RUB

Evgeny Epelbaum, Ruhr University Bochum

LEES2024 Workshop, Tohoku University, Sendai, Oct. 28 — Nov. 1, 2024

Electromagnetic structure of light nuclei from chiral EFT

In collaboration with: ***Arseniy Filin***, Daniel Möller, Vadim Baru, Christopher Körber, Hermann Krebs, Andreas Nogga and Patrick Reinert

- Introduction
- Nuclear forces and e.m. currents in χEFT
- E.m. structure of the deuteron
- Beyond the deuteron
- Summary

Updates since 2023:

— Error analysis
— Insights into scaling of MECs
— First results for the magnetic FFs

European Research Council
Bundesministerium für Bildung und Forschung
DFG Deutsche Forschungsgemeinschaft
Ministerium für Kultur und Wissenschaft des Landes Nordrhein-Westfalen
ERATO Exploratory Research for Advanced Technology

Experimental data

Number of protons Z

Number of neutrons N

^{3}He

$r = 1.97007(94)\mathrm{fm}$

0.05 %

CREMA 2023
arXiv:2305.11679

^{4}He

$r = 1.6786(12)\mathrm{fm}$

0.07 %

Krauth et al. 2021
Pachucki:2022tgl

Pohl:2013yb,
CREMA:2016idx,
Pohl:2016glp,
Schmidt:2018kjc,
Krauth:2021foz
...

^{1}H

$r_p = 0.8406(4)\mathrm{fm}$

0.05 %

Antognini et al. 2013
Pachucki:2022tgl

^{2}H

$r_d = 2.12758(78)\ \mathrm{fm}$

0.04 %

CREMA:2016idx
Pachucki:2022tgl

^{3}H

$r = 1.7xxx(\quad 2)\mathrm{fm}$

TREX Mainz
[in progress]

$r = 1.7550(860)\mathrm{fm}$

Amroun et al. 1994

5 %

Data from isotope shift

$(r_d^2 - r_p^2) = 3.82070(31) fm^2$

0.01 %

Pachucki et al. 2018
Jentschura et al. 2011

n

$r_n^2 = -0.1155(17)\ \mathrm{fm}^2$

1.5 %

PDG 2022

Connecting nucleon and nuclear structure requires
getting nuclear effects under control
with high accuracy

From QCD to nuclear physics

The Standard Model (QCD, ...)

Schwinger-Dyson , large-N_c, …

Lattice QCD

Approximate chiral $SU(2)_L \times SU(2)_R$ symmetry

effective chiral Lagrangian $\mathscr{L}_{\text{eff}}(\pi, N)$

Chiral perturbation theory $Q \in \{M_\pi/\Lambda_b,\ |\vec{p}\,|/\Lambda_b\}$

— S-matrix ($\pi\pi$, πN, $\pi\pi$N, …)
— nuclear forces and currents

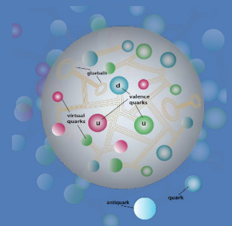
Few-body methods

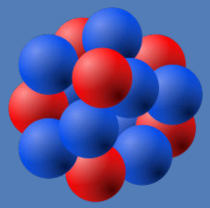
E_{FV}

EFT

finite-volume methods

Hadron/nuclear structure and dynamics


proton


nuclei

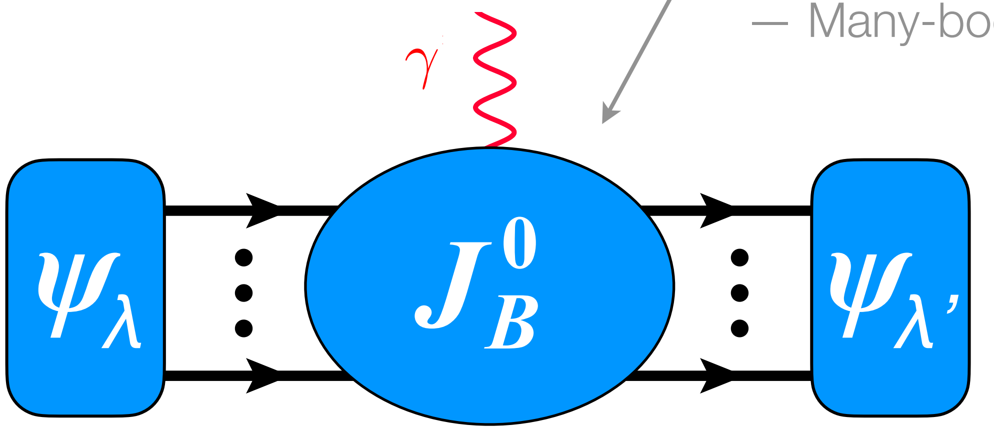

neutron stars

Anatomy of the calculation

Charge and current density

— 1N: experimental / empirical (FFs)

— Many-body: χEFT



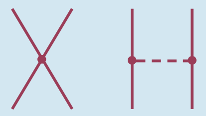
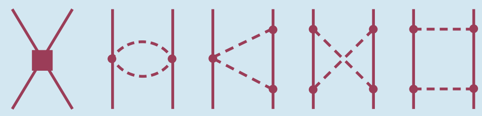
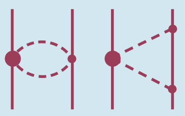
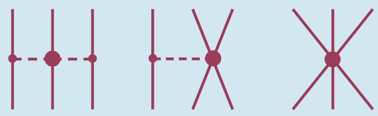
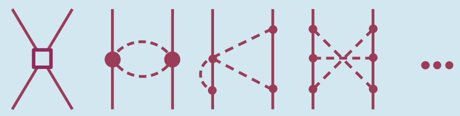
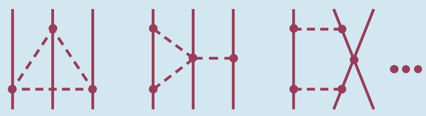
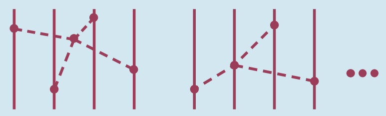
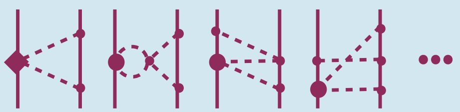
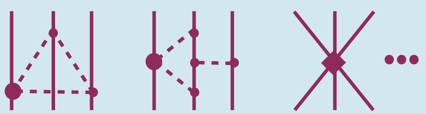
Nuclear Hamiltonian (χEFT)

+ numerical solution of the Faddeev-Yakubovsky equations

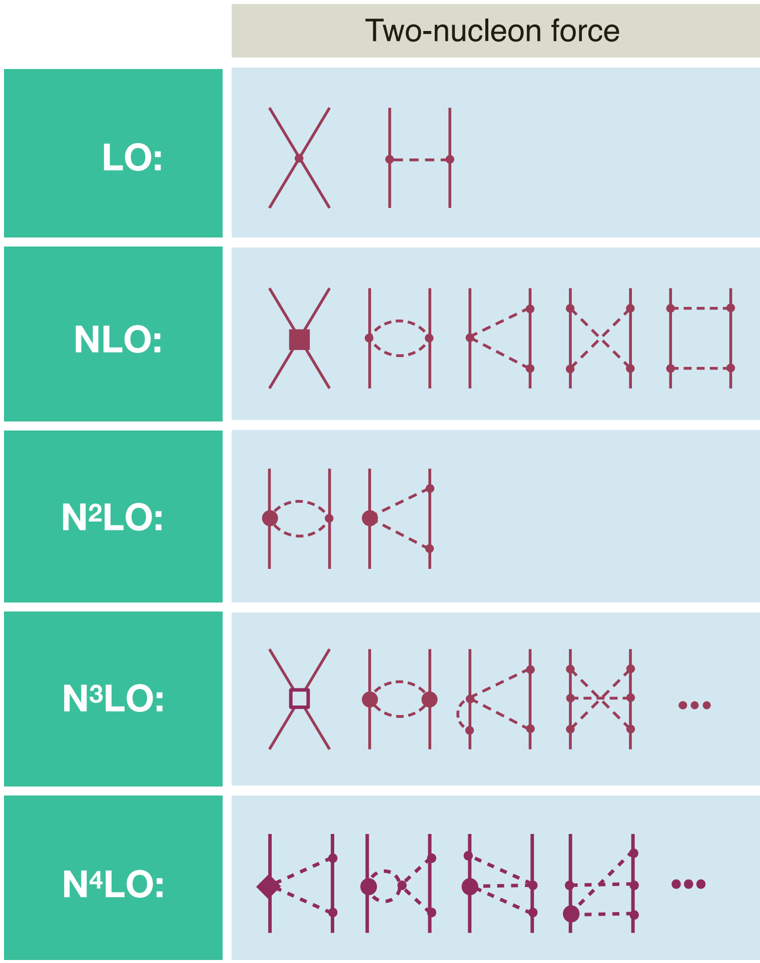
— data driven: Use np + pp data, ^{3}H(e) and ^{4}He BEs, up-to-date parametrizations of the nucleon FFs and ^{2}H, ^{4}He FF data (assuming 2γ effects are small…)

— no reliance on charge densities (everything expressed in terms of FFs)

Chiral expansion of nuclear forces

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO:		—	—
NLO:		—	—
N²LO:			—
N³LO:			
N⁴LO:			—

Chiral expansion of nuclear forces

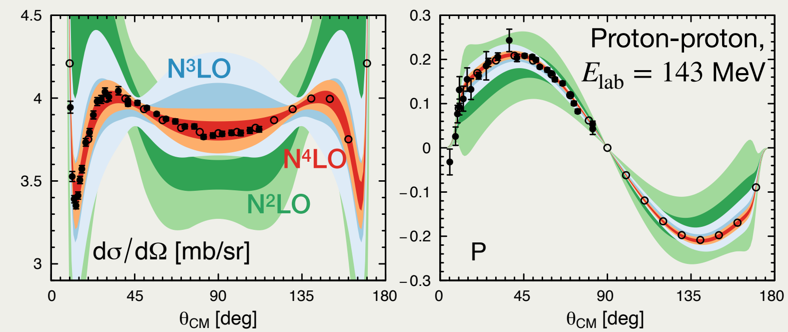


χEFT as a precision tool in the 2N sector

— N^4LO^+: currently most accurate and precise NN interactions on the market

— clear evidence of the TPEP from NN data

— Bayesian truncation-error estimation



— almost no residual cutoff dependence

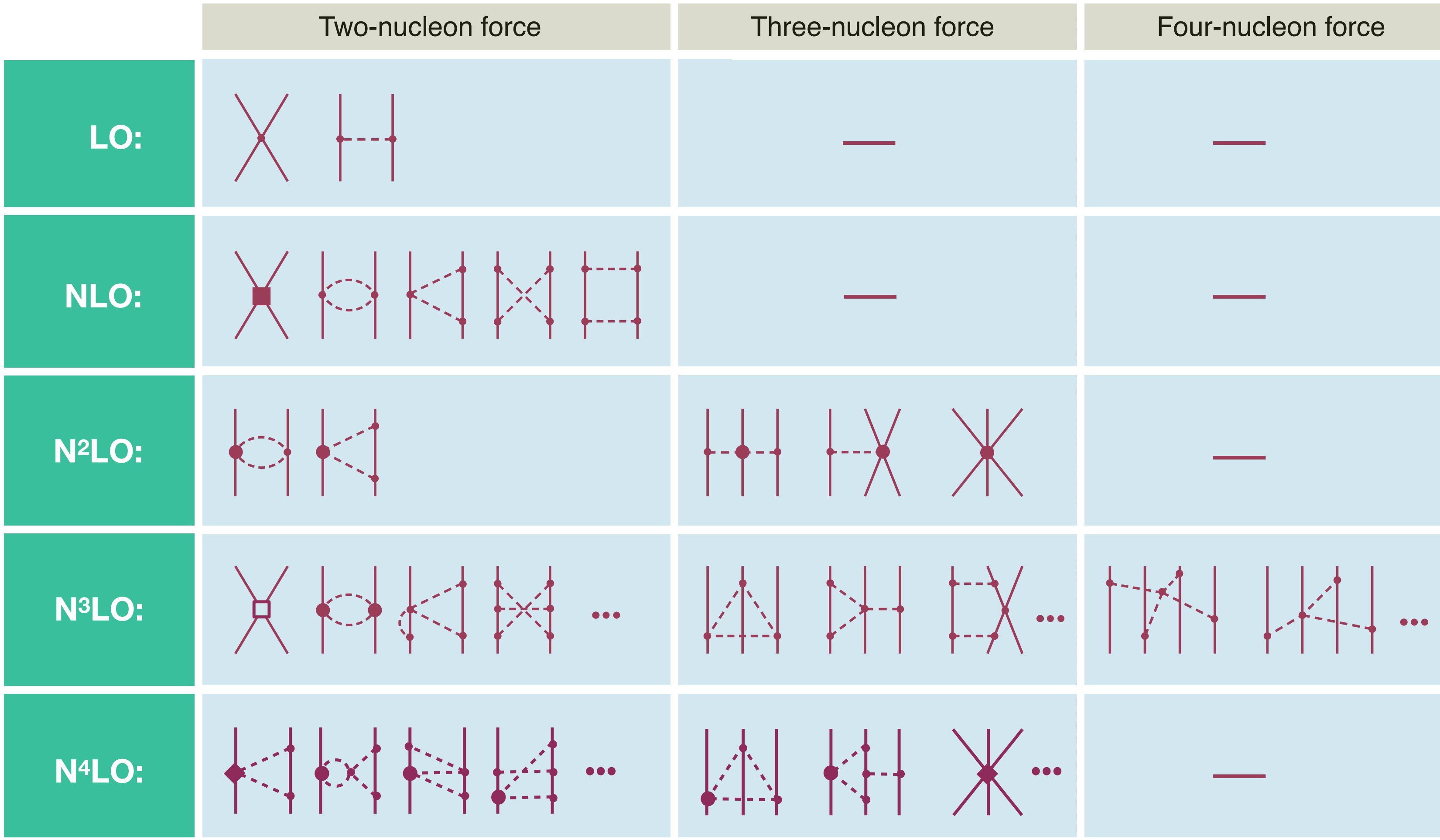
⇒ precision 2N physics from χEFT

Semi-local regularization in momentum space Reinert, Krebs, EE, EPJA 54 (2018) 86; PRL 126 (2021) 092501

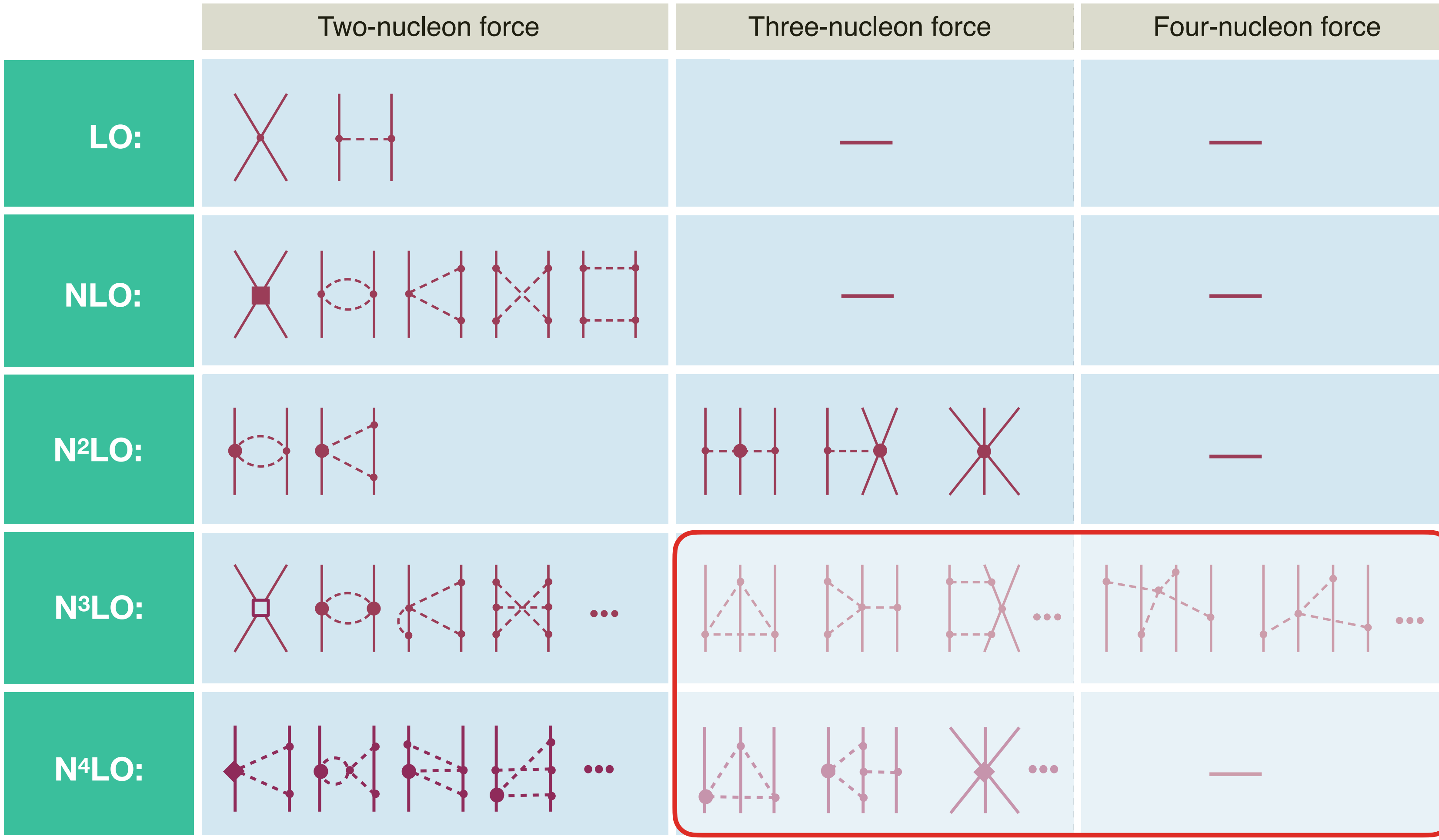
$$V_{1\pi}(q) = \frac{\alpha}{\vec{q}^2 + M_\pi^2}\, e^{-\frac{\vec{q}^2 + M_\pi^2}{\Lambda^2}} + \text{subtraction}, \qquad V_{2\pi}(q) = \frac{2}{\pi}\int_{2M_\pi}^{\infty} d\mu\, \mu \frac{\rho(\mu)}{\vec{q}^2 + \mu^2}\, e^{-\frac{\vec{q}^2 + \mu^2}{2\Lambda^2}} + \text{subtractions}$$

+ nonlocal (Gaussian) cutoff for contacts

Chiral expansion of nuclear forces



Chiral expansion of nuclear forces



Loop diagrams are calculated using DimReg, but the Schrödinger equation is regularized with a cutoff…

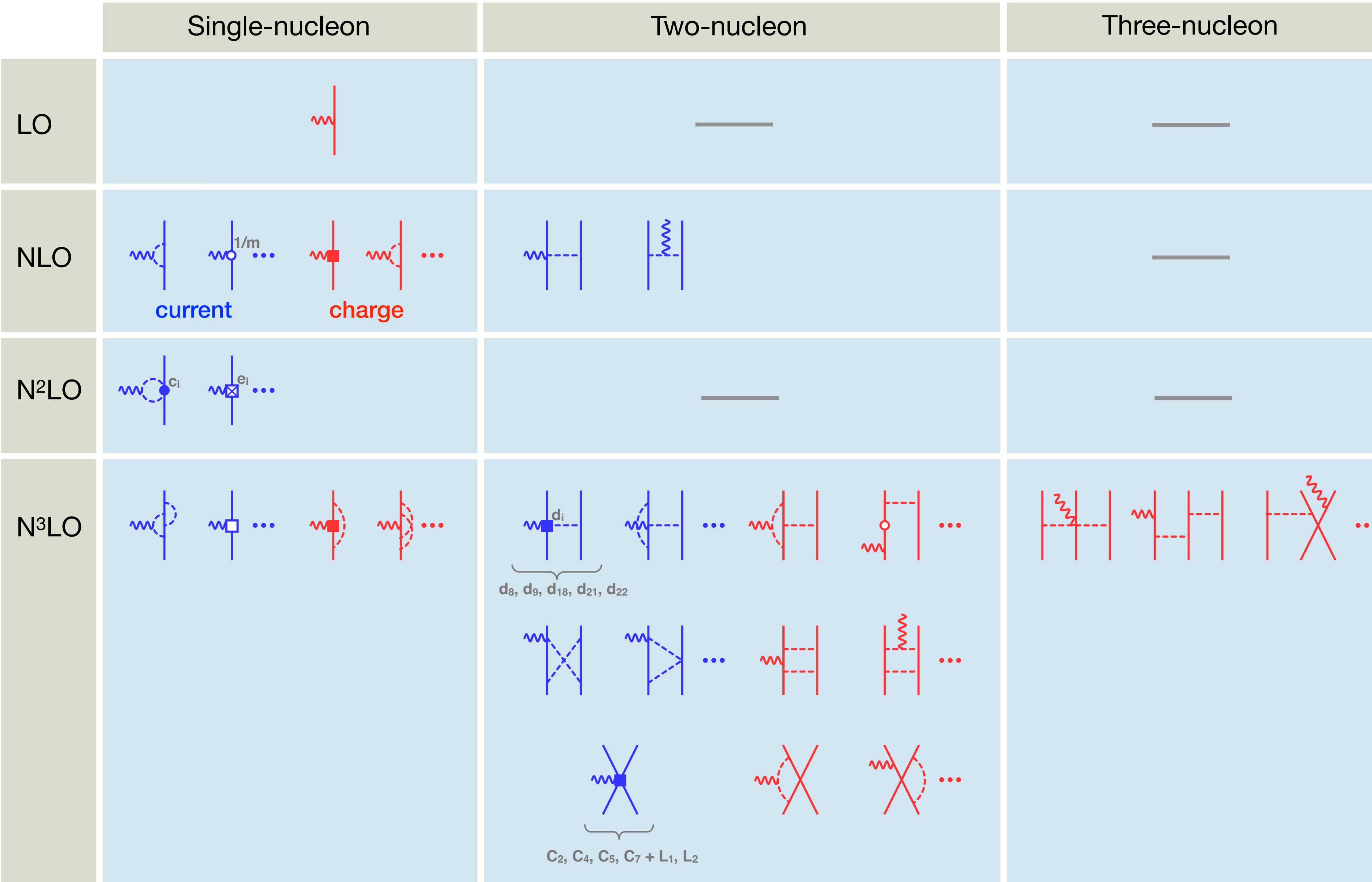
But: Mixing DimReg with CutoffReg violates χ-symmetry EE, Krebs, Reinert, Front. In Phys. 8 (20)

⇒ 3NF beyond N^{2}LO & 4NF must be re-derived using invariant CutoffReg

like, e.g., Gradient Flow Krebs, EE, PRC 110 (2024) 04403; 04404

Electromagnetic currents

Kölling, EE, Krebs, Meißner, PRC 80 (09) 045502; PRC 86 (12) 047001; Krebs, EE, Meißner, FBS 60 (2019) 31; Krebs, EPJA 56 (00) 240



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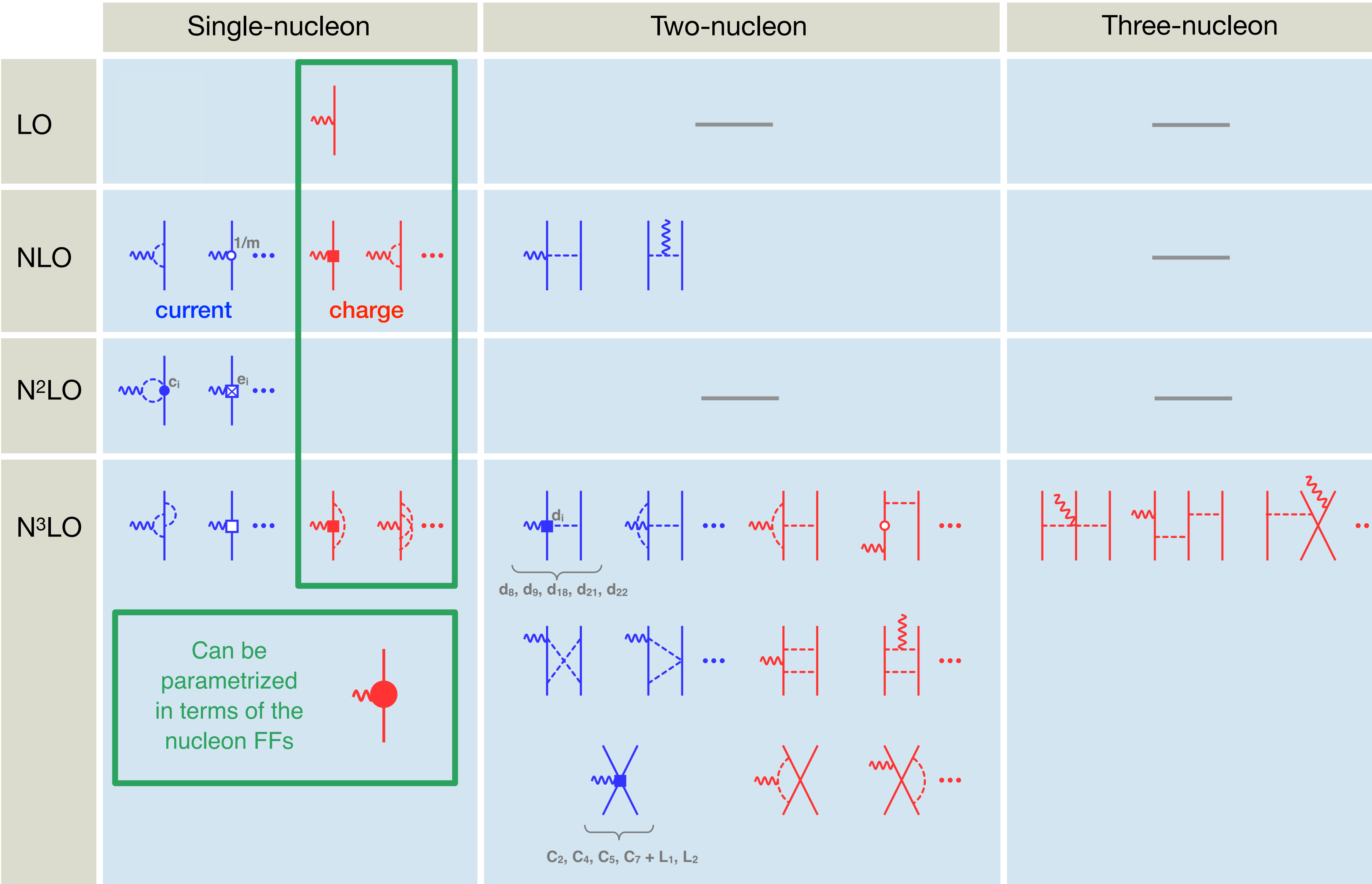

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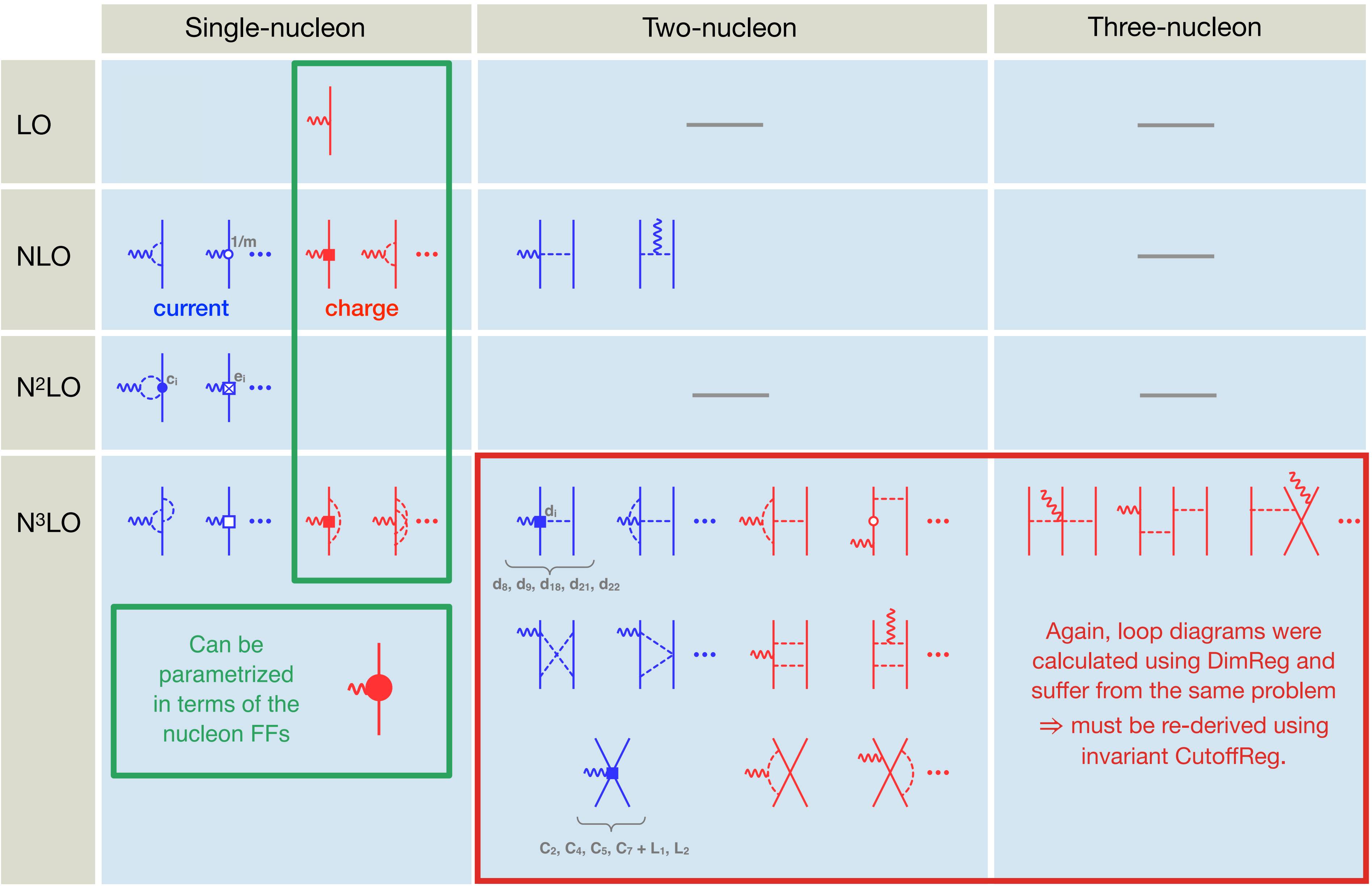

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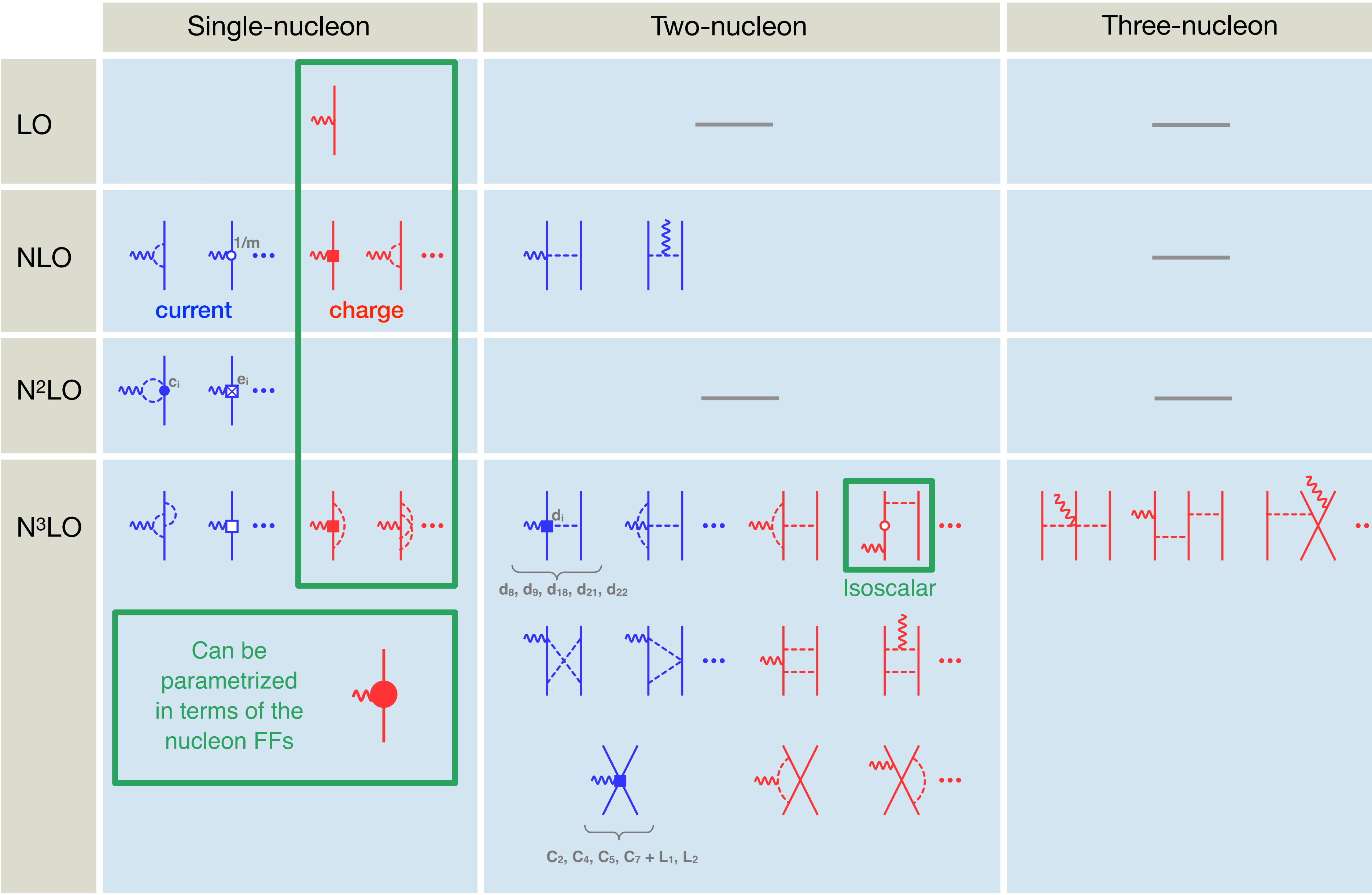

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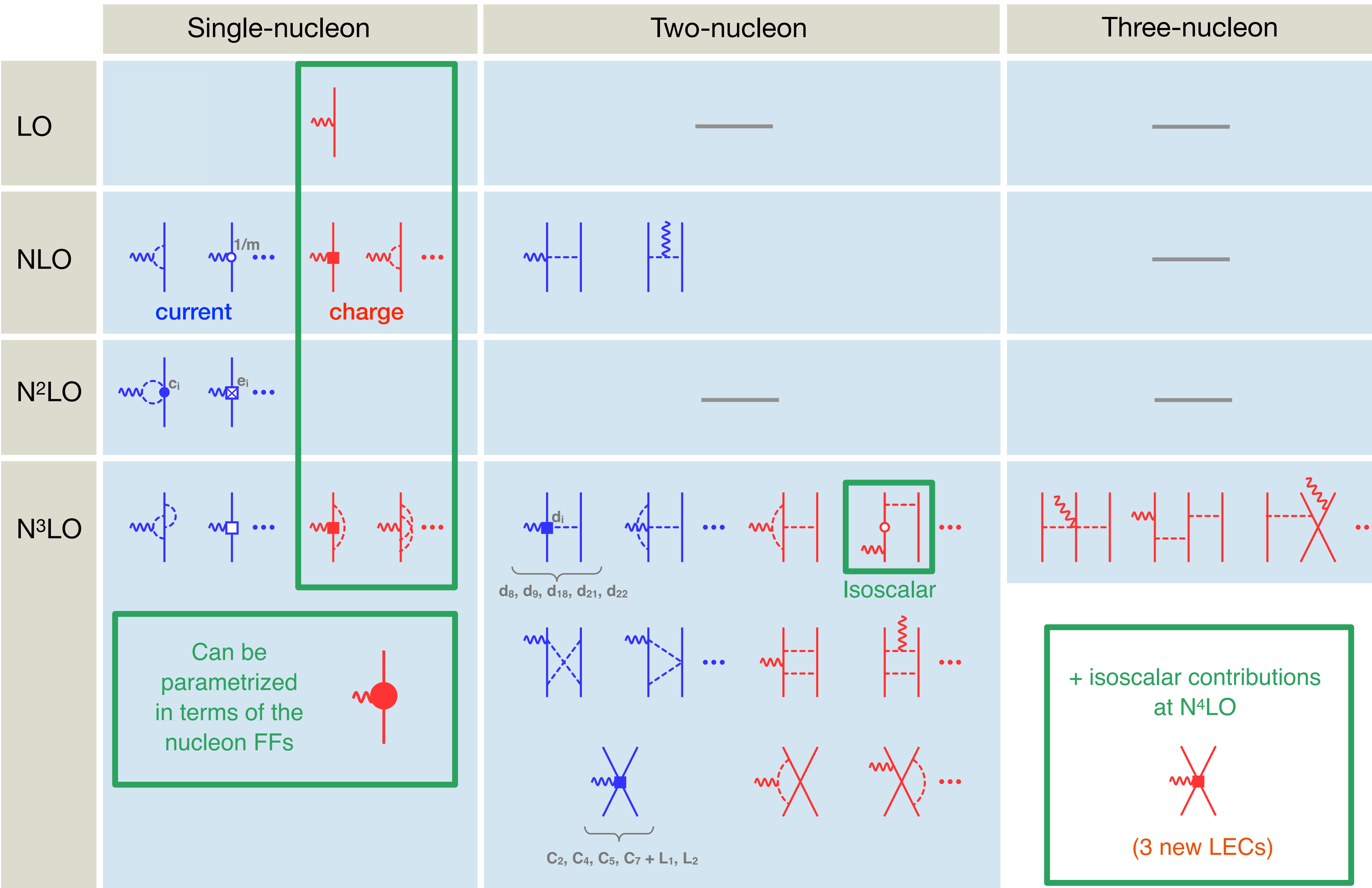
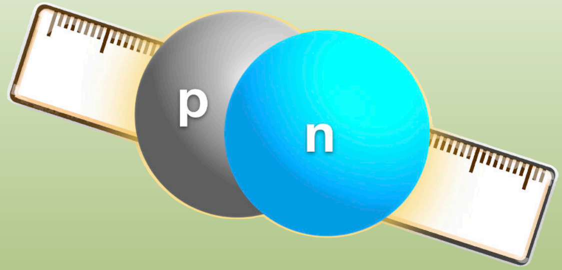




The deuteron

Arseniy Filin, Vadim Baru, EE, Hermann Krebs, Daniel Möller, Patrick Reinert, Phys. Rev. Lett. 124 (2020) 082501;
Phys. Rev. C103 (2021) 024313

$$\rho_{1N}^{\rm DF} = -e\frac{\mathbf{k}^2}{8m_N^2}G_E(\mathbf{k}^2)$$

$$G(Q^2) = G^{\rm Main}(Q^2) + G^{\rm DF}(Q^2) + G^{\rm SO}(Q^2) + G^{\rm Boost}(Q^2) + G^{1\pi}(Q^2) + G^{\rm Cont}(Q^2)$$

$$\rho_{1N}^{\rm Main} = eG_E(\mathbf{k}^2) \qquad \rho_{1N}^{\rm SO} = ie\frac{2G_M(\mathbf{k}^2) - G_E(\mathbf{k}^2)}{4m_N^2}\sigma\cdot\mathbf{k}\times\mathbf{p}$$

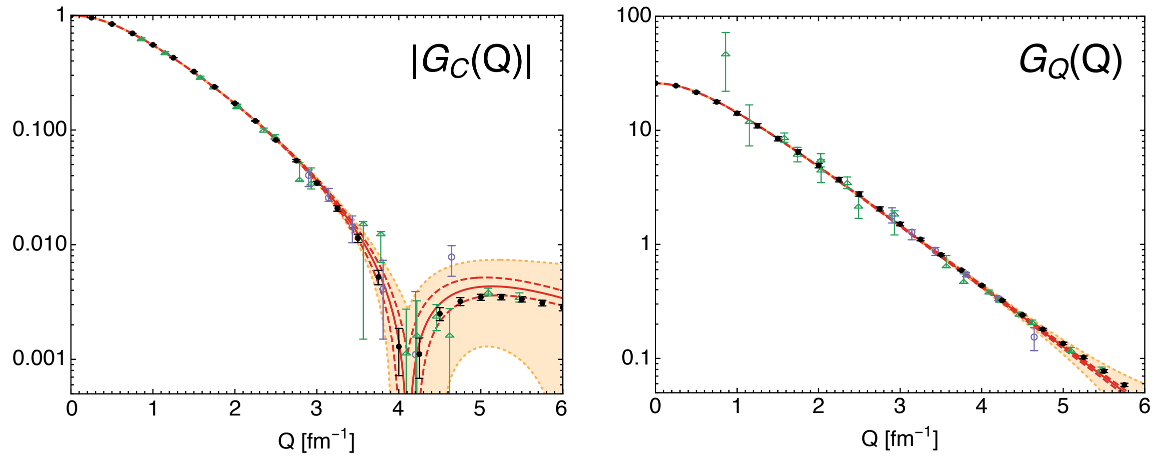
— Both the nuclear force and the 2N charge density are available to N^4LO

— Simple numerics

Electromagnetic FFs of the deuteron

Filin, Möller, Baru, EE, Krebs, Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313

Charge and quadrupole form factors of the deuteron at N⁴LO



Extracted quadrupole moment:

$$Q_\mathrm{d} = 0.2854^{+0.0038}_{-0.0017}\ \mathrm{fm}^2$$

EFT truncation, choice of fitting range, NN, πN and γNN LECs

to be compared with experiment

$$Q_\mathrm{d}^\mathrm{exp} = 0.285\,699(15)(18)\ \mathrm{fm}^2$$

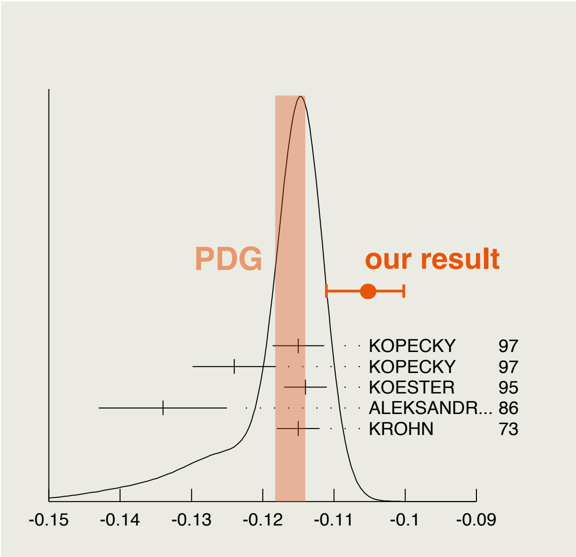
Puchalski et al., PRL 125 (2020)

The charge and structure radius:

$$r_d^2 = (-6)\frac{\partial G_C(Q^2)}{\partial Q^2}\bigg|_{Q^2=0} = r_{str}^2 + r_p^2 + r_n^2 + \frac{3}{4m_p^2}$$

Combining our result $r_\mathrm{str} = 1.9729^{+0.0015}_{-0.0012}$ fm with very precise isotope-shift spectroscopy data for $r_d^2 - r_p^2$, we determine the neutron m.s. charge radius:

$$r_n^2 = -\,0.105^{+0.005}_{-0.006}\ \mathrm{fm}^2$$



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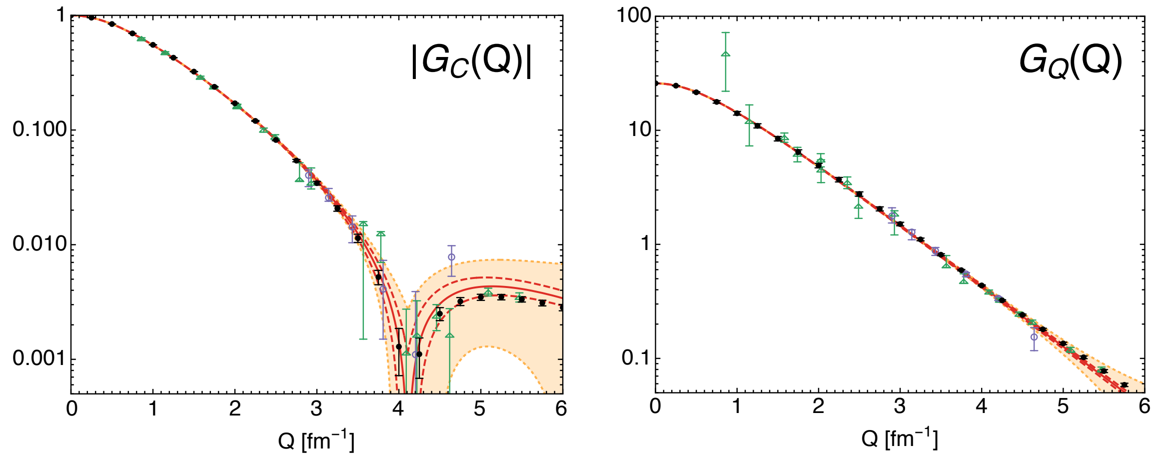

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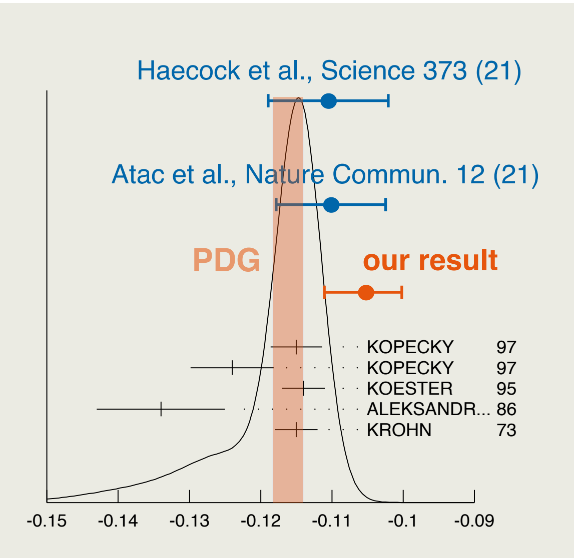

4^{th} moment of the charge distribution

Arseniy Filin et al., preliminary

The fourth-order moment $\langle r_d^4 \rangle := 60\, G_C''(0)$ can be measured in the ULQ2 exp Toshimi Suda et al.

$$\langle r_d^4 \rangle = r_{\rm str}^{(4)} + \frac{10}{3} r_{\rm str}^{(2)} \left(r_n^{(2)} + r_p^{(2)} + \frac{3}{4m^2} \right) + \left(r_n^{(4)} + \frac{5}{2m^2} r_n^{(2)} \right) + \left(r_p^{(4)} + \frac{5}{2m^2} r_p^{(2)} + \frac{45}{16m^4} \right)$$

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Results for $\Lambda = 500$ MeV:
(preliminary, likely to change)

$$r_{\rm str}^{(4)} = \underbrace{r_{\rm matter}^{(4)}}_{55.442} + \underbrace{r_{\rm boost}^{(4)}}_{0.215} + \underbrace{r_{\rm SO}^{(4)}}_{-0.007} + \underbrace{r_{\rm 2N,OPE}^{(4)}}_{0.025} + \underbrace{r_{\rm 2N,CT}^{(4)}}_{0.008} = 55.68(5)\ {\rm fm}^4$$

Error budget (preliminary):

Truncation (N^4LO): ± 0.035, πN LECs: ± 0.005, NN LECs: ± 0.04, other errors negligible

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$$\langle r_d^4 \rangle = 64.62 \pm 0.05_{\chi\rm EFT} \pm \underbrace{0.05_{r_N^{(2)}}}_{\text{CODATA + PDG}} \pm \underbrace{0.36_{r_N^{(4)}}}_{\substack{r_p^{(4)} = 1.3 \pm 0.3\ {\rm fm}^4 \\ r_n^{(4)} = -0.5 \pm 0.2\ {\rm fm}^4}}\ {\rm fm}^4$$

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Alternative determination of the nucleon isoscalar radius?

1% accuracy $\Rightarrow$ 8% accuracy for $r_n^{(2)} + r_p^{(2)}$ ...

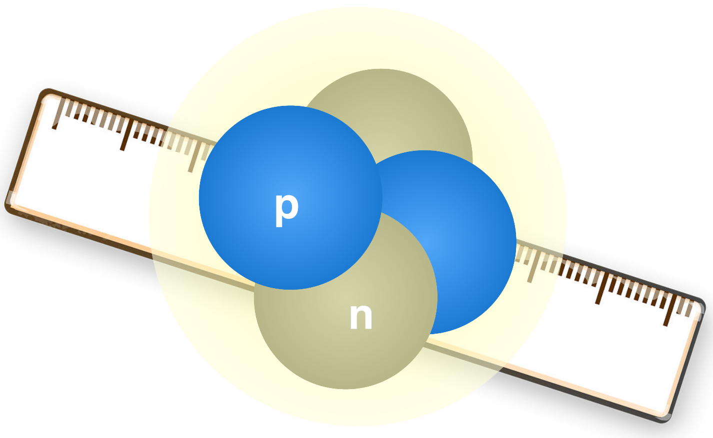
$$r_p^{(2)} + r_n^{(2)} = \frac{\overbrace{\left[\langle r_d^4 \rangle - r_p^{(4)} - r_n^{(4)}\right]} - r_{\rm str}^{(4)} - \frac{15}{16m^4}}{\frac{10}{3} r_{\rm str}^{(2)} + \frac{5}{2m^2}} - \frac{3}{4m^2}$$

Generalization to A = 3, 4 systems

Arseniy Filin, Christopher Körber, Hermann Krebs, Daniel Möller, Andreas Nogga, Patrick Reinert, in preparation

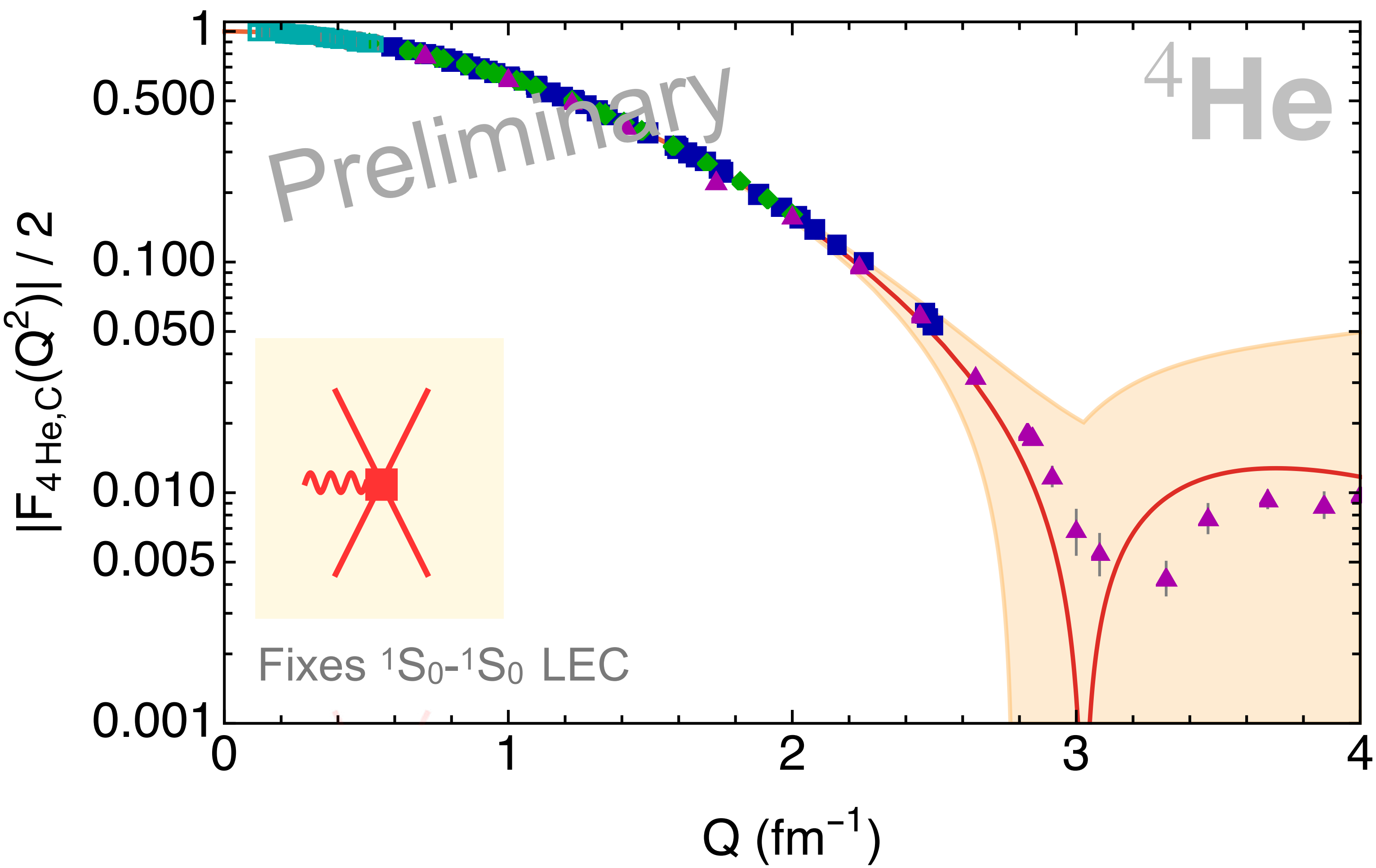
…is a difficult task:

- 3NF beyond N^{2}LO not yet available
  - → use the strong correlations between BEs & matter radii to *implicitly* include 3NF effects
- difficult to account for relativistic corrections beyond ^{2}H
  - → irreps of the Poincare group on A-particle Hilbert spaces a-la Polyzou et al.
- iso-vector NN charge density not yet available beyond N^{2}LO
  - → irrelevant for 4He; for $A = 3$ focus on the iso-scalar linear combination only
- computational accuracy for evaluating the $A = 3{,}4$ FFs
  - → semi-analytical methods that exploit the local + separable form of the 2N charge density
- many sources of uncertainty
  - → comprehensive error analysis



The charge FF of ^{4}He

Arseniy Filin, Vadim Baru, EE, Hermann Krebs, Daniel Möller, Andreas Nogga, Patrick Reinert, in preparation



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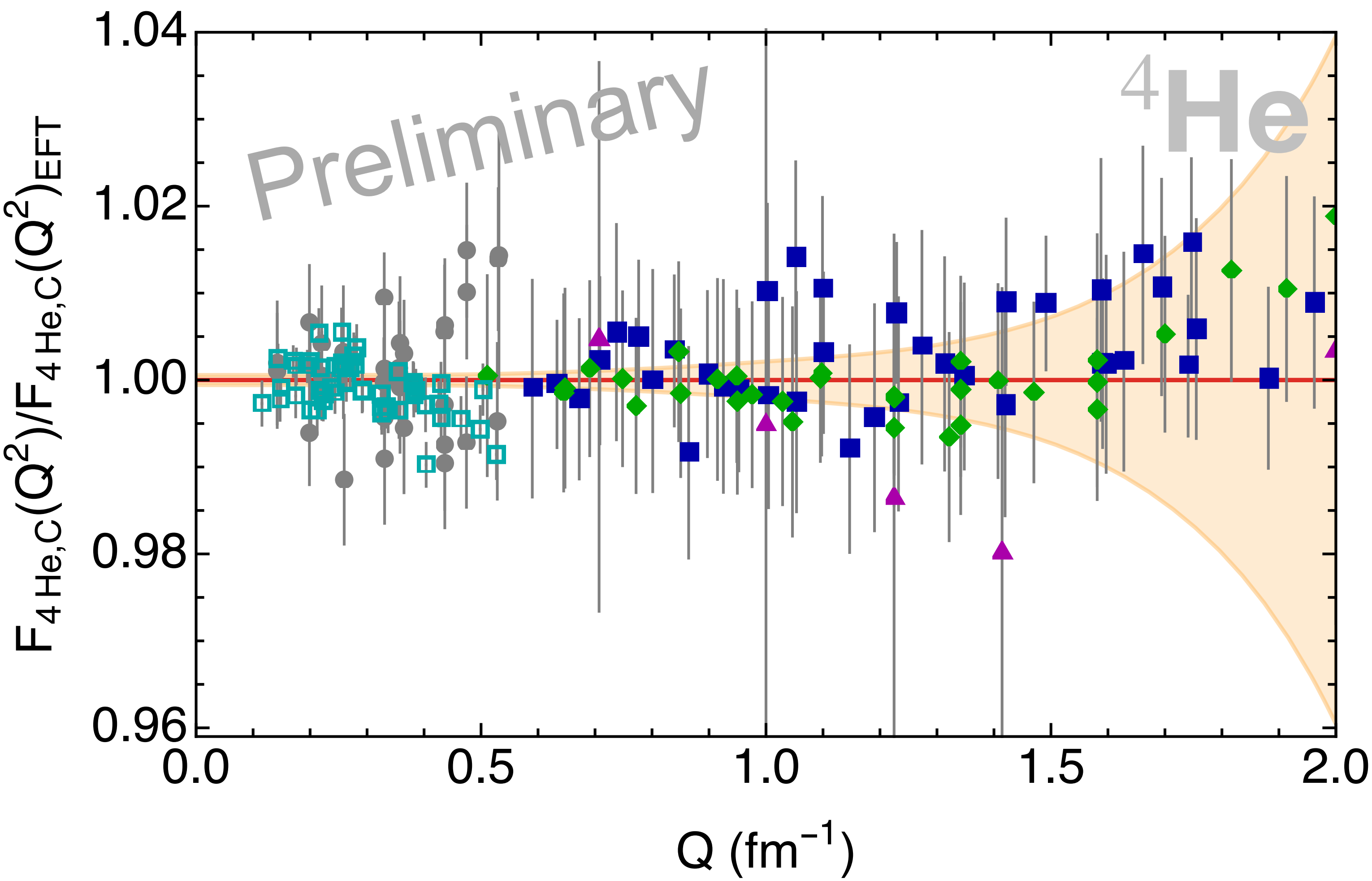


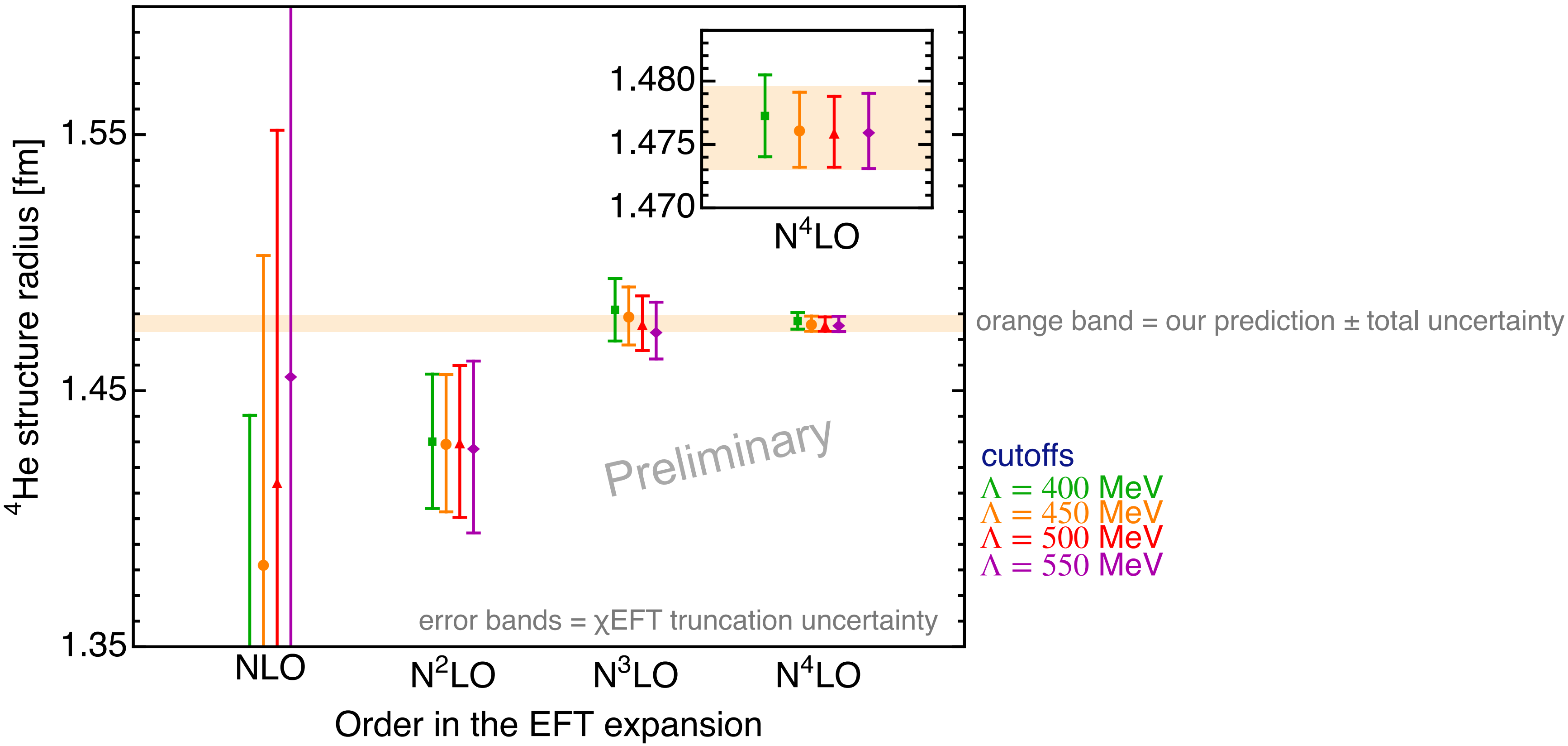
The structure radius of ^{4}He

Arseniy Filin, Vadim Baru, EE, Hermann Krebs, Daniel Möller, Andreas Nogga, Patrick Reinert, in preparation

Preliminary result for the ^{4}He structure radius:

$$r_{\rm str}(^4{\rm He}) = 1.4758 \pm 0.0028_{\rm trunc} \pm 0.0011_{\rm stat} \pm 0.0010_{\rm nucl-FF} \ {\rm fm}$$ (Preliminary)

Consistency check (residual cutoff dependence):



Nucleon size from the ^{4}He charge radius

Arseniy Filin, Vadim Baru, EE, Hermann Krebs, Daniel Möller, Andreas Nogga, Patrick Reinert, in preparation

Alternatively: Nucleon size from ^{4}He radius

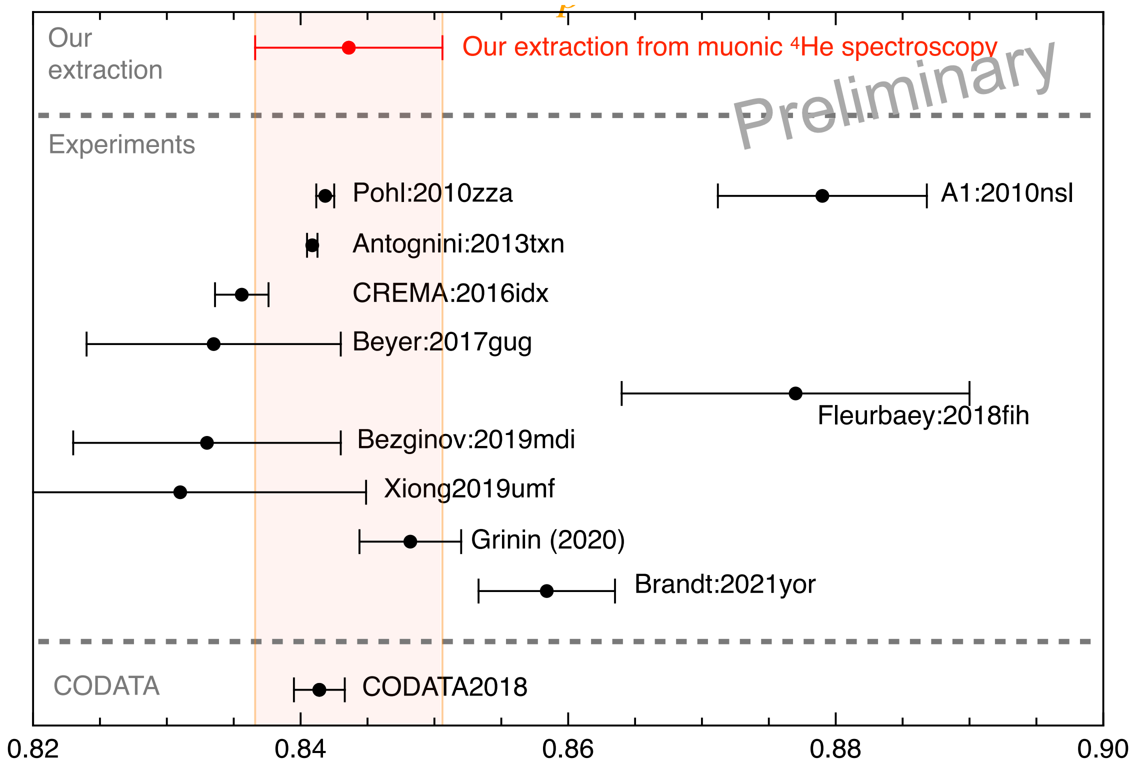
$$r_C^2(^4\text{He}) = r_{\text{str}}^2(^4\text{He}) + \left(r_p^2 + \frac{3}{4m_p^2}\right) + r_n^2 \quad \Rightarrow$$

$$r_p^2 + r_n^2 = (0.607 \pm 0.010)\ \text{fm}^2$$

$$r_p = (0.844 \pm 0.007)\ \text{fm}$$

preliminary (own determination of r_n)

Proton charge radius



Isoscalar charge radius of ^3H, ^3He

Arseniy Filin, Vadim Baru, EE, Hermann Krebs, Daniel Möller, Andreas Nogga, Patrick Reinert, in preparation

Predicted value of the isoscalar 3N charge radius $r_C^{\text{isoscalar}} = \sqrt{\frac{1}{3}\left(r_C^{3H}\right)^2 + \frac{2}{3}\left(r_C^{3He}\right)^2}$

$$r_C^{\text{isoscalar}} = (1.9060 \pm 0.0026)\ \text{fm}$$

preliminary (own determination of r_n)

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$$r_C^{\text{isoscalar}} = (1.9060 \pm 0.0026)\ \text{fm}$$

preliminary (own determination of r_n)

Experimental value: $r_{C,\,exp}^{\text{isoscalar}} = (1.9010 \pm 0.0260)\ fm$

error dominated by the ^{3}H datum

⇒ our prediction is 10x more precise than the current experimental value

The ongoing T-REX experiment in Mainz [Pohl et al.] aims at measuring the ^{3}H charge radius within ± 0.0002 fm (i.e., 400x more precise) ⇒ the isoscalar radius will be known within ± 0.0009 fm

⇒ precision test of nuclear chiral EFT

^{3}He

$r = 1.97007(94)\text{fm}$

0.05 %

CREMA 2023
arXiv:2305.11679

^{3}H

$r = 1.7\text{xxx}(\quad 2)\text{fm}$

TREX Mainz
[in progress]

$r = 1.7550(860)\text{fm}$

Amroun et al. 1994

Scaling of our results

Arseniy Filin, Vadim Baru, EE, Hermann Krebs, Daniel Möller, Andreas Nogga, Patrick Reinert, in preparation

$$r_C^2(^2H) = 1r_p^2 + 1r_{DF}^2 + 1r_n^2 + 3.8746 - 0.0161M_1 - 0.0047M_2$$

$$r_C^2(^3H) = 1r_p^2 + 1r_{DF}^2 + 2r_n^2 + 2.5148 - 0.0326M_1 - 0.0089M_2 - 0.0238M_3 + 0.0158M_4$$

$$2r_C^2(^3He) = 2r_p^2 + 2r_{DF}^2 + 1r_n^2 + 6.2821 - 0.0321M_1 - 0.0088M_2 - 0.0230M_3 - 0.0151M_4$$

$$2r_C^2(^4He) = 2r_p^2 + 2r_{DF}^2 + 2r_n^2 + 4.1117 - 0.0765M_1 - 0.0189M_2 - 0.0609M_3$$

nucleon size

structure radii

Slope of charge FF
assuming F(0) = Z

$$Z \times r_C^2$$

Cutoff 450 MeV

all numbers in fm^2

All LECs in dimensionless natural units

M4 is isovector contact interaction similar to isoscalar M3

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Arseniy Filin, Vadim Baru, EE, Hermann Krebs, Daniel Möller, Andreas Nogga, Patrick Reinert, in preparation

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Slope of charge FF assuming F(0) = Z: $Z \times r_C^2$

- $r_p^2 + r_{DF}^2$: proportional to the number of protons
- r_n^2: proportional to the number of neutrons
- constant, M_1, M_2: proportional to the number of pn pairs
- M_3: proportional to the number of (pp pairs + nn pairs)
- M_4: proportional to the number of (pp pairs - nn pairs)

Cutoff 450 MeV

all numbers in fm^2

All LECs in dimensionless natural units

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Slope of charge FF assuming F(0) = Z

$Z \times r_C^2$

proportional to the number of protons

proportional to the number of neutrons

proportional to the number of pn pairs

proportional to the number of (pp pairs + nn pairs)

proportional to the number of (pp pairs - nn pairs)

Cutoff 450 MeV

all numbers in fm^2

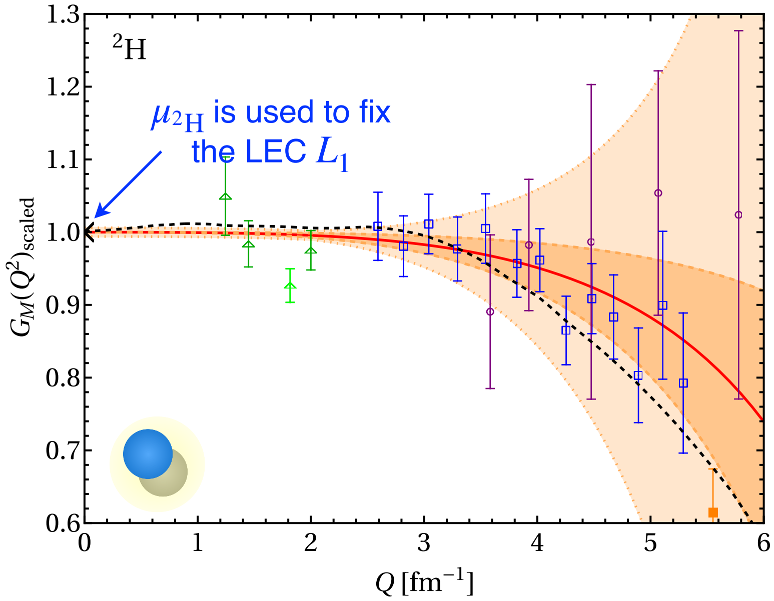
All LECs in dimensionless natural units

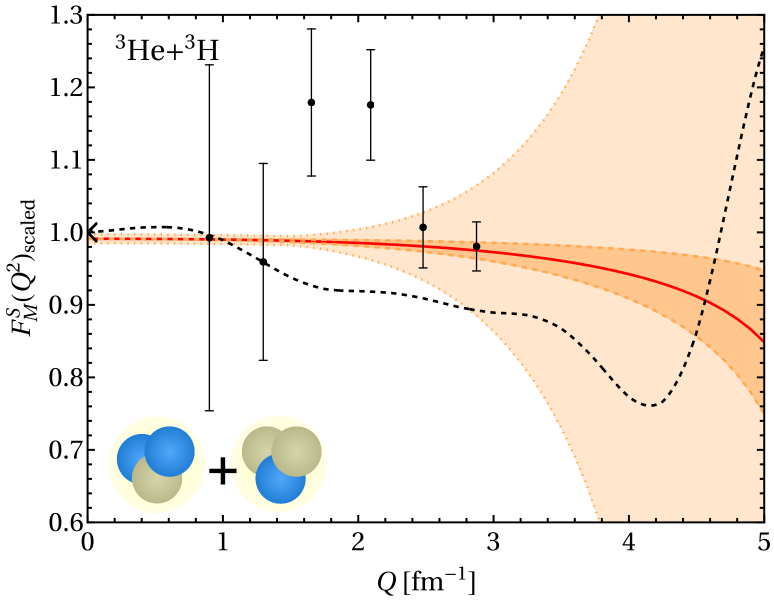
M4 is isovector contact interaction similar to isoscalar M3

The ^{4}He and the isoscalar A = 3 charge radii depend essentially on a single combination of LEC

Magnetic FFs of ^{2}H and ^{3}H(e)

Daniel Möller, PhD thesis, Bochum, 2024; Daniel Möller et al., in preparation





	$\mu^{\mathrm{iso-s}}_{^3\mathrm{H(e)}}$ [in μ_N]	$r^2_{^2\mathrm{H}}$ [fm^2]	$r^2_{^3\mathrm{H(e), iso-s}}$ [fm^2]
Theory (N^{4}LO)	$0.4219(25)_{\mathrm{tr}}$	$4.384(26)_{\mathrm{tr}}(138)_{\mathrm{nucl}}$	$2.280(14)_{\mathrm{tr}}(128)_{\mathrm{nucl}}$
Experiment	0.425669	4.29(7) Sick '01	2.1(2.4) Sick '14; Amroun et al. '94

Summary and outlook

- Charge & quadrupole FFs of ^2H are in good shape (N^4LO, high-precision)

- Other systems and processes are limited to N^2LO accuracy due to unavailability of (consistently regularized) many-body forces & exchange currents

  $\Rightarrow$ **symmetry-preserving gradient flow regularization** Krebs, EE '24

- Correlations between BEs and radii can be employed to obtain precise results for the charge FFs of ^4He & $^3H(e)_{isoscalar}$ already at this stage Arseniy Filin et al., in progress

- ^4He: Nuclear effects under control $\Rightarrow$ new source of information about 1N radii

- $^3He/^3H$: prediction for the isoscalar 3N charge radius is more precise than exp

Thank you for your attention

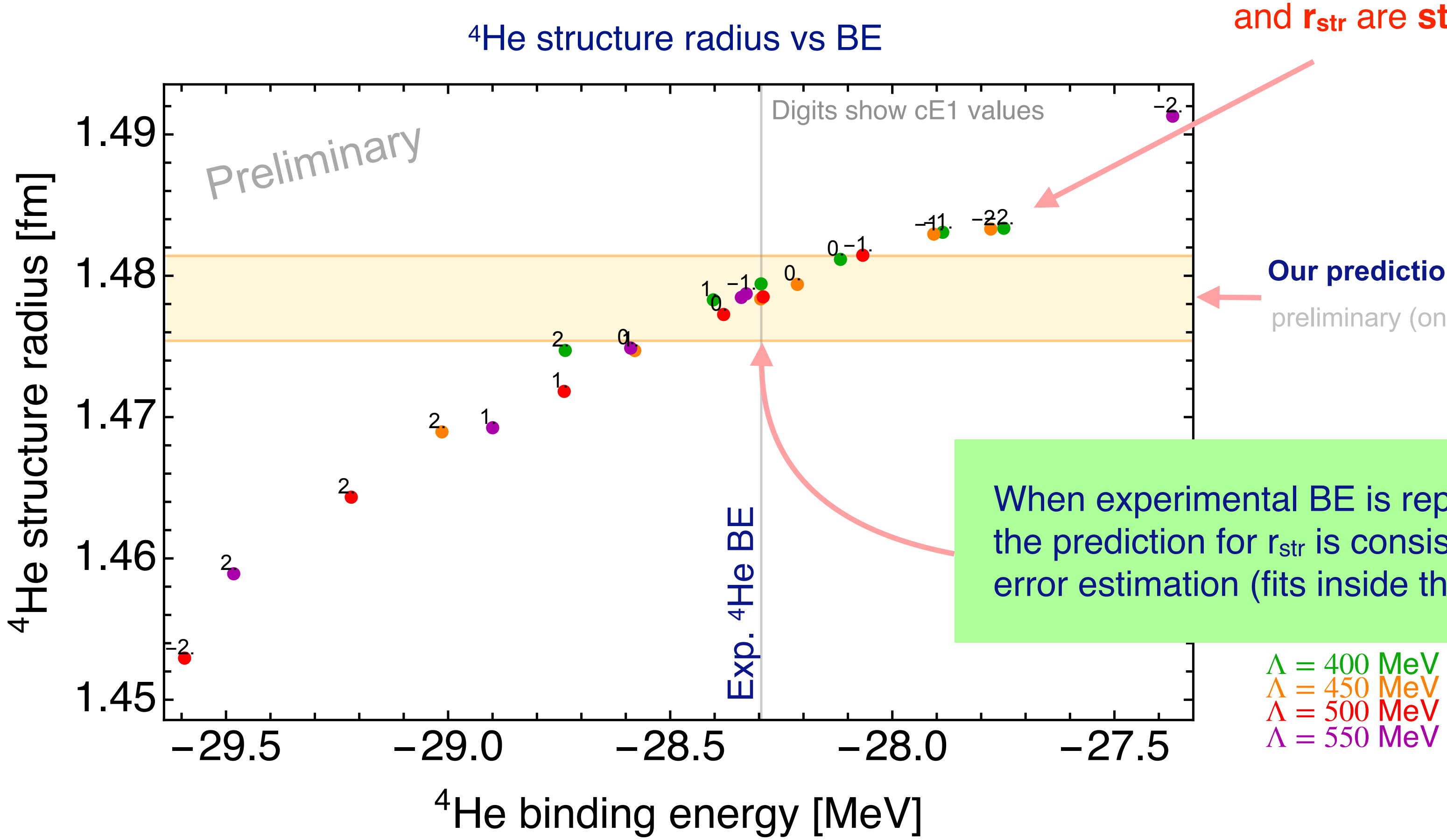
Error analysis for the radii (ongoing)

Type of error	$\delta r^2_{\mathrm{A=3,\,str}}$	$\delta r^2_{^4\mathrm{He,\,str}}$
Statistical uncertainties of LECs determined from πN, NN, p^{2}H, e^2H and e^4He data		
(a) NN LECs extracted from the database [23] of mutually consistent np and pp data	14	6
(b) πN LECs from matching χEFT to the Roy-Steiner analysis of πN scattering [79]	4	3
(c) LEC c_D in the 3NF extracted from p^{2}H differential cross section data of [52]	4	6
(d) LECs M_i in $\hat{\rho}_{2\mathrm{N}}$ extracted from ^{2}H [86, 87] and ^{4}He [69–75] form factor data	21	32
Total statistical uncertainty	26	33
Theory uncertainties		
(e) Truncation of the χEFT expansion	69	83
(f) Parametrizations of the nucleon form factors	XXX	30
Systematic uncertainties		
(g) Approximate treatment of relativistic effects	5	18

all numbers very preliminary…

Implicit treatment of higher-order 3NFs

^{4}He **binding energy** and **r_{str}** are **strongly correlated**!



Our prediction for ^{4}He structure radius:
preliminary (only truncation error is shown)

When experimental BE is reproduced the prediction for r_{str} is consistent with our error estimation (fits inside the orange band)