# γZ-exchange contributions in parity-violating ep scattering at low-energy

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"The Low-Energy Electron Scattering for Nucleon and Exotic Nucle"

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# Outline

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01 Introduction: FFs, $A_{PV}$, $Q_W$, γZ-exchange

02 γZ-exchange contribution at low energy

03 Compare with dispersion relations

04 Short summary

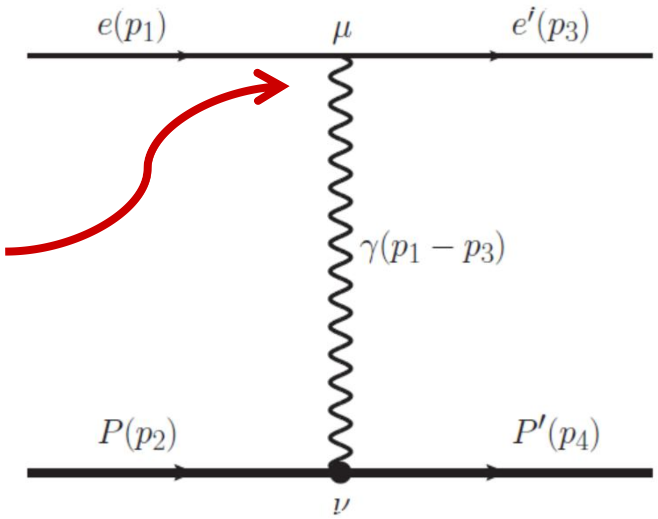
# Electromagnetic forms factors

The electromagnetic (EM) form factors (FFs) of proton are defined as

$$< P(p_4)\,|J_\mu^\gamma|\,P(p_2) > \equiv \overline{u}(p_4)[F_1(Q^2)\gamma_\mu + F_2(Q^2)\frac{i\sigma_{\mu\nu}}{2M_N}q^\nu]u(p_2)$$

EM current

$$\Gamma^\mu_{\gamma ee} = -ie\gamma^\mu$$




$$\Gamma^\mu_{\gamma pp} = ie[F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}}{2M_N}q_\nu]$$

two independent Lorentz invariant kinematical variables:

$$Q^2 = -q^2 = -(p_4 - p_2)^2;$$

$$\tau = \frac{Q^2}{4M_P^2};$$

$$\varepsilon = (1 + 2(1+\tau)\tan^2\frac{\theta}{2})^{-1}$$

$$G_E = F_1 - \tau F_2,\ \ G_M = F_1 + F_2$$

# Extract EM FFs from ep scattering

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Rosenbluth method: extract $F_1(Q^2), F_2(Q^2)$ from the unpolarized one-photon-exchange (OPE) cross section

$$\sigma_R^{Ex} = \sigma_R^{1\gamma} \equiv G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

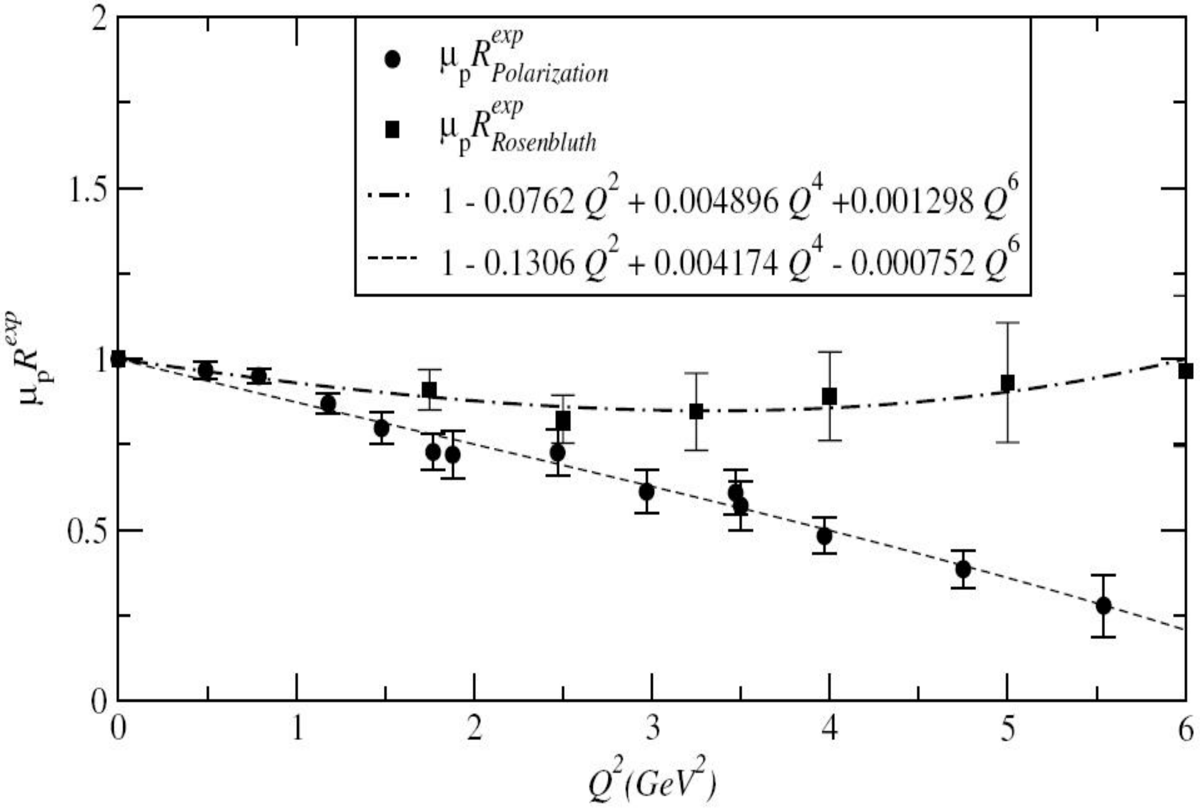
fixed

polarization transfer method: extract $\mu_p R$ from polarized ep scattering at fixed ε

$$R \equiv \mu_p \frac{G_E}{G_M} = -\mu_p \sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}} \frac{P_t^{(1\gamma)}}{P_l^{(1\gamma)}}$$

$$P_t^{(1\gamma)} = -\frac{1}{\sigma_R} \sqrt{\frac{2\varepsilon(1-\varepsilon)}{\tau}} G_E G_M, \quad P_l^{(1\gamma)} = \frac{1}{\sigma_R} \sqrt{1-\varepsilon^2} G_M^2$$

# Rosenbluth method vs. polarized method




**Rosenbluth method**

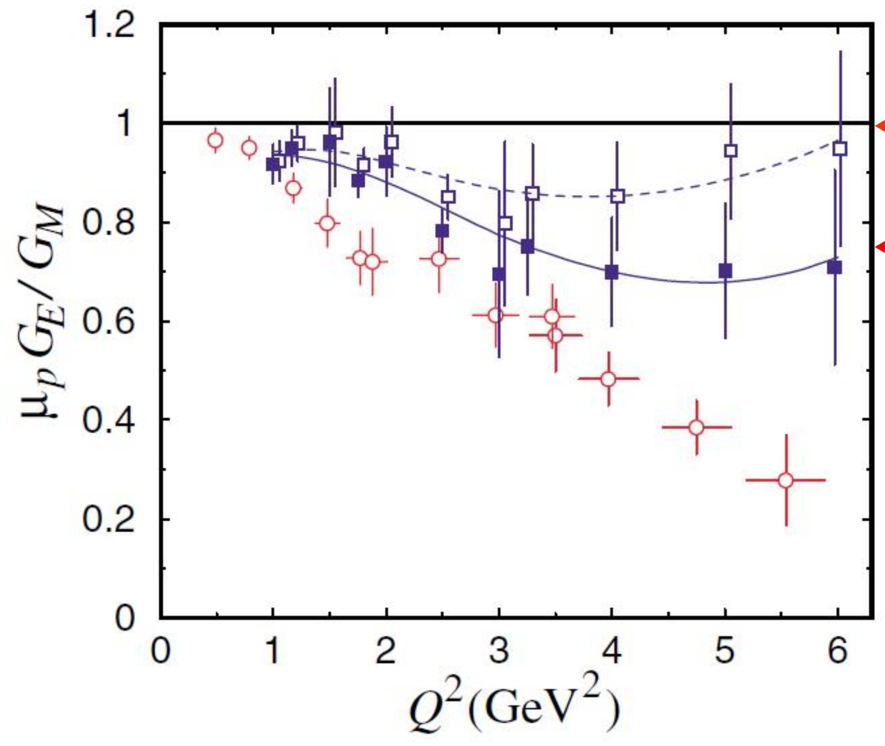
**Polarization method**

experimental values of $\mu_p R$ by Rosenbluth method and polarization method.

Phys. Rev. Lett. 91,142304(2003)

# Two-photon-Exchange (TPE) contributions

After considering the TPE contributions in ep scattering, the extracted results are better. For example, in 2003, one has




hollow:OPE

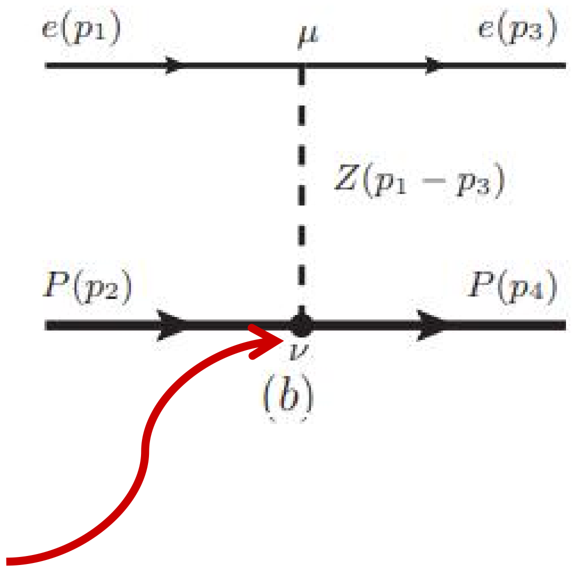
solid :OPE+TPE

P.G. Blunden, W. Melnitchouk and J. A. Tjon, Phys. Rev. Lett. 91,142304(2003)

# Weak FFs, $A_{PV}$ and $Q_W$

Similarly, weak FFs can be extracted from ex-measurement $A_{PV}$

$$A_{PV} = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-}$$




weak FFs of proton are defined as

$$< P(p_4) \, | \, J_\mu^Z \, | \, P(p_2) > \equiv \bar{u}(p_4)[F_1^{Z,P}(Q^2)\gamma_\mu + F_2^{Z,P}(Q^2)\frac{i\sigma_{\mu\nu}}{2M_N}q^\nu + G_A^Z\gamma_\mu\gamma_5]u(p_2)$$

also weak charge of proton $Q_W$ can be extracted when $Q^2$ goes to zero.

# $A_{PV}$ after considering radiative corrections

$$A_{PV} = \frac{G_F t}{4\sqrt{2}\pi\alpha_e}\left[(1+\Delta_\rho+\Delta_e)(1-4\sin^2\theta_W(0)+\Delta_e') + \square_{WW} + \square_{ZZ} + \square_{\gamma Z}\right] + \ldots$$

$Q_W$

$\theta_W(0)$: running weak mixing angle in the MS scheme at zero momentum transfer;

$\Delta_\rho$: radiative correction to the relative normalization of the neutral and charged current amplitudes;

$\Delta_e + \Delta_e'$: corrections to the axial vector Zee and $\gamma$ee couplings;

$\square_{WW} + \square_{ZZ}$: box graph corrections;.

...: terms that vanish with higher powers of t in the forward limit.

J. Erler, A. Kurylov, and M. J. Ramsey-Musolf, Phys. Rev. D. **68**, 016006 (2003).

# Weak charge $Q_w$

$$Q_W = (1 + \Delta_\rho + \Delta_e)(1 - 4\sin^2\theta_W(0) + \Delta_e^{'}) + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}(0)$$

depends on physics at low momentum scales

(1)In the low-energy limit, $Q_W$ is proportional to the asymmetry $A_{PV}$, which means that the accurate determination of $Q_W$ requires precise measurements and analysis of $A_{PV}$.

(2)Radiative corrections are important to precise extraction, especially γZ-exchange

J. Erler, A. Kurylov, and M. J. Ramsey-Musolf, Phys. Rev. D. **68**, 016006 (2003).

# γZ-exchange contributions

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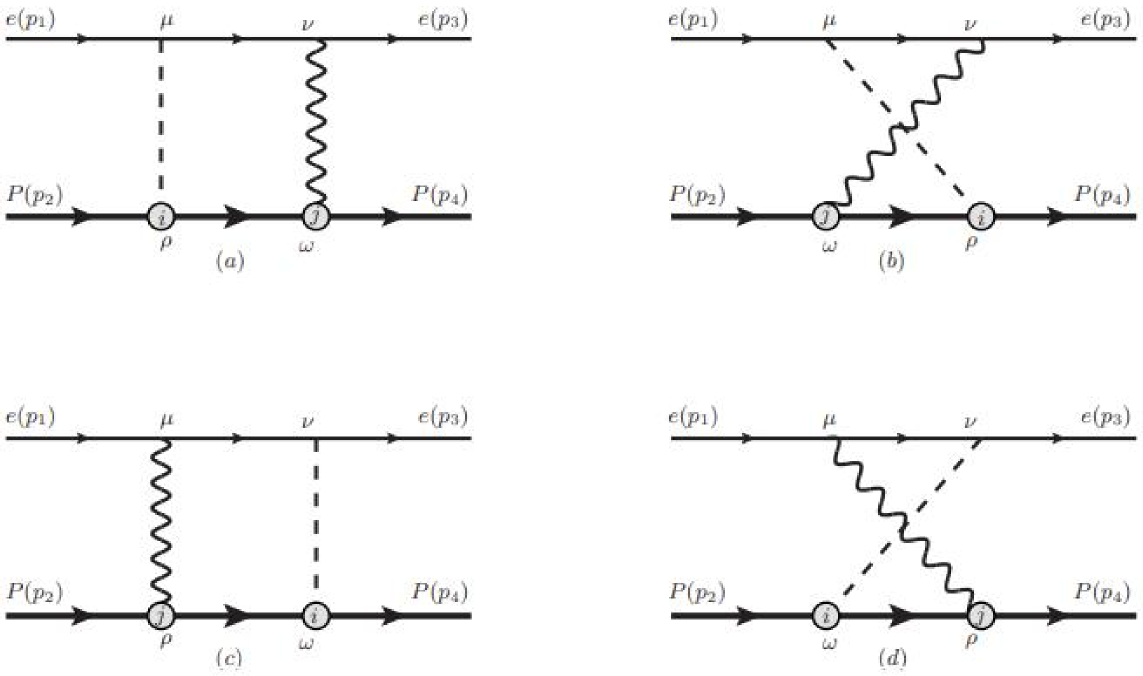
- In this talk, we limit our discussion in the low energy limit where the momentum transfer goes to zero and the center mass energy goes to the corresponding physical threshold at fixed momentum transfer.

- In the low energy limit, we discuss the contributions due to the following interactions with the leading-order and the next-to-leading-order of the momenta

$$\Gamma^{\mu}_{\gamma pp,0} = ieF_1\gamma^{\mu}, \quad \Gamma^{\mu}_{\gamma pp,1} = ieF_2\frac{i\sigma^{\mu\nu}}{2M_N}q_{\nu},$$

$$\Gamma^{\mu}_{Zpp,0} = -i[\bar{g}_1\gamma^{\mu} + \bar{g}_3\gamma^{\mu}\gamma_5], \quad \Gamma^{\mu}_{Zpp,1} = -i\bar{g}_2\frac{i\sigma^{\mu\nu}}{2M_N}q_{\nu},$$

# γZ-exchange contributions




$$\Gamma^{\mu}_{\gamma ee} =- ie\gamma^{\mu},$$

$$\Gamma^{\mu}_{Zee} =- i[\bar{g}^{V}_{e}\gamma^{\mu} + \bar{g}^{A}_{e}\gamma^{\mu}\gamma_5]$$

$$\Gamma^{\mu}_{\gamma pp,0} = ieF_1\gamma^{\mu}, \quad \Gamma^{\mu}_{\gamma pp,1} = ieF_2\frac{i\sigma^{\mu\nu}}{2M_N}q_{\nu}, \quad \Gamma^{\mu}_{Zpp,0} =- i[\bar{g}_1\gamma^{\mu} + \bar{g}_3\gamma^{\mu}\gamma_5], \quad \Gamma^{\mu}_{Zpp,1} =- i\bar{g}_2\frac{i\sigma^{\mu\nu}}{2M_N}q_{\nu}$$

$$s \equiv (p_1 + p_2)^2; Q^2 \equiv -(p_1 - p_3)^2; \nu \equiv 2s - 2M_N^2 - Q^2$$

# γZ-exchange contributions to the amplitudes

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To discuss the contributions at amplitude level, we separate the full amplitude into a parity conserved (PC) part and a parity violated (PV) part.

$$M_Z \equiv M_Z^{PC} + M_Z^{PV},$$
$$M_Z^{PV} \equiv \overline{g}_e^A M_Z^V + \overline{g}_e^V M_Z^A,$$

$$M_{\gamma Z}^{(a+b+c+d)} \equiv M_{\gamma Z}^{PC} + M_{\gamma Z}^{PV},$$
$$M_{\gamma Z}^{PV} \equiv \overline{g}_e^A M_{\gamma Z}^V + \overline{g}_e^V M_{\gamma Z}^A,$$

Taking the approximation $m_e$=0, the amplitudes $M_Z^{V,A}$ and $M_{\gamma Z}^{V,A}$ can be expressed as follows:

$$M_Z^X \equiv \sum_{i=1}^{3} \mathcal{F}_{Z,i}^X \mathcal{P}_i^X$$

$$M_{\gamma Z}^V \equiv \sum_{i=1}^{3} \mathcal{F}_{\gamma Z,i}^V \mathcal{P}_i^V \equiv \sum_{i=1}^{3}\sum_{j=1}^{2}\sum_{k=1}^{2} \mathcal{C}_{\gamma Z,ijk}^V F_j \overline{g}_k \mathcal{P}_i^V$$

$$M_{\gamma Z}^A \equiv \sum_{i=1}^{3} \mathcal{F}_{\gamma Z,i}^A \mathcal{P}_i^A \equiv \sum_{i=1}^{3}\sum_{j=1}^{2} \mathcal{C}_{\gamma Z,ij3}^A F_j \overline{g}_3 \mathcal{P}_i^A$$

# The invariant amplitudes



To deal with $\gamma_5$ in naive dimensional regularization (NDR) scheme, we project the amplitudes $M_{\gamma Z}^{V,A}$ to following invariant amplitudes

$$M_1^V \equiv [\bar{u}_3\gamma_\mu\gamma_5 u_1][\bar{u}_4\gamma^\mu u_2],$$
$$M_2^V \equiv \frac{1}{Q}[\bar{u}_3\gamma_\mu\gamma_5 u_1][\bar{u}_4 i\sigma^{\mu\nu}q_\nu u_2],$$
$$M_3^V \equiv \frac{1}{M_N Q}[\bar{u}_3 \not{P}\gamma_5 u_1][\bar{u}_4 \not{K} u_2],$$

$$M_1^A \equiv [\bar{u}_3\gamma^\mu u_1][\bar{u}_4\gamma_\mu\gamma_5 u_2],$$
$$M_2^A \equiv \frac{1}{Q}[\bar{u}_3\gamma^\mu u_1][\bar{u}_4\gamma_\mu \not{K}\gamma_5 u_2],$$
$$M_3^A \equiv \frac{1}{M_N Q}[\bar{u}_3 \not{P} u_1][\bar{u}_4 \not{K}\gamma_5 u_2]$$

$$where\ P = p_2 + p_4;\ \ K = p_1 + p_3;\ Q^2 =- q^2;\ q = p_4 - p_2 = p_1 - p_3$$

Note: only three of them are independent, in 4 dimension their relations can be easily gotten by project method.

# calculation of the coefficients

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1. using FeynCalc calculate the coefficients before the loop integrals in d-dimension.

2. using PackageX to do the loop integral.

(3)Expand the result in the low energy limit. Physically, when $Q^2$ is fixed, the physical $\nu$ has a minimum value given by

$$\nu_{phs} \geq \nu_{min} = Q\sqrt{4M_N^2 + Q^2}$$
$$\delta \equiv \nu - \nu_{min}$$

We expand the results on $Q = 0$ and $\delta = 0$ independently.

# expressions of coefficients

the expressions at the tree level: $\mathcal{F}^{X}_{Z,i}$ :

$$\mathrm{Re}[\mathcal{F}^{V}_{Z,1}] = -\frac{\bar{g}_1}{M_Z^2}, \quad \mathrm{Re}[\mathcal{F}^{V}_{Z,2}] = -\frac{\bar{g}_2}{M_Z^2}\frac{Q}{2M_N}, \quad \mathrm{Re}[\mathcal{F}^{V}_{Z,3}] = 0,$$

$$\mathrm{Re}[\mathcal{F}^{A}_{Z,1}] = -\frac{\bar{g}_3}{M_Z^2}, \quad \mathrm{Re}[\mathcal{F}^{A}_{Z,2}] = 0, \quad \mathrm{Re}[\mathcal{F}^{A}_{Z,3}] = 0.$$

the analytical expressions for γZ-exchange : $\mathcal{C}^{X}_{\gamma Z,ijk}$ :

$$\mathrm{Re}[\mathcal{C}^{V}_{\gamma Z,111}] = -\frac{\alpha_e}{\pi M_Z^2}\frac{2M_N Q+\delta}{Q^2}[\frac{5}{4} + 3\log\frac{M_Z}{M_N} + \frac{Q^2}{2M_N Q+\delta}\mathrm{R_{IR}}],$$

$$\mathrm{Re}[\mathcal{C}^{V}_{\gamma Z,112}] = -\frac{\alpha_e}{\pi M_Z^2}\frac{2M_N Q+\delta}{Q^2}[\frac{1}{2} + 2\log\frac{M_Z}{M_N}],$$

$$\mathrm{Re}[\mathcal{C}^{V}_{\gamma Z,121}] = \mathrm{Re}[\mathcal{C}^{V}_{\gamma Z,112}],$$

$$\mathrm{Re}[\mathcal{C}^{V}_{\gamma Z,122}] = \frac{\alpha_e}{\pi M_Z^2}\frac{2M_N Q+\delta}{Q^2}[\frac{1}{4} + \frac{9}{16}\frac{M_Z^2}{M_N^2} - \log\frac{M_Z}{M_N} + \frac{3M_Z^2}{8M_N^2}\mathrm{R_{UV}}],$$

$$\mathrm{Re}[\mathcal{C}^{V}_{\gamma Z,211}] = -\frac{\alpha_e}{\pi M_Z^2}\frac{Q}{M_N}[\frac{1}{4}\log\frac{\nu+Q^2}{\nu-Q^2} + \frac{2M_N Q+\delta}{8M_N^2}\log\frac{4M_N^4}{\nu^2-Q^2}],$$

# Parity-violating asymmetry



parity-violating asymmetry($A_{PV}$) is defined as

$$A_{\mathrm{PV}}=\frac{\sum_{helicity}(M^{+}M^{+*}-M^{-}M^{-*})}{\sum_{helicity}(M^{+}M^{+*}+M^{-}M^{-*})}$$

at the tree level,

$$A_{\mathrm{PV}}^{\gamma\otimes Z}=\frac{1}{e^{2}\sigma}[\sum_{i=1}^{2}\sum_{k=1}^{2}\mathcal{A}_{Z,ik}^{V}F_{i}\bar{g}_{k}\bar{g}_{e}^{A}+\sum_{i=1}^{2}\mathcal{A}_{Z,i3}^{A}F_{i}\bar{g}_{3}\bar{g}_{e}^{V}]$$

$$\sigma=4F_{1}^{2}M_{N}^{2}(\upsilon^{2}-4M_{N}^{2}Q^{2}+Q^{4})+F_{2}^{2}Q^{2}(\upsilon^{2}+4M_{N}^{2}Q^{2}-Q^{4})+16F_{1}F_{2}M_{N}^{2}Q^{4}$$

# Parity-violating asymmetry

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when considering the interference between the one-photon exchange diagram and γZ-exchange diagrams, we have

$$A_{\mathrm{PV}}^{\gamma\otimes\gamma Z}=\frac{1}{e^2\sigma}\left(\sum_{i=1}^{3}\mathcal{N}_i^V\mathrm{Re}[\mathcal{F}_{\gamma Z,i}^V]\overline{g}_e^A+\sum_{i=1}^{3}\mathcal{N}_i^A\mathrm{Re}[\mathcal{F}_{\gamma Z,i}^A]\overline{g}_e^V\right)$$

$$=\frac{1}{e^2\sigma}\left(\sum_{i=1}^{3}\sum_{j=1}^{2}\sum_{k=1}^{2}\mathcal{N}_i^V\mathrm{Re}[\mathcal{C}_{\gamma Z,ijk}^V]F_j^{\gamma Z}\overline{g}_k\overline{g}_e^A+\sum_{i=1}^{3}\sum_{j=1}^{2}\mathcal{N}_i^A\mathrm{Re}[\mathcal{C}_{\gamma Z,ij3}^A]F_j^{\gamma Z}\overline{g}_3\overline{g}_e^V\right)$$

$$=\frac{1}{e^2\sigma}\left(\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\mathrm{Re}[\mathcal{A}_{\gamma Z,ijk}^V]F_i^{\gamma}F_j^{\gamma Z}\overline{g}_k\overline{g}_e^A+\sum_{i=1}^{2}\sum_{j=1}^{2}\mathrm{Re}[\mathcal{A}_{\gamma Z,ij3}^A]F_i^{\gamma}F_j^{\gamma Z}\overline{g}_3\overline{g}_e^V\right)$$

$$=\frac{G_F t}{4\sqrt{2}\pi\alpha_e}\left(\mathrm{Re}[\square_{\gamma Z}^A]+\mathrm{Re}[\square_{\gamma Z}^V]\right)$$

# expressions of $\mathcal{A}^{X}_{\gamma Z,ijk}$

The partial expressions of $\mathcal{A}^{X}_{\gamma Z,ijk}$:

$$\mathrm{Re}\left[\mathcal{A}^{V}_{\gamma Z,111}\right] = -\frac{8\alpha_e}{\pi M_Z^2} M_N^2 Q^2 z^2 \left[\pi^2 + \log\frac{\nu + Q^2}{\nu - Q^2} + R_{\mathrm{IR}}\right],$$

$$\mathrm{Re}\left[\mathcal{A}^{V}_{\gamma Z,112}\right] = -\frac{2\alpha_e}{\pi M_Z^2} Q^2 \left[(2M_N Q + \delta)^3 + 16 M_N^2 Q^2 (2M_N Q + \delta)\log\frac{M_Z}{M_N} + 8 M_N^2 Q^4 R_{\mathrm{IR}}\right],$$

$$\mathrm{Re}\left[\mathcal{A}^{V}_{\gamma Z,121}\right] = -\frac{2\alpha_e}{\pi M_Z^2} Q^2 (2M_N Q + \delta)\left[(2M_N Q + \delta)^2 + 16 M_N^2 Q^2 \log\frac{M_Z}{M_N}\right],$$

$$\mathrm{Re}\left[\mathcal{A}^{V}_{\gamma Z,122}\right] = \frac{\alpha_e}{2\pi M_Z^2} Q^2 (2M_N Q + \delta)\left[2(4M_N^2 + 9M_Z^2)Q^2 - 7z^2 - 4(8M_N^2 Q^2 + z^2)\log\frac{M_Z}{M_N} + 12 M_Z^2 Q^2 R_{\mathrm{UV}}\right],$$

……

$$R_{\mathrm{IR}} = \log\frac{\nu + Q^2}{\nu - Q^2}\left(\log\frac{4M_N^2 \bar{\mu}_{\mathrm{IR}}^2}{\nu^2 - Q^4} + \frac{1}{\tilde{\epsilon}_{\mathrm{IR}}}\right), \qquad R_{\mathrm{UV}} = \log\frac{\bar{\mu}_{\mathrm{UV}}^2}{M_Z^2} + \frac{1}{\tilde{\epsilon}_{\mathrm{UV}}}$$

$$\frac{1}{\tilde{\epsilon}_{\mathrm{IR,UV}}} = \frac{1}{\epsilon_{\mathrm{IR,UV}}} - \gamma_E + \ln 4\pi$$

# Results by dispersion relations (DRs) in literatures

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1. Forward-limit DRs usually are used to estimate $\Box_{\gamma Z}^{V,A}(\mathrm{E}, Q^2)$

$$\mathrm{Re}[\Box_{\gamma Z}^{V}(\mathrm{E}, Q^2) \approx \mathrm{Re}[\Box_{\gamma Z}^{V}(\mathrm{E}, 0)] = \frac{2E}{\pi} P[\int_0^\infty \frac{\mathrm{Im}[\Box_{\gamma Z}^{V}(\bar{E}^+, 0)]}{\bar{E}^2 - E^2} d\bar{E}]$$

$$\mathrm{Re}[\Box_{\gamma Z}^{A}(\mathrm{E}, Q^2) \approx \mathrm{Re}[\Box_{\gamma Z}^{A}(\mathrm{E}, 0)] = \frac{2}{\pi} P[\int_0^\infty \frac{\bar{E}\,\mathrm{Im}[\Box_{\gamma Z}^{A}(\bar{E}^+, 0)]}{\bar{E}^2 - E^2} d\bar{E}]$$

$$\Box_{\gamma Z}(\mathrm{E}, Q^2) \approx \Box_{\gamma Z}(\mathrm{E}, 0) \frac{\exp(-\mathrm{B}|Q^2|/2)}{F_1^{\gamma P}(Q^2)}$$

only depends on $Q^2$

2. another naive approximation is also used to improve FW DRs

$$\mathrm{Re}[\Box_{\gamma Z}^{V}(\mathrm{E}, Q^2) \approx \mathcal{C}_{\gamma Z}^{V}(\mathrm{E}, Q^2) = \frac{2E}{\pi} P[\int_0^\infty \frac{\mathrm{Im}[\Box_{\gamma Z}^{V}(\bar{E}^+, Q^2)]}{\bar{E}^2 - E^2} d\bar{E}]$$

$$\mathrm{Re}[\Box_{\gamma Z}^{A}(\mathrm{E}, Q^2) \approx \mathcal{C}_{\gamma Z}^{A}(\mathrm{E}, Q^2) = \frac{2}{\pi} P[\int_0^\infty \frac{\bar{E}\,\mathrm{Im}[\Box_{\gamma Z}^{A}(\bar{E}^+, Q^2)]}{\bar{E}^2 - E^2} d\bar{E}]$$

$s \equiv (p_1 + p_2)^2; \nu \equiv 2s - 2M_N^2 - Q^2,\ E = (s - M_N^2)/(2M_N)$


[Phys. Rev. C. 84, 015502 (2011)]
[Phys. Rev. D 88, 013011 (2013)]
[Phys. Rev. Lett. 122, 211802 (2019)]
[Phys. Rev. C. 109, 014308 (2024)]

# DRs beyond FW Limit

3. one can approximately extent the FW limit DRs to finite Q² case in a naive form as

$$\mathrm{Re}[\Box_{\gamma Z}^{V}(\mathrm{E},Q^2)] \approx \mathcal{D}_{\gamma Z}^{V}(\mathrm{E},Q^2) = \frac{2\nu}{\pi}P[\int_{\nu_{th}}^{\infty}\frac{\mathrm{Im}[\Box_{\gamma Z}^{V}(\bar{E}^{+},Q^2)]}{\bar{\nu}^2-\nu^2}d\bar{\nu}]$$

$$\mathrm{Re}[\Box_{\gamma Z}^{A}(\mathrm{E},Q^2)] \approx \mathcal{D}_{\gamma Z}^{A}(\mathrm{E},Q^2) = \frac{2}{\pi}P[\int_{\nu_{th}}^{\infty}\frac{\bar{\nu}\mathrm{Im}[\Box_{\gamma Z}^{A}(\bar{E}^{+},Q^2)]}{\bar{\nu}^2-\nu^2}d\bar{\nu}]$$

$$\nu = 4M_N E - Q^2$$
$$\nu_{th} =- Q^2$$

4. our direct calculations show that the DRs at finite Q² should be modified as

$$\mathrm{Re}[\Box_{\gamma Z}^{A}(\mathrm{E},Q^2)] = \frac{c_A\nu}{\nu^2-\nu_p^2} + \frac{2\nu}{\pi}P[\int_{\nu_{th}}^{\infty}\frac{\mathrm{Im}[\Box_{\gamma Z}^{A}(\bar{\nu}^{+},Q^2)]}{\bar{\nu}^2-\nu^2}d\bar{\nu}]$$

$$\mathrm{Re}[\Box_{\gamma Z}^{V}(\mathrm{E},Q^2)] = \frac{c_V}{\nu^2-\nu_p^2} + \frac{2}{\pi}P[\int_{\nu_{th}}^{\infty}\frac{\bar{\nu}\mathrm{Im}[\Box_{\gamma Z}^{V}(\bar{\nu}^{+},Q^2)]}{\bar{\nu}^2-\nu^2}d\bar{\nu}]$$

$\nu_p$ is the zero point of $\sigma$

# Our results vs FW DRs

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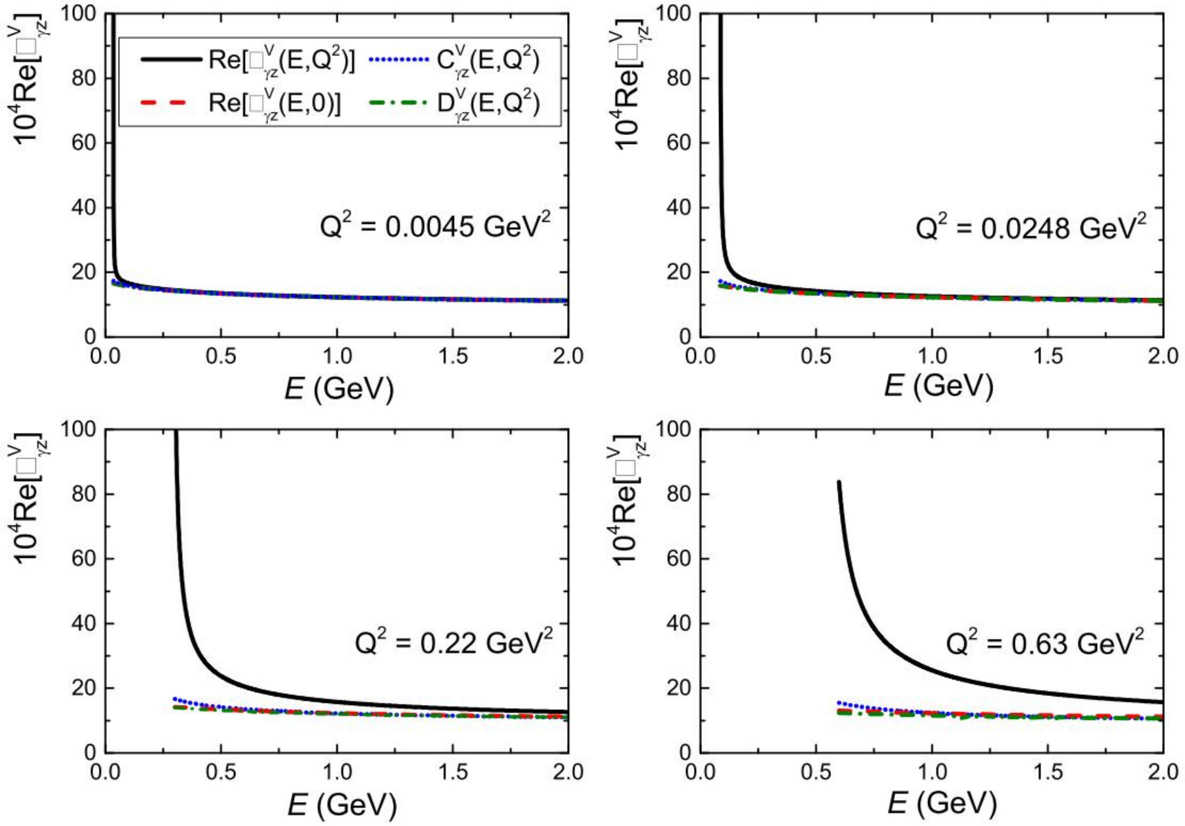
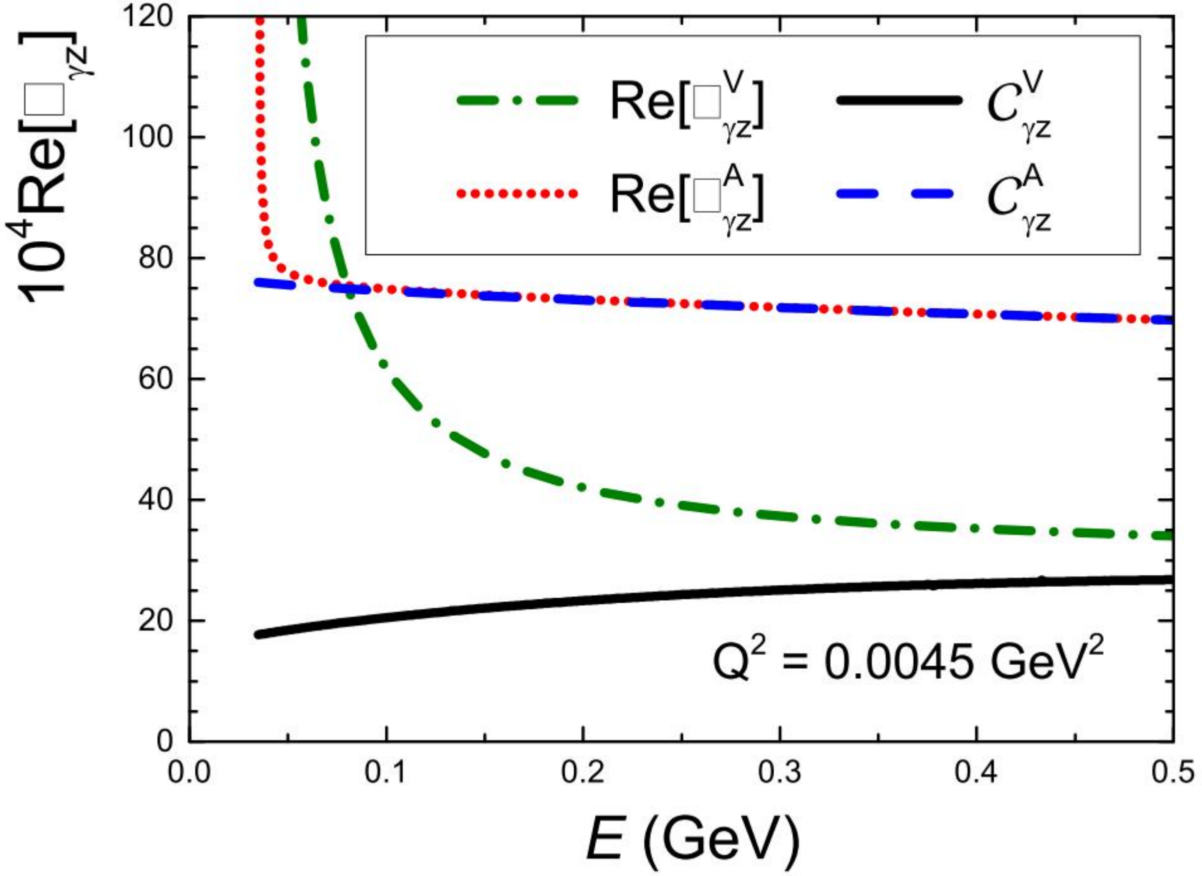
- Numerical results for $\mathrm{Re}[\Box^{V}_{\gamma Z}(\mathrm{E},Q^2)]$, $\mathrm{Re}[\Box^{V}_{\gamma Z}(\mathrm{E},0)]$, $\mathcal{C}^{V}_{\gamma Z}(\mathrm{E},Q^2)$ ,and $\mathcal{D}^{V}_{\gamma Z}(\mathrm{E},Q^2)$




- they are very similar to each other in almost the entire range.
- When E is small or $Q^2$ > 0.22 GeV², there are significant differences due to the double poles.
- $\mathrm{Re}[\Box^{V}_{\gamma Z}(\mathrm{E},Q^2)]/\ \mathrm{Re}[\Box^{V}_{\gamma Z}(\mathrm{E},0)]$ not only depends on $Q^2$ but also on E, particularly when E is small.

# Our results vs FW DRs

- Comparison of $\mathrm{Re}[\Box^V_{\gamma Z}(\mathrm{E},Q^2)]$ and $\mathcal{C}^V_{\gamma Z}(\mathrm{E},Q^2)$




- The result for $\mathrm{Re}[\Box^{\mathrm{A}}_{\gamma Z}(\mathrm{E},Q^2)]$ at small E such as 0.05 GeV is consistent with $\mathcal{C}^{\mathrm{A}}_{\gamma Z}(\mathrm{E},Q^2)$

- However, the behavior of $\mathrm{Re}[\Box^V_{\gamma Z}(\mathrm{E},Q^2)]$ at small physical E is much different with $\mathcal{C}^V_{\gamma Z}(\mathrm{E},Q^2)$

# Our results vs FW DRs

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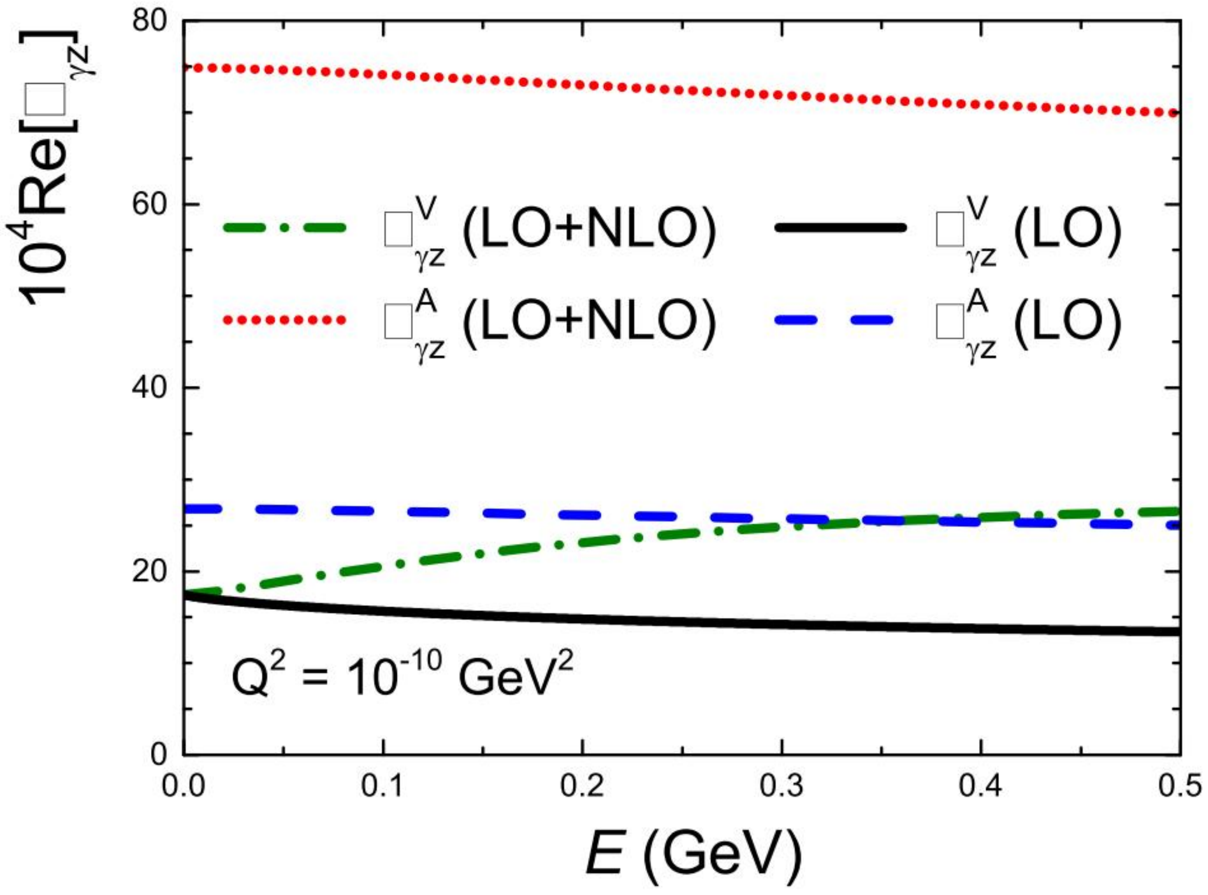
- $\mathrm{Re}[\Box_{\gamma Z}^{V,A}(\mathrm{E},Q^2)$ obtained with the LO and LO+NLO low-energy interactions, respectively.




The results indicate that, for very small values of E,

- the NLO interactions give a large contribution to $\mathrm{Re}[\Box_{\gamma Z}^{A}(\mathrm{E},Q^2)$ due to the nonzero $\mathrm{F}_2$,
- give a very small contribution to $\mathrm{Re}[\Box_{\gamma Z}^{V}(\mathrm{E},Q^2)$.

# Corrections to FW DRs for coming P2 experiment

For the upcoming P2 experiment($Q^2$ = 0.0045 GeV², E = 0.155 GeV), we get

the literature →

our results →

$$\frac{\mathcal{C}^{V}_{\gamma Z}(\mathrm{P2}) = 0.002221}{\mathrm{Re}\left[\Box^{V}_{\gamma Z}(\mathrm{P2})\right] = 0.004685} = 47.41\%$$

$$\frac{\mathcal{C}^{A}_{\gamma Z}(\mathrm{P2}) = 0.007370}{\mathrm{Re}\left[\Box^{A}_{\gamma Z}(\mathrm{P2})\right] = 0.007383} = 99.82\%$$

# Short summary

◆The full results reveal many interesting and important properties of the $\gamma Z$-exchange contributions at the amplitude level.

◆To estimate the $\gamma Z$-exchange contributions, both the LO and NLO interactions should be included.

◆For the upcoming P2 experiment, the numerical results show that the forward-limit DRs used in the literature may potentially underestimate $\mathrm{Re}[\Box^{V}_{\gamma Z}(\mathrm{P2})]$ by as much as 47%.

Thanks!