Status of lattice QCD determination of nucleon form factors at the physical point and its challenge for the proton charge radius

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PACS＝**P**rocessor **A**rray for **C**ontinuum **S**imulation

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PACS10 Nucleon Project

Nucleon structure

- **Proton radius puzzle (electron-nucleon scattering)**

  ➡ Electric/magnetic form factor (**rms radius**)

- **Precise knowledge of neutrino-nucleon scattering**

  ➡ Axial-vector from factor (axial charge & **axial radius**)

An important opportunity to develop our understanding of nucleon structure using lattice QCD simulations

Our Physics Targets

Nucleon structure = properties of single nucleon

✓ Nucleon matrix elements

Vector $\langle p'|V^\mu(q)|p\rangle = \overline{u}(p')\left[\gamma^\mu F_1(q^2) + i\sigma^{\mu\nu}\frac{q_\nu}{2M}F_2(q^2)\right]u(p)$ weak and elemag

$$= \overline{u}(p')\left[\frac{(p'+p)^\mu}{2M}\frac{G_E(q^2) - \frac{q^2}{4M^2}G_M(q^2)}{1-\frac{q^2}{4M^2}} + i\sigma^{\mu\nu}\frac{q_\nu}{2M}G_M(q^2)\right]u(p)$$

Axial-vector $\langle p'|A^\mu(q)|p\rangle = \overline{u}(p')\left[\gamma^\mu\gamma_5 F_A(q^2) + iq^\mu\gamma_5 F_P(q^2)\right]u(p)$ only weak

Two nucleon matrix elements can be described in terms of four types of nucleon elastic form factors

$$G_E(q^2), G_M(q^2), F_A(q^2), F_P(q^2)$$

Our Physics Targets

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Axial-vector $\langle p'|A^\mu(q)|p\rangle = \overline{u}(p')\left[\gamma^\mu\gamma_5 F_A(q^2) + iq^\mu\gamma_5 F_P(q^2)\right]u(p)$ only weak

axial form factor: $F_A(q^2) = F_A(0)\left(1 - \frac{q^2}{6}\langle r^2\rangle_A + \mathcal{O}(q^4)\right)$ $\quad F_A(0) = g_A$

axial rms radius: $\sqrt{\langle r_A^2\rangle} = 0.67(1)\ \text{fm} < \sqrt{\langle r_E^2\rangle_p} = 0.84 - 0.88\ \text{fm}$

→ Precise knowledge of neutrino-nucleon scattering

→ Search for $\nu_\mu \to \nu_e$ oscillation at NOvA and T2K

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➡ Five basic quantities: $g_A = F_A(0), \mu = G_M(0), G_{E,M}(q^2) = G_{E,M}(0)\left(1 - \frac{1}{6}r_{E,M}^2 q^2 + \mathcal{O}(q^4)\right)$

axial charge (g_A), magnetic moment (μ), charge radius (r_E), magnetic radius (r_M), axial radius (r_A)

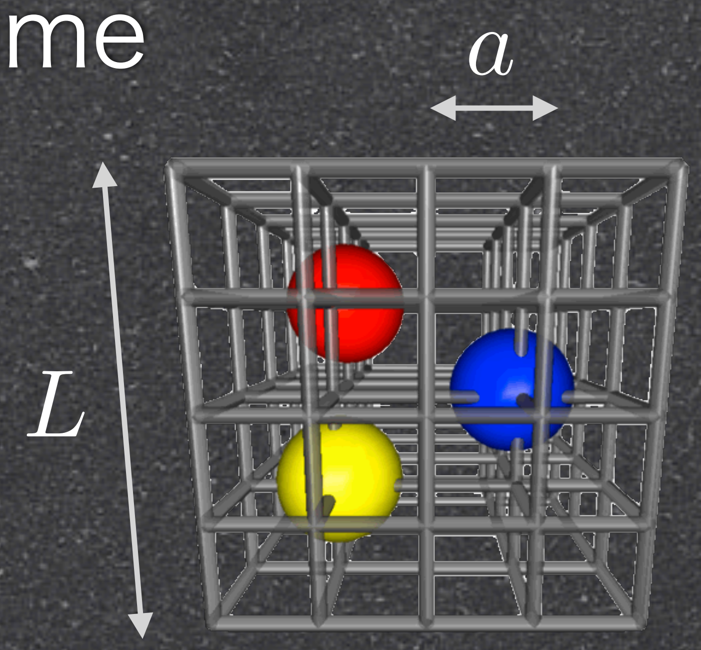
R. Tsuji et al., PRD109 (2024) 094505

Lattice Quantum Chromodynamics

- Fields defined on discrete space-time

  ➡ introduce cutoff in a gauge-invariant way

$$U_\mu(n) = e^{iagA_\mu(n)}$$



- Euclidean space : imaginary time $\tau =$ it

- Path integral quantization

  ➡ compute the quantum expectation value of a physical observable using a Monte Carlo method

$$\langle O \rangle = \frac{1}{Z} \int D[U] \int D[\psi] D[\overline{\psi}] O(U, \psi, \overline{\psi}) e^{-S_{\mathrm{QCD}}(U,\psi,\overline{\psi})}$$

Lattice QCD action

Uncertainties in lattice QCD

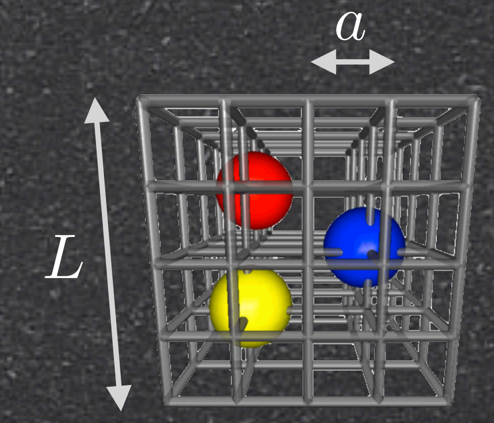
Statistical uncertainties (Monte Carlo Method)

$$\langle O \rangle \approx \langle O \rangle_N + \delta\langle O \rangle_N \qquad \langle O \rangle_N \equiv \frac{1}{N}\sum_{k=1}^{N} O(\{U_k\}), \quad \delta\langle O \rangle_N \equiv \sqrt{\frac{\langle O^2 \rangle_N - \langle O \rangle_N^2}{N-1}}$$

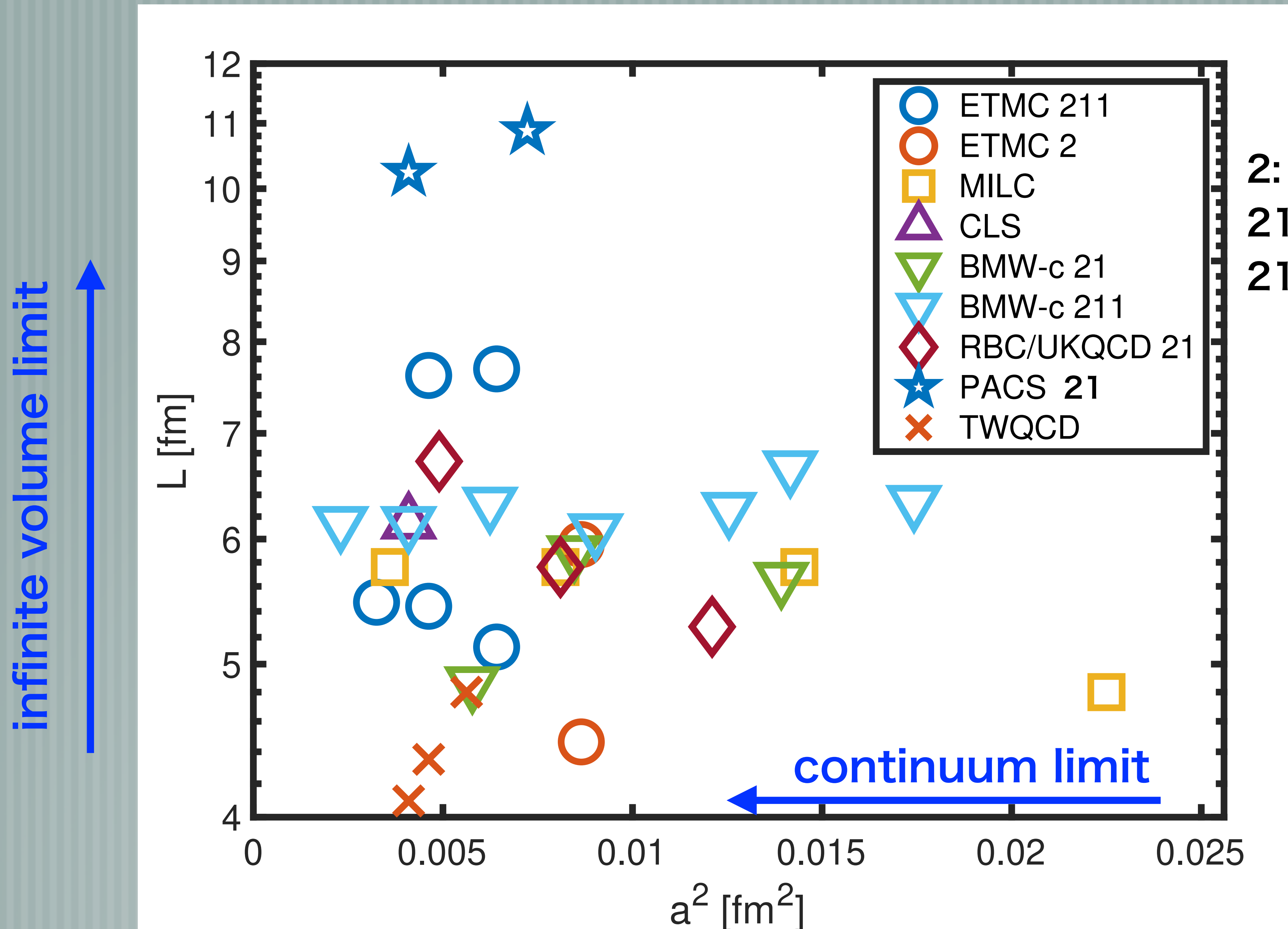
Four main systematic uncertainties

- **Quark vacuum polarization** : the number of dynamical quarks (Nf)
- **Finite lattice spacing** (a) : regularization of UV divergence: Λ~1/a
- **Finite volume** (L) : finite number of lattice grids: $(Ns)^3$ x Nt = L^3 x T
- **Chiral (quark mass) extrapolation** to the physical point: M_π

Limited by the size of computing resources



World status of lattice QCD projects near the physical point



2: $m_u=m_d$

21: $m_u=m_d\neq m_s$

211: $m_u=m_d\neq m_s\neq m_c$

Plenary talk at Lattice 2022 given by Finkenrath

World status of lattice QCD projects
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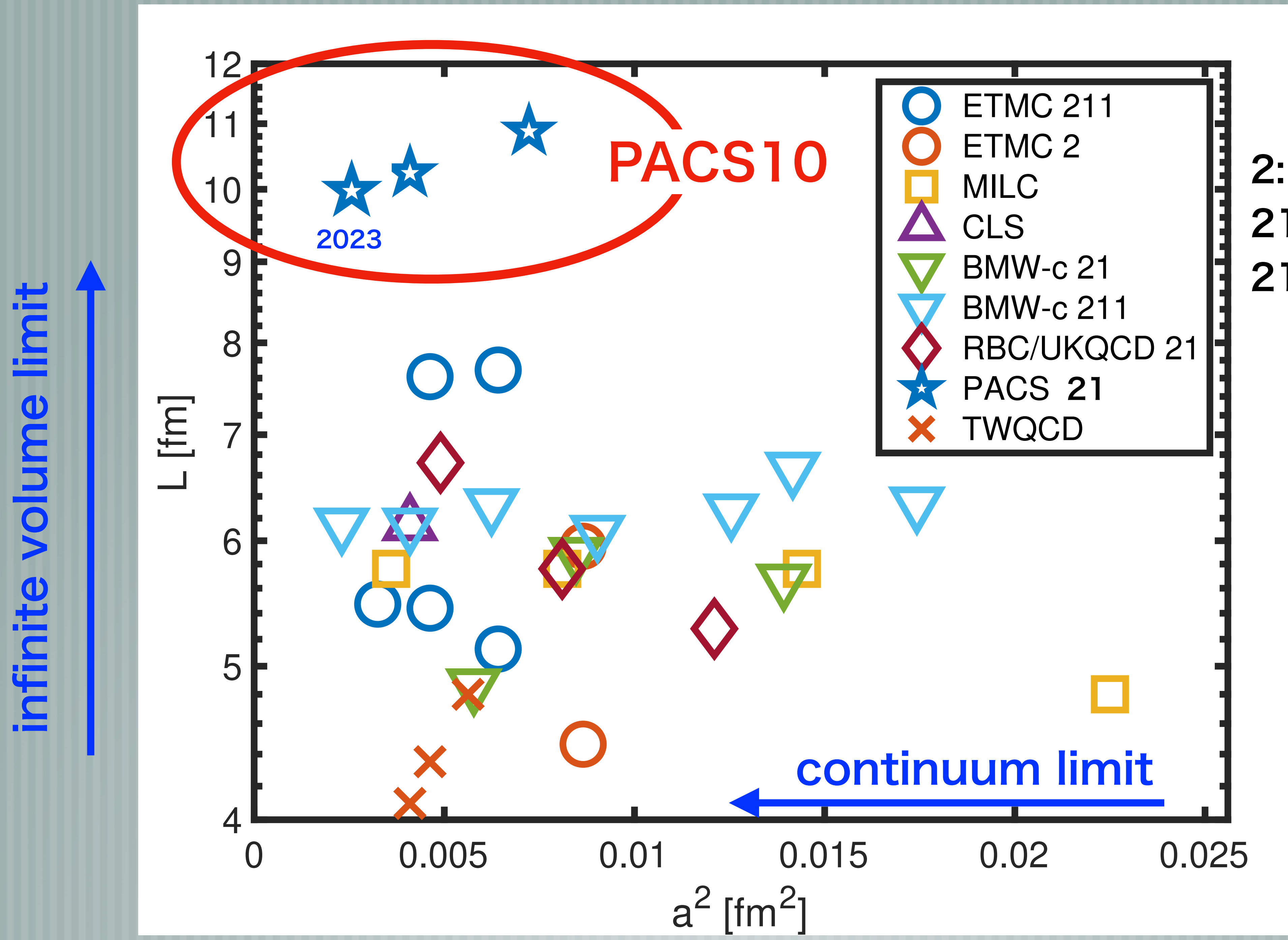

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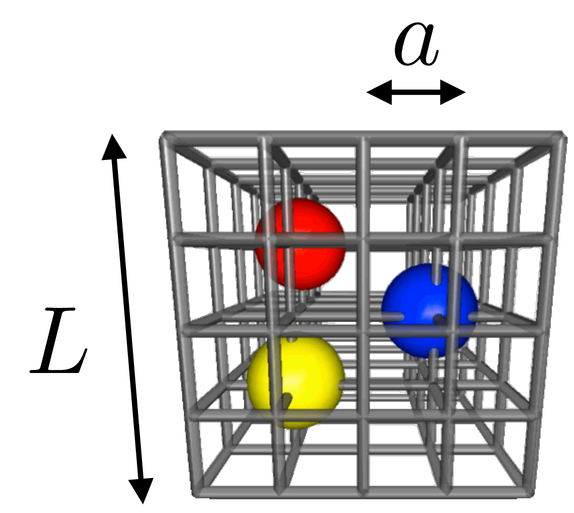
Plenary talk at Lattice 2022 given by Finkenrath

Our strategy



✓ Use **2+1 flavor PACS10** gauge configurations

- **Physical point** → No chiral extrapolation
- **Very large spatial volume (L)** → No finite size effect & **Low q^2 physics**
- **3 different lattice cut-offs (a)** → Continuum limit (currently not available)

✓ **All-mode averaging technique** → **High precision** measurements

✓ **Highly tuned smearing** → **Suppression of excited-state contributions**

✓ **Model-independent Q^2 fit** by z-Expansion method

✓ **Disclaimer:** only **iso-vector** quantities are considered

3 points to note

1. Why is such a large spatial size (~10 fm) is necessary?

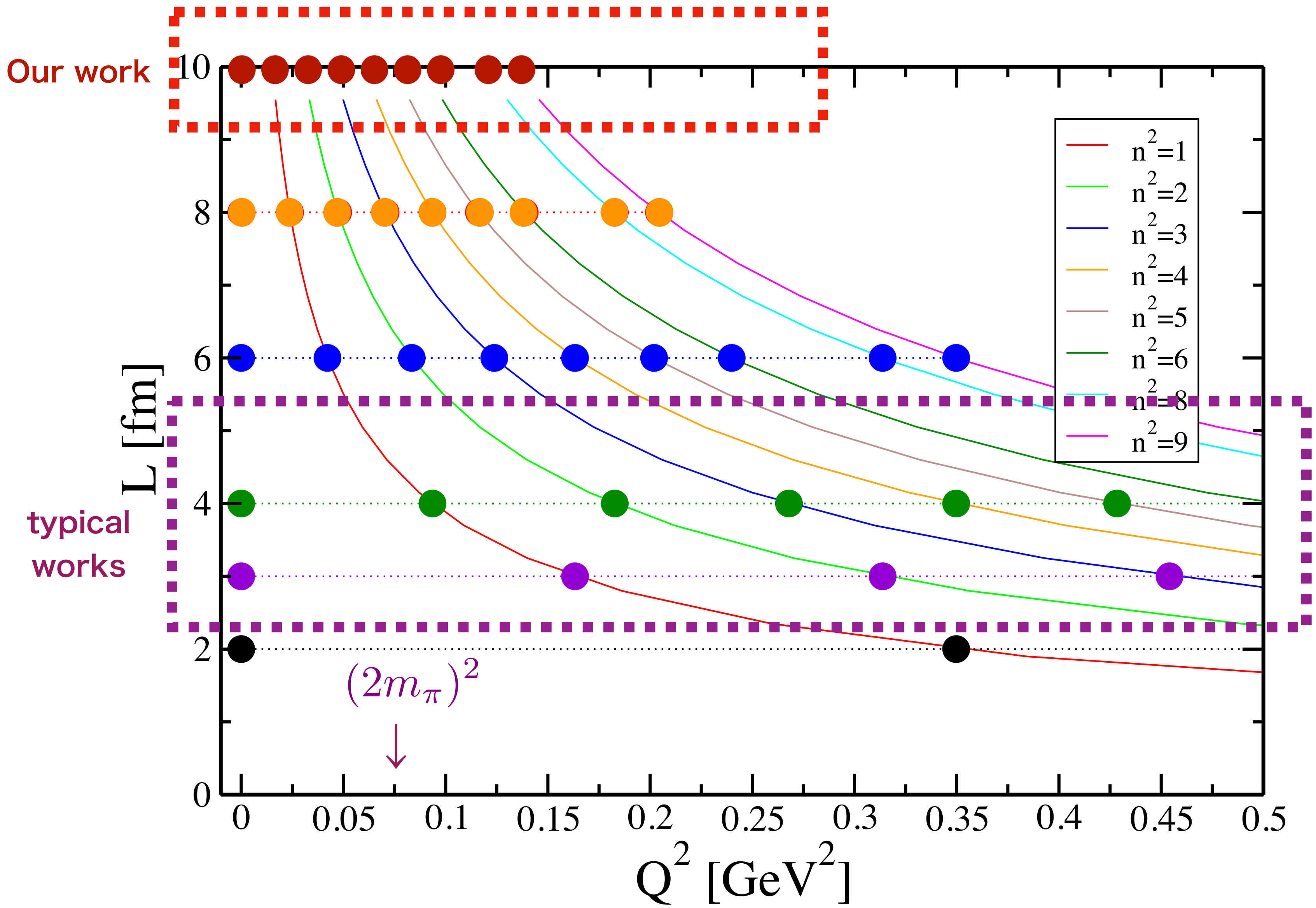
2. What is the significance of the suppression of excited-state contributions?

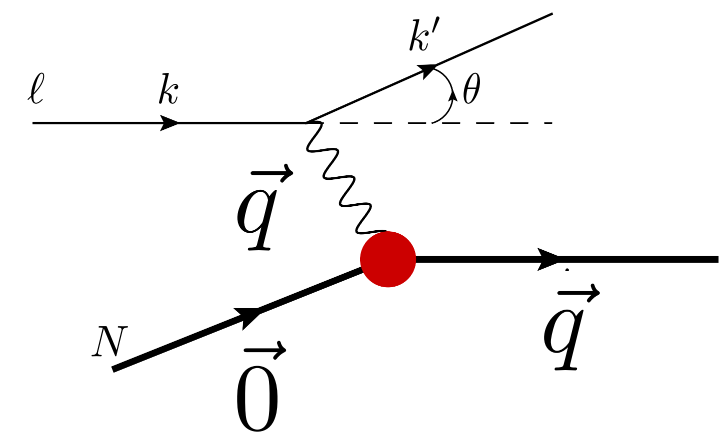
3. Why only iso-vector quantities are considered?

Why is such a large spatial size (~10 fm) is necessary?

How Large Spatial Size is Necessary?

Discrete momenta on the lattice are related to the size of the spatial extent L





$$Q^2 = -q^2 = 2M_N(E(\vec{q}) - M_N)$$

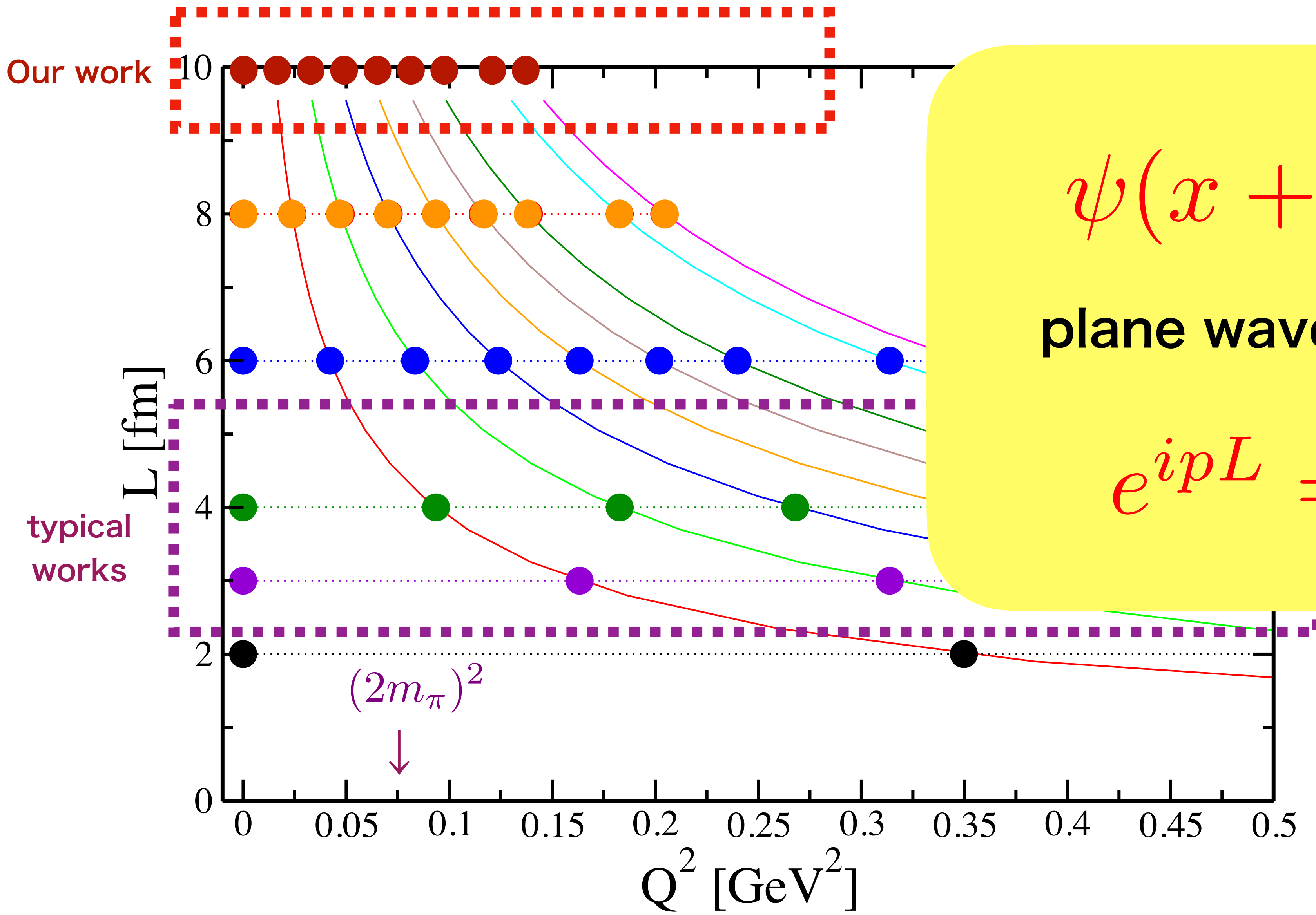
$$E(\vec{q}) = \sqrt{M_N^2 + \vec{q}^2}$$

$$\vec{q}^2 = \left(\frac{2\pi}{L}\right)^2 \vec{n}^2$$

✓can access the **small** momentum transfer **up to 0.01 [GeV^2] for L=10 fm**

How Large Spatial Size is Necessary?

Discrete momenta on the lattice are related to the size of the spatial extent L



$$\psi(x+L) = \psi(x)$$

plane wave $\psi(x) = e^{ipx}$

$$e^{ipL} = 1 \rightarrow p = \frac{2\pi}{L} n$$

$$\vec{q}^2 = \left(\frac{2\pi}{L}\right)^2 \vec{n}^2$$

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How Large Spatial Size is Necessary?

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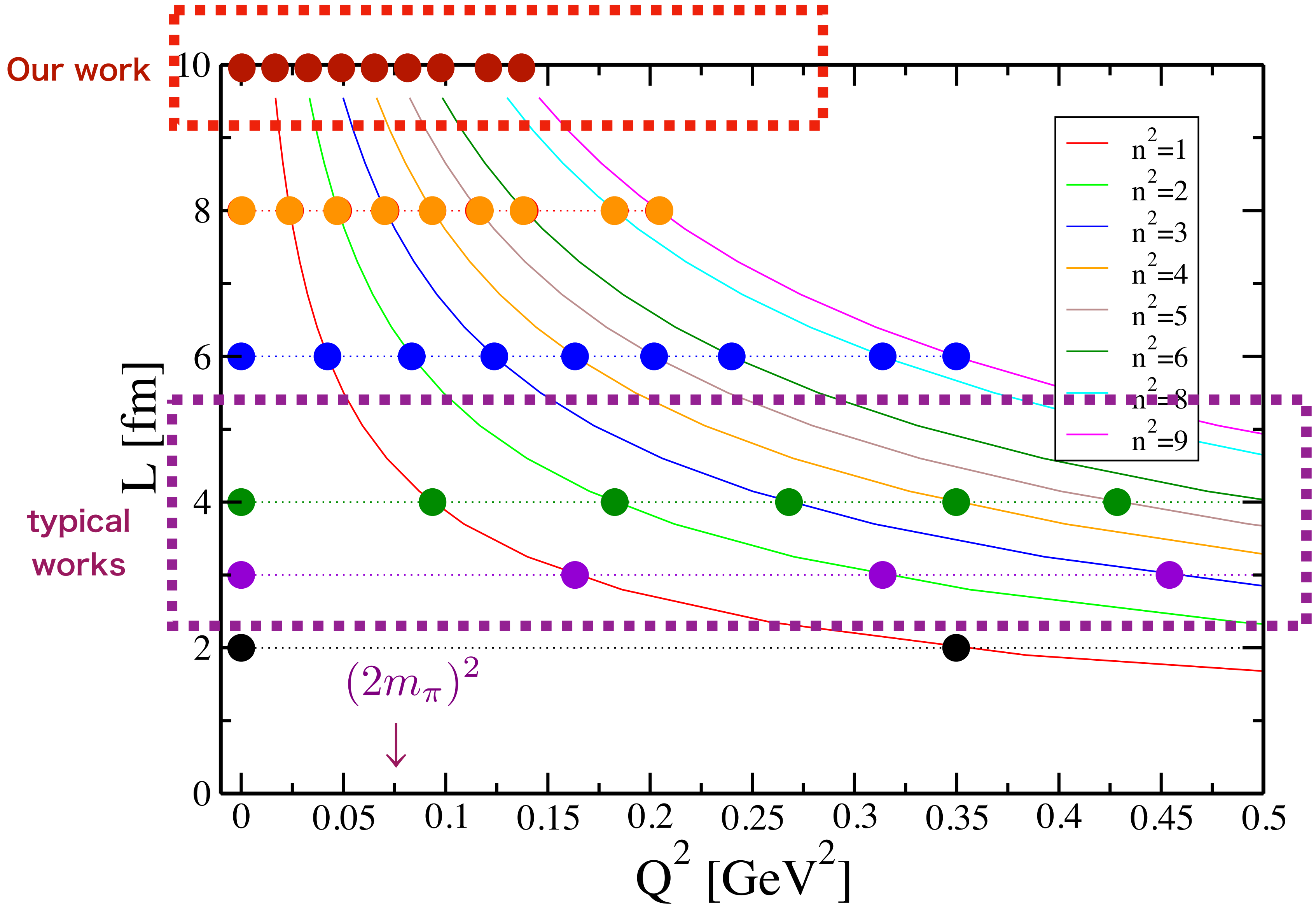


$$\langle r_E^2 \rangle = -\frac{6}{G_E(0)} \frac{dG_E(q^2)}{dq^2}\bigg|_{q^2=0}$$

Root-Mean-Square radius

$$R = \sqrt{\langle r_E^2 \rangle} = r_E$$

✓can access the **small** momentum transfer **up to 0.01 [GeV^2] for L=10 fm**

What is the significance of the suppression of excited-state contributions?

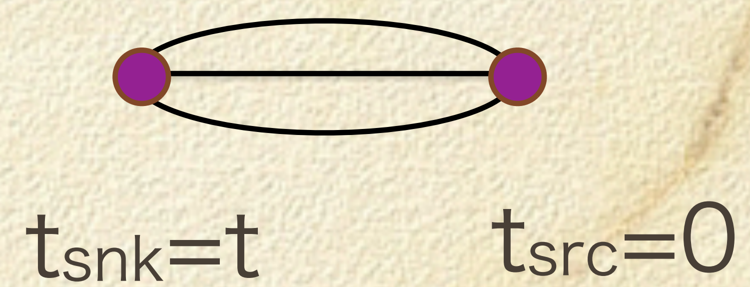
Nucleon correlation functions

- Compute **2-pt** and **3-pt** functions, using nucleon interpolator $\mathcal{H}$ and operator insertion $\mathcal{O}$

$$\sum_i |i\rangle\langle i| = 1$$

$$\langle \mathcal{H}(t)\mathcal{H}^\dagger(0)\rangle = \sum_i |\langle 0|\mathcal{H}(0)|i\rangle|^2 e^{-M_i t}$$

$$\rightarrow |\langle 0|\mathcal{H}|N\rangle|^2 e^{-M_N t}$$



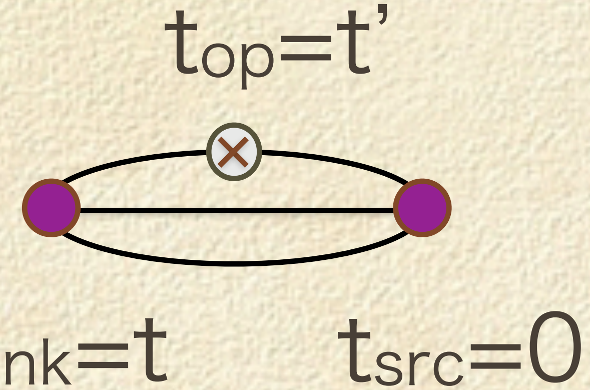
a sum of exponentials

$$\langle \mathcal{H}(t)\mathcal{O}(t')\mathcal{H}^\dagger(0)\rangle = \sum_{i,j} e^{-M_i(t-t')}\langle 0|\mathcal{H}|i\rangle\langle i|\mathcal{O}|j\rangle\langle j|\mathcal{H}^\dagger|0\rangle e^{-M_j t'}$$

$$\rightarrow |\langle 0|\mathcal{H}|N\rangle|^2 \langle N|\mathcal{O}|N\rangle e^{-M_N t}$$

$t - t' \gg 0$
and
$t' \gg 0$

no t'-dependence

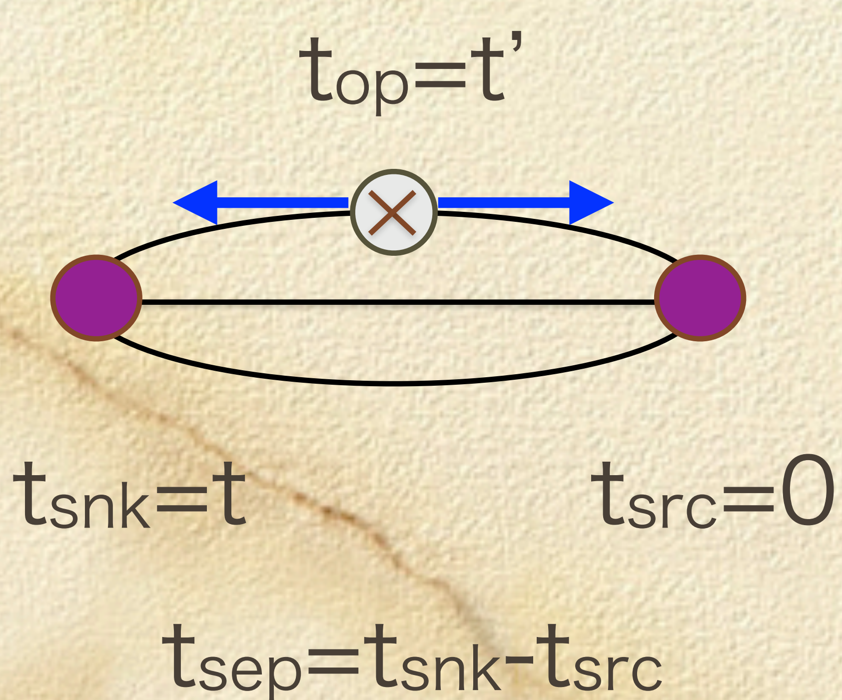


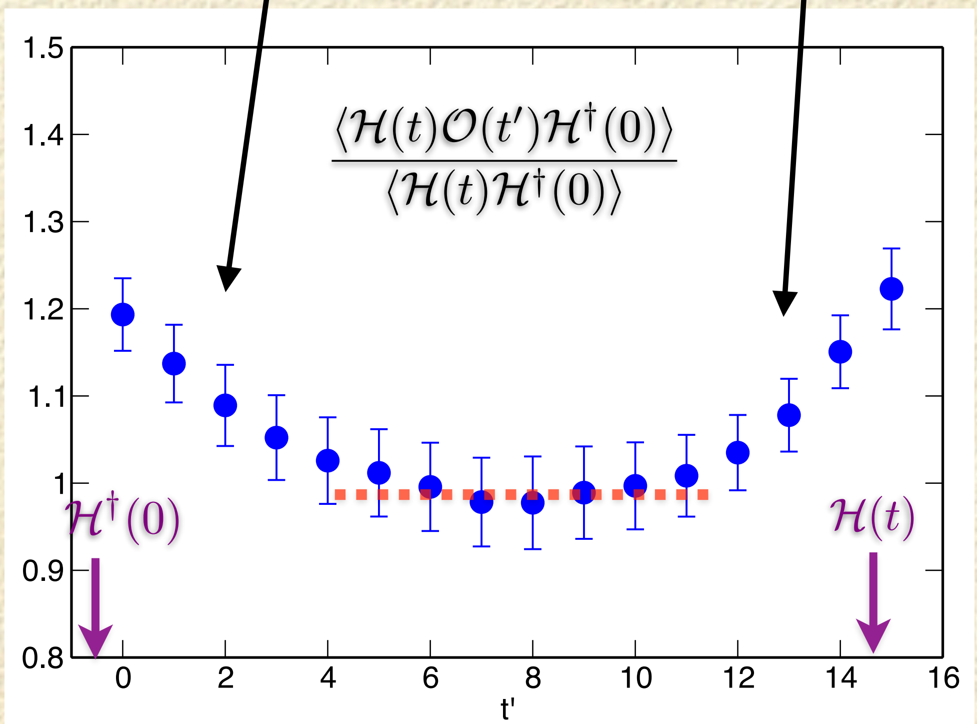
Ratio of 2-pt and 3-pt functions

✓matrix elements (form factors) can be determined from ratios of the 3-pt and 2-pt functions

$$\frac{\langle \mathcal{H}(t)\mathcal{O}(t')\mathcal{H}^\dagger(0)\rangle}{\langle \mathcal{H}(t)\mathcal{H}^\dagger(0)\rangle} \to \langle N|\mathcal{O}|N\rangle + Be^{-\Delta E(t'-t_{\rm src})} + Ce^{-\Delta E(t_{\rm snk}-t')}$$

$\Delta E = E_1 - M_N$ $\quad E_1$ (Excited-state energy)





$t = 15$ is fixed

Ratio of 2-pt and 3-pt functions

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Highly tuned smearing

$$\frac{\langle \mathcal{H}(t)\mathcal{O}(t')\mathcal{H}^\dagger(0)\rangle}{\langle \mathcal{H}(t)\mathcal{H}^\dagger(0)\rangle} \to \langle N|\mathcal{O}|N\rangle + Be^{-\Delta E(t'-t_{\rm src})} + Ce^{-\Delta E(t_{\rm snk}-t')}$$

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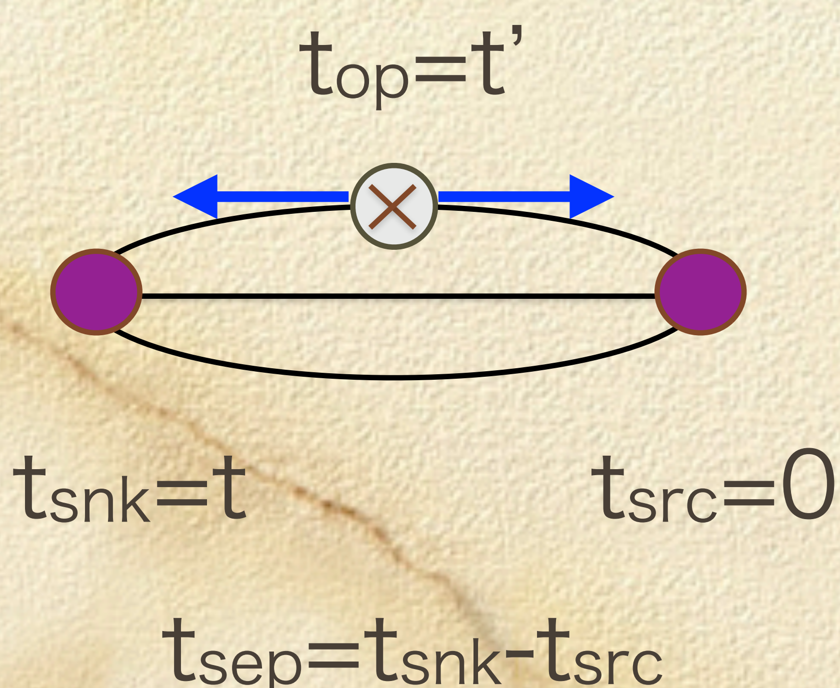
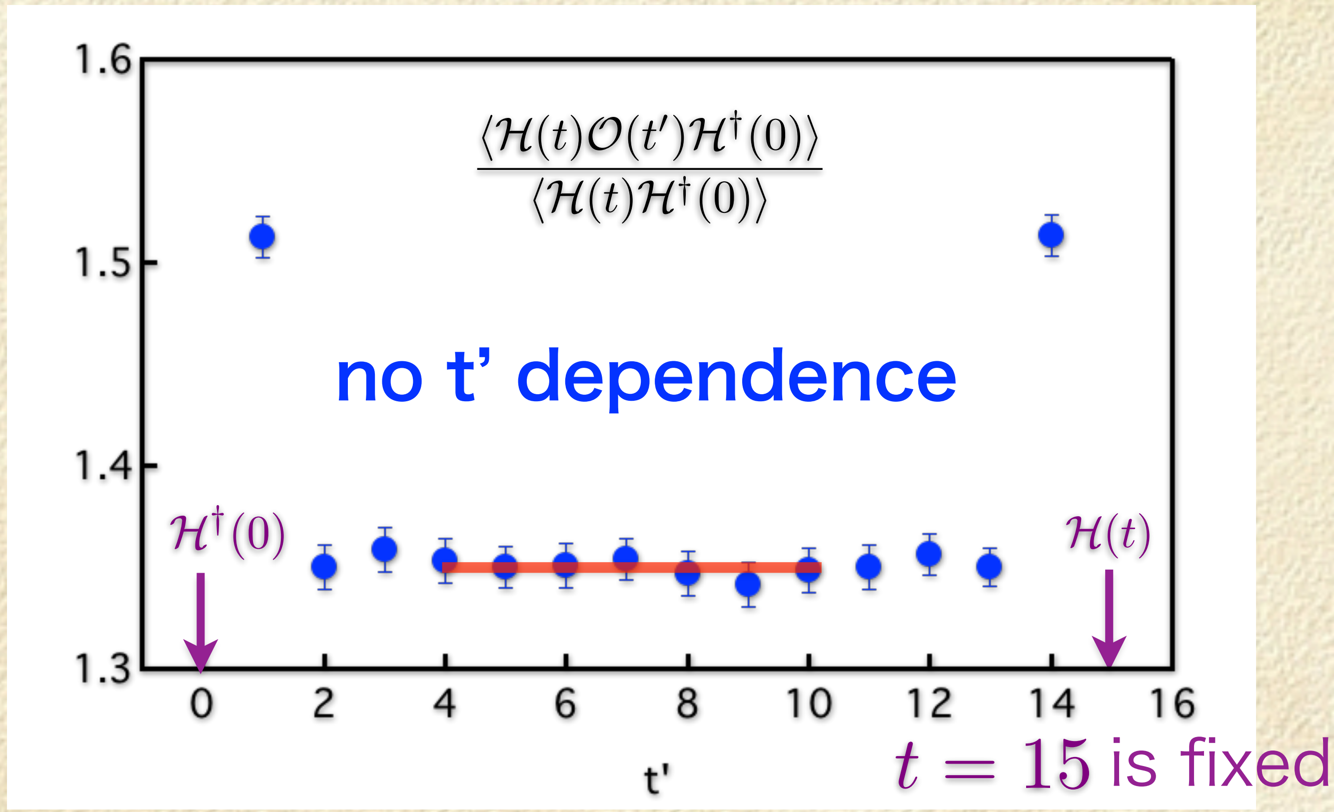




Why only iso-vector quantities are considered?

Iso-vector quantities

electromagnetic current

$$J_\mu^{\rm em} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d + \cdots$$

$$= \frac{1}{2}\boxed{(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)} + \frac{1}{6}\boxed{(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d)} = J_\mu^V + \frac{1}{3}J_\mu^S$$

iso-vector iso-scalar

matrix element (ME)

proton $\langle p|J_\mu^{\rm em}|p\rangle = \langle p|J_\mu^V|p\rangle + \frac{1}{3}\langle p|J_\mu^S|p\rangle$

neutron $\langle n|J_\mu^{\rm em}|n\rangle = \langle n|J_\mu^V|n\rangle + \frac{1}{3}\langle n|J_\mu^S|n\rangle$

iso-spin symmetry

$$\langle p|J_\mu^S|p\rangle = \langle n|J_\mu^S|n\rangle$$

$$\langle p|J_\mu^V|p\rangle = -\langle n|J_\mu^V|n\rangle$$

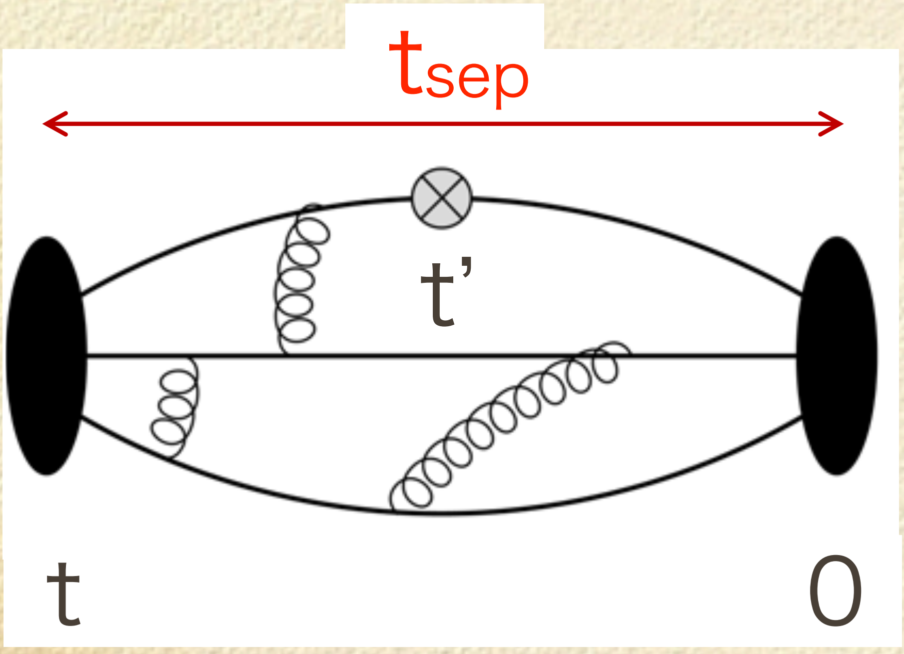
$$\langle p|J_\mu^{\rm em}|p\rangle - \langle n|J_\mu^{\rm em}|n\rangle = \boxed{\langle p|\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d|p\rangle} = \boxed{\langle p|\bar{u}\gamma_\mu d|n\rangle}$$

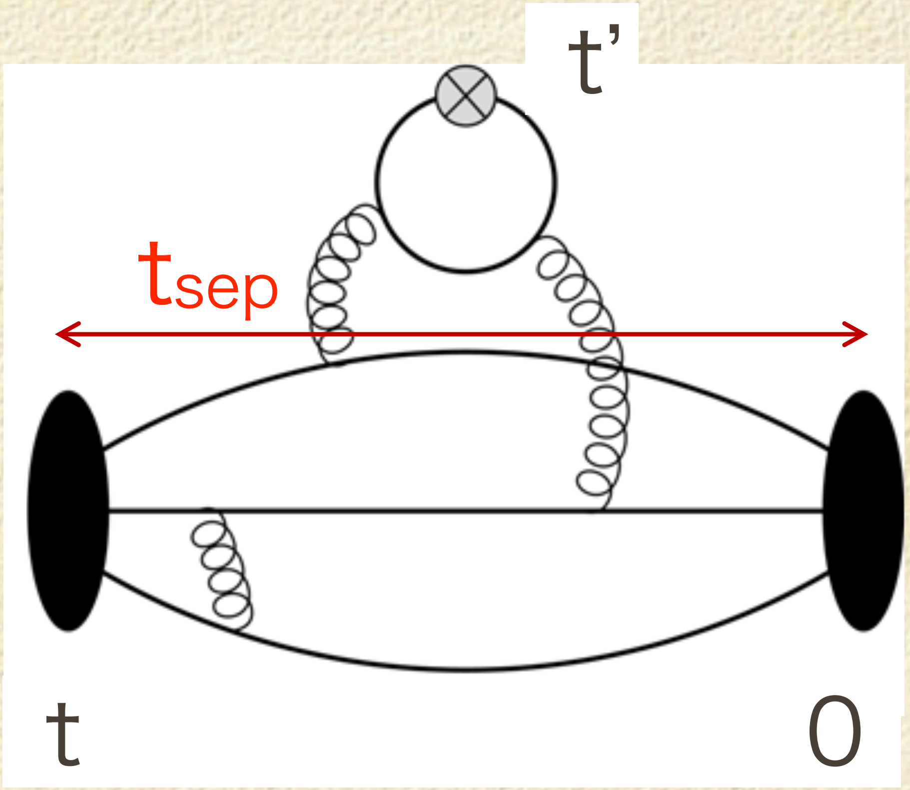
proton ME **neutron ME** **iso-vector** Weak process

Iso-vector part receives NO disconnected contribution in 2+1 flavor QCD

Connected/disconnected diagrams

$\langle \mathcal{H}(t)\mathcal{O}(t')\mathcal{H}^\dagger(0)\rangle$ has **two** types of quark contraction diagrams (Wick contractions)





✓ **iso-vector** quantities

✓ β-decay (**weak matrix** elements)

✓ **iso-scalar** quantities

✓ **electro-magnetic** matrix elements

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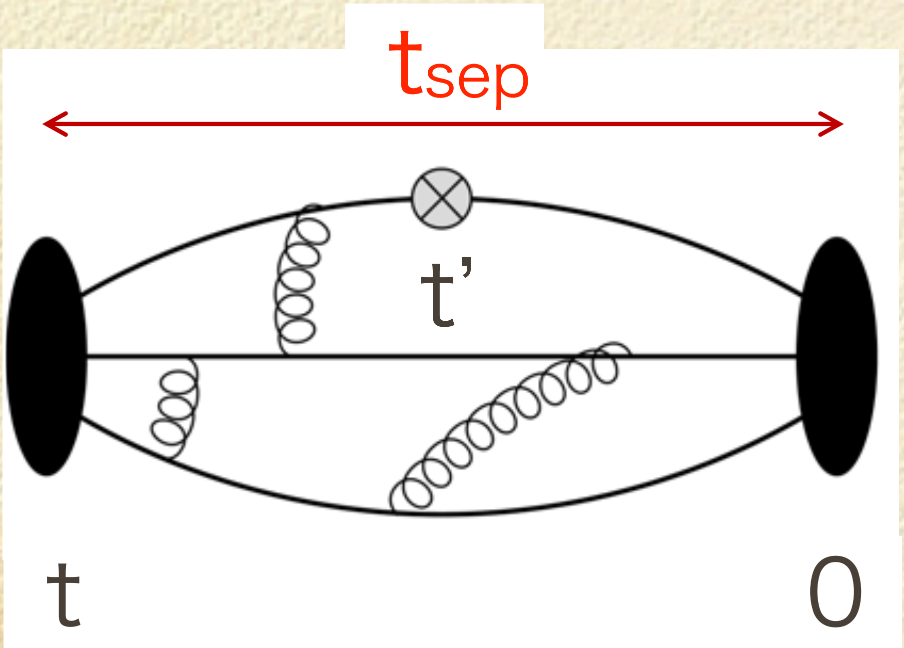

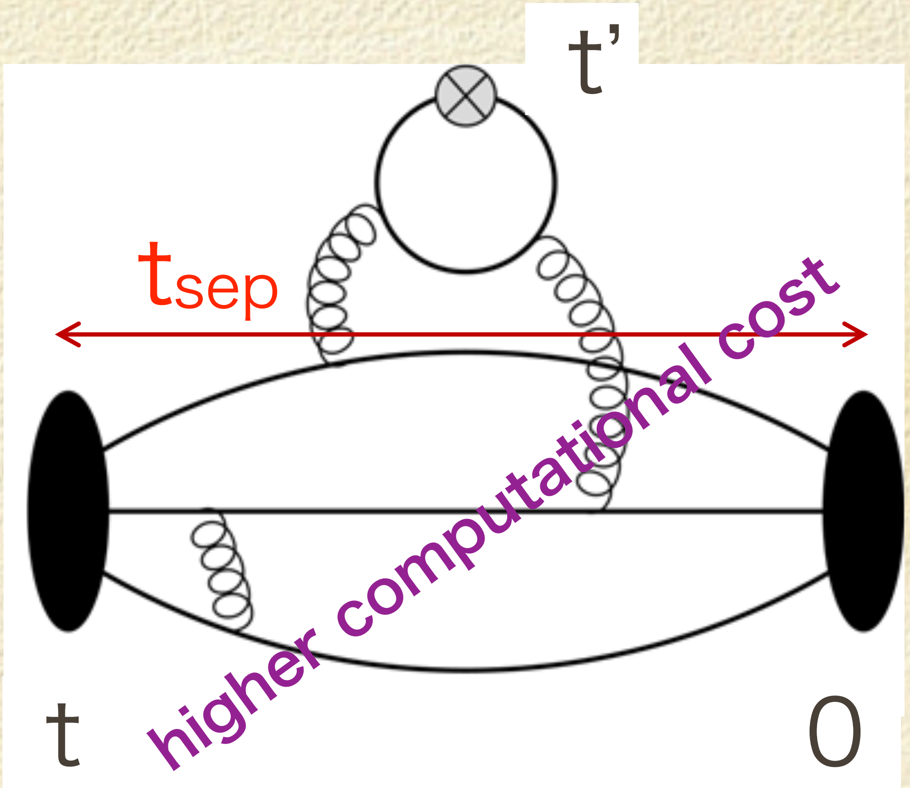

Typically **10 to 100 times** higher

✓ **iso-vector** quantities

✓ β-decay (**weak matrix** elements)

✓ **iso-scalar** quantities

✓ **electro-magnetic** matrix elements

Numerical results

Status of PACS10 projects

Configuration	PACS10			HPCI
Resource	Oakforest-PACS → Fugaku			K-computer
N_f	2+1			2+1
m_π [MeV]	135	138	142	146
L [fm]	10 fm			8.1 fm
L^3 x T	128^4(64^4)	160^4	256^4	96^4
a [fm]	0.085	0.063	0.041	0.085
Status	done	done	done	done
Nucleon FF	done	done	running	done
Renorm (SF, NPR)	done	partly done	planning	done

- K.I. Ishikawa et al., Phys. Rev. D98 (2018) 074510. (HPCI)
- E. Shintani et al., Phys. Rev. D99 (2019) 014510. (PACS10)
- K.I. Ishikawa et al., Phys. Rev. D104 (2021) 074514. (PACS10)
- R. Tsuji et al., Phys. Rev. D106 (2022) 094505. (PACS10)
- R. Tsuji et al., Phys. Rev. D109 (2024) 094505. (PACS10)

m_π [MeV]	135	138	142	146
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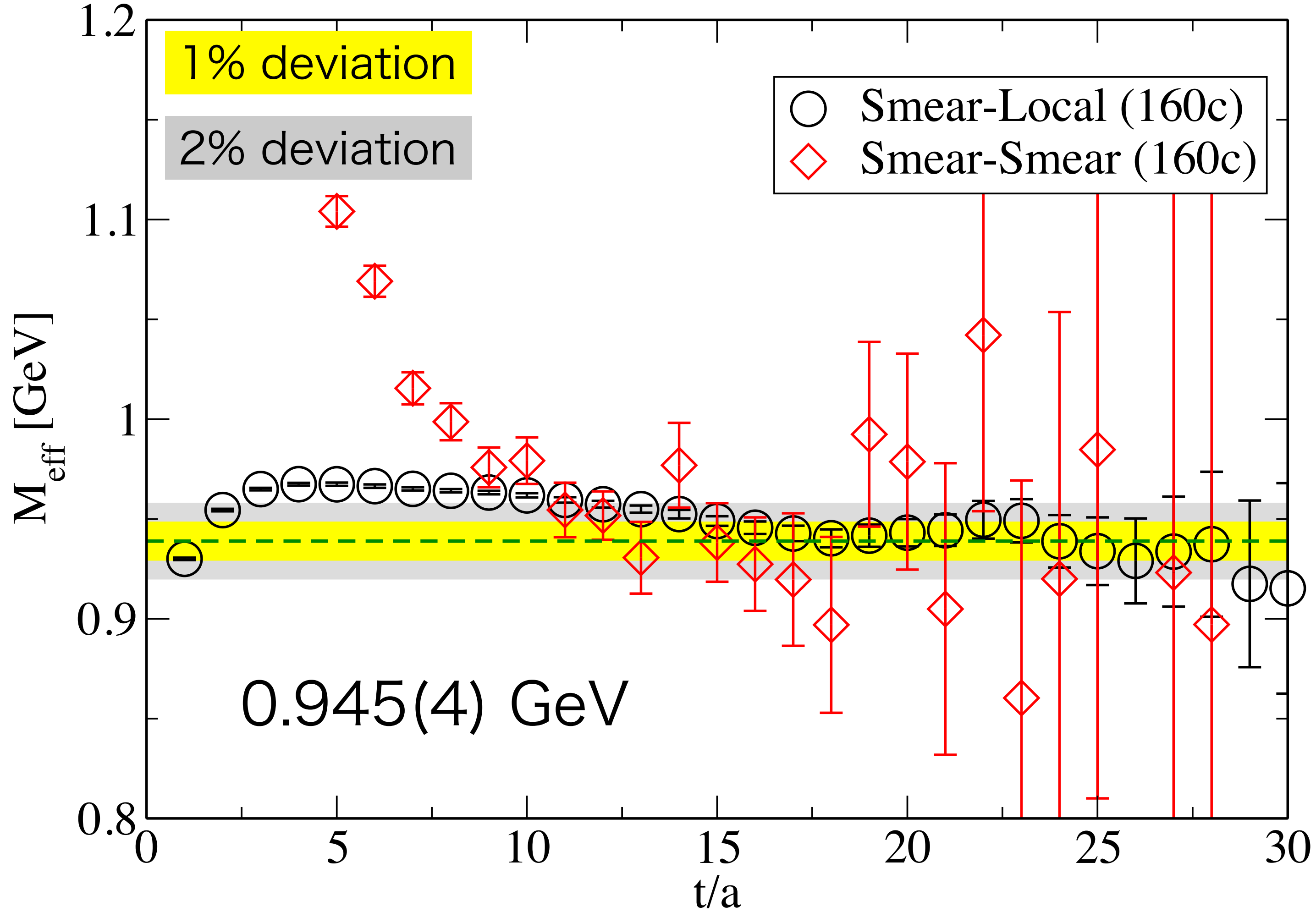
Nucleon mass

M_N

Effective mass plot for M_N

Smearing parameters are **highly tuned to maximize the ground-state dominance**.



$L^4=160^4$, a=0.063 fm

$$G(t) = \sum_i A_i \exp(-M_i t)$$

A sum of exponential func.

$$M_0 < M_1 < \cdots$$

$$M_{\text{eff}}(t) = \ln\{G(t)/G(t+1)\}$$

$$\xrightarrow[t\to\infty]{} M_0$$

$$A_0 \gg A_{i>0}$$

Achieving a percent level precision on the nucleon mass

Axial charge

g_A

Ratio for axial charge g_A

R. Tsuji et al., PRD109 (2024) 094505

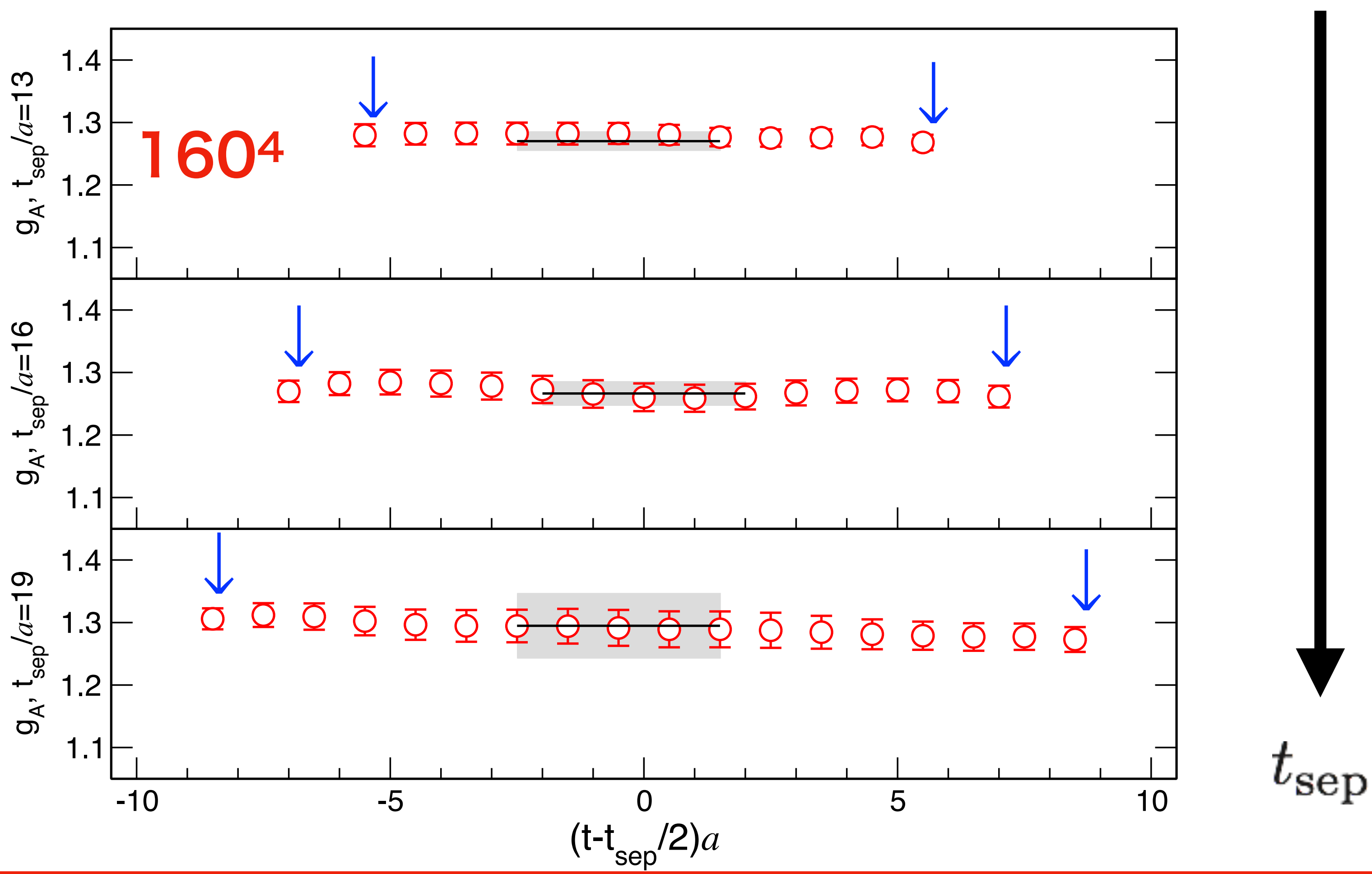
✴**Highly tuned smearing**

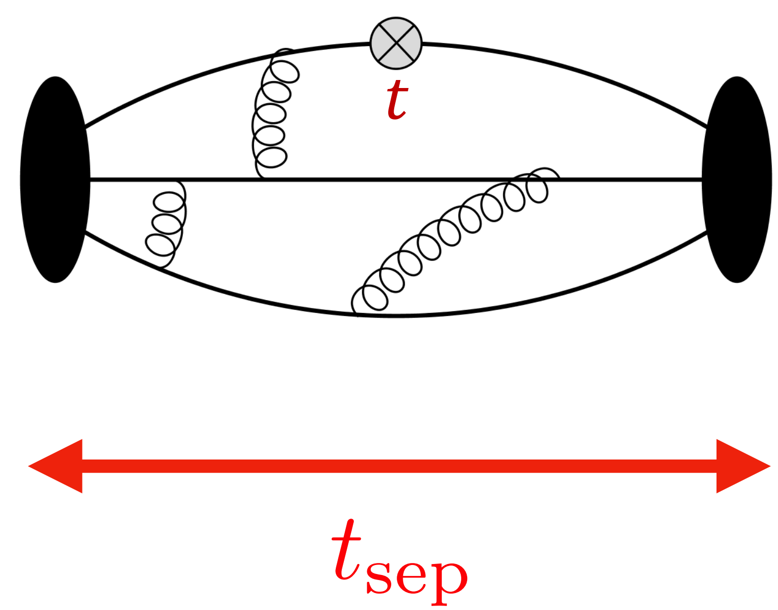
→ Suppression of excited-state contributions

Ratio method

$$\frac{\langle \mathcal{H}(t)\mathcal{O}(t')\mathcal{H}^\dagger(0)\rangle}{\langle \mathcal{H}(t)\mathcal{H}^\dagger(0)\rangle} \to \langle N|\mathcal{O}|N\rangle$$

GS contribution

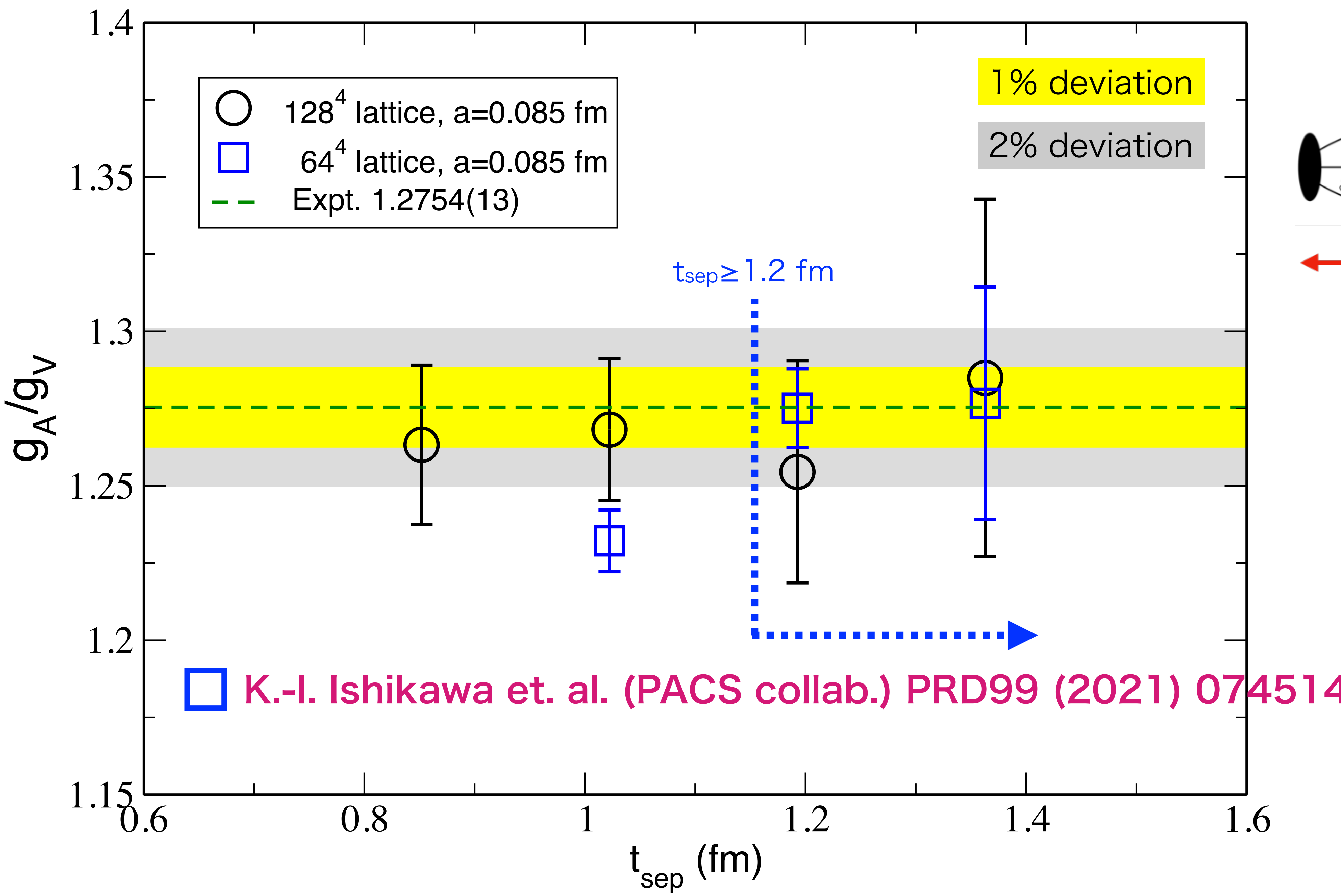


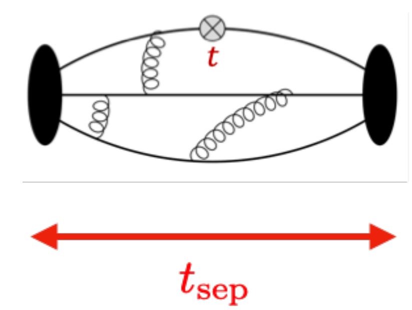


Effect of excited-state contamination is negligible

Good plateau for t_{sep}=19, 16, 13 and no t_{sep} dependence

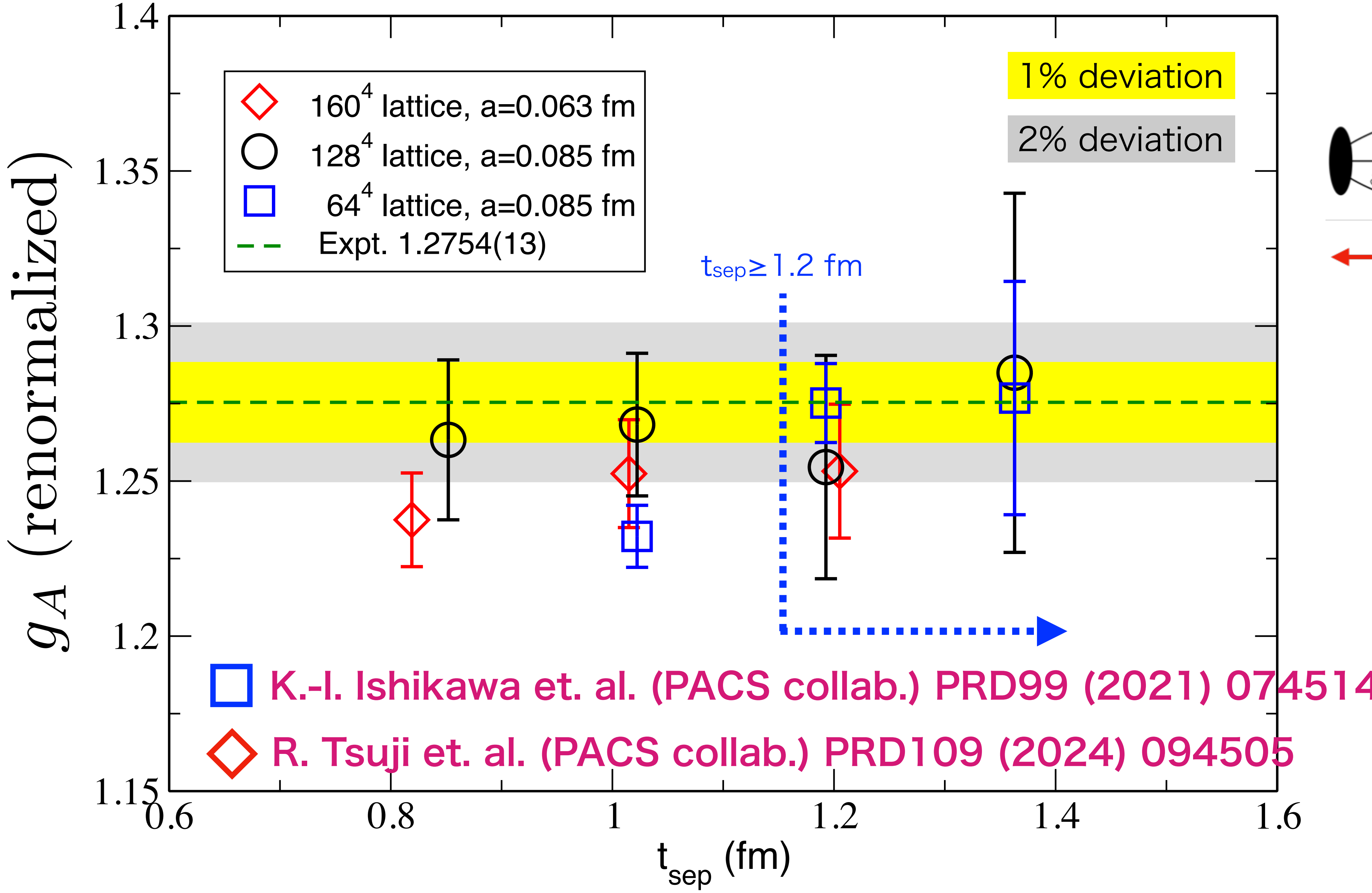
A precent-level determination of g_A

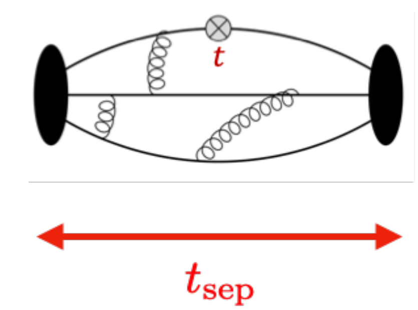




Effect of excited state contamination is negligible for $t_{sep}\geq$1.2 fm.
Finite volume error is less than 1%.

A precent-level determination of g_A





Effect of excited state contamination is negligible for t_{sep}≥1.2 fm.

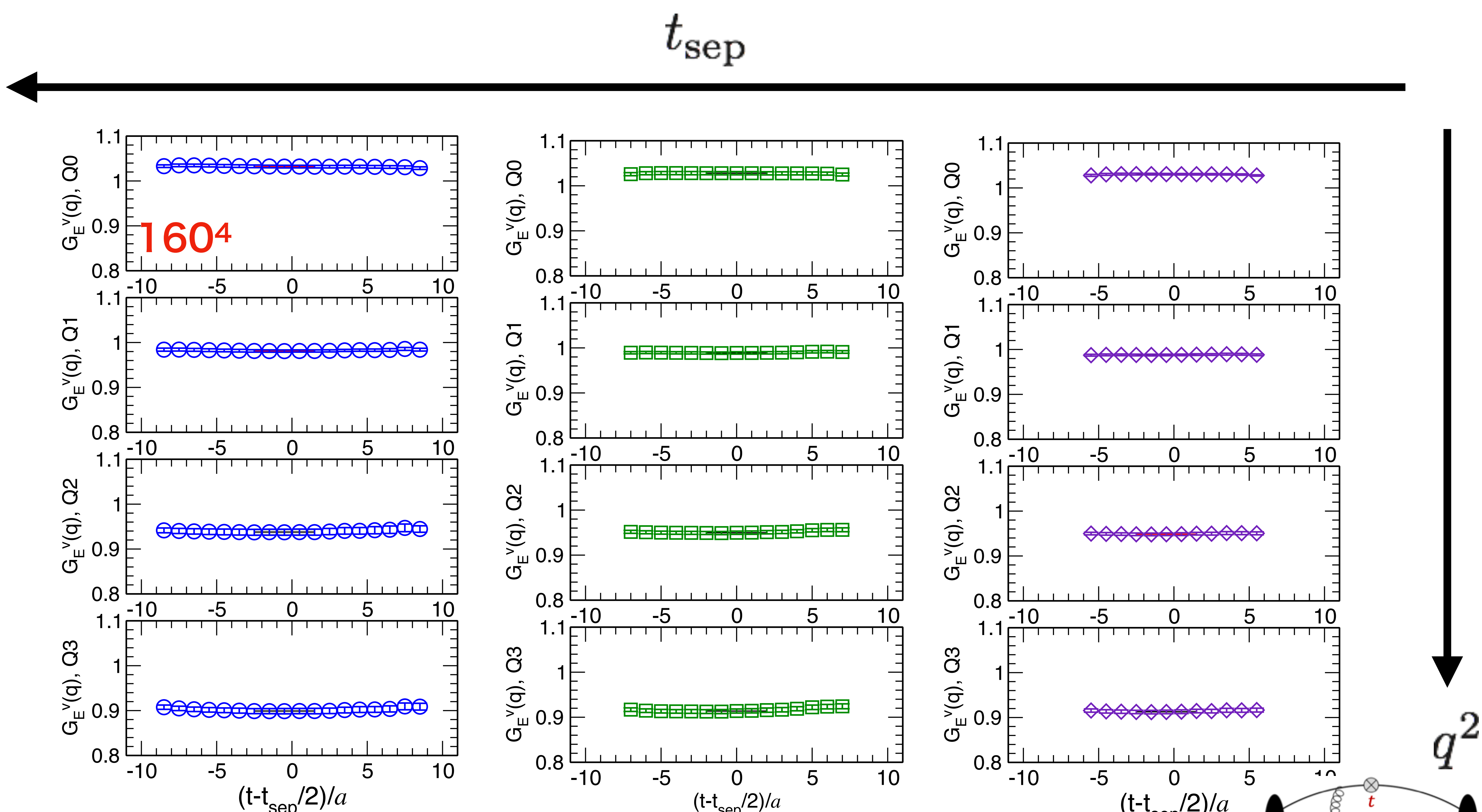
Finite volume error is less than 1%.

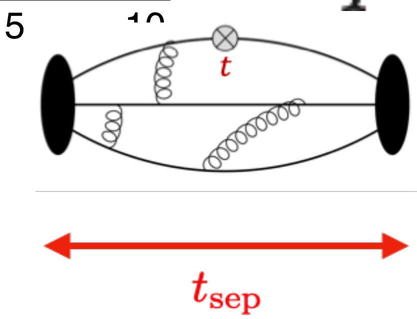
Discretization error is less than 1%.

Electric form factor

G_E

Ratio for iso-vector $G_E(q^2)$

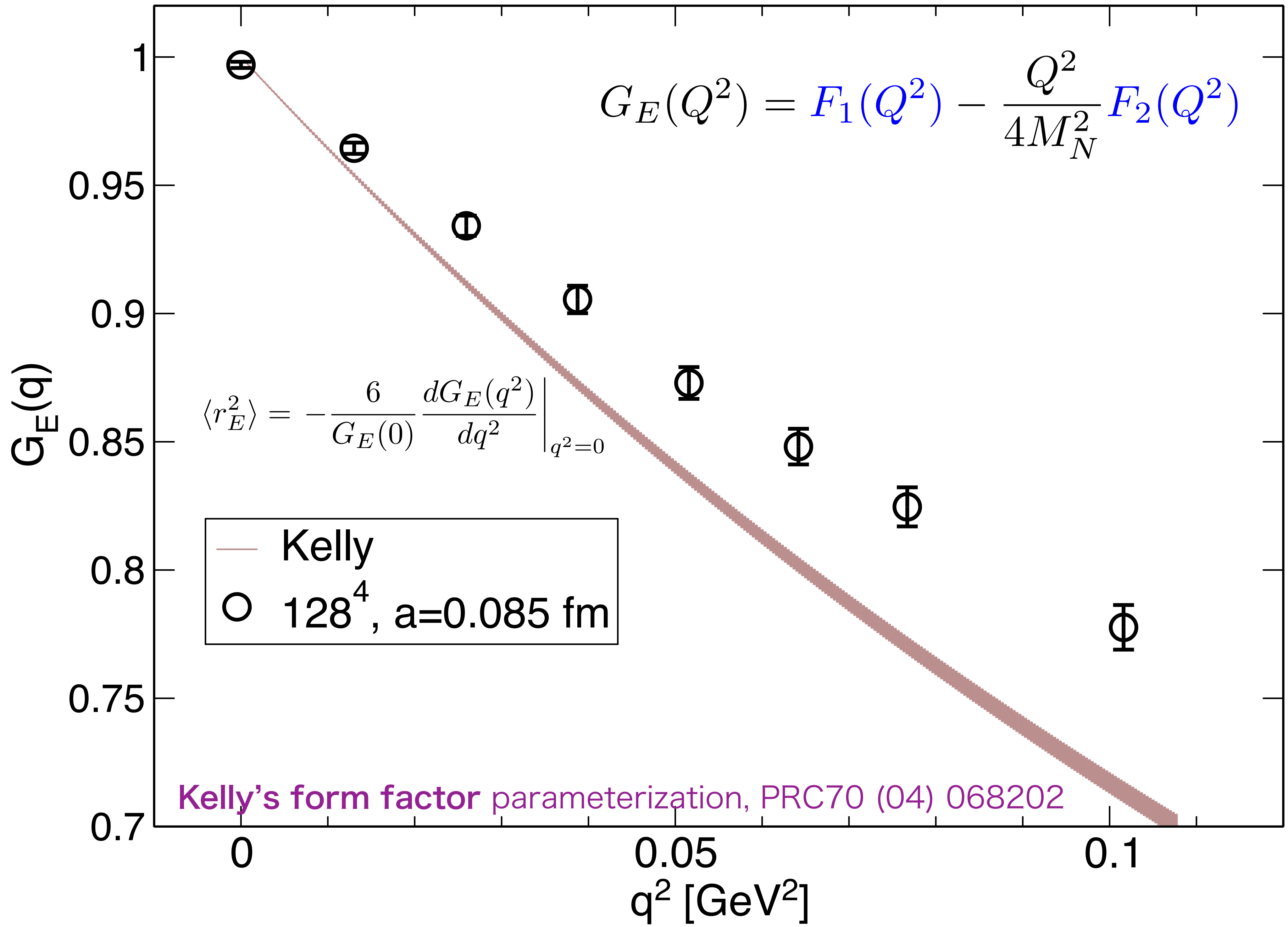




Good plateau for t_{sep}=13, 16, 19

Iso-vector electric form factor G_E

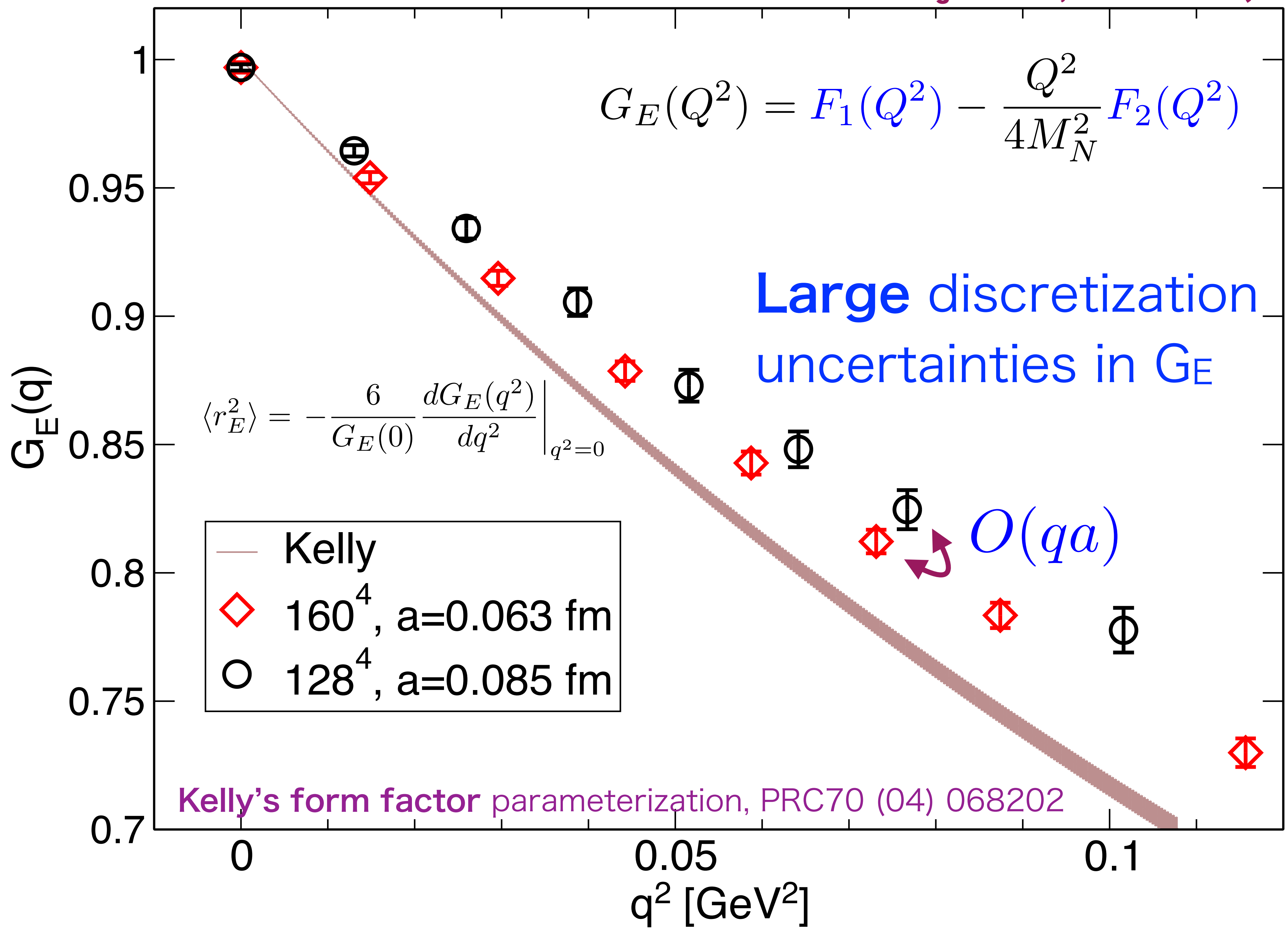
E. Shintani et al., Phys. Rev. D99 (2019) 014510



The previous results disagree with the Kelly's curve

Iso-vector electric form factor G_E

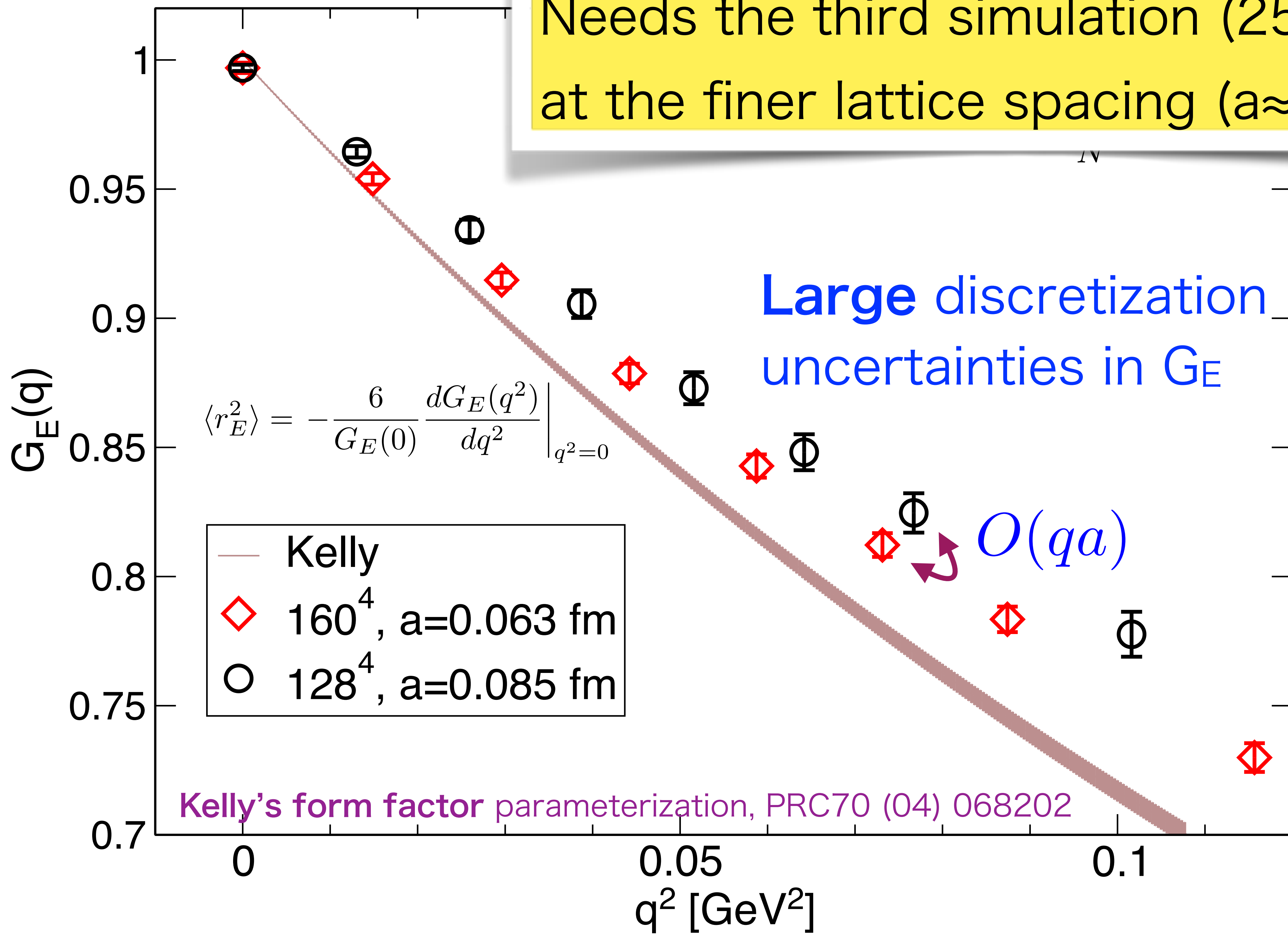
R. Tsuji et al., PRD109 (2024) 094505



The new results obtained with the fine lattice spacing (a≈0.06 fm) approaches to the Kelly's curve

Iso-vector electric form factor G_E

Needs the third simulation (256^4) at the finer lattice spacing (a≈0.04 fm)

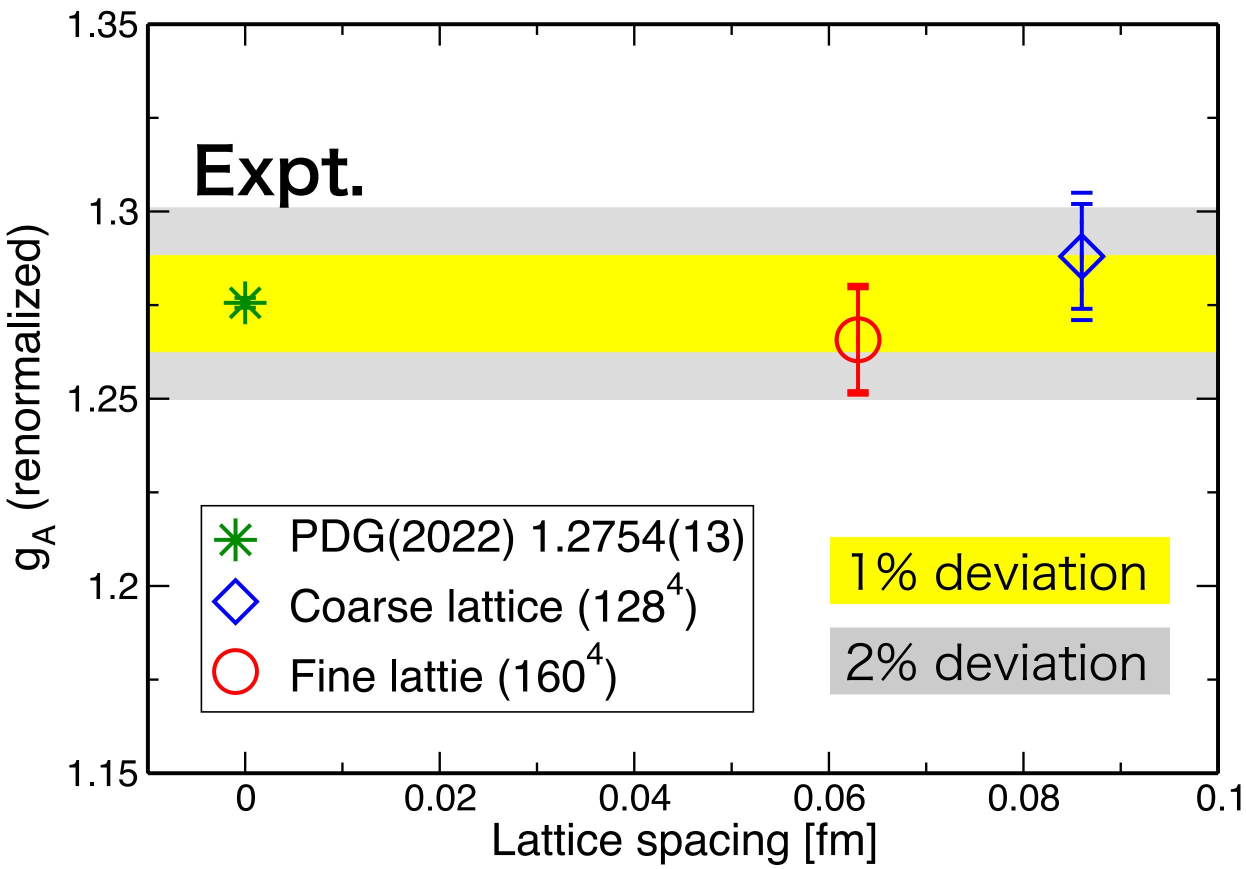


The new results obtained with the fine lattice spacing (a≈0.06 fm) approaches to the Kelly's curve

Lattice discretization uncertainties

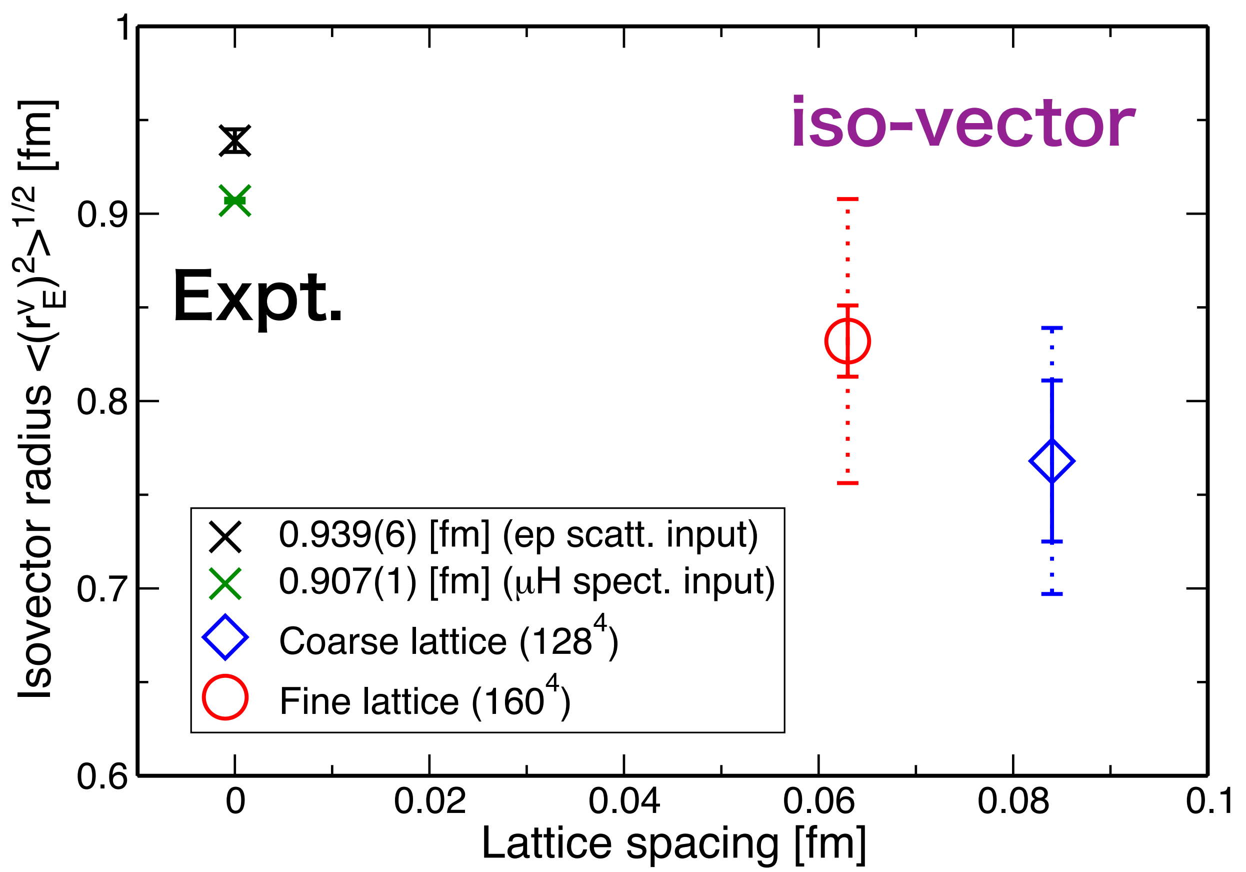
- Lattice discretization uncertainties on g_A and r_E

axial charge g_A



Negligibly small (< 1%)

rms charge radius r_E



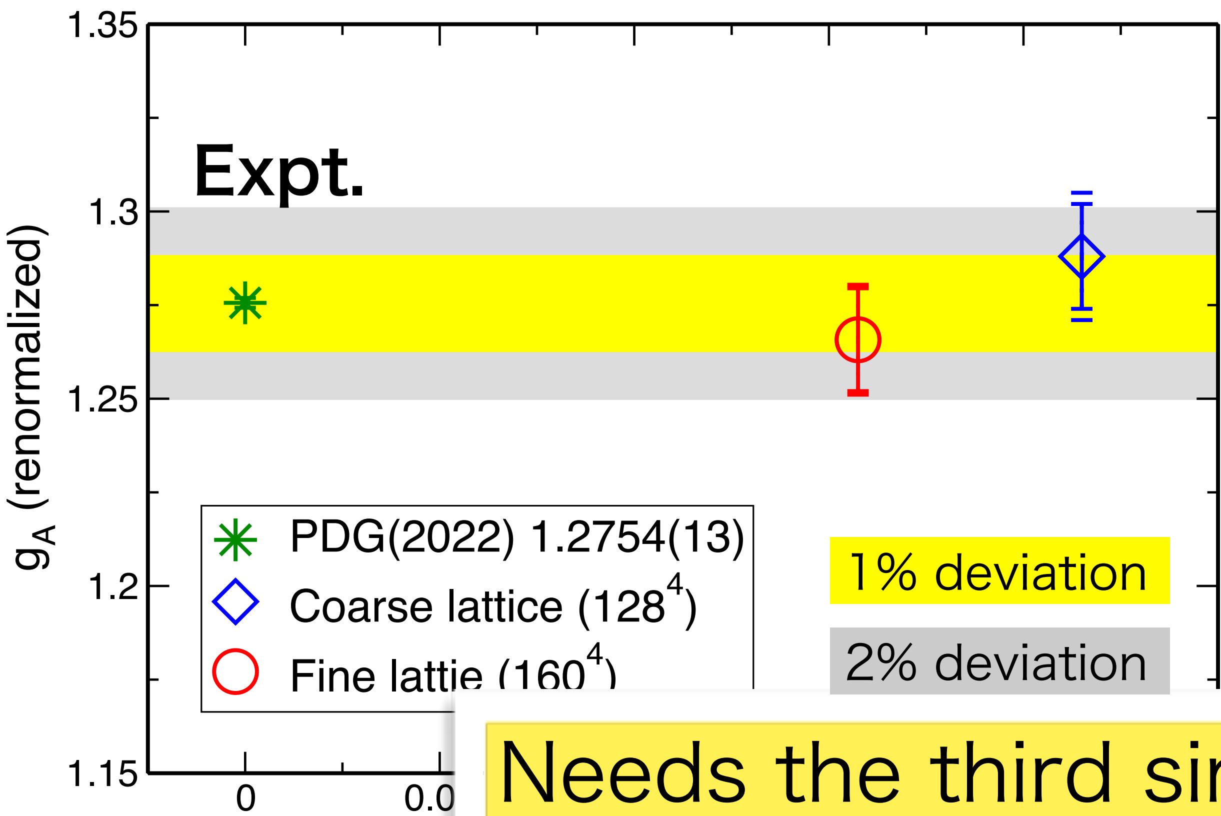
Significantly large(~10%)

R. Tsuji et al., PRD109 (2024) 094505

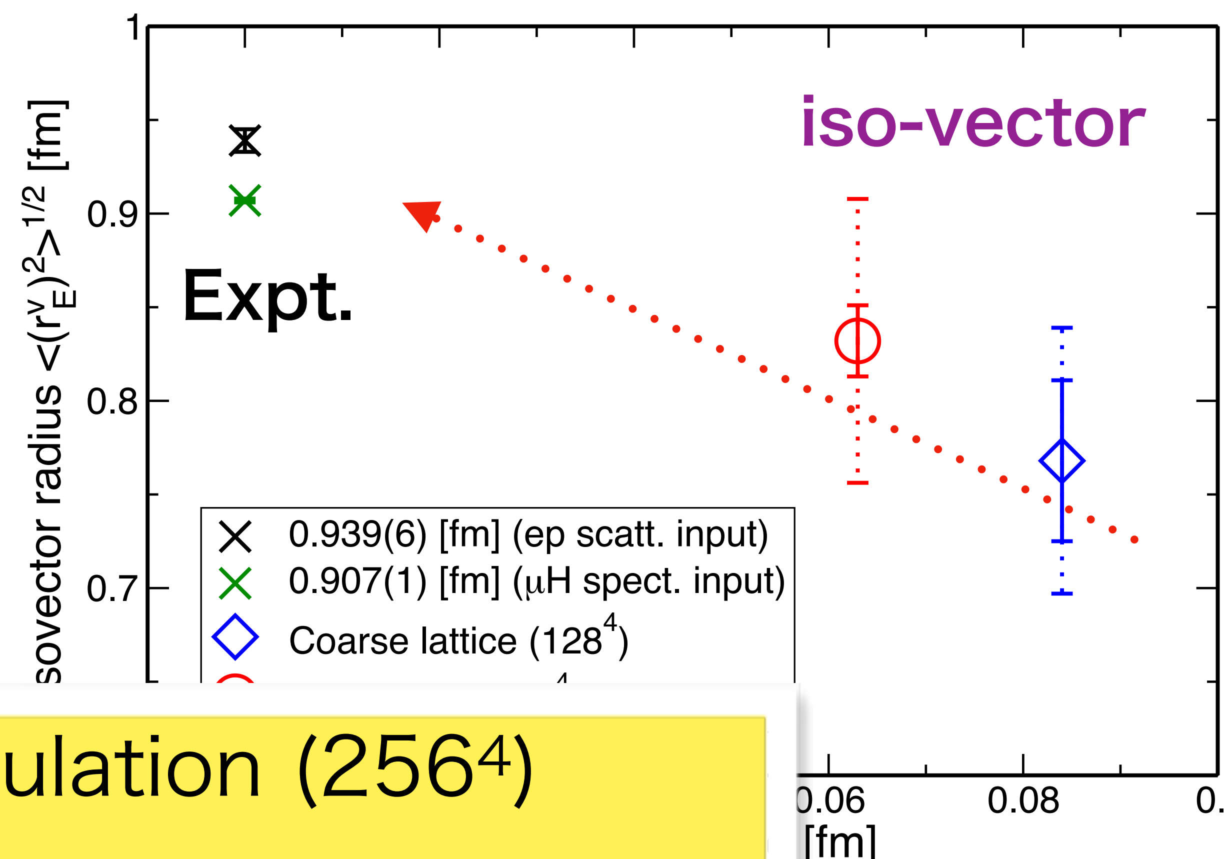
Lattice discretization uncertainties

- **Lattice discretization uncertainties** on g_A and r_E

axial charge g_A



rms charge radius r_E



Negl… …ge(~10%)

Needs the third simulation (256^4)
at the finer lattice spacing (a≈0.04 fm)

R. Tsuji et al., PRD109 (2024) 094505

Summary

- We have studied **iso-vector** nucleon form factors calculated in 2+1 flavor QCD **at the physical point** on **$(10\ \mathrm{fm})^4$** lattice at **two lattice spacings (a=0.085 and 0.063 fm)**

✓ Large spatial volume allows investigation in the **small momentum transfer region**, $0.01 < q^2 < 0.1$ [GeV^2] with $q^2=0$

✓ t_{sep} dependence is systematically investigated

➡ g_A and G_E, G_M show **no t_{sep} dependence**

➡ excited-state contributions are negligible for $t_{sep} \geq 1.2$ fm

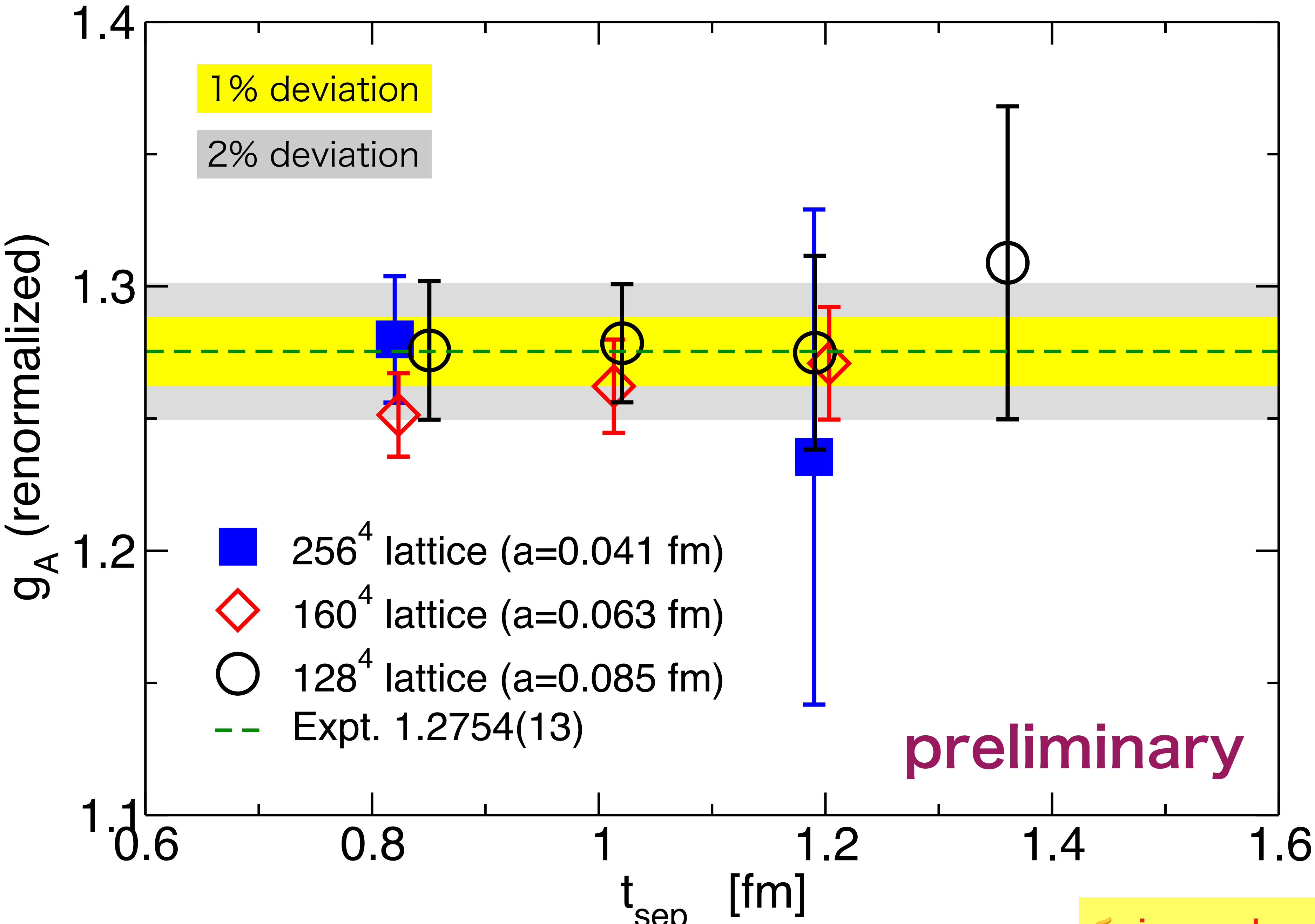
✓ **Large** discretization uncertainty is **observed in G_E, but not in g_A**

➡ needs the third simulation at the **finer** lattice spacing

3rd simulation performed on Fugaku

2023 16M node-hour@Fugaku
2024 10M node-hour@Fugaku

iso-vector

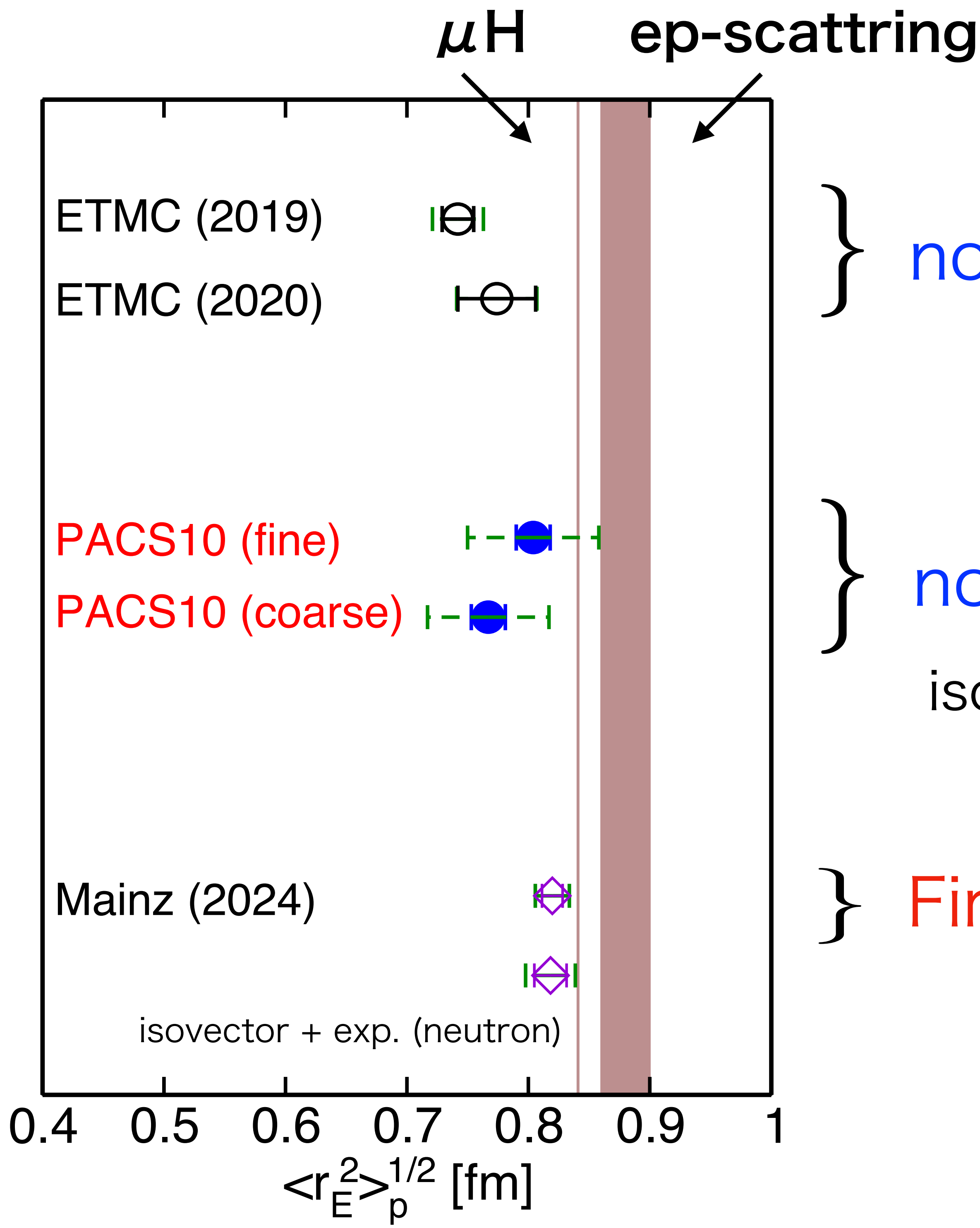


R. Tsuji et. al. (PACS collab.) Lattice 2024

👉 iso-scalar requires 10 to 100 times higher computational costs

Current status for proton's charge radius from lattice QCD

ETMC: PRD100 (2019) 014509
ETMC: PRD101 (2020) 114504
Mainz: PRD109 (2024) 094510



} not satisfy 2)

} not satisfy 1) and 2)

isovector + exp. (neutron)

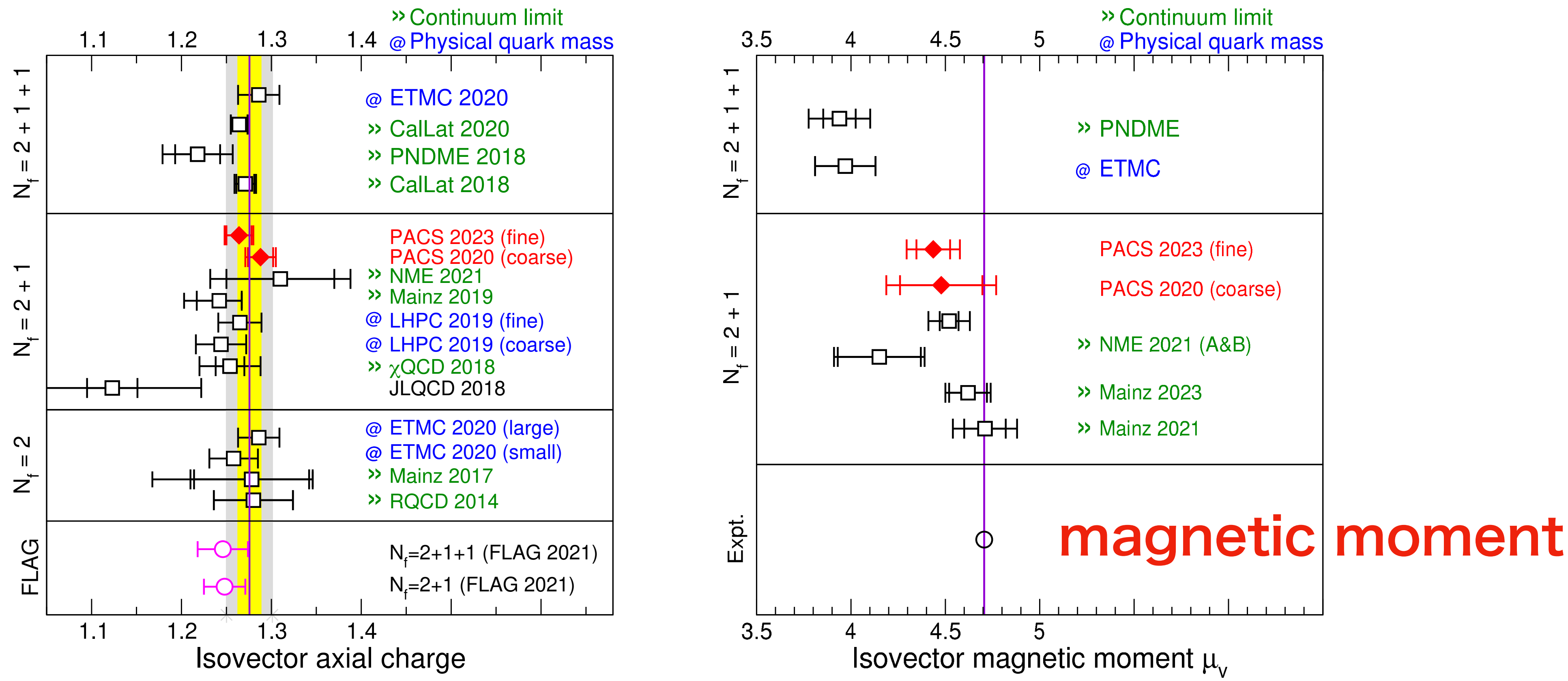
} First complete calculation

1) including the disconnected contribution

2) taking the continuum limit

Back up slides

axial charge



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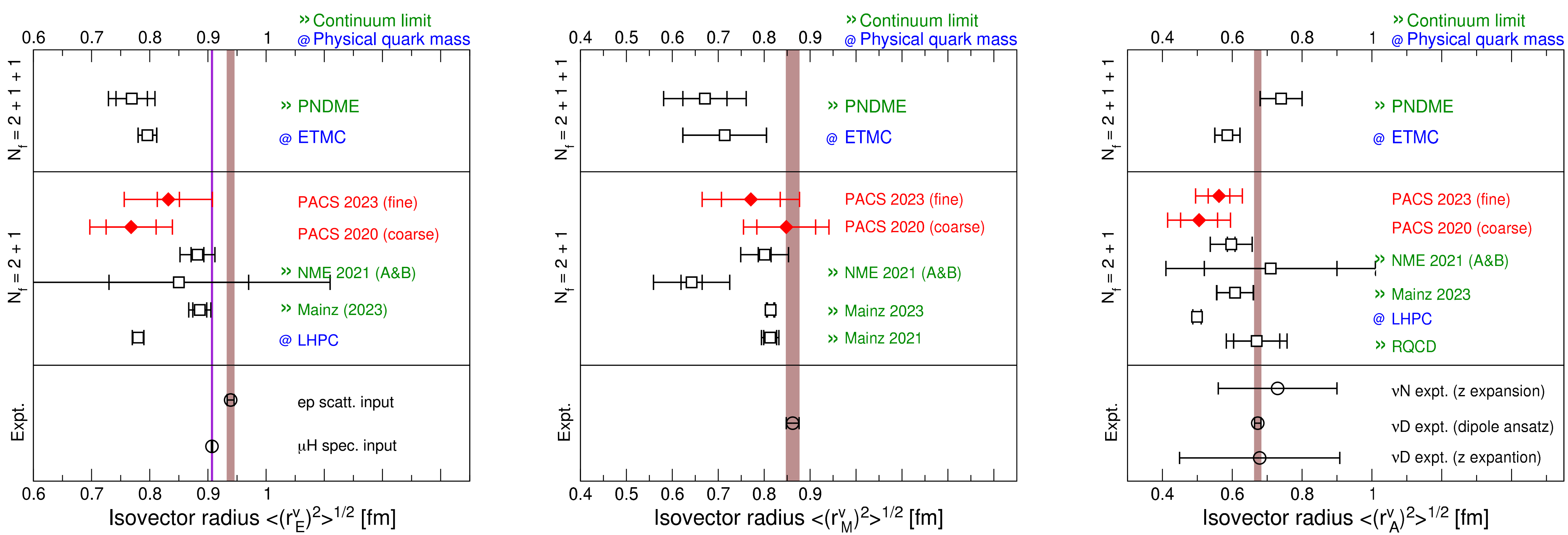

electric rms radius magnetic rms radius axial rms radius

Mainz Group

N_f	2+1			
a [fm]	0.086	0.073	0.064	0.050
m_π [MeV]	227, 283	218, 289	**130**, 207, 281, 295	176, 266
L [fm]	4.1, 2.8	4.9, 3.7	**6.1**, 4.1 3.1, 2.0	4.8, 3.2

Discretization effects are statistically less precise, especially for RMS radii.

Our Group (PACS)

N_f	2+1		
a [fm]	0.085	0.063	0.04
m_π [MeV]	135	138	138
8	10.8	10.1	10

Discretization effects are observed at fixed m_π with a fixed physical volume.