $G_E^p$  at  $Q^2$  between  $1 \cdot 10^{-5}$  and  $0.03 \text{ GeV}^2$

- Expected statistical uncertainty  $\sim 0.1 \%$ .
- Expected systematical uncertainty  $< 0.5 \%$ .
- Measurement of  $G_M^p$  using double-polarized experiments.



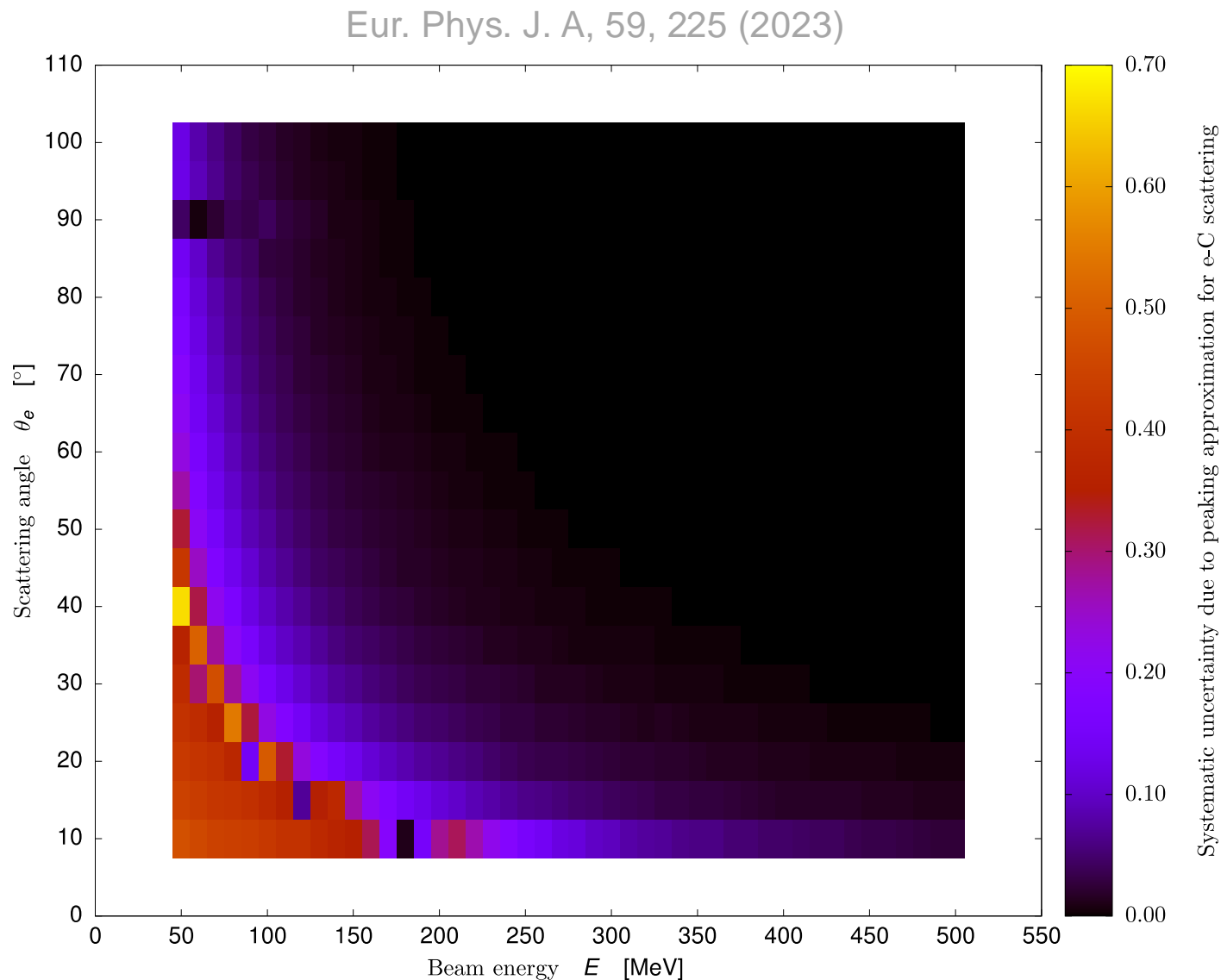
# Potential experiments with plastic targets

- Uncertainty of experiments dominated by the target-related systematics.
- Desired target is **thin** with **known and constant density** and **background**, that can be clearly **subtracted**.
- Plastic (-CH<sub>2</sub>-) target an effective hydrogen target with carbon background.

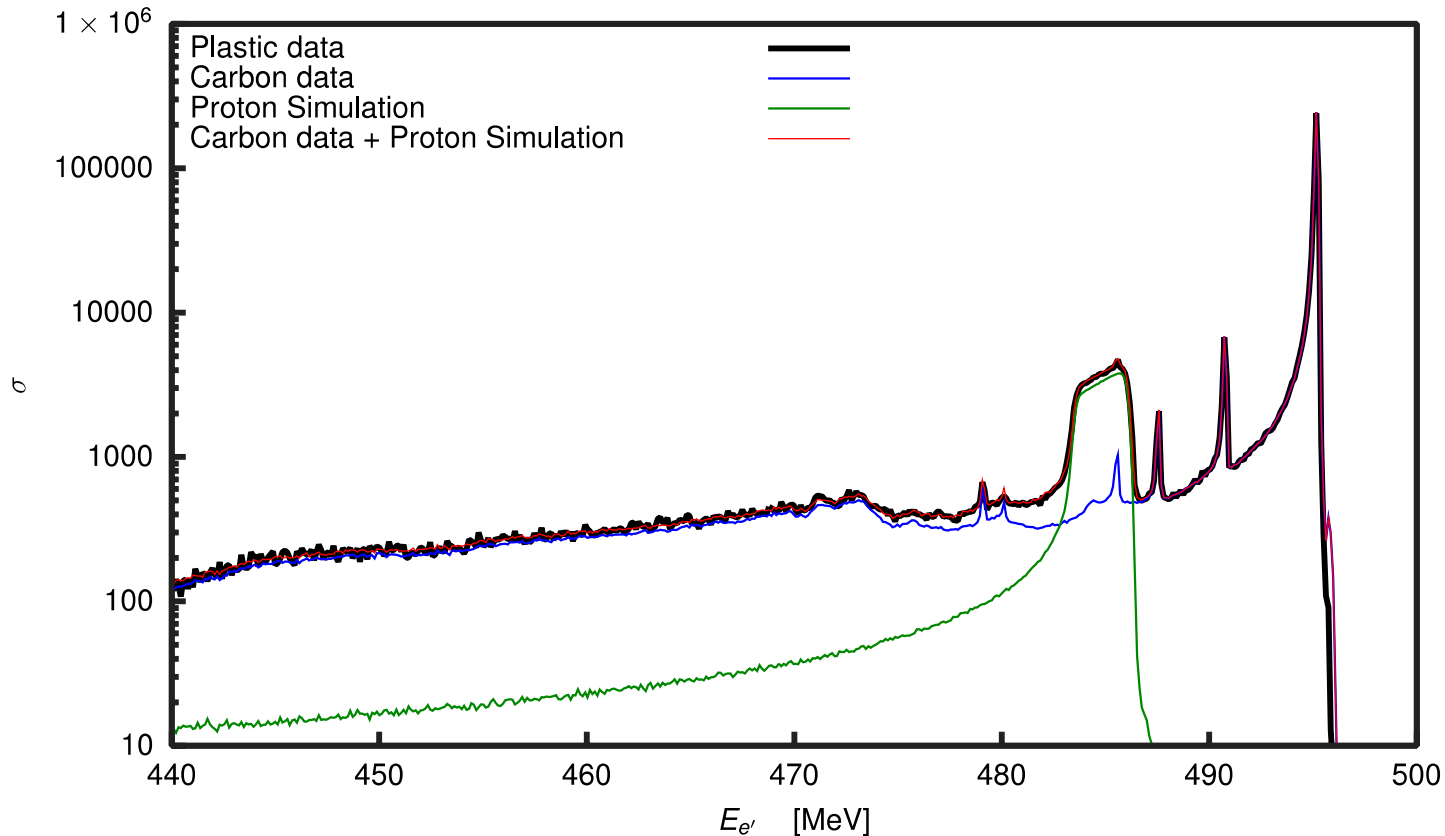


# Findings of tests with plastic target

- Peaking approximations **insufficient** for describing carbon background.



# Findings of tests with plastic target



- Peaking approximations insufficient for describing carbon background.
- Measurements with thin carbon targets are necessary due to the presence of inelastic contributions for adequate background description.
- External radiative corrections need to be applied to match plastic spectra.