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IJCLab Irène Joliot-Curie
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Spatial Moments (r^{λ}) of the Proton Charge Density

Dominique Marchand

On behalf of: *M. Atoui, M. Barbaro, M. Hoballah, C. Kairouz, M. Lassaut, D.M., G. Quéméner, E. Voutier*

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STRONG 2020

Spatial Moments and the Proton charge radius

$\mathbf{R_p} = \sqrt{< \mathbf{r}^2 >}$ $\mathbf{R_p}$ related to the **moment of order 2** of the **radial density** ρ

$\mathbf{R_p}$ *«experimentally» extracted from*

- (ordinary and muonic) **Hydrogen spectroscopy**
- **Lepton** (**e** or $\boldsymbol{\mu}$) **proton** elastic scattering (indirect measurement)

June 2019 ➞ July 2024

Networking activity

D. M., Prof. R. Pohl (Mainz)
(22 institutions, 11 countries)

To **stimulate** a **real synergy** between all the physicists involved in the world-wide **experimental** and **theoretical** effort, from **atomic spectroscopy** and **lepton scattering,** in order to investigate / understand persistent discrepancies on the **value of the proton charge radius.**

International Workshops

June 20-23 », 2022, Sorbonne Université, Paris
https://indico.mitp.uni-mainz.de/e/pren2022

June 26-30, 2023, J.G. University & HIM, Mainz
https://indico.him.uni-mainz.de/event/172/

Nuclear Radius Extraction Collaboration (NREC), J. Bernauer *et al.*
Kick-off meeting, 6-10 mai, 2024, CFNS, Stony Brook University, New-York

Some selected PREN publications

STRONG 2020 — PREN Proton Radius European Network

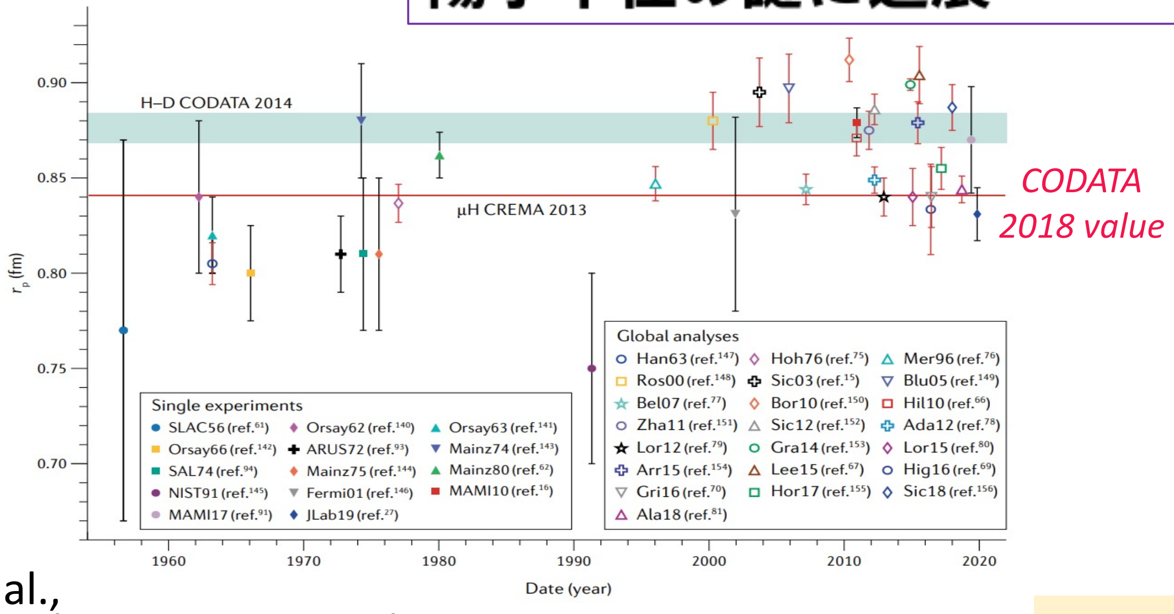
General review / outreach:

- J.-P. Karr, D. M., « **Progress on the proton-radius puzzle** », *Nature*, « New & views », Nature Publishing Group, **2019**, 575 (7781), pp.61-62. ⟨10.1038/d41586-019-03364-z⟩. ⟨hal-02392917⟩

News & views
https://www.natureasia.com/ja-jp/ndigest/v17/n2
実験物理学
陽子半径の謎に進展

- J.-P. Karr, D. M., E. Voutier, « **The proton size** », *Nature Rev. Phys.*, **2020**, 2 (11), pp.601-614. ⟨10.1038/s42254-020-0229-x⟩. ⟨hal-03011020⟩

  https://rdcu.be/cyFp5 (open access -read only- shared link)



Phenomenological activities:

- M. Hoballah, M.B. Barbaro, R. Kunne., M. Lassaut, D. M., et al., « **Connecting spatial moments and momentum densities** », *Physics Letters B*, Elsevier, **2020**, 808, pp.135669. ⟨10.1016/j.physletb.2020.135669⟩. ⟨hal-02917232⟩

Method

- M. Atoui, M.B. Barbaro, M. Hoballah, C. Keyrouz, M. Lassaut, D. M., G. Quéméner, E. Voutier, « **Determination of the moments of the proton charge density** », *Phys. Rev. C 110 (2024) 015207*

Application

OUTLINE

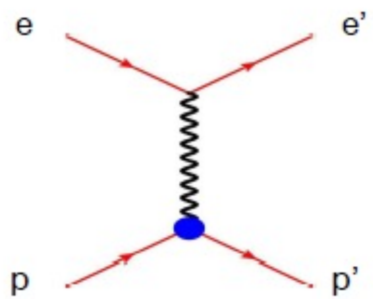
- Motivation
- Novel approach to extract the moments of the charge distribution from electron-proton scattering data: the **Integral Method**
- Application to proton electric form factor data taking into account statistical & systematics uncertainties
- Conclusions / Perspectives

Proton Radius extracted from electron-proton elastic scattering

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[\frac{G_{Ep}^2(Q^2) + \tau G_{Mp}^2(Q^2)}{1+\tau} + 2\tau \tan^2\left(\frac{\theta}{2}\right) G_{Mp}^2(Q^2)\right]$$

$\tau \equiv \frac{Q^2}{4M_p^2}$ G_E and G_M Sachs' electric and magnetic form factors

e+p -> e+p



$$G_{Ep}(Q^2) = 1 - \frac{Q^2}{6} < r_p^2 > + \frac{Q^4}{120} < r_p^4 > + \cdots$$

$$R_p = \sqrt{< r^2 >} = \sqrt{-6 \left.\frac{dG_{Ep}(Q^2)}{dQ^2}\right|_{Q^2=0}}$$

« Derivative method »

« Standard method »: proton charge radius (r_p)
≡ moment of order 2 of the radial charge density ρ
is extracted from Rosenbluth separation
or very low q^2 data, where GE dominates.

Slope of G_E at Q^2=0 ($Q^2 = - q^2$)

BUT the Q^2=0 limit cannot be reached in lepton scattering

=> Relying on interpolation (parametrization) of G_E data
to extrapolate to Q^2=0

Indirect determination of R_p

Extraction of Spatial Moments through the « Derivative Method »

$$< r^{\lambda} > = \int_{\mathbb{R}^3} d^3\mathbf{r}\, r^{\lambda}\, \rho_E(\mathbf{r})$$

More generally all **positive even** order moments can be determined based on the « **derivative method** »:

$$< r^{2j} > = (-1)^j \frac{(2j+1)!}{j!} \frac{d^j G_E(k^2)}{d(k^2)^j}\Bigg|_{k^2=0}$$

considering

$$\rho_E(\mathbf{r}) = \frac{1}{(2\pi)^3} \int_{\mathbb{R}^3} \mathrm{d}^3\mathbf{k}\, e^{i\mathbf{k}\cdot\mathbf{r}} G_E(k^2)$$

Inverse of the Fourrier Transform of the Sach's form factor G_E

(Valid for non-relativistic description; Breit frame)

All **even positive order moments** of the radial distribution can be related to zero 4-momentum squared derivatives of the electric form factor and **provide complementary information on the charge distribution inside the proton**, *e.g.* **high order moments** describe the **tail of the charge distribution.**

Issues faced relying on the « derivative method »:

- **Based on the zero 4-momentum squared extrapolation of the G_E *q^2* dependency**
- **Sensitivity to the functional form used for the parametrization (interpolation)**
- **Sensitivity to the *q^2 range used for the interpolation***
- ***Give only access to positive even moments***

Novel approach: Integral Method

Spatial Moments of the charge density: Integral Method (IM)

The **moment of any real order λ** can be expressed as an **integral over the full momentum space**

$$< \boldsymbol{r}^{\boldsymbol{\lambda}} >= (r^{\lambda}, \rho_E) = \frac{1}{(2\pi)^3} \int_{\mathbb{R}^3} d^3\mathbf{k}\, G_E(\mathbf{k}) \int_{\mathbb{R}^3} \boldsymbol{d^3 r}\, \boldsymbol{r^{\lambda}}\, \boldsymbol{e^{ik\cdot r}}$$

Spatial Moments of the charge density: Integral Method (IM)

The **moment of <u>any real</u> order** λ can be expressed as an **<u>integral over the full momentum space</u>**

$$< \boldsymbol{r}^{\lambda} >= \left(r^{\lambda}, \rho_E\right) = \frac{1}{(2\pi)^3} \int_{\mathbb{R}^3} d^3\mathbf{k}\, G_E(\mathbf{k}) \underbrace{\int_{\mathbb{R}^3} \boldsymbol{d^3 r\, r^{\lambda}\, e^{ik\cdot r}}}_{g_\lambda(\mathbf{k})}$$

Finite

Physical observable

Divergent
needs to be regularised

$g_\lambda(\mathbf{k}) = \int d^3\mathbf{r}\, r^{\lambda} e^{i\mathbf{k}\cdot\mathbf{r}}$ can be treated as a **distribution**

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<u>Regularization methods:</u>

1) Counter-terms subtraction **(IM1)**

2) Exponential regularization **(IM2)** → Faster convergence

« Connecting spatial moments and momentum densities », M. Hoballah et al, Phys. Lett. B 808 (2020) 135669

Exponential regularization (IM2)

$< r^\lambda >= \frac{1}{(2\pi)^3} \int_{\mathbb{R}^3} d^3\mathbf{k}\, G_E(\mathbf{k})\, g_\lambda(\mathbf{k})$

$$g_\lambda(\mathbf{k}) = \int_{\mathbb{R}^3} d^3\mathbf{r}\, r^\lambda\, e^{i\mathbf{k}\cdot\mathbf{r}} = \lim_{\epsilon\to 0+} \int d^3\mathbf{r}\, r^\lambda \mathbf{e}^{-\epsilon \mathbf{r}} e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$g_\lambda(\mathbf{k}) \equiv \lim_{\epsilon\to 0+} I_\lambda(\mathbf{k},\epsilon)$$

Standard technique to regularize the Fourier transform of the Coulomb potential

The distribution $g_\lambda(\mathbf{k})$ is the weak limit for $\epsilon \to 0$ of a convergent integral $I_\lambda(\mathbf{k},\epsilon)$ which can be evaluated analytically

$g_\lambda(\mathbf{k})$ analytical integration $\longrightarrow$ $\mathcal{I}_\lambda(k,\epsilon) = \dfrac{4\pi\,\Gamma(\lambda+2)\, \sin\left[(\lambda+2)\mathrm{Arctan}\,(k/\epsilon)\right]}{k(k^2+\epsilon^2)^{\frac{\lambda}{2}+1}}$

$$\boxed{\langle r^\lambda\rangle = \frac{2}{\pi}\Gamma(\lambda+2)\lim_{\epsilon\to 0+}\int_0^\infty \mathrm{d}k\, G_E(k^2)\, \frac{k\sin\left[(\lambda+2)\mathrm{Arctan}\,(k/\epsilon)\right]}{(k^2+\epsilon^2)^{\lambda/2+1}}} \quad (\lambda > -3)$$

Exponential regularization (IM2)

$$\langle r^\lambda \rangle = \frac{2}{\pi}\Gamma(\lambda+2) \lim_{\epsilon\to 0+} \int_0^\infty \mathrm{d}k\, G_E(k^2) \frac{k \sin\left[(\lambda+2)\mathrm{Arctan}\,(k/\epsilon)\right]}{(k^2+\epsilon^2)^{\lambda/2+1}}$$

$(\lambda > -3)$

- For $\lambda = m$ **positive integer**: $< r^m > = \frac{2}{\pi}(m+1)! \lim_{\epsilon\to 0+} \epsilon^{m+2} \int_0^\infty dk\, k\, G_E(k^2) \frac{\Phi_m\left(\frac{k}{\epsilon}\right)}{(k^2+\epsilon^2)^{m+2}}$ with $\Phi_m(x) \equiv \sum_{j=0}^{m+2} \sin\left(\frac{j\pi}{2}\right)\binom{m+2}{j} x^j$
- For **even** m, the IM recovers formally the same quantities as the derivative method

Advantages of the IM over the derivative method:

1. can access **moments of any real order >-3, including odd and negative,** carrying complementary information on the charge distribution inside the proton
2. **data** over the **full** $\mathbf{Q^2}$ **range** is used, **without relying on the zero-momentum extrapolation**
3. takes into account the **full experimentally probed momentum** region, **improving the statistics**

- **Short-distance** behaviour of the charge distribution is encoded in the **negative order** moments particularly sensitive to the **large k^2-dependence** of GE (k^2)
- **Long-distance** behaviour is encoded in the high **positive order** moments sensitive to the **low k^2-dependence**.

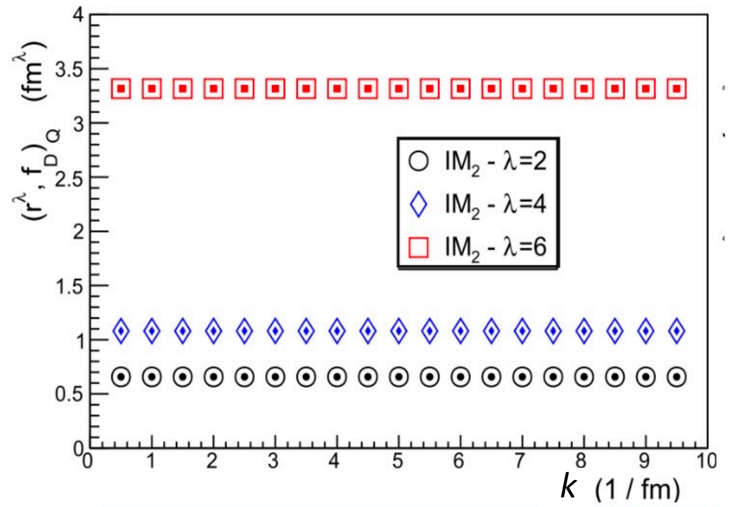
Truncated Moments

Experimental measurements of the Form Factor do not extend to infinite k^2:

- **But**: Integrals are most likely to saturate at a squared four-momentum transfer value well below infinity.
- **Hence**: **Cut-off Q** replaces the infinite integral boundary : **truncated moments**.

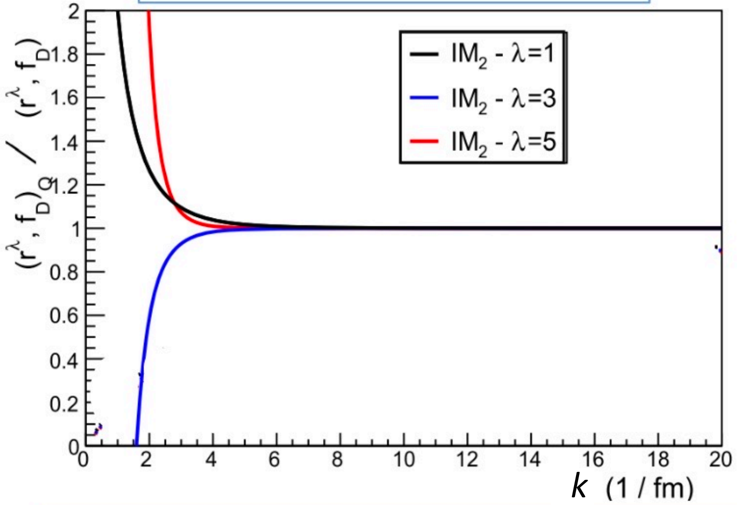
Use the polynomial ratio parametrization $\mathbf{G_E}(k) = \frac{1+a_1k^2}{1+b_1k^2+b_2k^4+b_3k^6}$ **J.J. Kelly, Phys. Rev. C 70 (2004) 068202.**

Positive even truncated moments



Q-independence

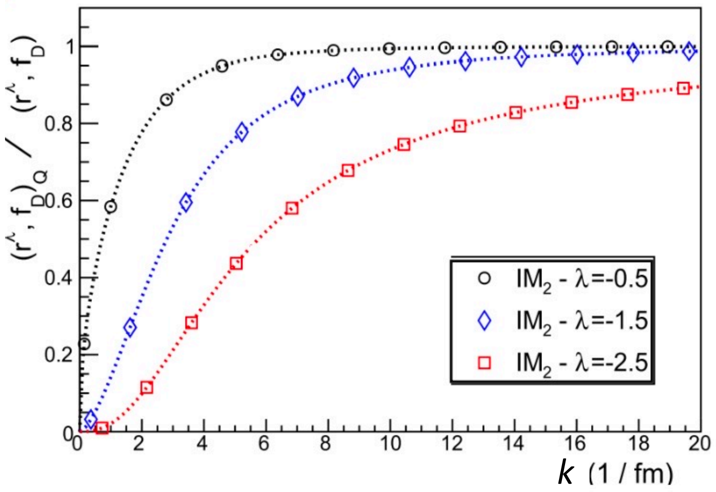
Odd truncated moments



The method rapidly saturates about 6 fm^{-1}, in a momentum region well covered by proton electromagnetic FF data.

Q^2 ~ 1.4 [(GeV/c^2)^2]

Negative truncated moments



Convergence is not guaranteed within the domain covered by experimental data

Moment determination from G_E experimental data

- Select G_E from electron scattering experiments

➢ Rosenbluth Separation : Measure σ at a given k^2 for different values of beam energy and scattering angle

➢ G_M contribution is strongly suppressed at very low k^2

➢ 21 data sets: $5.51 \times 10^{-3} \leq k^2(\mathrm{fm}^{-2}) \leq 226$
$2.15 \times 10^{-4} \leq Q^2\,[(\mathrm{GeV}/c^2)] \leq 8.8$

Fit simultaneously the different datasets using the functional form

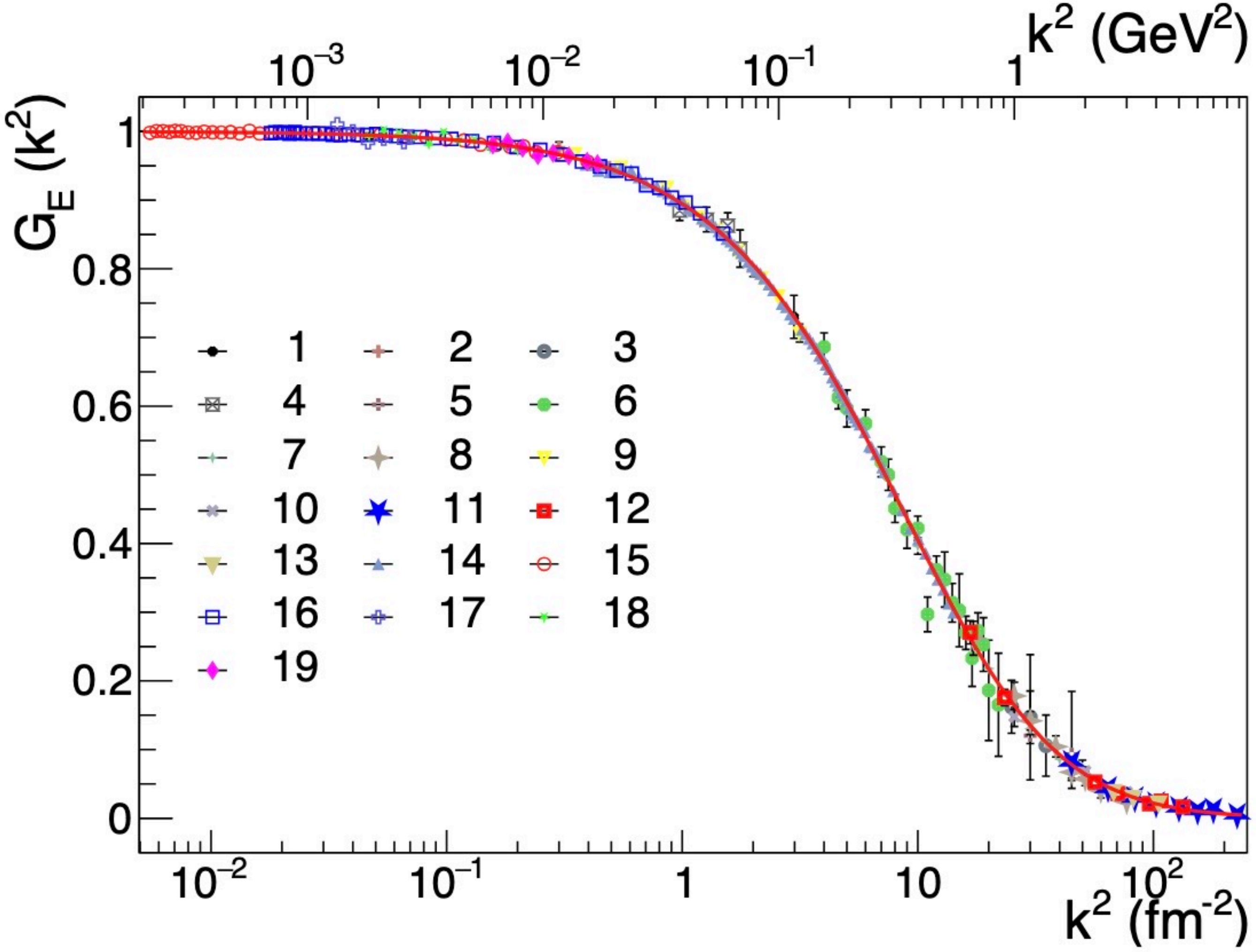
$$G_E(k) = \eta_i \frac{1 + a_1 k^2}{1 + b_1 k^2 + b_2 k^4 + b_3 k^6}$$

➢ The **same functional behavior** is assumed for each dataset

➢ A **separate normalization parameter** η_i is considered for each dataset number i

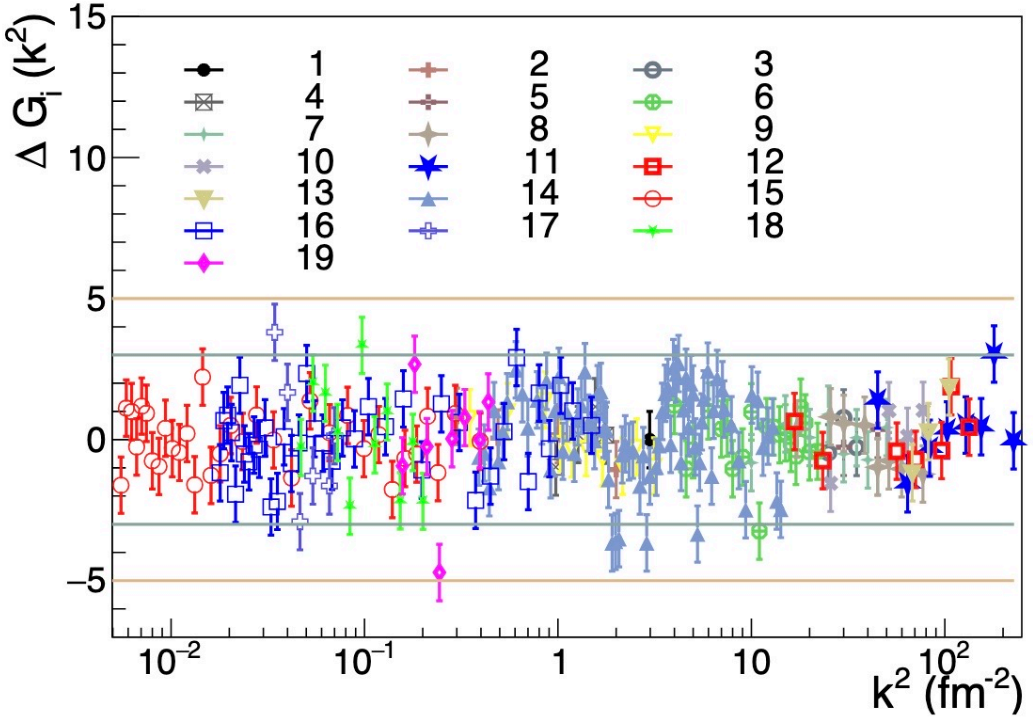
Data Set Number	Year	Authors	Ref.	Number of data points	k^2-range k^2_{min} [fm^{-2}]	k^2_{max} [fm^{-2}]
1	1962	Lehmann *et al.*	[19]	1	2.98	2.98
2	1963	Dudelzak *et al.*	[20]	4	0.30	2.00
3	1963	Berkelman *et al.*	[21]	3	25.0	35.0
4	1966	Frèrejacque *et al.*	[22]	4	0.98	1.76
5	1966	Chen *et al.*	[23]	2	30.0	45.0
6	1966	Janssens *et al.*	[24]	20	4.00	22.0
7	1971	Berger *et al.*	[25]	9	1.00	50.0
8	1973	Bartel *et al.*	[26]	8	17.2	77.0
9	1975	Borkowski *et al.*	[27]	10	0.35	3.15
10	1994	Walker *et al.*	[28]	4	25.7	77.0
11	1994	Andivahis *et al.*	[29]	8	44.9	226.
12	2004	Christy *et al.*	[30]	7	16.7	133.
13	2005	Qattan *et al.*	[31]	3	67.8	105.
*A1 Rosenbluth 14	2014	Bernauer *et al.*	[10]	77	0.39	14.2
PRad 15	2019	Xiong *et al.* - 1.1 GeV	[11]	33	5.51×10^{-3}	3.96×10^{-1}
PRad 16	2019	Xiong *et al.* - 2.1 GeV	[11]	38	1.79×10^{-2}	1.49
ISR 17	2021	Mihovilovič *et al.* - 195 MeV	[32]	6	3.43×10^{-2}	6.99×10^{-2}
ISR 18	2021	Mihovilovič *et al.* - 330 MeV	[32]	11	4.69×10^{-2}	2.00×10^{-1}
ISR 19	2021	Mihovilovič *et al.* - 495 MeV	[32]	8	1.57×10^{-1}	4.37×10^{-1}

Simultaneous Fit of G_E experimental data



$\chi_r{}^2$ = 1.97

$$\Delta G_i(k^2) = \frac{G_E^i(k^2) - \eta_i G_E(k^2)}{\delta G_E^i(k^2)}$$



Residuals within $\pm 3\ \sigma$ with some outlyers

Fit Parameters

Functional Form: polynomial ratio parametrisation

$$G_E(k^2) = \eta_i \frac{1 + a_1 k^2}{1 + b_1 k^2 + b_2 k^4 + b_3 k^6}$$

	a1 [x 10^{-1} fm^2]	b1 [x 10^{-1} fm^2]	b2 [x 10^{-1} fm^4]	b3 [x 10^{-3} fm^6]
	8.8008	**9.9570**	**1.0285**	**2.9252**
Statistical	**0.0054**	**0.0113**	**0.0058**	**0.0666**
Systematic	**0.0095**	**0.0019**	**0.0003**	**0.0219**

Systematic errors of the fit parameters reflect the experimental data systematics:

1. The data are shifted (upwards or downwards the central point) with their respective systematic errors: $\mathbf{2^{19}}$ configurations
1. For each configuration a fit is performed and parameters are extracted
2. Systematics are evaluated from the difference of the parameter value WRT the reference fit

Normalization parameters η_i:

Recent experiments (2010-21):
Deviation from unity is smaller than 1%

Old Experiments:
Deviations up to 17% (still reasonable at large k^2 where *GE* gives small contribution to the cross section)

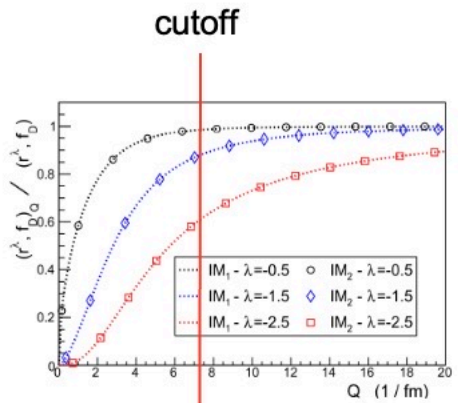
Evaluation of Moments

- Evaluations are compatible for positive valued order moments

λ	Truncated Moments $Q_c = 2$ $(GeV/c^2)^2$ $\langle r^\lambda \rangle_Q$ $[\mathrm{fm}^\lambda]$	Moments $k^2 \to \infty$ $\langle r^\lambda \rangle$ $[\mathrm{fm}^\lambda]$	Moments Derivative Method $\langle r^{2p} \rangle_d$ $[\mathrm{fm}^\lambda]$
-2	6.5245	8.7226	—
-1	1.9681	2.0906	—
1	0.7206	0.7179	—
2	0.6937	0.6937	0.6937
3	0.8697	0.8701	—
4	1.4728	1.4728	1.4728
5	3.8139	3.8137	—
6	16.405	16.405	16.405
7	104.64	104.64	—

Advantage of the approach WRT the derivative method (Positive odd, negative)

- Negative order moments show discrepancy when a cut-off is applied (as predicted)



Positive even order moments: Same as obtained by derivative method

Moment Statistical Errors

Procedure:

- ➢Make **replicas of parameters (50 000)** following the assumption of statistical errors associated to data
- ➢The **moments are estimated from each replica**
- ➢A dedicated study of the **variance of the replicas** is performed from which the statistical errors on the moments are obtained

λ	$\langle r^\lambda \rangle_Q$ $[\mathrm{fm}^\lambda]$	$\langle r^\lambda \rangle$ $[\mathrm{fm}^\lambda]$	$\langle r^{2p} \rangle_d$ $[\mathrm{fm}^\lambda]$	$\delta\left[\langle r^\lambda \rangle_Q\right]$ $[\mathrm{fm}^\lambda]$
-2	6.5245	8.7226	–	0.0172
-1	1.9681	2.0906	–	0.0024
1	0.7206	0.7179	–	0.0020
2	0.6937	0.6937	0.6937	0.0094
3	0.8697	0.8701	–	0.0457
4	1.4728	1.4728	1.4728	0.2461
5	3.8139	3.8137	–	1.4822
6	16.405	16.405	16.405	10.058
7	104.64	104.64	–	76.676

Larger statistical errors for high order positive moments (probing the large distance behavior of the charge density): lack of measurements at ultra low k^2

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Moment Systematics Errors

Sources of **systematic errors**:

1. Originating from the systematic error that is reported by each considered **experiment on GE**

2. **Discrepancy** between **truncated** and **exact moments**

3. Bias that could be generated on the fit parameters from **the fitting model itself**

4. Bias induced by the normalisation parameter η_i

5. Error coming from the **choice of the fitting model** **(ex:** Polynomial ratios, polynomials, Inverse polynomials, Constant Fraction (CF) expansion

λ	$\langle r^\lambda \rangle_Q$ [fm$^\lambda$]	$\langle r^\lambda \rangle$ [fm$^\lambda$]	$\langle r^{2p} \rangle_d$ [fm$^\lambda$]	$\delta\left[\langle r^\lambda \rangle_Q\right]$ [fm$^\lambda$]	Systematic Error: Dat. [fm$^\lambda$]	Int. [fm$^\lambda$]	Fun. [fm$^\lambda$]	Nor. [fm$^\lambda$]	Mod. [fm$^\lambda$]
-2	6.5245	8.7226	—	0.0172	0.0106	2.1981	0.0001	0.0095	0.3731
-1	1.9681	2.0906	—	0.0024	0.0019	0.1225	0.0001	0.0029	0.0278
1	0.7206	0.7179	—	0.0020	0.0025	0.0027	0.0001	0.0011	0.0029
2	0.6937	0.6937	0.6937	0.0094	0.0111	0	0.0001	0.0010	0.0116
3	0.8697	0.8701	—	0.0457	0.0494	0.0004	0.0007	0.0013	0.0633
4	1.4728	1.4728	1.4728	0.2461	0.2474	0	0.0065	0.0022	0.3805
5	3.8139	3.8137	—	1.4822	1.4297	0.0002	0.0343	0.0056	2.6276
6	16.405	16.405	16.405	10.058	9.4985	0	0.1871	0.0240	21.531
7	104.64	104.64	—	76.676	71.727	0.0001	2.8169	0.1528	212.48

Data | Method | fit fct. intrinsic | Normalisation η_i | choice of fit fct.

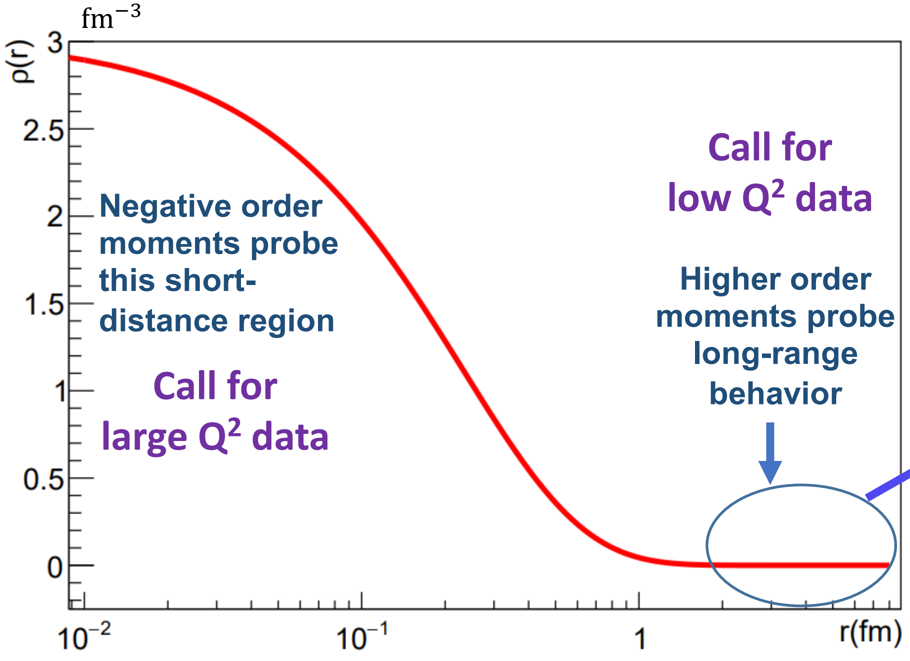
Truncation effects are dominant for
Negative order moments

Fitting model choice is the **most significant systematic** error in the IM method.
This error is sometimes **omitted in "proton radius puzzle" analyses.**
Using a mathematical function obtained from a physics model is the only way to minimize this error.

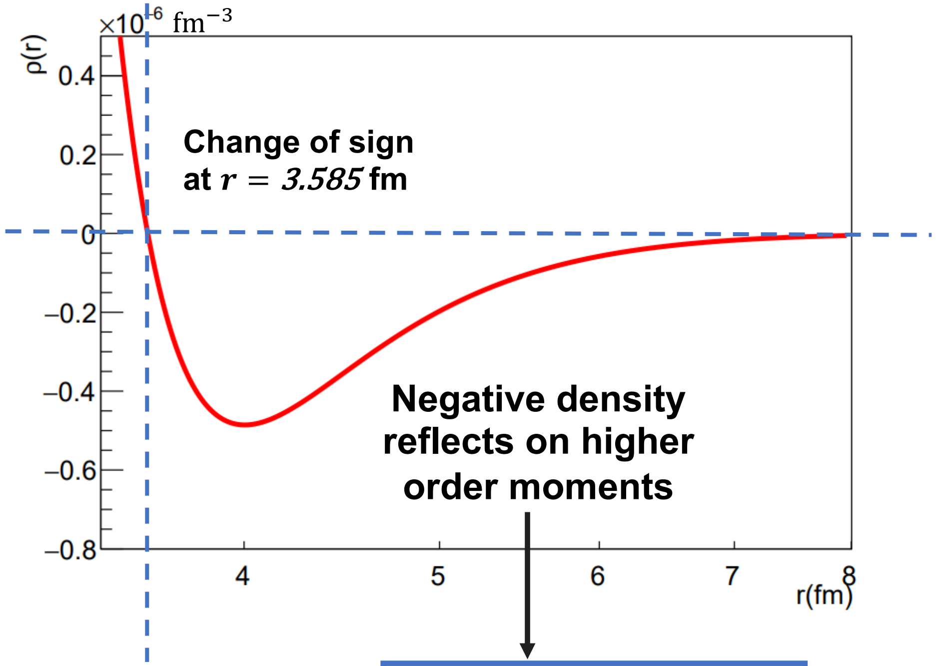
Moments and charge distribution

Spatial density $\rho(r) = \frac{1}{(2\pi)^3}\int_{R^3} d\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{r}} G_E(k^2)$

Fourier transform of $\mathbf{G_E}(\boldsymbol{k}) = \frac{1+a_1k^2}{1+b_1k^2+b_2k^4+b_3k^6}$



Zoom



High positive order moments describe the tail of the charge distribution

$< r^6 >= -3.68\text{fm}^6$

$< r^7 >= -49.68\text{fm}^7$

Proton Charge Radius

λ	$\langle r^\lambda \rangle_Q$ [fm$^\lambda$]	$\langle r^\lambda \rangle$ [fm$^\lambda$]	$\langle r^{2p} \rangle_d$ [fm$^\lambda$]	Statistical Error $\delta[\langle r^\lambda \rangle_Q]$ [fm$^\lambda$]	$\delta[\langle r^{2p} \rangle_d]$ [fm$^\lambda$]	Systematic Error Dat. [fm$^\lambda$]	Int. [fm$^\lambda$]	Fun. [fm$^\lambda$]	Nor. [fm$^\lambda$]	Mod. [fm$^\lambda$]
-2	6.5245	8.7226	–	0.0172	–	0.0106	2.1981	0.0001	0.0095	0.3731
-1	1.9681	2.0906	–	0.0024	–	0.0019	0.1225	0.0001	0.0029	0.0278
1	0.7206	0.7179	–	0.0020	–	0.0025	0.0027	0.0001	0.0011	0.0029
2	0.6937	0.6937	0.6937	0.0094	0.0105	0.0111	0	0.0001	0.0010	0.0116
3	0.8697	0.8701	–	0.0457	–	0.0494	0.0004	0.0007	0.0013	0.0633
4	1.4728	1.4728	1.4728	0.2461	0.2365	0.2474	0	0.0065	0.0022	0.3805
5	3.8139	3.8137	–	1.4822	–	1.4297	0.0002	0.0343	0.0056	2.6276
6	16.405	16.405	16.405	10.058	10.839	9.4985	0	0.1871	0.0240	21.531
7	104.64	104.64	–	76.676	–	71.727	0.0001	2.8169	0.1528	212.48

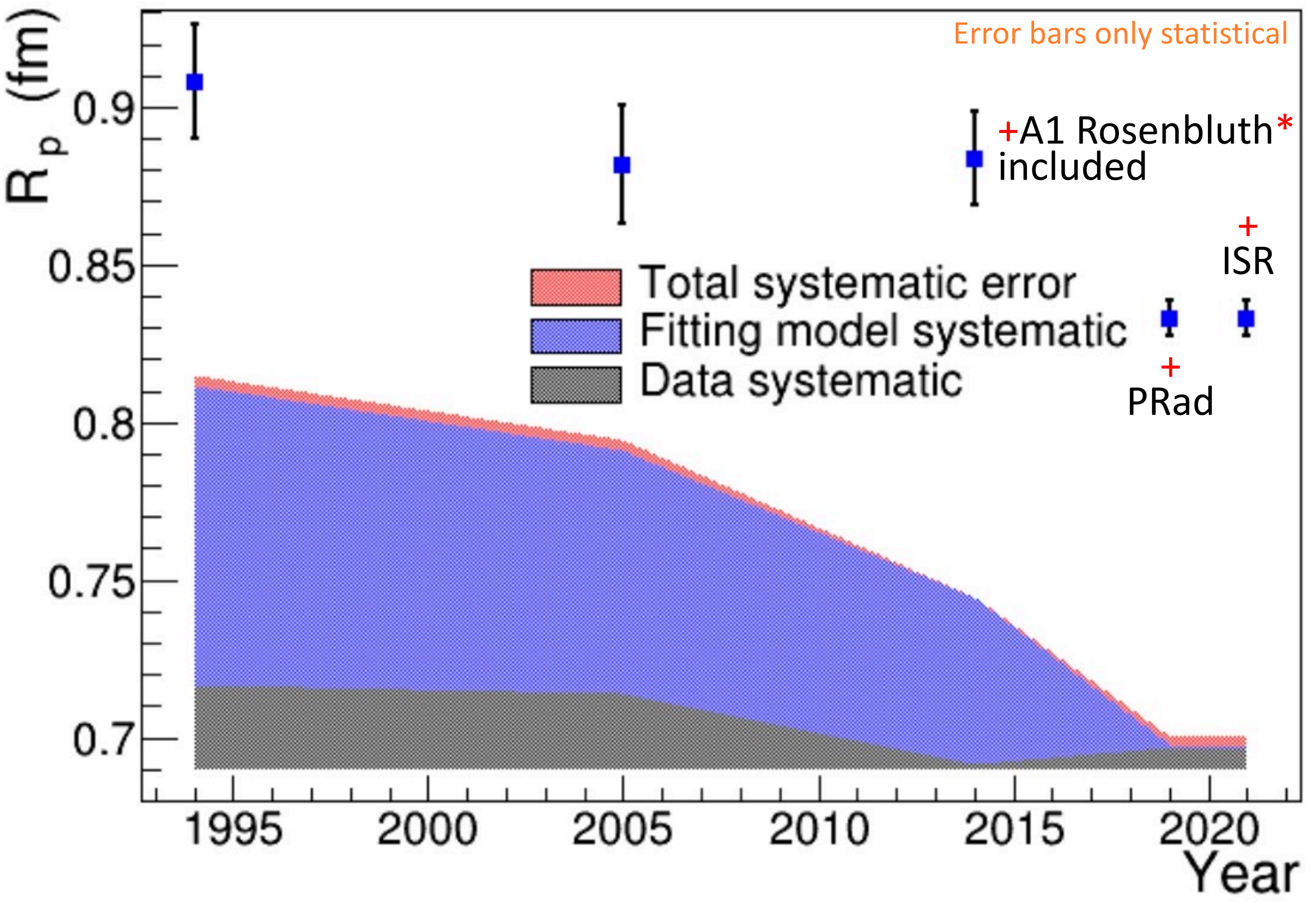
$$\mathbf{R_p} = \sqrt{<\mathbf{r^2}>} = \mathbf{0.8329 \pm 0.0056}(\boldsymbol{stat}) \pm \mathbf{0.0097}(\boldsymbol{syst})\boldsymbol{fm}$$

- **consistent with Codata 2018 value 0.8414(19) fm**
- within 2σ **from** μH **spectroscopic measurement**
- 1% accuracy: remarkable for e-p measurement but still far from atomic measurements precision

Proton Charge Radius

- Data are grouped into **5 time periods**, from 1961 to 2021
- For each period we have performed the IM analysis estimating the **radius** and the **systematic errors**

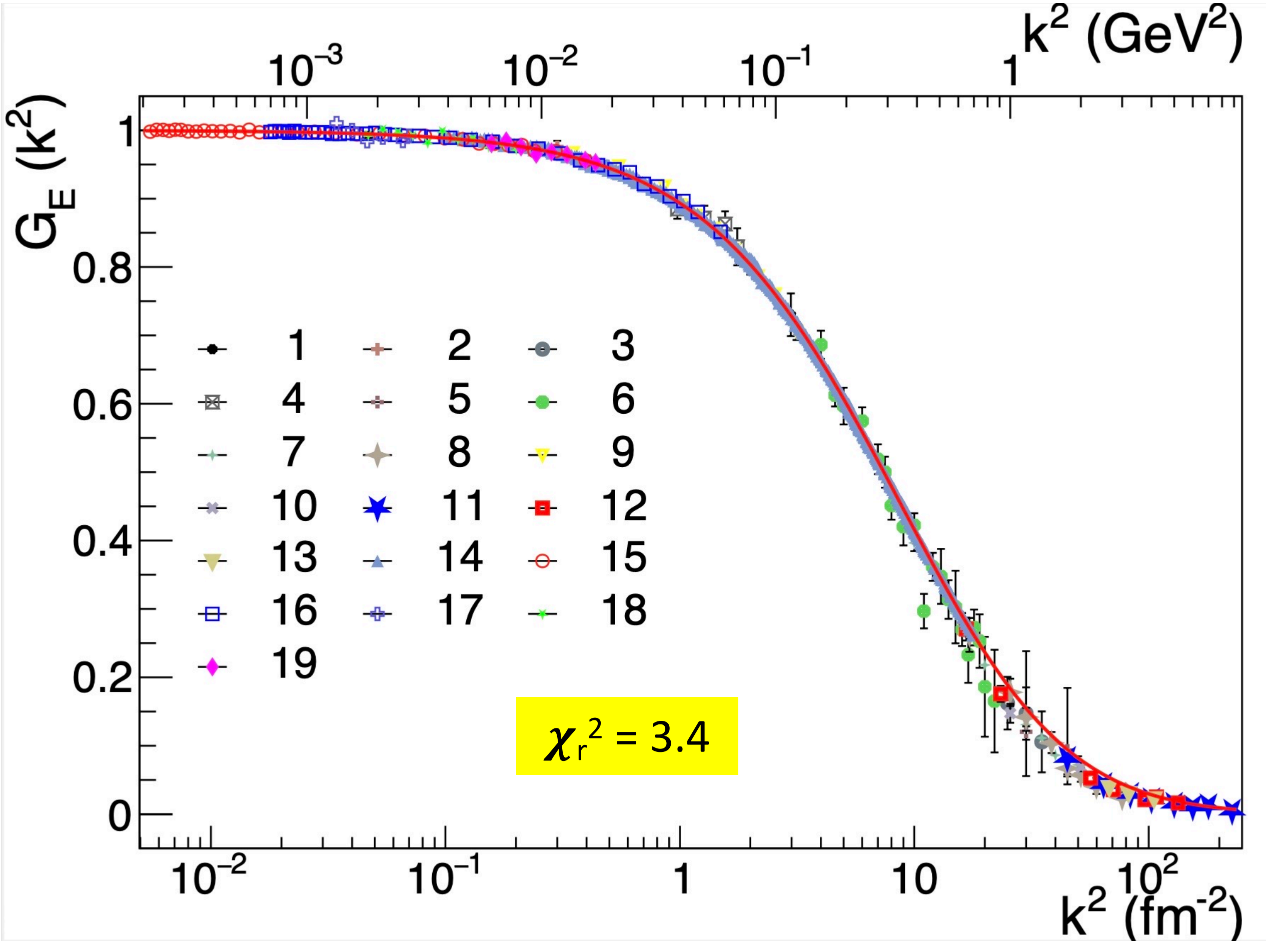
Time period	Data set range	R_p [fm]	$(\delta R_p)_{Sta.}$ [fm]	$(\delta R_p)_{Sys.}$ [fm]
1962-1994	1 - 11	0.9081	0.0178	0.1249
1962-2005	1 - 13	0.8813	0.0191	0.1044
1962-2014	1 - 14	0.8837	0.0148	0.0544
1962-2019	1 - 16	0.8329	0.0057	0.0102
1962-2021	1 - 19	0.8329	0.0056	0.0097

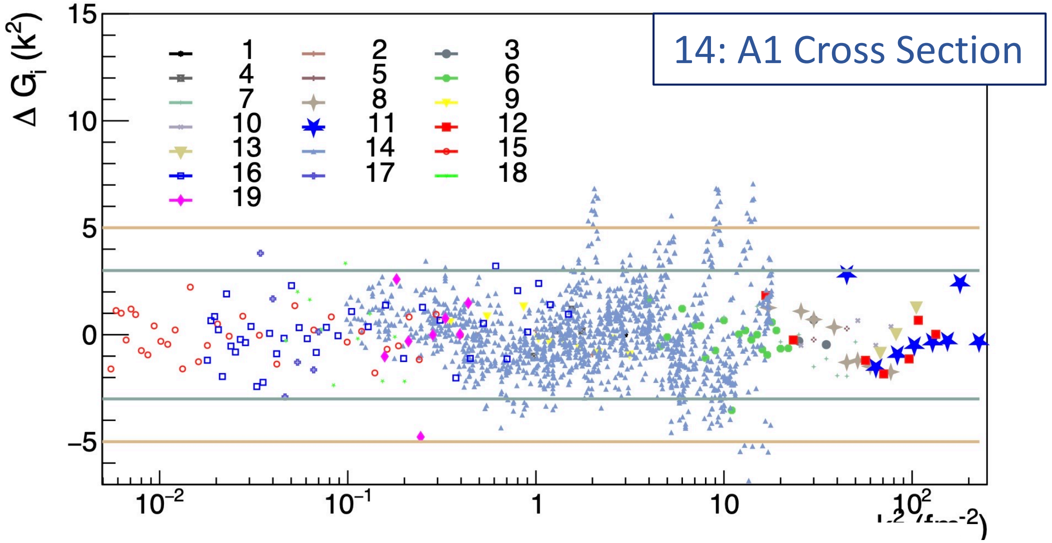


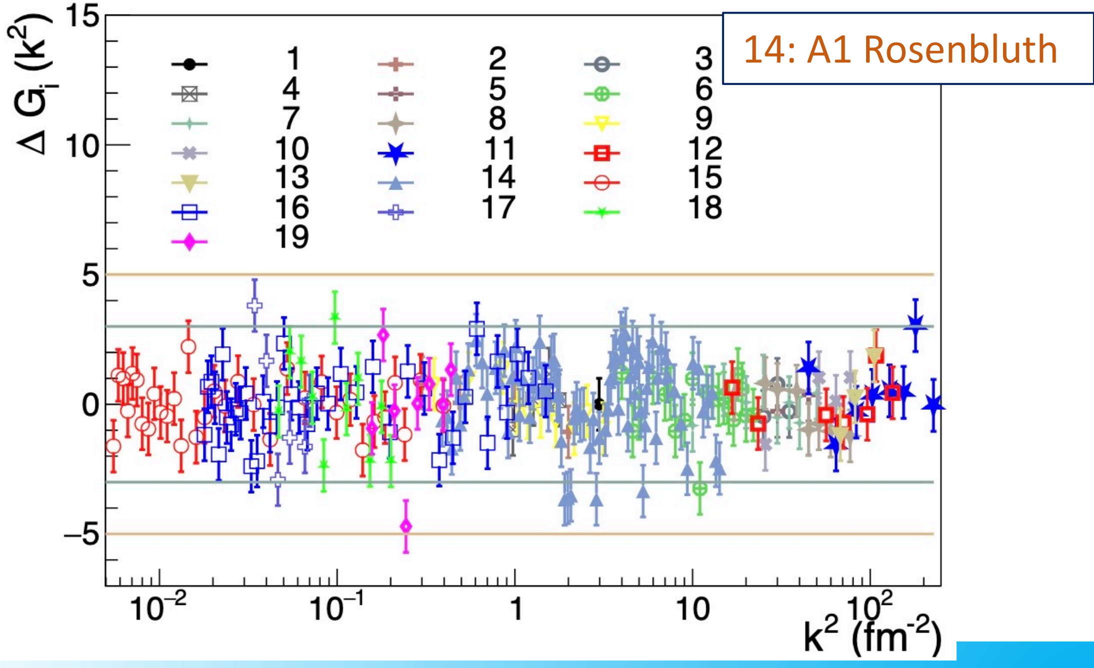
Up to 2014 the **dominant** source of **systematic** uncertainty is the choice of the **fitting model** (blue band)

With data at very low Q2 (Mainz A1, PRAD, ISR) constraints on the fit model are reinforced and this systematics is strongly reduced: the **data systematics** becomes **dominant** (grey band)

Simultaneous Fit of G_E Data: A1 Cross section Data (1360 points)







To be compared with χ_r^2 = 1.97 (A1 Rosenbluth)

Number of points: 1539 (256)

Rp = 0.828876 +- 0.00176695 fm (Rp= 0.832892 +- 0.00573039 fm)

Discussion

Attempt to identify data set(s) increasing the χ_r^2

Comment	N points	χ_r^2	Rp [fm]
19 exp. (A1 Rosenbluth	256	1.97	0.832892 +- 0.00573039
19 exp. (A1 cross section)	1539	**3.40**	0.828876 +- 0.00176695
18 exp. (Not A1)	179	1.56	0.823496 +- 0.00911815
16 exp. (Not A1, nor ISR)	154	1.20	0.821268 +- 0.00922865

- ❖ A1 & ISR @ Mainz
- ❖ A1 data very precise
- ❖ ISR fixed higher orders in their fit based on A1 → bias

❖ Impact of fixing $G_E(0) = 1$ in the fit:

* F. Borkowski et al., Nuclear Physics A Vol. 222 (1974) Issue 2:
found a « small radius » (0.82 fm), then fixed $G_E(0) = 1$ and got Rp = 0.87 fm

* PRad decided not to fix $G_E(0) = 1$ ⇨ « small » radius ; if $G_E(0) = 1$ is fixed ⇨ « large radius »

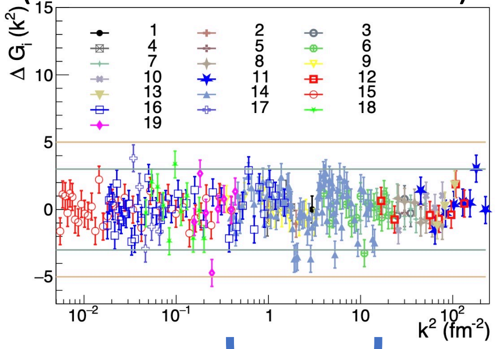
* A1 data alone: if $G_E(0) = 1$ is fixed ⇨ « large radius » ; if not ⇨ « small radius value.

Fixing $G_E(0) = 1$ results in adding a data point with an infinite precision

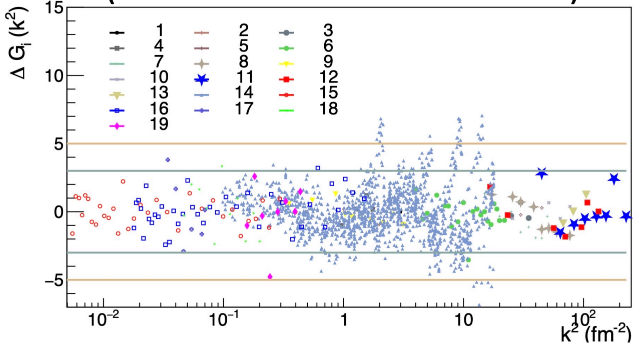
Discussion of A1 data

Residuals from G_E simultaneous fit

(with A1 Rosenbluth data)

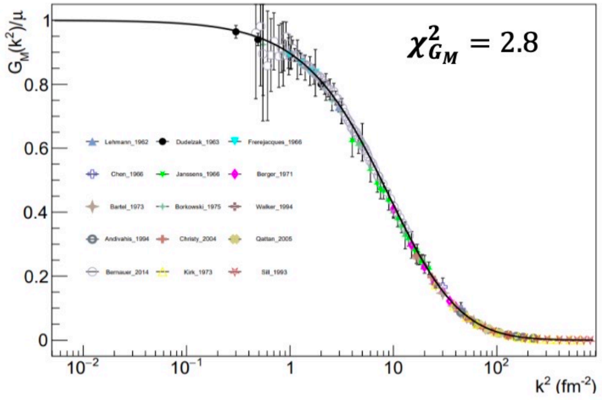


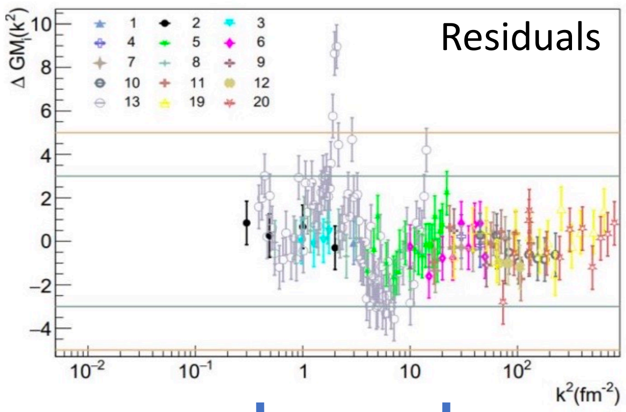
(with A1 cross section data)



Oscillation?

G_M simultaneous fit (A1 Rosenbluth data)





Oscillation?

Precision of G_M data?

Computation of Negative Moments based on 2nd regularization method

« Negative moments as the signature of the radial density at small distances » arXiv:2410.13340 [nucl-th] (Oct. 17th, 2024)

M. Atoui, M. Hoballah, M. Lassaut, J. Van de Wiele

- Robust evaluation of any radial density at small distances using **negative-order** radial moments evaluated **in momentum space.**

- **Valuable insight into the behavior of a given radial density in the vicinity of r=0** ⇔ Strong emphasis to measure FF at large squared-momentum transfers ≡ essential domain to determine negative order moments.

- Special attention paid to the regularization scheme directly affetcting the numerical determination of the density’s parametrization.

- Application to non-relativistic study cases: **(GE_n,GE_p) magnetic and *GMp* form factors**.

- Application to the relativistic Dirac form factor **F1**.

Summary (1/2)

- ✓ **Novel method** to extract the **moments of the charge density** via an **integral** over the **full experimentally probed momentum range** of **any G_E data « physical » functional form**, which has **advantages** compared to the usual **derivative** method:
  - **- does not rely on zero squared-momentum extrapolation**
  - **- uses data in a wide range of Q^2 improving the statistics**
  - **- gives access to moments of any order** (even, odd, negative, non-integer) **[$\lambda > -3$]**

- ✓ Moments of **even positive order** **formally equal** to those obtained with the **« derivative method »**

- ✓ **High positive order** moments probe **large-distance effects**, require **G_E data at low Q^2**
- ✓ **Higher positive order** moments = inputs for atomic spectroscopy corrections
- ✓ **Negative order** moments **[$\lambda > -3$]** probe **short-range effects**, **require** **G_E data at large Q^2**

Summary & Perspectives

✓ **Application of the integral method:**

* to available G_E data sets (choice of Rosenbluth data and low Q^2)
* Simultaneaous fit: choice of polynomial ratio parametrisation as the functional form
* Normalisation parameter (η_i) associated to each data set
* Extraction of moments considering errors from exp. data (stat. & syst.)
* $\mathrm{Rp} =$ 0.832892 +/- 0.00573039 fm (A1 Ros. data) // 0.828876 +/- 0.00176695 fm (A1 cross section)
* Discrepancies betw. Rp values from ep scattering seems to originate from underestimated syst. err

.

* Investigation of reason(s) for χ_r^2 increase
* Impact of fixing $G_E(0)=1$ for G_E data parametrisation

✓ Recent publication presenting method to access any negative order moments [(non-) relativistic ; (GE) F1]

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Perspectives

❖ Simultaneous Fit of G_M data ⇒ Moments of the magnetization density

❖ Evaluation of the Zemach moments (convolution of G_E & G_M)

https://www.youtube.com/watch?v=8iuBwdBxp3M

In French

Subtitles in English should be available shortly


ご清聴ありがとう
ございました
Thank you for
your attention

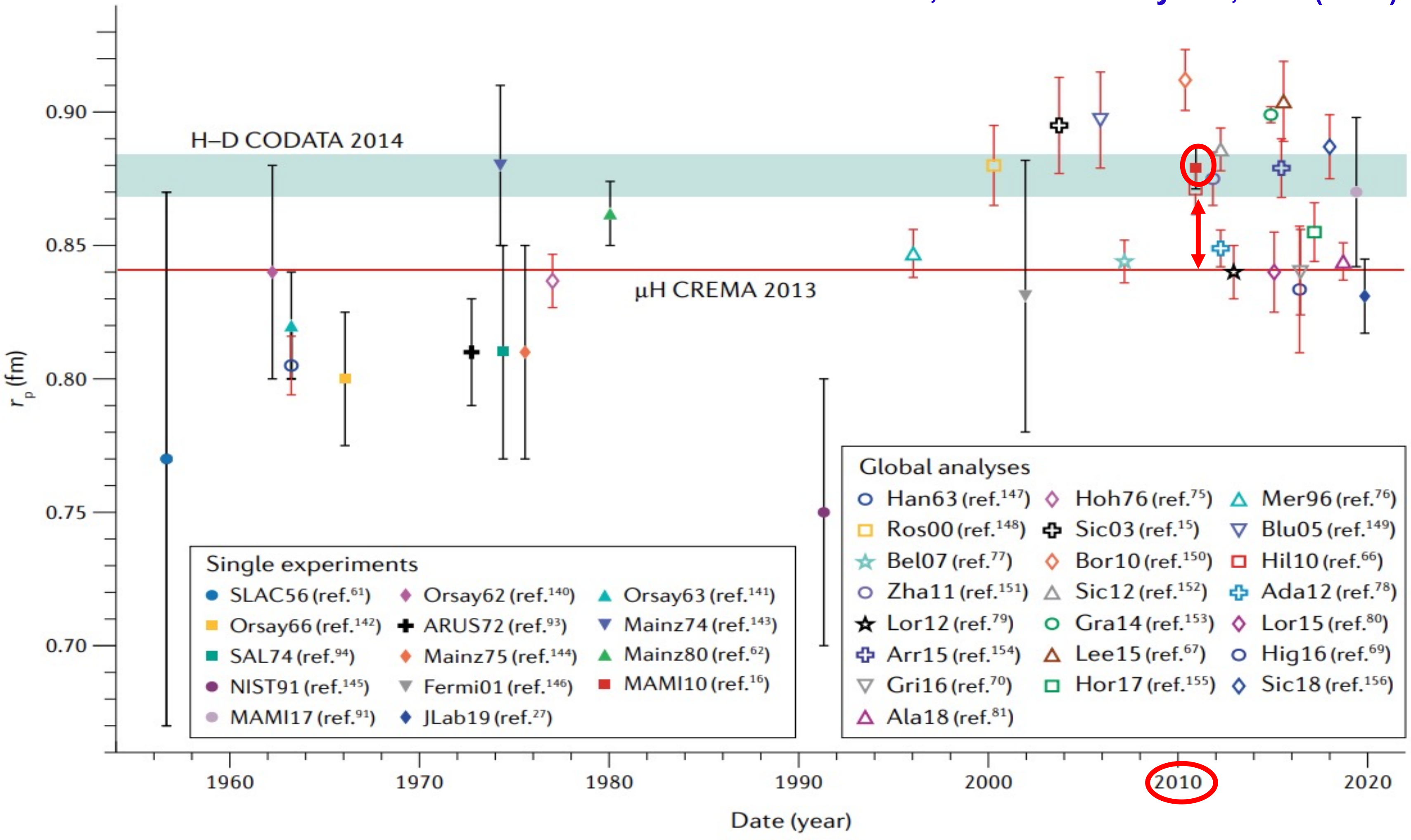
Back-up

η_i normalisation parameter

Data Set Number	Ref.	η_i	$(\delta\eta_i)_{Sta.}$ $(\times 10^{-2})$	$(\delta\eta_i)_{Sys.}$ $(\times 10^{-2})$
1	[19]	0.993	3.020	9.922
2	[20]	0.982	0.505	0.752
3	[21]	2.441	15.87	12.20
4	[22]	0.991	0.917	0.208
5	[23]	0.922	30.43	4.612
6	[24]	1.004	1.132	0.803
7	[25]	1.001	1.333	2.001
8	[26]	1.025	4.490	1.077
9	[27]	0.981	0.254	1.766
10	[28]	1.170	4.902	0.469
11	[29]	0.972	2.144	6.812
12	[30]	1.042	3.513	0.506
13	[31]	1.072	3.509	0.584
14	[10]	0.991	0.083	0.993
15	[11]	1.000	0.022	0.215
16	[11]	0.998	0.018	0.119
17	[32]	1.001	0.113	0.370
18	[32]	1.000	0.097	0.365
19	[32]	0.998	0.066	0.442

Chronological overview of the proton radius values from electron scattering experiments

J.-P. Karr et al., Nature Rev. Phys. 2 , 601 (2020)

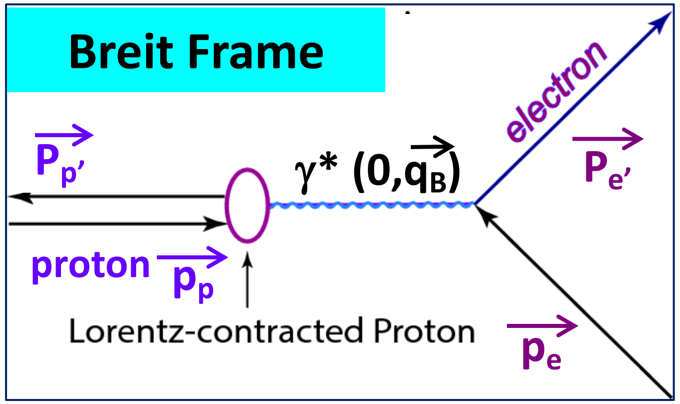


Framework of lepton proton scattering experiments

$$G(\boldsymbol{k}) = \int_0^\infty d^3\boldsymbol{r}\,\rho(\boldsymbol{r})\,\exp(-i\,\boldsymbol{k}\cdot\boldsymbol{r}) \qquad (1)$$

Electric Form Factor
Measured

Fourier Transform
of the charge probability density



Breit Frame	Laboratory
$\vec{P}_{p'} = -\vec{P}_p$	$\vec{P}_p = \vec{0}$
$\gamma^*\,(0, \vec{q}_B);\ q_B^2$	$\gamma^*\,(q^0, \vec{q}_L);\ q_L^2$
$J^0 \propto G_E$	$J^0 = f(G_E, G_M)$
$J^{1,2,3} \propto G_M$	

Invariant $q_L^2 = q_B^2 = -\vec{q}_B^{\,2} = -\vec{k}^2 = K^2 \qquad (K^2 < 0)$

No global Lorentz Transform (LT) to the lab frame: 1 LT per $|K^2|$ point

$|K^2|\,[\mathrm{fm}^{-2}] = 25{,}7 \times Q^2\ [(\mathrm{GeV}/c^2)^2]$