J-PARCにおけるΣ陽子散乱実験の結果

T. Nanamura for the J-PARC E40 collaboration Kyoto University, JAEA

Baryon-Baryon interaction

- Hyperon-nucleon (YN) interaction
 - A extension of NN interaction
 - Mass of s quark is similar to u,d quarks
 - can be treated under the $SU(3)_f$ symmetry
 - Existence of hyperon in neutron star (Hyperon puzzle)
- B-B interaction between the octet baryons (n,p, Λ , Σ , Ξ)

 $\mathbf{8}\otimes\mathbf{8}=\mathbf{27}\oplus\mathbf{8}_s\oplus\mathbf{1}\oplus\mathbf{10}^*\oplus\mathbf{10}\oplus\mathbf{8}_a.$

- Some multiplets may have different futures from NN
 - Due to Quark-Pauli effect and color-magnetic interaction
 - 10, 8s-plets: strongly repulsive?
 - 1-plet: attractive core? (H-dybaryon?)
- ∑N(I=3/2) is suitable to investigate 10-plet

BB channel $\left(I\right)$	$^{1}\mathrm{Even}$ or $^{3}\mathrm{Odd}$	$^3\mathrm{Even}$ or $^1\mathrm{Odd}$
NN(I = 0)	-	(10 *)
NN(I = 1)	(27)	-
$\Lambda N(I=\frac{1}{2})$	$\frac{1}{\sqrt{10}}[(8_s) + 3(27)]$	$\frac{1}{\sqrt{2}}[-(8_{a})+(10^{*})]$
$\Sigma N(I = \frac{1}{2})$	$\frac{1}{\sqrt{10}}[3(8_s) - (27)]$	$\frac{1}{\sqrt{2}}[(8_{a}) + (10^{*})]$
$\Sigma N(I = \frac{3}{2})$	(27)	(10)

Σ^+ p interaction

- Strong repulsive core is expected
 - Pauli exclusive principle in quark level
 - In ³S₁ (S=1, L=0) state, 2 u quarks
 - have same spin, color with a high probability
 - Some circumstantial evidences from Σ -nucleus interaction
 - Spin-isospin averaged potential is repulsive
 - Isospin dependence in A=4 system (I=1/2:bound I=3/2:unbound)
 - HAL QCD calculation
- However, the strength of "strong repulsion" was ambiguous.



γEFT NLO19

pp scattering

J-PARC E40 experiment Measurement of $d\sigma/d\Omega$ of Σp scatterings

- Physics motivations
 - Verification of repulsive force due to quark Pauli effect in the Σ +p channel
 - Determination of the strength of the repulsive force is also important.
 - Systematic study of the ΣN interaction



I=3/2, ³Even and ¹Odd: 10-plet of SU(3)_f B-B interaction ³S₁:Almost Pauli forbidden →strong repulsive force?

BB channel $\left(I\right)$	1 Even or 3 Odd	$^3\mathrm{Even}$ or $^1\mathrm{Odd}$
NN(I = 0)	-	(10 *)
$\mathrm{NN}(I=1)$	(27)	-
$\Lambda N(I=\frac{1}{2})$	$\frac{1}{\sqrt{10}}[(8_s) + 3(27)]$	$\frac{1}{\sqrt{2}}[-(8_a) + (\mathbf{10^*})]$
$\Sigma N(I = \frac{1}{2})$	$\frac{1}{\sqrt{10}}[3(8_s) - (27)]$	$\frac{1}{\sqrt{2}}[(8_{a}) + (10^{*})]$
$\Sigma N(I = \frac{3}{2})$	(27)	(10)

- Purpose of experiments
 - Measurement of $d\sigma/d\Omega$ with high statistics
 - Σ -p elastic, Σ -p $\rightarrow \Lambda$ n inelastic scattering (Σ data)
 - Σ +p elastic scattering (Σ + data)
 - Data taking had been finished on June 2020.

Difficulties of **Sp** scattering experiment

- Generally, hyperon-nucleon scattering experiment is difficult.
 - Short life time of hyperons : 10⁻¹⁰ s
 - Difficulty of producing plenty of hyperon beam
 - Difficulty of detection and identification of scattering hyperon
 - Previous Σp scattering experiments could identify only a few tens of events.
- Other experimental methods to extract ΣN interaction
 - Hypernuclei \rightarrow only ${}^{4}{}_{\Sigma}$ He is observed.
 - Large isospin dependence in A=4 system: attractive I=1/2 and repulsive I=3/2 state
 - Spin-isospin averaged potential was evaluated to be (V,W)=(30,-40) MeV
 - Femtoscopy by ALICE collaboration (for low relative momentum)
 - Results on \sum^{o} p interaction has been reported phys.lett.B 805 (2020) 135419
 - Now, statistical error is large. LHC Run3, 4 data is awaited.
 - \rightarrow Scattering experiment is difficult, but it is necessarry.
- How do we overcome these difficulties?
 - High rate π beam and large acceptance spectrometer
 - Producing and tagging large amount of Σ beam
 - LH2 target and Surrounding detector system
 - Large acceptance for the recoil proton
 - Reconstructing reactions from two body kinematics

K1.8 beamline @hadron hall





Analysis:Σ⁺ production

- Σ⁺ identification
 - Missing mass of $\pi^+p \rightarrow K^+X$ reaction
- Momentum of Σ^+
 - Missing momentum of π⁺, K⁺
 - Σ+p scattering analysis was performed for three separated momentum region

8

• Low (0.44-0.55 GeV/c), Middle (0.55-0.65 GeV/c), High (0.65-0.80 GeV/c)



Analysis: CATCH part

• Tracking by CFT

dE/dx in CFT [MeV/mm]

- Particle trajectories are reconstructed.
- Particle identification
 - Using energy loss correlation between CFT & BGO
 - Protons are well distinguished.
 - Kinetic energy of protons are fully measured by BGO.



CFT dE/dx Total E



CFT tracking efficiency as a function of Ep @54°



Kinematical identification of Σ +p scattering events

- Hereafter, we concentrate on events with 2 protons in final state.
 - Σ^+ p scattering followed by $\Sigma^+ \rightarrow p\pi^o$ decay



Checking a kinematical consistency for recoil proton

 $\cdot E_{meas}$: measured kinetic energy with CATCH

 $\cdot E_{calc}$: calculated kinetic energy from incident Σ^+ momentum and recoil angle

 $\Delta E(\Sigma^+ p) = E_{meas} - E_{calc}$

• For Σ^+ p scattering events, ΔE distributes around 0.



Background reduction

• Background reactions are also generate in a Monte Carlo simulation and distribution in $\Delta E(\Sigma^+p)$ histogram is estimated.

signal





Background reduction

• Background reactions are also generate in a Monte Carlo simulation and distribution in $\Delta E(\Sigma^+p)$ histogram is estimated.

backgrounds





Background reduction

• Background reactions are also generate in a Monte Carlo simulation and distribution in $\Delta E(\Sigma^+p)$ histogram is estimated.









Kinematical identification of Σ +p scattering events

- Hereafter, we concentrate on events with 2 protons in final state.
 - Σ^+ p scattering followed by $\Sigma^+ \rightarrow p\pi^o$ decay <u>Kinetic energy of recoil proton</u>



Checking a kinematical consistency for recoil proton

 $\cdot E_{meas}$: measured kinetic energy with CATCH $\cdot E_{calc}$: calculated kinetic energy from incident Σ^+ momentum and recoil angle

$$\Delta E(\Sigma^+ p) = E_{meas} - E_{calc}$$

• For Σ^+ p scattering events, ΔE distributes around 0.



Differential cross sections

- Differential cross sections were derived from \sim 2400 Σ^+ p scattering events.
 - The data quality has beem significantly improved!
 - Main sources of systematic error: background estimation, efficiency for low momentum proton.
 - FSS and fss2 are obviously larger. On the other hand, ESC08, NSC97f are consistent to some extent.
 - Note:NSC97f suggests the attractive ${}^{3}S_{1}$ interaction, which does not agree with the current common understanding of ΣN interaction.



Results from Σ^{-} data

• We have already reported the differential cross sections of the Σ -p elastic scattering and Σ -p \rightarrow An inelastic scattering.



K. Miwa et al., PRC 104, 045204 (2021)

K. Miwa et al., PRL, 128, 072501 (2022)



fss2 and χ EFT well reproduced Σ - data. Anyway, together with Σ +p data, our data will be essential input to establish realistic BB interaction models.

Phase shift analysis for Σ^+p

- Extracting the contribution of the ${}^{3}S_{1}$ is important to study the repulsive nature of $\Sigma^{+}p$ system due to the quark Pauli effect.
- Referring to formalism of NN scattering, the differential cross section was calculated as a function of phase shifts and we tried to fit data.



Phase shift analysis

• We considered contribution by D wave(L<=2), and Coulomb effects were merely ignored.(bar phase shifts were regarded as nuclear bar phase shifts)

$$I_0 = \frac{1}{4} |M_{0,0}^{0,0}|^2 + \frac{1}{2} |M_{1,1}^{1,1}|^2 + \frac{1}{4} |M_{0,0}^{1,1}|^2 + \frac{1}{2} |M_{0,1}^{1,1}|^2 + \frac{1}{2} |M_{1,0}^{1,1}|^2 + \frac{1}{2} |M_{1,-1}^{1,1}|^2$$
(5.3)

$$M_{0,0}^{0,0} = h_{1S_0} + 3h_{1P_1}\cos\theta + 5h_{1D_2} \times \left(\frac{3\cos^2\theta - 1}{2}\right),\tag{5.4}$$

$$M_{1,1}^{1,1} = (h_{3S_{1}} - \frac{\sqrt{2}}{2}h^{3S_{1}-3D_{1}}) + \left(\frac{3}{2}h_{3P_{2}} + \frac{3}{2}h_{3P_{1}}\right)\cos\theta + \left(2h_{3D_{3}} + \frac{5}{2}h_{3D_{2}} + \frac{1}{2}h_{3D_{1}} - \frac{\sqrt{2}}{2}h^{3S_{1}-3D_{1}}\right) \times \frac{3\cos^{2}\theta - 1}{2},$$
(5.5)

$$M_{0,0}^{1,1} = (h_{3S_{1}} + \sqrt{2}h^{3S_{1}-3D_{1}}) + (2h_{3P_{2}} + h_{3P_{0}})\cos\theta + (3h_{3D_{3}} + 2h_{3D_{1}} + \sqrt{2}h^{3S_{1}-3D_{1}}) \times \frac{3\cos^{2}\theta - 1}{2},$$
(5.6)

$$\begin{split} M_{0,1}^{1,1} &= \left(-\frac{3}{2\sqrt{2}} h_{^{3}P_{2}} + \frac{3}{2\sqrt{2}} h_{^{3}P_{1}} \right) \times (-\sin\theta) \\ &+ \left(-\frac{4}{3\sqrt{2}} h_{^{3}D_{3}} + \frac{5}{6\sqrt{2}} h_{^{3}D_{2}} + \frac{1}{2\sqrt{2}} h_{^{3}D_{1}} - \frac{1}{\sqrt{2}} h^{^{3}S_{1}-^{3}D_{1}} \right) \times (-3\cos\theta\sin\theta), \end{split}$$
(5.7)
$$M_{1,0}^{1,1} &= \left(\frac{1}{\sqrt{2}} h_{^{3}P_{2}} - \frac{1}{\sqrt{2}} h_{^{3}P_{0}} \right) \times (-\sin\theta) + \left(\frac{1}{\sqrt{2}} h_{^{3}D_{3}} - \frac{1}{\sqrt{2}} h_{^{3}D_{1}} - \frac{1}{\sqrt{2}} h^{^{3}S_{1}-^{3}D_{1}} \right) \times (-3\cos\theta\sin\theta),$$
(5.7)

$$M_{1,-1}^{1,1} = \left(\frac{1}{6}h_{^{3}D_{3}} - \frac{5}{12}h_{^{3}D_{2}} + \frac{1}{4}h_{^{3}D_{1}} - \frac{1}{2\sqrt{2}}h^{^{3}S_{1}-^{3}D_{1}}\right) \times (3\sin^{2}\theta), \tag{5.9}$$

where partial wave amplitude h were defined as

$$h_{2S+1L_J} = \begin{cases} \frac{1}{2ik} (\cos(2\bar{\epsilon}_1) \exp(2i\bar{\delta}_{2S+1L_J}) - 1) & ({}^3S_1 \text{ and } {}^3D_1 \text{ case}) \\ \frac{1}{2ik} (\exp(2i\bar{\delta}_{2S+1L_J}) - 1) & (\text{else}) \end{cases}$$
(5.10)
$$h^{{}^3S_1 - {}^3D_1} = \frac{1}{2k} \sin(2\bar{\epsilon}_1) \exp(i\bar{\delta}_{3S_1} + i\bar{\delta}_{3D_1}).$$
(5.11)

Phase shift analysis

- The function $I_0(\theta, p, \delta[27](p), \delta[10](p))$ has 11 phase shift parameters.
 - $\ \ \, \boldsymbol{\delta}[27] = \{ \delta_{1S0}, \delta_{3P2}, \delta_{3P1}, \delta_{3P0}, \delta_{1D2} \}, \, \boldsymbol{\delta}[10] = \{ \delta_{3S1}, \delta_{1P1}, \delta_{3D3}, \delta_{3D2}, \delta_{3D1}, \epsilon_1 \}$
- δ [27] are well constrained from NN data and are regarded as constants taken from
 - <u>pp scattering</u> based on complete SU(3)f symmetry.
 - Less realistic, but independent from baryon-baryon interaction model.
 - <u>NSC97f or ESC16</u> in order to approximately consider the effect of the flavor symmetry breaking and the difference of meson exchange potential.
- δ [10] are to be investigated, but 6 parameters are still too much to perform meaningful fitting.
 - only δ_{3S1} and δ_{1P1} were treated as free parameters.
 - Rest 4 parameters $(\delta_{3D_3}, \delta_{3D_2}, \delta_{3D_1}, \text{ and } ε_1)$ are fixed at 0 or NSC97f and ESC16.

Note : the sign of δ_{3S1} cannot be determined. Positive and negative cases are examined.

BB channel (I)	¹ Even or ³ Odd	³ Even or ¹ Odd
NN(I = 0)	-	(10 *)
NN(I = 1)	(27)	-
$\Lambda N(I = \frac{1}{2})$	$\frac{1}{\sqrt{10}}[(8_s) + 3(27)]$	$\frac{1}{\sqrt{2}}[-(8_a) + (10^*)]$
$\Sigma N(I=\frac{1}{2})$	$\frac{1}{\sqrt{10}}[3(8_s) - (27)]$	$\frac{1}{\sqrt{2}}[(8_{a}) + (10^{*})]$
$\Sigma N(I = \frac{3}{2})$	(27)	(10)

Fitting results

• Fixed phase shifts are taken from ESC16

20

- δ_{3S1} <0 case
- χ^2 /ndf is approximately 1.



Obtained phase shifts

- ${}^{3}S_{1}$:almost consistent with ESC16 ($\delta < 0$) or NSC97f ($\delta > 0$).
 - $|\delta|: 28.3 \pm 1.5 \pm 2.1$ (low), $23.4 \pm 2.0 \pm 3.0$ (mid), $32.5 \pm 2.5 \pm 2.5$ (high)
 - Fitting error and effect of the different sets of the fixed parameters
 - The interaction in this channel is moderately repulsive.
- ¹P₁:ambiguous.
 - They may support the prediction of the fss2, ESC16, NSC97f in which the interaction of 1P1 channel is weakly attractive.



Comparison with HAL QCD

- $\delta^3 S_1$ can be compared with HAL QCD!
- Our results are consistent with HAL QCD calculation with larger t-to.



H. Nemura AIP Conf. Proc. 2130, 040005(2019)

xEFT N²LO

- Recently, the $\chi EFT~N^2LO$ calculation for $\Lambda N\mathchar`-\Sigma N$ interaction is presented.
 - J. Haidenbauer, HYP 2022 presentation
 - ^o J. Haidenbauer, EPJ Web Conf., 271 (2022) 05001
- In χEFT N²LO, Our data were used to determine LECs in P-wave
 - LECs in S-wave were determined by low-energy Σp scattering data.
 - From NLO to N²LO, there are no new additional LECs in the two body sector.

Comparison with xEFT N²LO

Σ⁺p scattering

- χEFT N²LO well agrees with our data in the low momentum region. However, small differential cross sections in the middle momentum region were not reproduced.
- □ In our data, δ3S1 in the middle momentum region were smaller than in other momentum region. Influence of the $\Lambda \pi p$ threshold (pΣ=0.62 GeV/c)? $\Sigma^{+}p \rightarrow \Sigma^{+}p$ J. Haidenbauer,



Comparison with xEFT N²LO

• Σ -p elastic, Σ -p \rightarrow An scattering

- χEFT N²LO well agrees with our data, as the χEFT NLO.
- To determine the P-wave LECs uniquely, data for additional channel (Ap elastic, Σ -p \rightarrow $\Sigma^{o}n$,) or observables are needed.

25

• J-PARC E86 experiment (Λp scattering @K1.1 beam line)



Comparison with xEFT N²LO

- Total cross section
 - χ EFT N²LO well agrees with our data for p $\Sigma \sim 0.5$ GeV/c.

26

In this plot, total cross sections from experiment were calculated as

 $2 \times \sigma_{-0.5 < \cos\theta < 0.5}$. I think angular dependence (mainly come from contribution of P waves) should be considered for good comparison.



Future ∑⁺p scattering experiments?

• Higher momentum?

- To understand the short-range force and behavior of quark-Pauli effect, data for higher momentum is desired.
 - In present data, the distance of two particles were 0.5-0.8 fm.
- As long as contributions of D- and higher waves are small (or well estimated), our phase shift analysis method will work. The phase shifts of the ${}^{3}S_{1}$ and ${}^{1}P_{1}$ could be determined only from $d\sigma/d\Omega$.
 - Most of theoretical models for BB interaction are constructed using below 1GeV/c data... Will they be reliable?
- Different spectrometer setup for (π^+, K^+) reaction?
 - + E40: 1.41 GeV/c $\pi^{\scriptscriptstyle +}$ and 3 $^\circ~<\!\theta_K\!\!<\!\!25^\circ~$, more backward angle?
- Experiment $@\pi 20$ beam line will be possible?
 - LOI for Ap scattering (R. Honda et al., J-PARC LoI 2020-8)
- Additional observable?
 - If ${}^{3}D_{1}$ and ϵ_{1} can be determined with a aid of observables, our understanding of ${}^{3}S_{1}$ - ${}^{3}D_{1}$ state will be deepened.
 - Analyzing power can be derived even from E40 data. I will study phase shift analysis using analyzing power together with $d\sigma/d\Omega$

Future ∑⁺p scattering experiments?

- Wider angular acceptance?
 - More forward angle
 - cosθcm<0.8, 20<Ep [MeV] <30
 - Important to resolve a ambiguity of $\delta^{1}P_{1}$
 - Recoil proton can be measured by major modification of CATCH?
 - (e.g. SSD tracker instead of CFT)
 - Ultra-forward angle
 - $\cos\theta CM \sim 0.95$, Ep <5 MeV
 - By checking Coulomb interference, the sign of δ_3 S1 will be determined.
 - Recoil proton would stop in LH2 target.
 - Low-density active target is needed.
 - TPC with H2 gas?





Summary

- Hyperon-nucleon scattering experiment gives us very important information for B-B interaction, especially quark Pauli effect.
- J-PARC E40 Experiment
 - High-statistics Σp scattering experiment
 - Σ +p elastic scattering, Σ -p elastic scattering, Σ -p \rightarrow An inelastic scattering
 - Data taking was finished by June 2020.
- $d\sigma/d\Omega$ were derived by about 2,400 Σ^+ p scattering events.
 - We successfully performed difficult YN scattering experiment!
- By not only comparison with the existing theoretical calculations but also derivation of the phase shifts of the ${}^{3}S_{1}$ and ${}^{1}P_{1}$ channels, the nature of $\Sigma^{+}p$ interaction was investigated.
 - The absolute value (and the strength of interaction) of the ³S₁ is much smaller than fss2 and FSS expected.
- Recent χEFT N²LO calculation using our data was introduced.

Back up

30

- There are many backgrounds w/o cuts.
- Spatial consistency cut
 - At scattering and decay point
 - Vertex cut, closest distances cut
- Kinematical consistency cut
 - Missing mass cut for decay proton
 - pp scattering consistency
 - πp elastic scattering cut







- Spatial consistency cut
 - At scattering and decay point
 - Effective to cut backgrounds derived from accidental coincidences
- Vertex cut
 - Scattering vertex should be in the LH2 target, decay vertex after the scattering should not be far from the target.
- Closest distances cut

Simulated closest distance



Simulated closest distance at decay vertex after scattering



- Kinematical cuts
- Missing mass cut for decay proton
 - □ π^{o} missing reaction, Σ^{+} p scattering followed by Σ^{+} ·→ $p\pi^{o}$ decay and secondary pp scattering events is selected.
- Elastic πp scattering cut
 - Proton from accidental π +p scattering induced by accidental 1.41 GeV/c π + beam was rejected.

Missing mass for



Correlation between θ lab and E of proton



- Kinematical cuts
- Kinematical consistency cut for secondary pp scattering
 - The momentum of decay proton (incident proton for pp scattering)
 - Can be reconstructed by two ways:
 - Sum of the momenta of two detected proton
 - **p**_{sum}=p1+p2
 - Calculation from Σ + momentum and direction of \mathbf{p}_{sum}
 - p_{calc}
 - Consistency $\Delta p = p_{sum} p_{calc}$
 - In pp scattering, opening angle of 2 proton should be $\sim 90^{\circ}$
 - Secondary pp scattering events
 - Concentrates on around
 - $(\Delta p, \alpha) = (0, 90^{\circ}).$



b

 π^+

Background estimation

- ΔE spectra were fitted by the sum of the simulated distribution of considered reactions.
 Low-mommentum
- Parameter: scale factor of each reaction
- Fitting was performed for each scattering angle and momentum independently.
- Uncertainty from binning of ΔE spectra and various constraints on parameters are considered.



Background estimation

- ΔE spectra were fitted by the sum of the simulated distribution of middle mommentum
- Parameter: scale factor of each reaction
- Fitting was performed for each scattering angle and momentur independently.
- Uncertainty from binning of ΔE spectra and various constraints on parameters are considered.



Background estimation

- ΔE spectra were fitted by the sum of the simulated distribution of considered reactions.
 high momentum
- Parameter: scale factor of
- each reaction
- Fitting was performed for ea
- scattering angle and mom
- independently.
- Uncertainty from binning of
- spectra and various const
- on parameters are conside



The derivation of the differential cross sections $\frac{d\sigma}{d\Omega}(p,\cos\theta_{\rm CM}) = \frac{N(p,\cos\theta_{\rm CM})}{\varepsilon(p,\cos\theta_{\rm CM})\rho \cdot N_A \cdot L_{\rm tot}(p) \cdot \Delta\Omega}$

38

- N:the event number of Σp scattering
- ε:(averaged)efficiency evaluated by simulation
- L_{tot} : Total flight length of Σ + in LH₂ target



The derivation of the differential cross sections $\frac{d\sigma}{d\Omega}(p,\cos\theta_{\rm CM}) = \frac{N(p,\cos\theta_{\rm CM})}{\varepsilon(p,\cos\theta_{\rm CM})\rho \cdot N_A \cdot L_{\rm tot}(p) \cdot \Delta\Omega}$

- N:the event number of ∑p scattering
- ε:(averaged)efficiency evaluated by simulation
- L_{tot} : Total flight length of Σ + in LH₂ target



- Proton detection efficiency of CATCH consists of
 - Energy measurement efficiency
 CFT tracking efficiency
- They depends on (E_p , θ_p , z_{source})
- CATCH efficiency was evaluated from simulation and pp scattering data.
 - In simulation, proton with arbitrary (E_p , θ_p , z_{source}) can be generated.
 - In pp scattering data, (E_p, θ_p) is restricted by the kinematics of pp scattering.

- Energy measurement efficiency
 - Simulated efficiency well agree with the data.
 - Simulated efficiency is used.



- Data-based efficiency is less than simulation-based efficiency
 - Because of zig-zag structure of CATCH spiral layer
 - Difficult to reproduce in the Geant4
- CFT tracking efficiency is formulated as Fermi function and parameters are determined from pp scattering data.

$$\varepsilon_{\rm CFT}(\theta, E, z) = \frac{\varepsilon_{\rm max}(\theta, z)}{1 + \exp\left(\frac{E - E_{\rm half}(\theta)}{d(\theta)}\right)},$$

42

• Note; Because CFT tracking requires at least 6 layers hit in the fiber tracker, CATCH cannot analyze (detect) low energy protons.



- To evaluate uncertainty of CFT tracking efficiency for low energy protons, two possible highest and lowest CFT efficiencies were considered.
 - Except for low energy protons, $d\sigma/d\Omega$ for pp scattering using proton beam in calibration data are well derived.
- The angular dependence of secondary pp scattering events is sandwiched by two efficiency-corrected simulations.



Total flight length of Σ

- Total flight length of Σ^+ particles in the LH2 target was estimated by a Monte Carlo simulation.
 - Σ⁺ with analyzed momentum was generated at analyzed vertex in the LH2 target
 - Flight length were summed up until Σ decayed or exited LH2 target.



Region	Low	Middle	High
All events [cm]	$3.69 imes 10^7$	1.13×10^7	$6.70 imes 10^6$
Sideband BG [cm]	0.27×10^7	0.12×10^7	0.86×10^6
Σ^+ [cm]	3.42×10^7	1.00×10^7	5.84×10^{6}