ノックアウト反応で探る原子核クラスター構造
Nulcear cluster structure probed by knockout reaction

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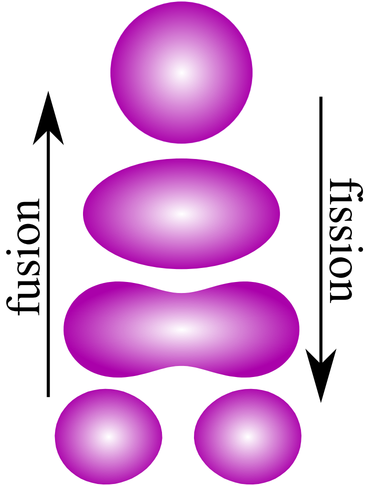
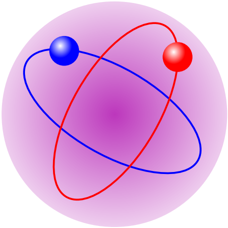
Japan Atomic Energy Agency

Various aspects of nucleus

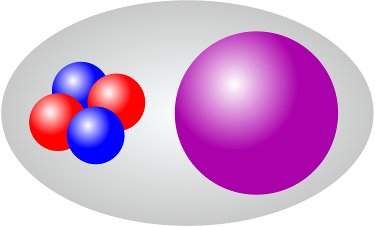
Liquid drop (fission/fusion)



single-particle orbital in the nuclear mean field

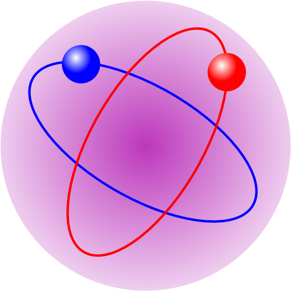


Cluster structure

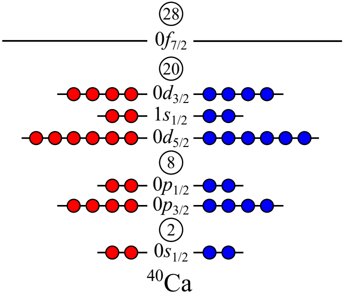


Shell structure and Magic number

Nucleon single-particle motion in the mean field

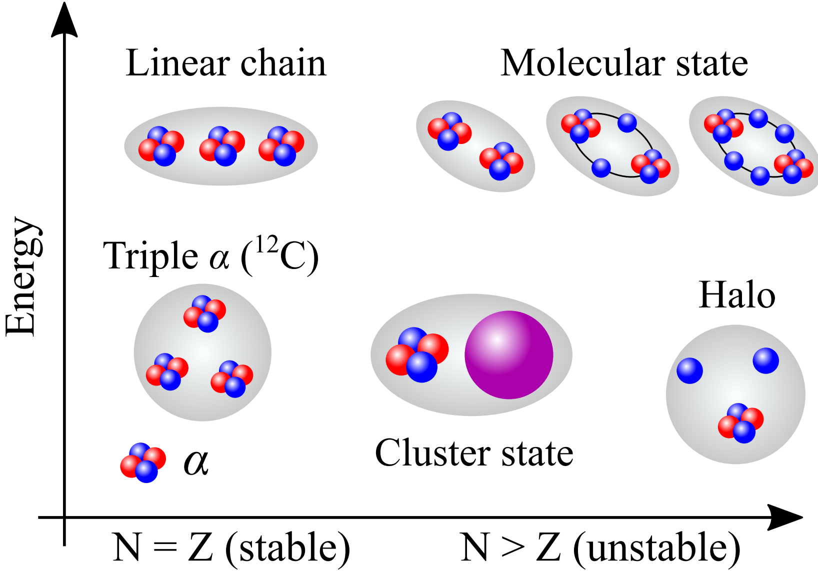


Shell orbit and magic numbers: ^{40}Ca case



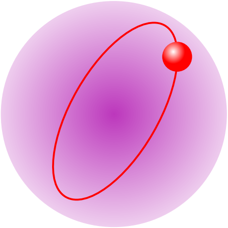
- Nucleons (protons and neutrons) occupy their single-particle orbits in the mean-filed potential
- Magic numbers (shell closure) 2, 8, 20, 28, 50, 82, 126 appear, similarly as in electron orbitals.

α-clustering of nuclei



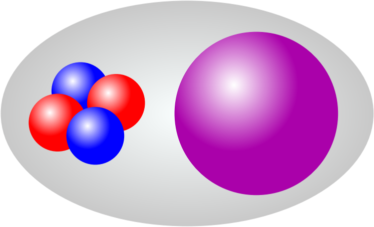
Few body aspect of nucleon many body system

Single-particle



- Independent nucleon picture
- Bound in a single-particle orbital
- Mean-filed potential made by all nucleons

Clusters



- Two or more subunits
- Nucleons are tightly bound to form a cluster
- Coupling between clusters are weak like a molecule

Note: These two states are not orthogonal, they overlap [1]

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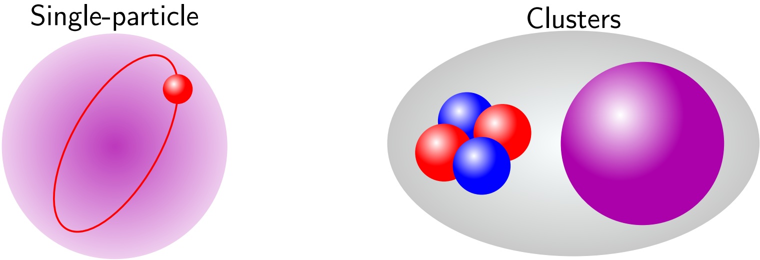
[1] B. Bayman and A. Bohr, Nuclear Physics **9**, 596 (1958).

Few body aspect of nucleon many body system



Single-particle and cluster orbitals are characterized by their radial (n), orbital ($\boldsymbol{\ell}$), spin ($\boldsymbol{s}$) and total $\boldsymbol{j} = \boldsymbol{\ell} + \boldsymbol{s}$ quantum numbers. Its amplitude

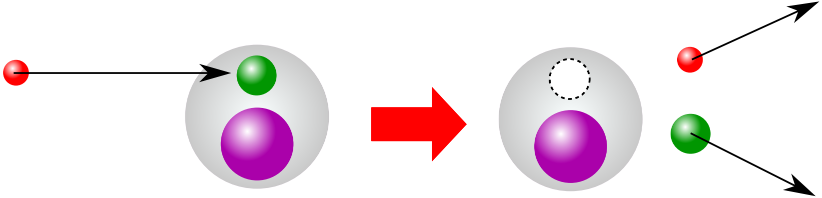
$$\varphi_{n\ell j}(R) = \langle \Phi_{\mathrm{B}} \,|\, \hat{a}_{n\ell j}(R) \,|\, \Phi_{\mathrm{A}} \rangle = \left\langle [\Phi_{\mathrm{B}} \otimes \phi(R)]_{nlj} \,\middle|\, \Phi_{\mathrm{A}} \right\rangle$$

and the spectroscopic factor

$$S_{n\ell j} = |\langle \Phi_{\mathrm{B}} \,|\, \hat{a}_{n\ell j} \,|\, \Phi_{\mathrm{A}} \rangle|^2$$

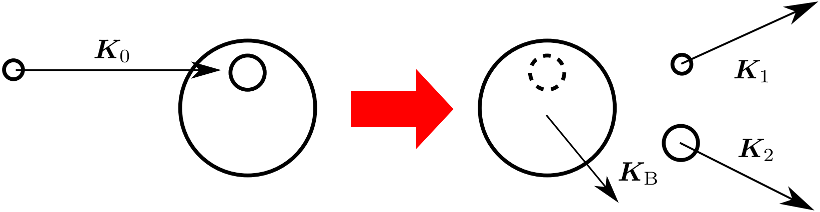
exhibit the few-body likeness of the system.

Particle knockout reaction



- One-step direct reaction with hundreds MeV incident energy
- Particle (nucleon or α) is knocked out by a impulse collision
- Reaction probability (cross section) is proportional to the particle probability
- Particle component only in the ground state of the target is probed
    - Little contribution from excited (resonance) states

Knockout cross section



$$d\sigma = \frac{(2\pi)^4}{\hbar v} d\boldsymbol{K}_1 d\boldsymbol{K}_2 d\boldsymbol{K}_\mathrm{B} \underbrace{\delta(\boldsymbol{K}_i - \boldsymbol{K}_f)\delta(E_i - E_f)}_{\text{conservation law}} \quad \underbrace{\times |T|^2}_{\text{Transition amp.}}$$

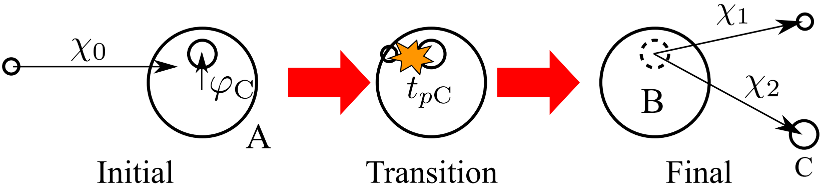
Knockout cross section (Triple differential cross section)

$$\frac{d\sigma}{dE_1 d\Omega_1 d\Omega_2} = \frac{(2\pi)^4}{\hbar v} F_\mathrm{kin} |T|^2$$

F_kin: Kinematical factor

$$F_\mathrm{kin} = \frac{E_1 K_1 E_2 K_2}{(\hbar c)^4} \left[1 + \frac{E_2}{E_\mathrm{B}} + \frac{E_2}{E_\mathrm{B}} \frac{(\boldsymbol{K}_1 - \boldsymbol{K}_0 - \boldsymbol{K}_\mathrm{A}) \cdot \boldsymbol{K}_2}{\boldsymbol{K}_2^2} \right]^{-1}$$

Reaction model: Distorted Wave Impulse Approximation



Transition matrix

$$T = \langle \chi_1 \chi_2 \Phi_{\rm C} \Phi_{\rm B} \,|\, t_{p\rm C} \,|\, \chi_0 \Phi_{\rm A} \rangle = \langle \chi_1 \chi_2 \,|\, t_{p\rm C} \,|\, \chi_0 \varphi_{\rm C} \rangle$$

χ_i: Distorted waves under optical potentials

$t_{p\rm C}$: p-C effective interaction in free space

$\varphi_{\rm C}$: Single-particle or cluster wave function $\langle [\Phi_{\rm C} \otimes \Phi_{\rm B}] \,|\, \Phi_{\rm A} \rangle$

Knockout cross section (Triple differential cross section)

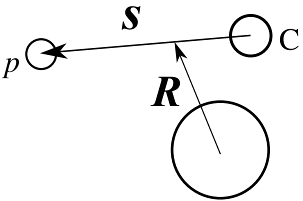
$$\frac{d^3\sigma}{dE_1 d\Omega_1 d\Omega_2} \propto |T|^2$$

Plane-wave limit (PWIA)

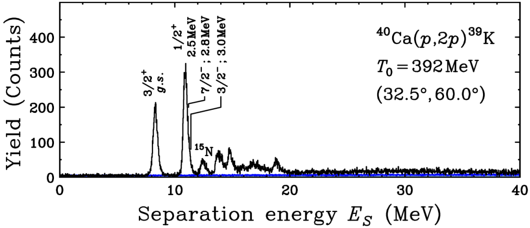
$$
\begin{aligned}
T &\approx \langle \chi_1\chi_2 \,|\, t_{p\mathrm{C}} \,|\, \chi_0\varphi_{\mathrm{C}} \rangle \\
&\xrightarrow[\text{(P.W.)}]{} \left\langle \boldsymbol{\kappa}' \,\middle|\, t_{p\mathrm{C}} \,\middle|\, \boldsymbol{\kappa} \right\rangle_{\boldsymbol{s}} \langle \boldsymbol{K}_1 + \boldsymbol{K}_2 - \boldsymbol{K}_0 \,|\, \varphi_{\mathrm{C}} \rangle_{\boldsymbol{R}} \\
&= \left\langle \boldsymbol{\kappa}' \,\middle|\, t_{p\mathrm{C}} \,\middle|\, \boldsymbol{\kappa} \right\rangle_{\boldsymbol{s}} \langle -\boldsymbol{K}_{\mathrm{B}} \,|\, \varphi_{\mathrm{C}} \rangle_{\boldsymbol{R}} \qquad (\boldsymbol{K}_{\mathrm{B}} = \boldsymbol{K}_0 - \boldsymbol{K}_1 - \boldsymbol{K}_2) \\
&= \underbrace{\left\langle \boldsymbol{\kappa}' \,\middle|\, t_{p\mathrm{C}} \,\middle|\, \boldsymbol{\kappa} \right\rangle_{\boldsymbol{s}}}_{p\text{-C collision}} \; \underbrace{\tilde{\varphi}_{\mathrm{C}}(\boldsymbol{k}_C)}_{\text{Structure}} \qquad (-\boldsymbol{K}_{\mathrm{B}} \approx \boldsymbol{k}_C)
\end{aligned}
$$

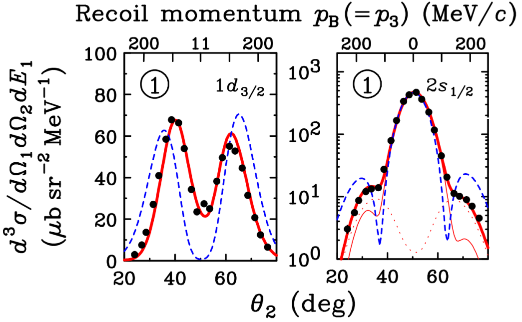
Knockout cross section

$$
|T|^2 \to \frac{d\sigma_{pC}}{d\Omega_{pC}} \, |\tilde{\varphi}_C(\boldsymbol{k}_C)|^2
$$



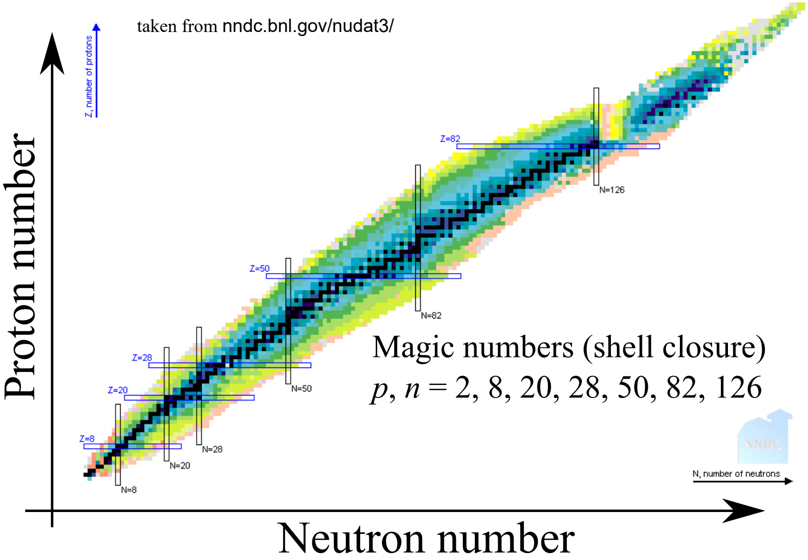
Single-particle orbital and shape of cross section





Figs.: T. Wakasa *et al.*, Progress in Particle and Nuclear Physics **96**, 32 (2017)

Nuclear shell and magic numbers



Magic numbers (shell closure)

$p, n = 2, 8, 20, 28, 50, 82, 126$

Nuclear shell and its evolution

Theoretical prediction: absence of the protons in the $1f_{7/2}$ orbit changes the neutron $1f_{5/2}$ orbit due to the lack of the tensor force

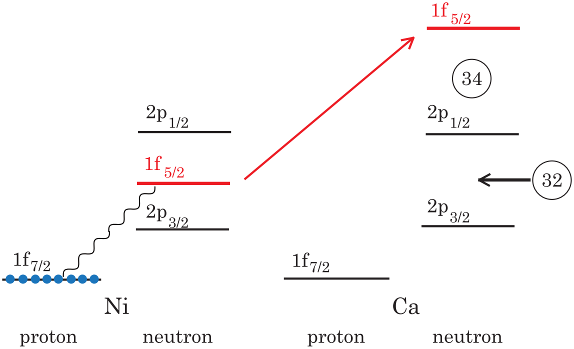
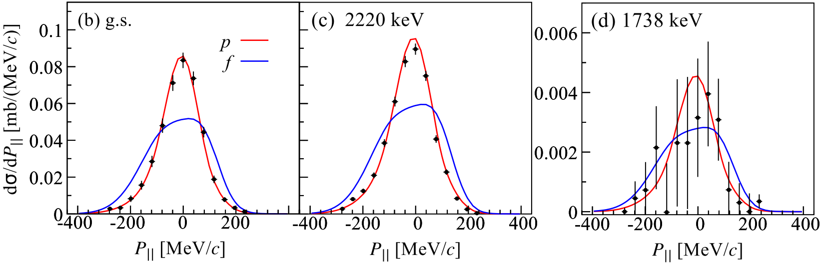


Figure from T. Otsuka and Y. Tsunoda
J. Phys. G: Nucl. Part. Phys. 43 024009 (2016).

Which orbit do neutrons occupy in ^{54}Ca ($Z = 20$, $N = 34$) system?

Shell Evolution And Search for Two-plus energies At RIBF

Neutron knockout reaction from ^{54}Ca



$P_{\parallel}$: Longitudinal (beam direction) component of the momentum $\boldsymbol{Q}$

- Neutron is knocked out mostly from the p-orbit
- Six neutrons occupy the p-orbit about $85\ \%$
- f-orbit is far above p-orbit, making the $N = 34$ shell gap

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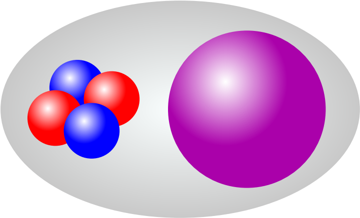
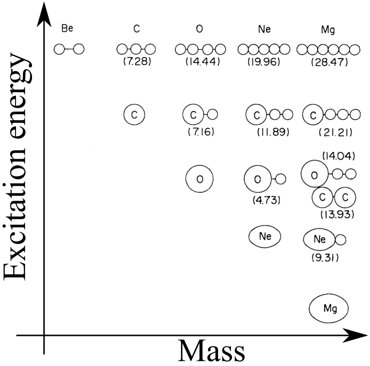
[3] S. Chen *et al.*, Phys. Rev. Lett. **123**, 142501 (2019).

α clustering in light nuclei



- Strong nucleon correlations makes a cluster
- Weak coupling between clusters
- What about the ground state?

Ikeda diagram [4]



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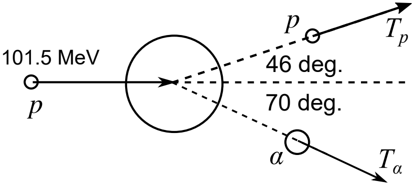
[4] K. Ikeda *et al.*, Prog. Theor. Phys. Supplement **E68**, 464 (1968).

Search for α clustering in the ground state

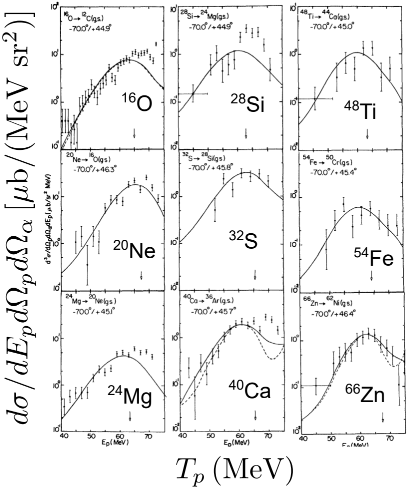
Systematic experiment of (p,$p\alpha$) reaction (Carey *et al.* 1984 [5])



Kinematical setup

Targets

- ^{16}O, ^{20}Ne, ^{24}Mg, ^{28}Si, ^{32}S, ^{40}Ca, ^{48}Ti, ^{54}Fe, ^{66}Zn



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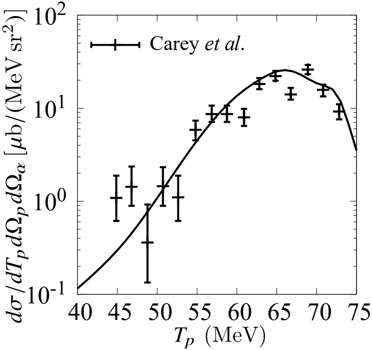
[5] T. A. Carey *et al.*, Phys. Rev. C **29**, 1273 (1984).

α+^{16}O cluster state in ^{20}Ne$_{g.s.}$

Carey *et al.* (1984) [5].

- α cluster wave function by a Woods-Saxon potential
- $S_\alpha = 0.54$ (exp. + reaction)
- $S_\alpha = 0.18 – 0.23$ (Structure theory [6–8])

Inconsistent by a factor of two



K. Yoshida *et al.* (2019) [9].

- DWIA + AMD wave function [10]
- $S_\alpha = 0.26$ (Consistent)

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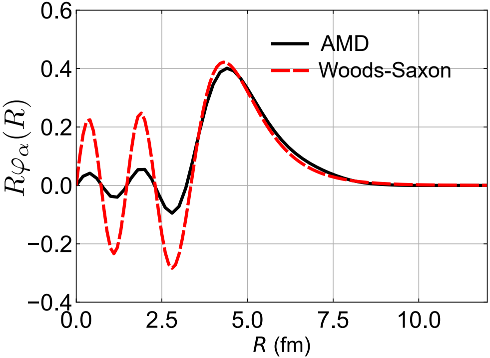
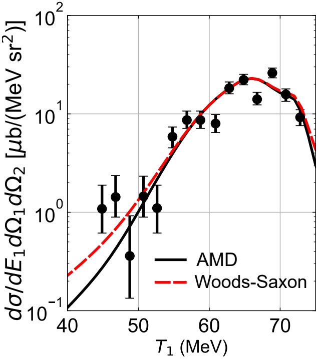
[5] T. A. Carey *et al.*, Phys. Rev. C **29**, 1273 (1984). [6] W. Chung *et al.*, Physics Letters B **79**, 381 (1978). [7] J. Draayer, Nuclear Physics A **237**, 157 (1975). [8] K. Hecht and D. Braunschweig, Nuclear Physics A **244**, 365 (1975). [9] K. Yoshida *et al.*, Phys. Rev. C **100**, 044601 (2019). [10] Y. Chiba and M. Kimura, Prog. Theor. Exp. Phys. **2017**, 053D01 (2017).

Peripherality of reaction and surface α amplitude

Cluster wave function



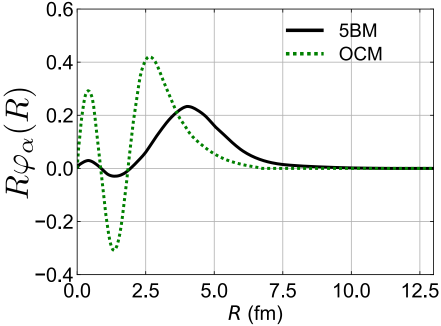
Knockout cross section



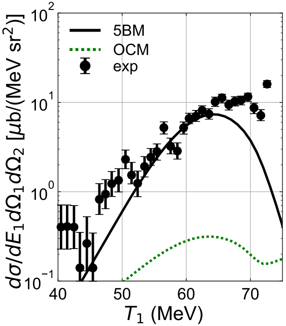
- Pauli principle is taken into account within the Antisymmetrized Molecular Dynamics (AMD) framework
- Both wave functions agree on the surface
- Knockout cross section is determined by the surface α amplitude, not the whole region (S-factor).

^{16}O(p,$p\alpha$)^{12}C reaction

Cluster wave function



Knockout cross section



- The 5-body model [11] ($S_\alpha = 0.105$) amplitude reproduces the data. Significant difference between the 5BM and OCM [12, 13] ($S_\alpha = 0.29$)
- Consistent with the α-transfer reaction analysis [14]

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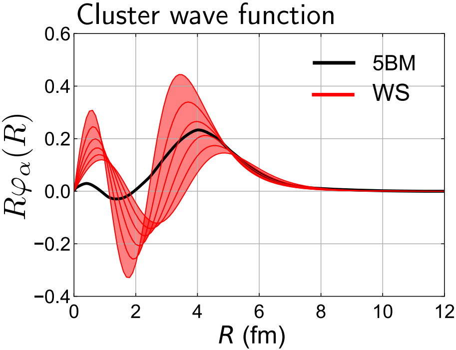
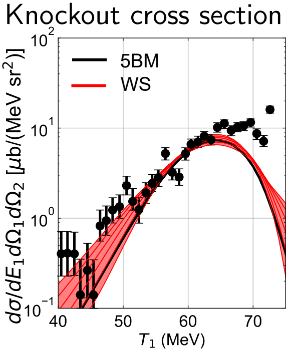
[11] W. Horiuchi and Y. Suzuki, Phys. Rev. C **89**, 011304(R) (2014). [12] Y. Suzuki, Prog. Theor. Phys. **55**, 1751 (1976). [13] Y. Suzuki, Prog. Theor. Phys. **56**, 111 (1976). [14] T. Fukui *et al.*, Nuclear Physics A **983**, 38 (2019).

What do we really know from reaction observables?

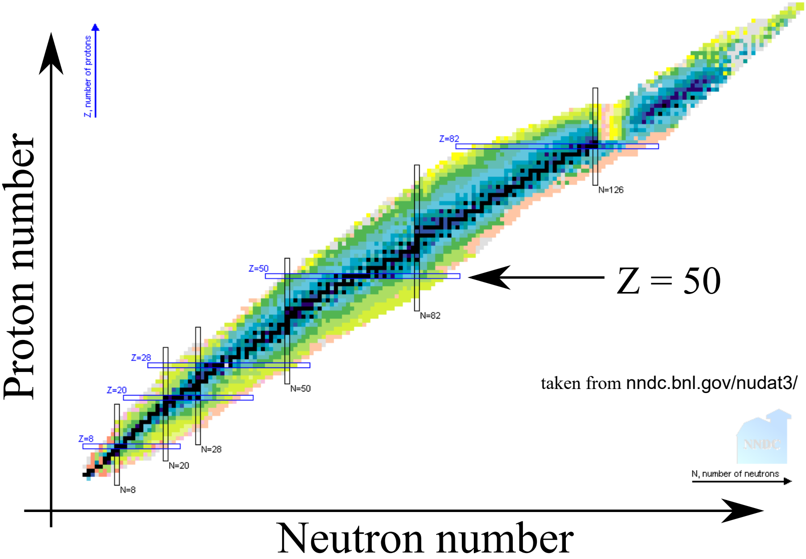
A test using phenomenological cluster w.fn. (Woods-Saxon pot.)





- Cross section put a strong constraint on the surface amplitude
- This is because of the absorption effect of the reaction
  - Short mean free path of α
  - Loss of probability flux to the Q-space of the Feshbach projection

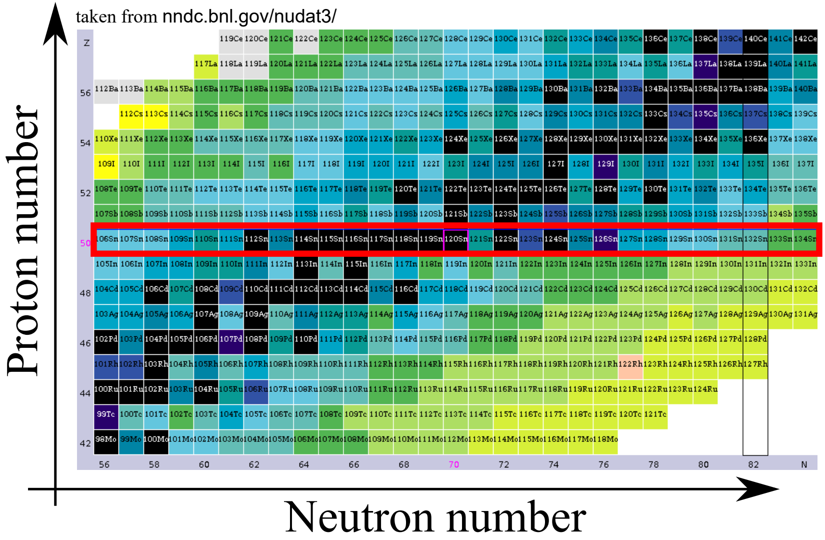
α particle on Sn surface



taken from nndc.bnl.gov/nudat3/

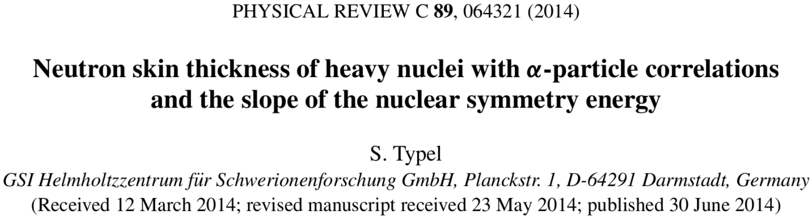
α particle on Sn surface

taken from nndc.bnl.gov/nudat3/



Z						119Ce	120Ce	121Ce	122Ce	123Ce	124Ce	125Ce	126Ce	127Ce	128Ce	129Ce	130Ce	131Ce	132Ce	133Ce	134Ce	135Ce	136Ce	137Ce	138Ce	139Ce	140Ce	141Ce	142Ce
					117La	118La	119La	120La	121La	122La	123La	124La	125La	126La	127La	128La	129La	130La	131La	132La	133La	134La	135La	136La	137La	138La	139La	140La	141La
56	112Ba	113Ba	114Ba	115Ba	116Ba	117Ba	118Ba	119Ba	120Ba	121Ba	122Ba	123Ba	124Ba	125Ba	126Ba	127Ba	128Ba	129Ba	130Ba	131Ba	132Ba	133Ba	134Ba	135Ba	136Ba	137Ba	138Ba	139Ba	140Ba
		112Cs	113Cs	114Cs	115Cs	116Cs	117Cs	118Cs	119Cs	120Cs	121Cs	122Cs	123Cs	124Cs	125Cs	126Cs	127Cs	128Cs	129Cs	130Cs	131Cs	132Cs	133Cs	134Cs	135Cs	136Cs	137Cs	138Cs	139Cs
54	110Xe	111Xe	112Xe	113Xe	114Xe	115Xe	116Xe	117Xe	118Xe	119Xe	120Xe	121Xe	122Xe	123Xe	124Xe	125Xe	126Xe	127Xe	128Xe	129Xe	130Xe	131Xe	132Xe	133Xe	134Xe	135Xe	136Xe	137Xe	138Xe
	109I	110I	111I	112I	113I	114I	115I	116I	117I	118I	119I	120I	121I	122I	123I	124I	125I	126I	127I	128I	129I	130I	131I	132I	133I	134I	135I	136I	137I
52	108Te	109Te	110Te	111Te	112Te	113Te	114Te	115Te	116Te	117Te	118Te	119Te	120Te	121Te	122Te	123Te	124Te	125Te	126Te	127Te	128Te	129Te	130Te	131Te	132Te	133Te	134Te	135Te	136Te
	107Sb	108Sb	109Sb	110Sb	111Sb	112Sb	113Sb	114Sb	115Sb	116Sb	117Sb	118Sb	119Sb	120Sb	121Sb	122Sb	123Sb	124Sb	125Sb	126Sb	127Sb	128Sb	129Sb	130Sb	131Sb	132Sb	133Sb	134Sb	135Sb
50	106Sn	107Sn	108Sn	109Sn	110Sn	111Sn	112Sn	113Sn	114Sn	115Sn	116Sn	117Sn	118Sn	119Sn	120Sn	121Sn	122Sn	123Sn	124Sn	125Sn	126Sn	127Sn	128Sn	129Sn	130Sn	131Sn	132Sn	133Sn	134Sn
	105In	106In	107In	108In	109In	110In	111In	112In	113In	114In	115In	116In	117In	118In	119In	120In	121In	122In	123In	124In	125In	126In	127In	128In	129In	130In	131In	132In	133In
48	104Cd	105Cd	106Cd	107Cd	108Cd	109Cd	110Cd	111Cd	112Cd	113Cd	114Cd	115Cd	116Cd	117Cd	118Cd	119Cd	120Cd	121Cd	122Cd	123Cd	124Cd	125Cd	126Cd	127Cd	128Cd	129Cd	130Cd	131Cd	132Cd
	103Ag	104Ag	105Ag	106Ag	107Ag	108Ag	109Ag	110Ag	111Ag	112Ag	113Ag	114Ag	115Ag	116Ag	117Ag	118Ag	119Ag	120Ag	121Ag	122Ag	123Ag	124Ag	125Ag	126Ag	127Ag	128Ag	129Ag	130Ag	131Ag
46	102Pd	103Pd	104Pd	105Pd	106Pd	107Pd	108Pd	109Pd	110Pd	111Pd	112Pd	113Pd	114Pd	115Pd	116Pd	117Pd	118Pd	119Pd	120Pd	121Pd	122Pd	123Pd	124Pd	125Pd	126Pd	127Pd	128Pd		
	101Rh	102Rh	103Rh	104Rh	105Rh	106Rh	107Rh	108Rh	109Rh	110Rh	111Rh	112Rh	113Rh	114Rh	115Rh	116Rh	117Rh	118Rh	119Rh	120Rh	121Rh	122Rh	123Rh	124Rh	125Rh	126Rh	127Rh		
44	100Ru	101Ru	102Ru	103Ru	104Ru	105Ru	106Ru	107Ru	108Ru	109Ru	110Ru	111Ru	112Ru	113Ru	114Ru	115Ru	116Ru	117Ru	118Ru	119Ru	120Ru	121Ru	122Ru	123Ru	124Ru				
	99Tc	100Tc	101Tc	102Tc	103Tc	104Tc	105Tc	106Tc	107Tc	108Tc	109Tc	110Tc	111Tc	112Tc	113Tc	114Tc	115Tc	116Tc	117Tc	118Tc	119Tc	120Tc	121Tc						
42	98Mo	99Mo	100Mo	101Mo	102Mo	103Mo	104Mo	105Mo	106Mo	107Mo	108Mo	109Mo	110Mo	111Mo	112Mo	113Mo	114Mo	115Mo	116Mo	117Mo	118Mo								
	56		58		60		62		64		66		68		70		72		74		76		78		80		82		N

α particle on Sn surface



PHYSICAL REVIEW C **89**, 064321 (2014)

Neutron skin thickness of heavy nuclei with α-particle correlations and the slope of the nuclear symmetry energy

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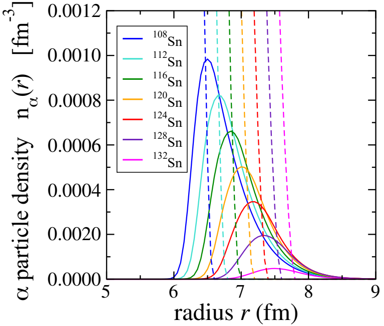
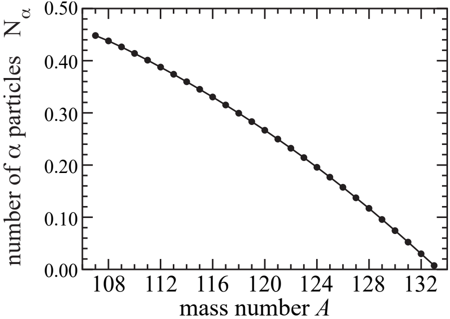
(Received 12 March 2014; revised manuscript received 23 May 2014; published 30 June 2014)

α particle on Sn surface

α particle density

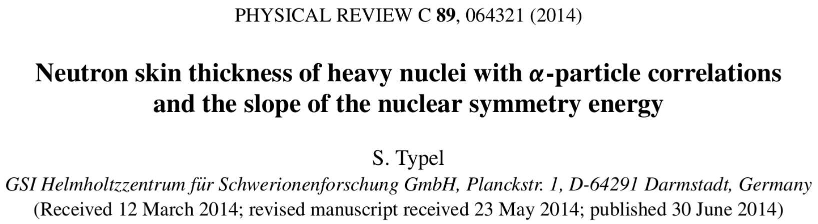


α particle number



- A theoretical prediction of the α density distribution on Sn isotopes by the generalized relativistic mean-field model with explicit α degrees of freedom.
- The α density decreases as neutron number increases.

α particle on Sn surface



PHYSICAL REVIEW C **89**, 064321 (2014)

Neutron skin thickness of heavy nuclei with α-particle correlations and the slope of the nuclear symmetry energy

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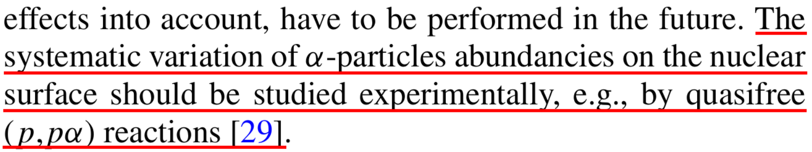
In the end of Summary

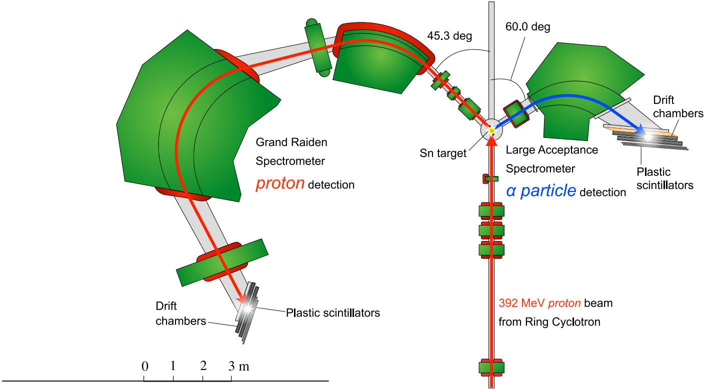


effects into account, have to be performed in the future. The systematic variation of α-particles abundancies on the nuclear surface should be studied experimentally, e.g., by quasifree $(p,p\alpha)$ reactions [29].

α-knockout experiment from Sn isotopes

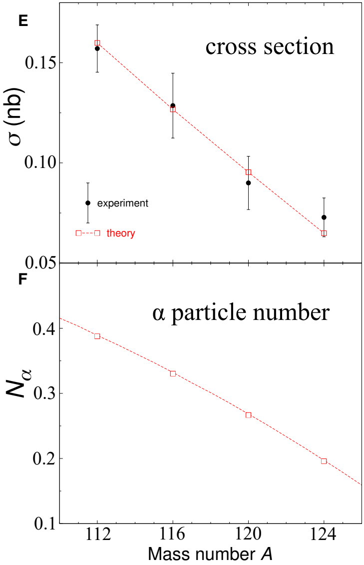
Formation of α clusters in dilute neutron-rich matter

Junki Tanaka[1,2,3]*, Zaihong Yang[3,4]*, Stefan Typel[1,2], Satoshi Adachi[4], Shiwei Bai[5], Patrik van Beek[1], Didier Beaumel[6], Yuki Fujikawa[7], Jiaxing Han[5], Sebastian Heil[1], Siwei Huang[5], Azusa Inoue[4], Ying Jiang[5], Marco Knösel[1], Nobuyuki Kobayashi[4], Yuki Kubota[3], Wei Liu[5], Jianling Lou[5], Yukie Maeda[8], Yohei Matsuda[9], Kenjiro Miki[10], Shoken Nakamura[4], Kazuyuki Ogata[4,11], Valerii Panin[3], Heiko Scheit[1], Fabia Schindler[1], Philipp Schrock[12], Dmytro Symochko[1], Atsushi Tamii[4], Tomohiro Uesaka[3], Vadim Wagner[1], Kazuki Yoshida[13], Juzo Zenihiro[3,7], Thomas Aumann[1,2,14]



[15] J. Tanaka *et al.*, Science **371**, 260 (2021).

α-knockout experiment from Sn isotopes



- The experiment was performed at RCNP, Osaka Univ.
- Theoretical α-knockout cross sections are obtained by introducing a scaling factor of 0.148 for the imaginary potential depth (for all isotopes).
- An isotopic trend of the cross sections is well reproduced by the theoretical calculation using predicted α-particle density distribution.

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[15] J. Tanaka *et al.*, Science **371**, 260 (2021).

Heavy and unstable: α formation on decaying nuclei

α-decay lifetime and its width (independent of channel radius R)

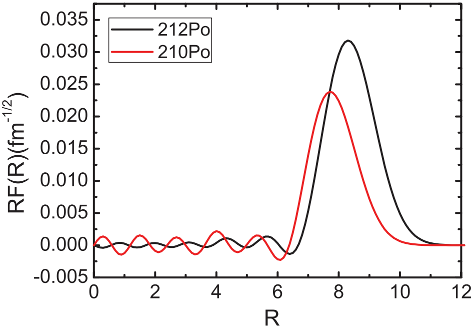
$$T_{1/2} = \frac{\hbar \ln 2}{\Gamma_l},$$

$$\Gamma = 2 \underbrace{\frac{kR}{F^2(kR) + G^2(kR)}}_{\text{Penetrability}} \underbrace{\frac{\hbar^2}{2\mu R} \left|RF(R)\right|^2}_{\text{reduced width}}$$

$$= 2\, P(R)\; \gamma^2(R)$$

Heavy and unstable: α formation on decaying nuclei

$\alpha+^{206,208}$Pb amplitude of ^{210,212}Po by C. Qi [16]



$$F(R;^{212}\mathrm{Po}) = \int d\boldsymbol{R}d\xi_\alpha\phi_\alpha(\xi_\alpha)\Psi(\boldsymbol{r}_1,\boldsymbol{r}_2;^{210}\mathrm{Pb})\Psi(\boldsymbol{r}_3,\boldsymbol{r}_4;^{210}\mathrm{Po})$$

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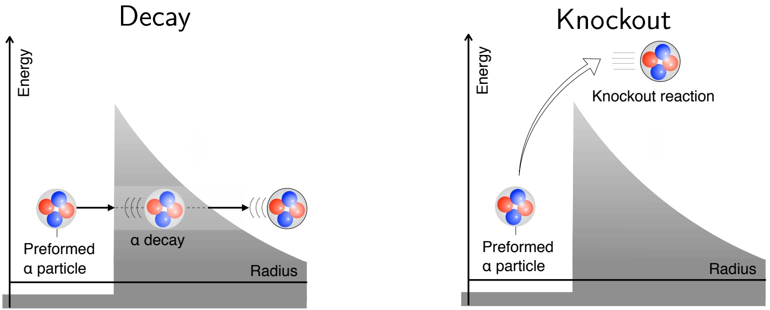
[16] C. Qi *et al.*, Phys. Rev. C **81**, 064319 (2010).

α knockout reaction from decaying nuclei

K. Yoshida and J. Tanaka, Phys. Rev. C **106**, 014621 (2022)



Decay

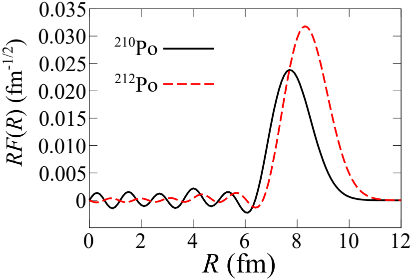
Knockout

Knockout an α before it penetrates the barrier

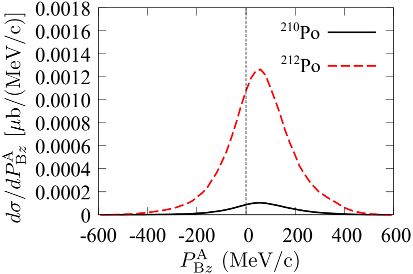
- Timescale: $T_{\text{decay}} \sim 0.3\ \mu s$ (^{212}Po), $T_{\text{knockout}} \sim 10^{-22}\ s$.
- Free from the penetration process, direct access to α amplitude
- Clean probe for the α component in the g.s.

α knockout from ^{210,212}Po case

α amplitude from decay study [16]



momentum distribution



Recoil momentum of the residue

	S-factor	peak height	Cross section	$\lvert RF(R)\rvert^2$
^{212}Po / ^{210}Po	1.92	12.1	11.9	10.2

Difference is magnified by the peripherality of the reaction

→ sensitive probe for preformed α particle on the surface

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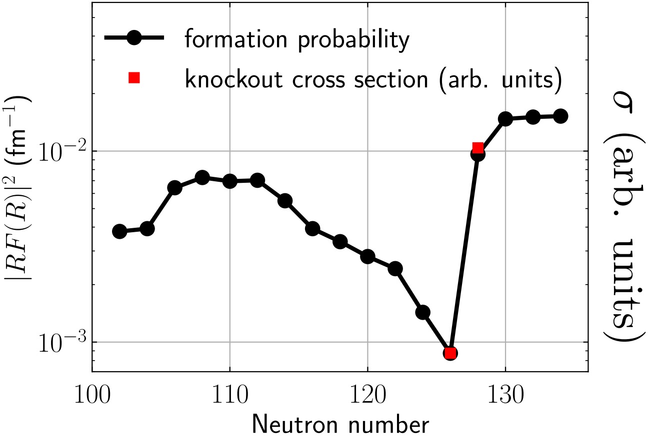
[16] C. Qi *et al.*, Phys. Rev. C **81**, 064319 (2010).

α knockout from ^{210,212}Po case



Data: A. N. Andreyev *et al.*, Phys. Rev. Lett. **110**, 242502 (2013)

Difference is magnified by the peripherality of the reaction
→ sensitive probe for preformed α particle on the surface

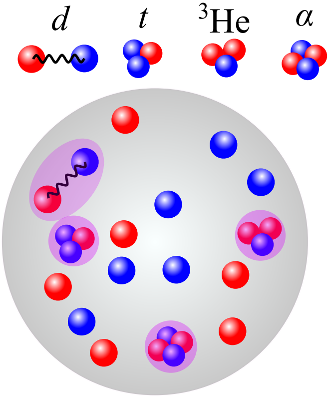
Beyond the α correlation: Onokoro project

ONOKORO

πάντα ῥεῖ

Led by Uesaka-san, FY2021–2025

- Cluster knockout experiments to investigate
  - Non-uniformity in nuclear mean-field and nuclear matter (clustering)
  - d, t, ^{3}He cluster formation and (local) spin symmetry breaking
  - Clustering in the sparse nuclear matter and the nuclear surface



Summary

- Knockout reaction is a powerful probe for both the nucleon single-particle state and the α cluster state of a nucleus
- Recoil momentum distribution of the knockout cross section tells us the orbital angular momentum of the knocked out particle
- α knockout reaction is sensitive to the surface α amplitude because of the strong absorption (short mean free path) of the α particle
- If the time scale of the reaction is short enough compared to the decay lifetime, the knockout reaction may be a probe for a resonant state (this is true in the case of α decay, but. . . ?)