Spectrum of the fully-charmed tetraquark state

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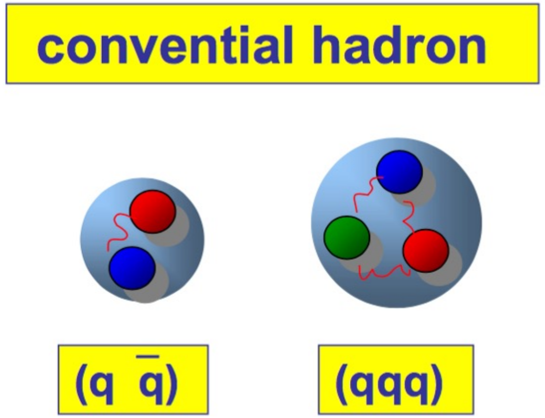
Based on Phys. Rev. D. 100, 096013, Phys. Rev. D 104, 036016, Phys. Rev. D 106, 096005

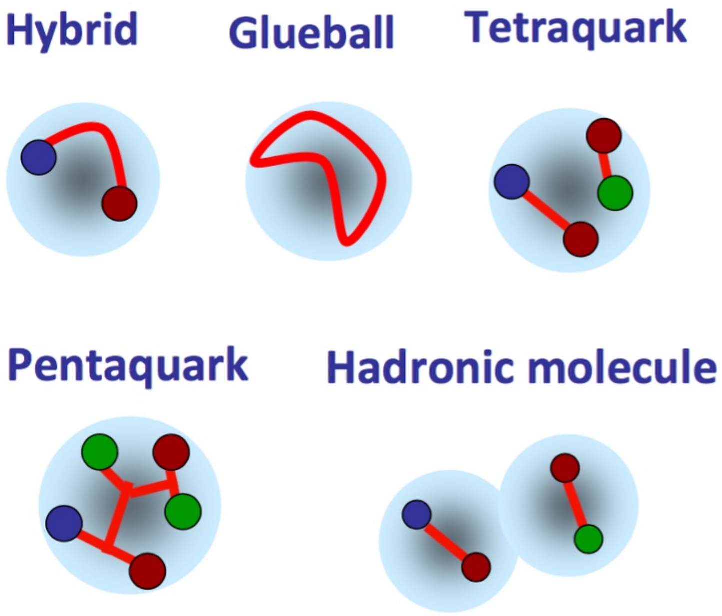
.

QM VS QCD

- Classical Quark model (QM):



- The quantum chromodynamics (QCD):

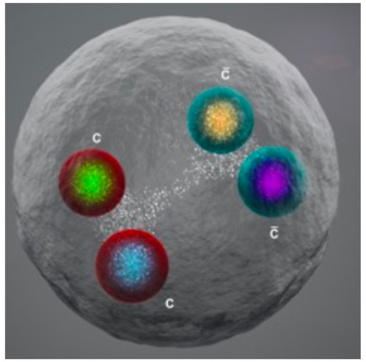


More complicated hadron structures.

$D_{s0}^{*}(2317)$ &X(3872)@2003,…, P_c @2019, X(6900)@2020, T_{cc}^{+}@2021

Fully-heavy tetraquark

- The fully heavy tetraquark state $T_{Q_1Q_2\bar{Q}_3\bar{Q}_4}(Q=c,b)$ is a good candidate for a **compact** tetraquark state.



- Theoretical works started in1970s. (*More details are referred to Prog.Part.Nucl.Phys. 107 (2019) 237-320.*)
  PRL 36 (1976) 1266, Z.Phys.C7 (1981) 317, PRD 25 (1982) 2370

✓ ***The tension in the existence*** of the stable (bound) fully heavy tetraquark state:

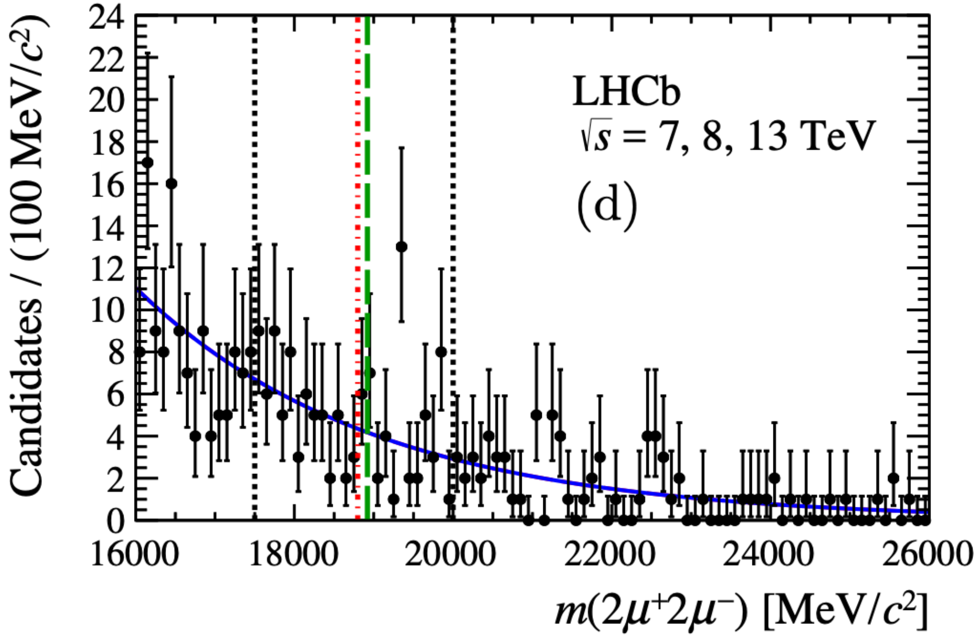
✦ Stable $QQ\bar{Q}\bar{Q}$ states exist: $bb\bar{b}\bar{b}\sim 18-20$ GeV, $cc\bar{c}\bar{c}\sim 5-7$ GeV (compared with theoretical di- $Q\bar{Q}$ channels)
  arXiv:1612.00012, Eur. Phys. J. C 78, 647, EPJ Web Conf. 182, 02028,
  Phys. Lett. B 718, 545, Phys. Rev. D 70, 014009 ...

✦ Negative: no stable $QQ\bar{Q}\bar{Q}$ states exist.
  Phys. Rev. D 97, 094015, Phys. Rev. D.97.054505, Phys. Rev. D. 100, 096013, ...

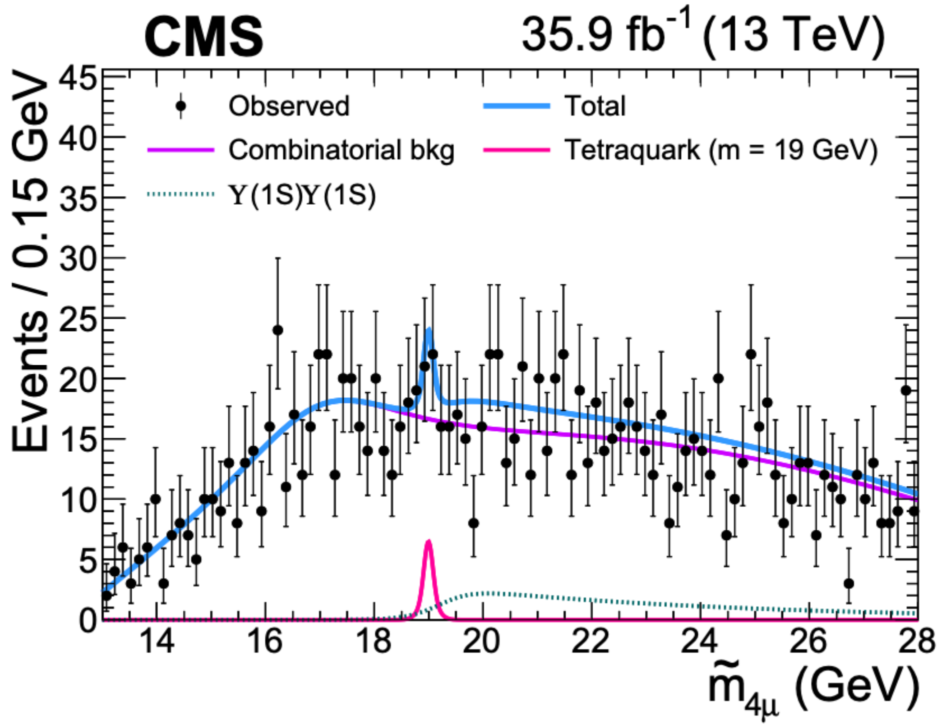
✓ ***Existence*** of the the ***resonant*** $T_{Q_1Q_2\bar{Q}_3\bar{Q}_4}$ and ***the mass spectrum.***

Experimental search for $T_{QQ\bar{Q}\bar{Q}}$

- ***No significant excess observed for $T_{bb\bar{b}\bar{b}}$.***



LHCb, JHEP 1810, 086 (2018).



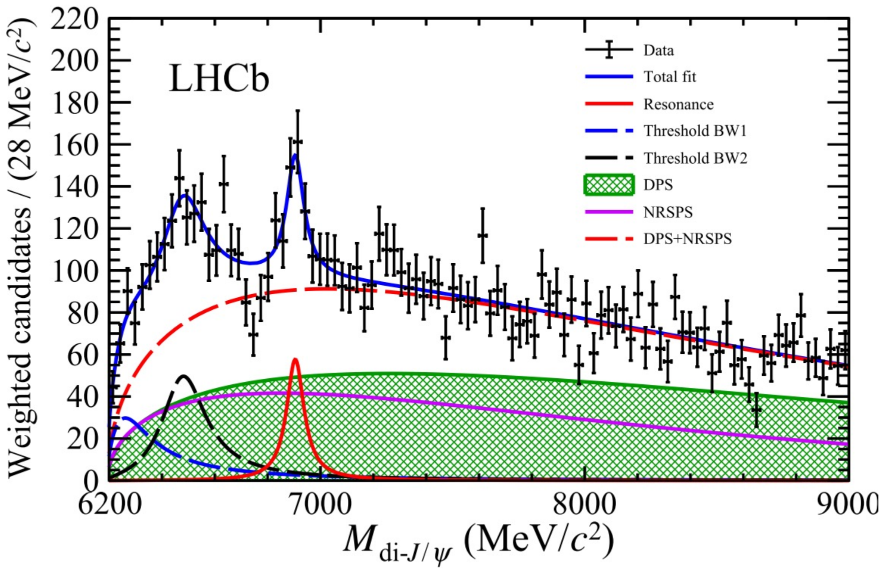
CMS, arXiv: 2002.06393

Experimental search for $T_{cc\bar{c}\bar{c}}$

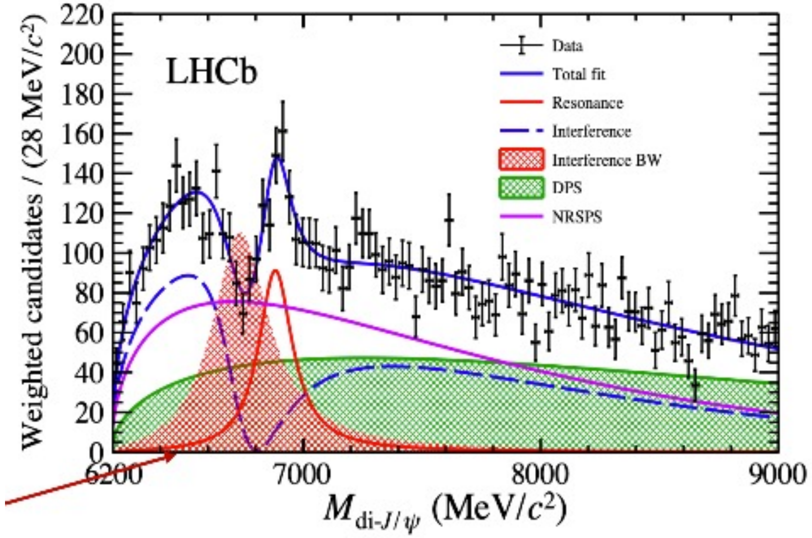
- Observation of $T_{cc\bar{c}\bar{c}}$ structure in di-J/ψ channel:

LHCb, Science Bulletin 65 (2020) 198 3.

Modeling I



Modeling II



- *A* broad structure ranging $(6.2, 6.8)$ GeV.

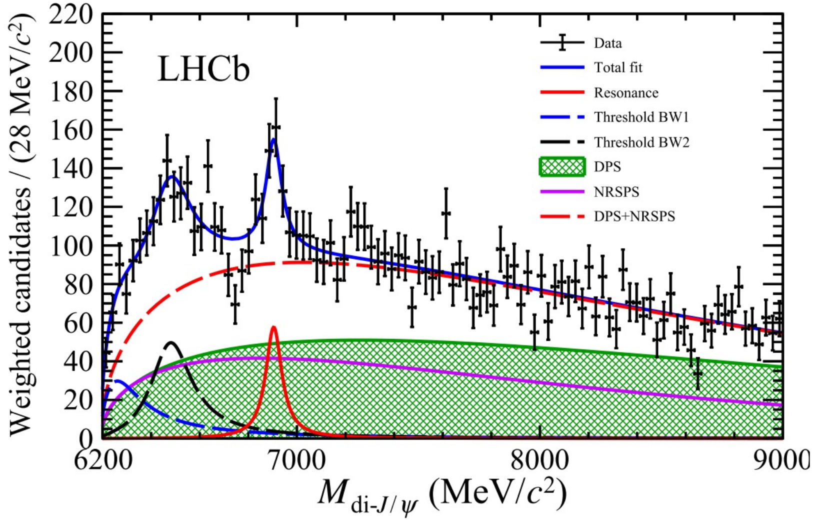
- Narrow structure $X(6900)$:

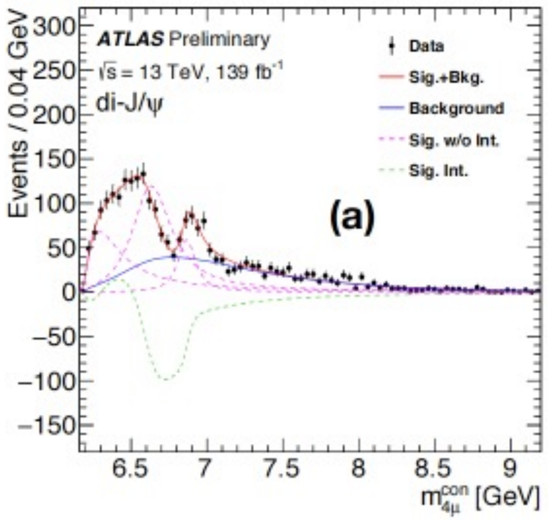
  $m = 6905 \pm 11$ MeV/c^2, $\Gamma = 80 \pm 19$ MeV or $m = 6886 \pm 11$ MeV/c^2, $\Gamma = 168 \pm 33$ MeV.

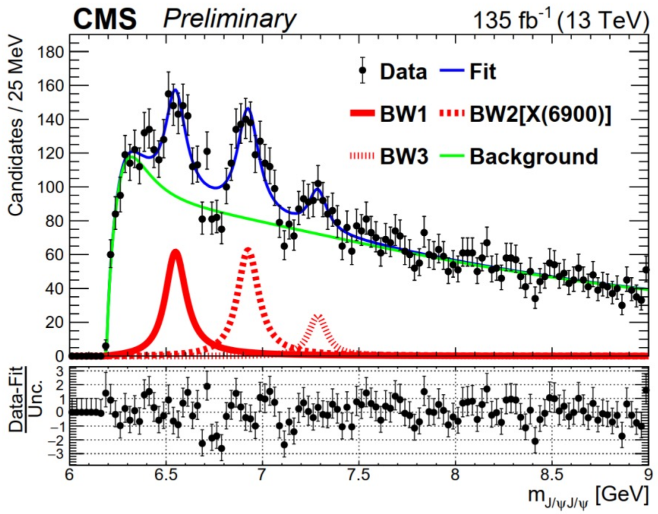
- Hint for $X(7200)$ but not important.

Experimental search for $T_{cc\bar{c}\bar{c}}$

• Observation of structure $T_{cc\bar{c}\bar{c}}$ in di-J/ψ channel





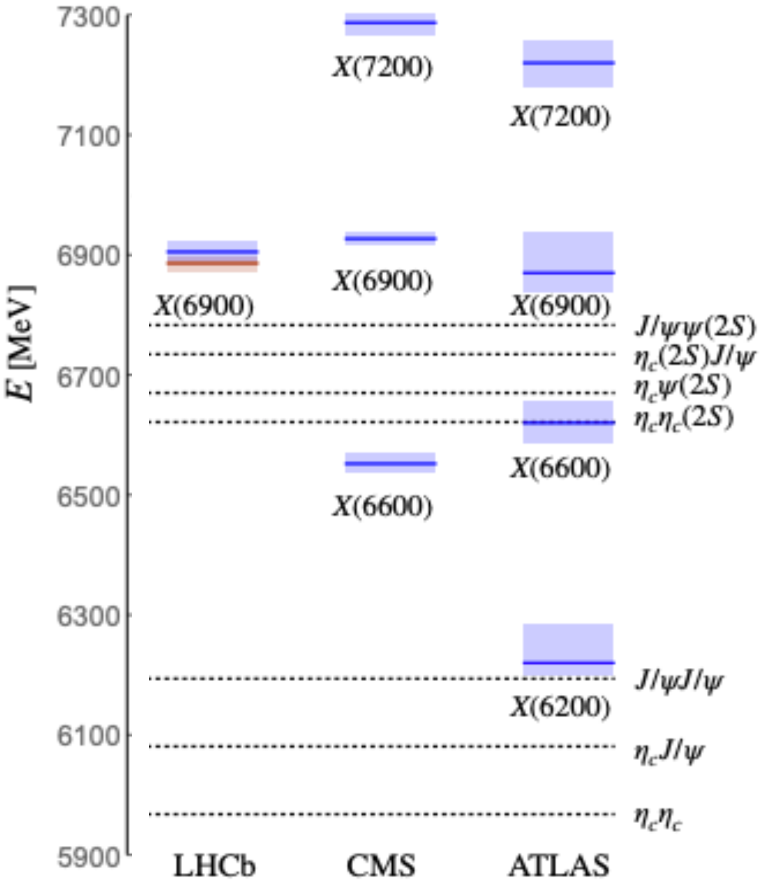


LHCb, Science Bulletin 65 (2020) 198 3.

K. Y. on behalf of the CMS Collaboration, https: //agenda.infn.it/event/28874/contributions/ 170300/.

E. B.-T. on behalf of the ATLAS Collaboration, https://agenda.infn.it/event/28874/ contributions/170298/.

J/ψ-J/ψ resonances observed in experiments



Theoretical interpretations

- The predicted ground S-wave $T_{cc\bar{c}\bar{c}}$: $(6.3, 6.5)$ GeV.

- X(6900): Radial & P-wave excitation?

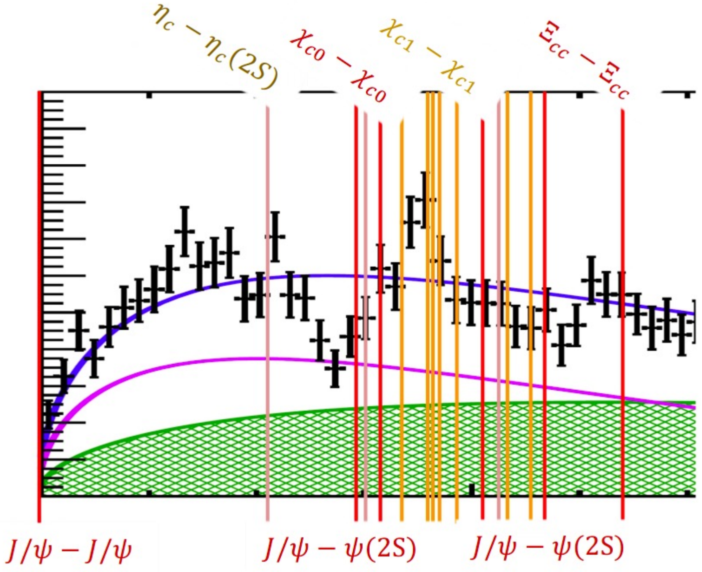
Phys. Rev. D 104, 116029 (2021).
arXiv:2207.07537 [hep-ph].
Phys. Rev. D 105, 014006 (2022).
Phys. Rev. D 104, 036016 (2021).
Phys. Rev. D 104, 014020 (2021).
arXiv:2104.08814 [hep-ph].
….

	J^{PC}	$M^1_{\rm th}$	$M^2_{\rm th}$	[43]	[44]	[47]	[34]	[33]	[41]	[49]	[37,57]
$cc\bar{c}\bar{c}$	0^{++}	6.377	6.371	5.966	6.192 ± 0.025	6.001	···	···	6.038	6.470	6.44 ± 0.15
		6.425	6.483							6.558	
	1^{+-}	6.425	6.450	6.051	···	6.109	···	···	6.101	6.512	6.37 ± 0.18
	2^{++}	6.432	6.479	6.223	···	6.166	···	···	6.172	6.534	6.37 ± 0.19
$bb\bar{b}\bar{b}$	0^{++}	19.215	19.243	18.754	18.826 ± 0.025	18.815	18.72 ± 0.02	18.69 ± 0.03	···	19.268	18.45 ± 0.15
		19.247	19.305							19.305	
	1^{+-}	19.247	19.311	18.808	···	18.874	···	···	···	19.285	18.32 ± 0.17
	2^{++}	19.249	19.325	18.916	···	18.905	···	···	···	19.295	18.32 ± 0.17
$bb\bar{c}\bar{c}(cc\bar{b}\bar{b})$	0^{++}	12.847	12.886	···	···	12.571	···	···	···	12.935	···
		12.866	12.946							13.023	
	1^{+-}	12.864	12.924	···	···	12.638	···	···	···	12.945	···
	2^{++}	12.868	12.940	···	···	12.673	···	···	···	12.956	···

- The dynamical rescattering mechanism of double-charmonium.

Phys. Rev. D 103, 034024 (2021).
Phys. Rev. Lett. 126, 132001 (2021) .
arXiv:2011.11374 [hep-ph].
Phys. Rev. D 103, 071503 (2021) .
Sci. Bull. 66, 2462 (2021).
arXiv:2206.13867 [hep-ph].
…

From Zhang's talk



Fully-heavy tetraquark

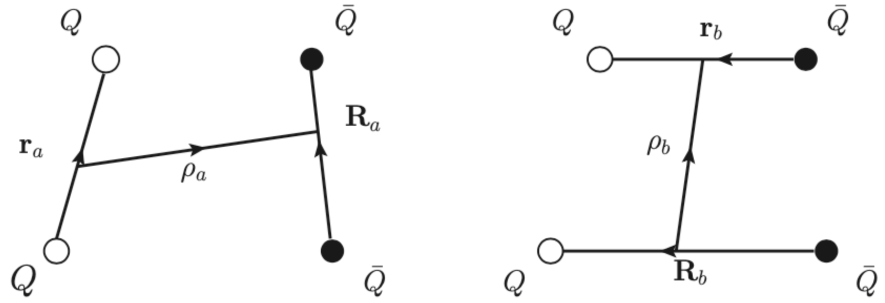
- Four body system: ***two color singlet states***

$$\mathbf{3}\otimes\mathbf{3}\otimes\overline{\mathbf{3}}\otimes\overline{\mathbf{3}} = 2\times\mathbf{1}\oplus 4\times\mathbf{8}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$$

✓ Diquark-antidiquark: (QQ)-$(\bar{Q}\bar{Q})$:

$\bar{3}_c \otimes 3_c = 1_c$ and $6_c \otimes \bar{6}_c = 1_c$.

✓ Meson-Meson: $(Q\bar{Q})$-$(Q\bar{Q})$



$$|\mathbf{1}\rangle \equiv |(Q_1\bar{Q}_3)_{\mathbf{1}}(Q_2\bar{Q}_4)_{\mathbf{1}}\rangle,$$
$$|\mathbf{8}\rangle \equiv |(Q_1\bar{Q}_3)_{\mathbf{8}}(Q_2\bar{Q}_4)_{\mathbf{8}}\rangle,$$

Or

$$|\mathbf{1}\rangle \equiv |(Q_1\bar{Q}_3)_{\mathbf{1}}(Q_2\bar{Q}_4)_{\mathbf{1}}\rangle,$$
$$|\mathbf{1}'\rangle \equiv |(Q_1\bar{Q}_4)_{\mathbf{1}}(Q_2\bar{Q}_3)_{\mathbf{1}}\rangle,$$

Quark-(anti)quark interaction

- Hamiltonian:

$$H = H_0 + \sum_i \sum_j \frac{\lambda_i}{2} \cdot \frac{\lambda_j}{2} \left[V_{\text{cen}}^{(0)}(r_{ij}) + V_{\text{so}}^{(1)}(r_{ij}) + V_{\text{tens}}^{(1)}(r_{ij}) \right]$$

$$H_0 = \sum_{i=1}^{4} \frac{\mathbf{p}_i^2}{2m_i} + \sum_i m_i - T_G.$$

- $V_{cen}^{(0)}$: Color Coulomb + linear confinement + hyperfine

$$V_{\text{cen}}^{(0)}(r_{ij}) = \frac{\alpha_s}{r_{ij}} - \frac{3}{4} b r_{ij} - \frac{8\pi\alpha_s}{3m_i m_j} \left(\frac{\sigma}{\sqrt{\pi}} \right)^3 e^{-\sigma^2 r_{ij}^2} \mathbf{s}_i \cdot \mathbf{s}_j.$$

Phys. Rev. D 72 (2005) 054026

- $V_{so}^{(1)} + V_{tens}^{(1)}$: spin-orbital and tensor interactions.

$$V_{\text{so}}^{(1)}(r_{ij}) = V_{\text{so}}^{v}(r_{ij}) + V_{\text{so}}^{s}(r_{ij}).$$

$$V_{\text{so}}^{v}(r_{ij}) = \frac{1}{r_{ij}} \frac{dV_{\text{Coul}}}{dr_{ij}} \frac{1}{4} \left[\left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4}{m_i m_j} \right) \mathbf{L}_{ij} \cdot \mathbf{S}_{ij} + \left(\frac{1}{m_i^2} - \frac{1}{m_j^2} \right) \mathbf{L}_{ij} \cdot (\mathbf{s}_i - \mathbf{s}_j) \right]$$

$$V_{\text{so}}^{s}(r_{ij}) = -\frac{1}{r_{ij}} \frac{dV_{\text{lin}}}{dr_{ij}} \left(\frac{\mathbf{L}_{ij} \cdot \mathbf{s}_i}{2m_i^2} + \frac{\mathbf{L}_{ij} \cdot \mathbf{s}_j}{2m_j^2} \right)$$

$$V_{\text{tens}}^{(1)}(r_{ij}) = -\left(\frac{\partial^2}{\partial r_{ij}^2} - \frac{1}{r_{ij}} \frac{\partial}{\partial r_{ij}} \right) \frac{V_{\text{Coul}}}{3m_i m_j} \mathcal{S}_{ij}$$

Phys. Rev. D 32, 189

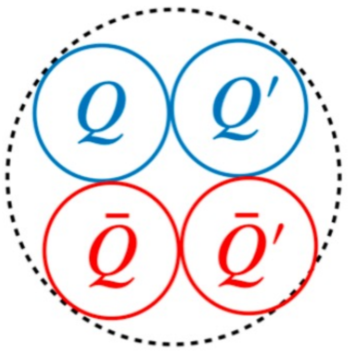
Charmonium state $\bar{c}c$

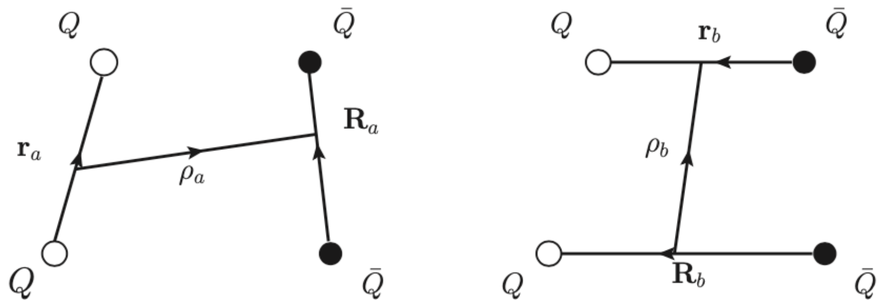
Mass spectrum (MeV)			
$^{2S+1}L_J$	Meson	EXP	THE
$^1S_0$	η_c	2983.9	2984
$^3S_1$	J/ψ	3096.9	3092
$^3P_0$	χ_{c0}	3414.7	3426
$^3P_1$	χ_{c1}	3510.7	3506
$^1P_1$	$h_c(1P)$	3525.4	3516
$^3P_2$	χ_{c2}	3556.2	3556
$^1S_0$	$\eta_c(2S)$	3637.5	3634
$^3S_1$	$\psi(2S)$	3686.1	3675
$^3S_1$	$\psi(3S)$	4039.0	4076
$^3S_1$	$\psi(4S)$	4421.0	4412

PDG 2020

Gaussian Expansion Method

- **Gaussian Expansion Method:** few-body calculation





$$\psi_{JM} = \sum_{C=a,b} \sum_{\alpha} A_{12}A_{34} \sum_{\alpha} \mathcal{B}_{\alpha}^{(C)} \chi_C^{(C)} \times \left[[\phi_{nl}(\mathbf{r}_C) \otimes \phi_{NL}(\mathbf{R}_C) \otimes \phi_{\nu\lambda}(\rho_C)]_{J_L} \otimes \chi_S^{(C)} \right]^{JM}$$

Antisymmetric Color w.f. Spatial w.f. Spin w.f.

$$\phi_{nlm}(\mathbf{r}_C) = N_{nl} r^l e^{-(r/r_n)^2} Y_{lm}(\hat{\mathbf{r}}_C)$$

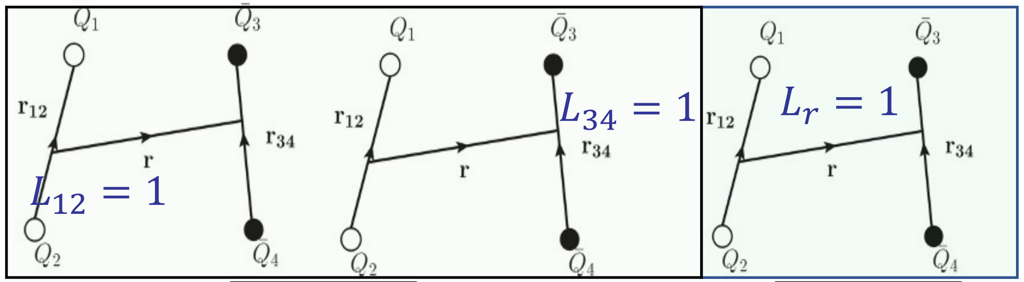
G.-J. Wang et al, Phys. Rev. D 100, 096013,
G.-J. Wang et al, Phys. Rev. D 104, 036016,

Diquark-antidiquark configuration

- The S-wave $T_{cc\bar{c}\bar{c}}$ state: $L_{12} = L_{34} = L_r = 0$.

0^{++}	$[[QQ]^1_{\bar{3}_c}[\bar{Q}\bar{Q}]^1_{3_c}]^0_{1_c}$	1^{+-}	$[[QQ]^1_{\bar{3}_c}[\bar{Q}\bar{Q}]^1_{3_c}]^1_{1_c}$
	$[[QQ]^0_{6_c}[\bar{Q}\bar{Q}]^0_{\bar{6}_c}]^0_{1_c}$	2^{++}	$[[QQ]^1_{\bar{3}_c}[\bar{Q}\bar{Q}]^1_{3_c}]^2_{1_c}$

- P-wave state: λ-and ρ- mode excitations.



ρ-mode **λ-mode**

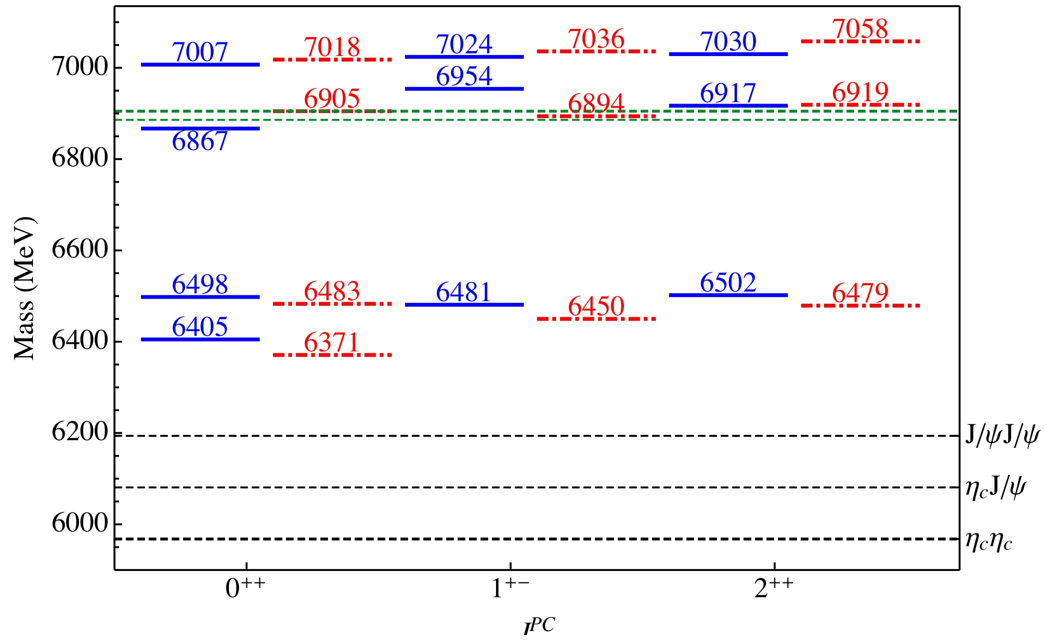
TABLE II. The color-flavor-spin configurations of the QQ ($\bar{Q}\bar{Q}$) diquark (antidiquark). The scripts "S" and "A" represent the exchange symmetry and antisymmetry for the identical particles, respectively.

Flavor	S-wave ($L=0$)	Spin	Color		J^P
S	S	S($S_{QQ}=1$)	$\bar{3}_c$(A)	$[QQ]^1_{\bar{3}_c}$	1^+
S	S	A($S_{QQ}=0$)	6_c(S)	$[QQ]^0_{6_c}$	0^+
Flavor	P-wave ($L=1$)	Spin	Color		
S	A	S($S_{QQ}=1$)	6_c(S)	$[[QQ]^1_{6_c},\rho]^0_{6_c}$	0^-
				$[[QQ]^1_{6_c},\rho]^1_{6_c}$	1^-
				$[[QQ]^1_{6_c},\rho]^2_{6_c}$	2^-
S	A	S($S_{QQ}=0$)	$\bar{3}_c$(A)	$[[QQ]^0_{\bar{3}_c},\rho]^1_{\bar{3}_c}$	1^-

Phys. Rev. D. 100, 096013

S-wave $T_{cc\bar{c}\bar{c}}$

- The S-wave $T_{cc\bar{c}\bar{c}}$ state: $L_{12} = L_{34} = L_r = 0$.
- The coupling with non S-wave orbital excitations is neglected.



J^{PC}	Decay Modes
0^{++}	$\eta_c\eta_c$, $\boldsymbol{J/\psi J/\psi}$, $\chi_{c1}\eta_c$(P-wave), $J/\psi h_c(1P)$(P-wave), $J/\psi\psi(2S)$, $\chi_{c0}\chi_{c0}$
1^{+-}	$\eta_c J/\psi$, $h_c\eta_c$(P-wave), $J/\psi\chi_{c1}$(P-wave), $\eta_c\psi'$, $h_c\chi_{c0}$
2^{++}	$\boldsymbol{J/\psi J/\psi}$, $\eta_c\chi_{c1}$(P-wave), $\eta_c\chi_{c2}$(P-wave), $J/\psi h_c$(P-wave), $J/\psi\psi(2S)$, $\chi_{c0}\chi_{c2}$

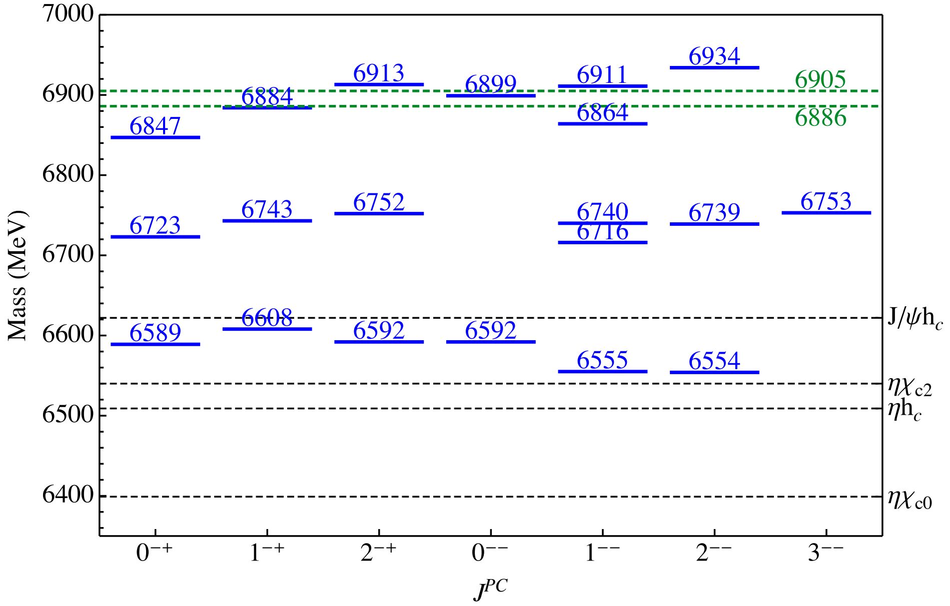
TABLE IV. The mass spectrum (MeV), the percentage of different color configurations, and the root mean square radius (fm) of the *S*-wave tetraquark states.

0^{++}	Mass	$\bar{3}_c\otimes 3_c$	$6_c\otimes\bar{6}_c$	$1_c\otimes 1_c$	$8_c\otimes 8_c$	r_{12}/r_{34}	r	r_{13}/r_{24}	r'
1S	6405	31.9%	68.1%	96.9%	3.13%	0.52	0.31	0.48	0.37
	6498	67.7%	32.3%	5.7%	94.3%	0.51	0.36	0.51	0.36
2S	6867	10.6%	89.4%	80.6%	19.4%	0.65	0.35	0.58	0.46
	7007	89.7%	10.3%	26.0%	74.0%	0.49	0.47	0.59	0.35
1^{+-}	Mass	$\bar{3}_c\otimes 3_c$	$6_c\otimes\bar{6}_c$	$1_c\otimes 1_c$	$8_c\otimes 8_c$	r_{12}/r_{34}	r	r_{13}/r_{24}	r'
1S	6481	100%	0%	33.3%	66.7%	0.48	0.37	0.51	0.34
2S	6954	100%	0%	33.3%	66.7%	0.61	0.44	0.61	0.43
3S	7024	100%	0%	33.3%	66.7%	0.66	0.42	0.62	0.46
2^{++}	Mass	$\bar{3}_c\otimes 3_c$	$6_c\otimes\bar{6}_c$	$1_c\otimes 1_c$	$8_c\otimes 8_c$	r_{12}/r_{34}	r	r_{13}/r_{24}	r'
1S	6502	100%	0%	33.3%	66.7%	0.49	0.39	0.53	0.35
2S	6917	100%	0%	33.3%	66.7%	0.55	0.60	0.72	0.39
3S	7030	100%	0%	33.3%	66.7%	0.64	0.46	0.64	0.45

- 0^{++} state: an admixture of $\bar{3}_c - 3_c$ and $6_c - \bar{6}_c$ configurations.
- 0^{++} ground state: $6_c - \bar{6}_c$ component is lighter and dominates.
- No bound states exist.
- Wide S-wave $T_{cc\bar{c}\bar{c}}$: *di-* J/ψ, *di-* η_c, $\eta_c J/\psi$.
- $X(6900)$: wide S-wave states $J^{PC} = 0^{++}$ or 2^{++}.

Phys. Rev. D. 100, 096013

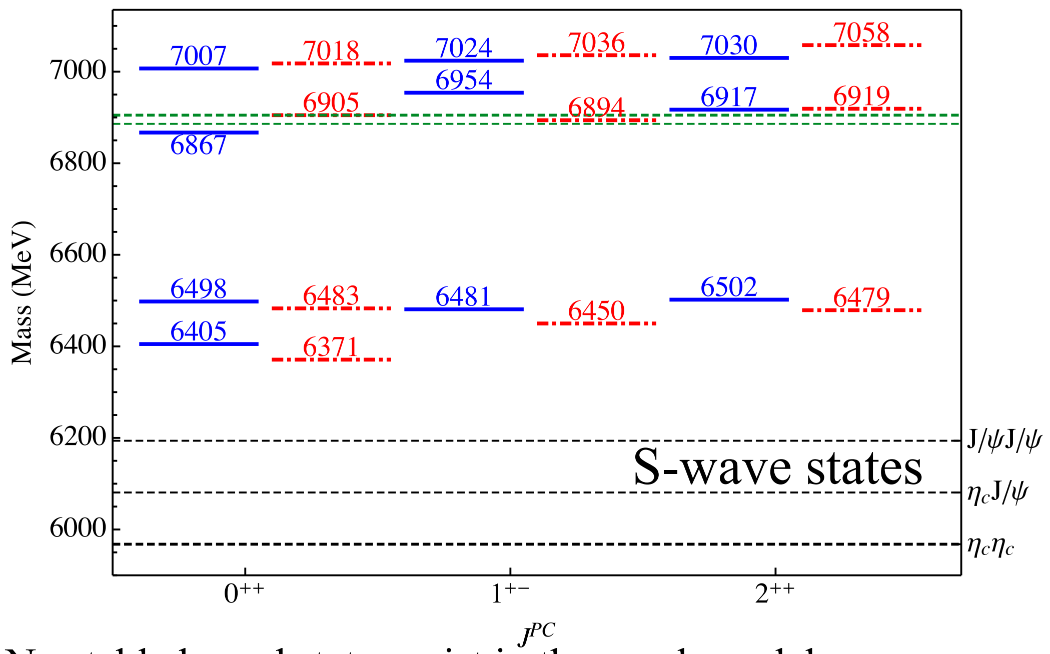
P-wave $T_{cc\bar{c}\bar{c}}$

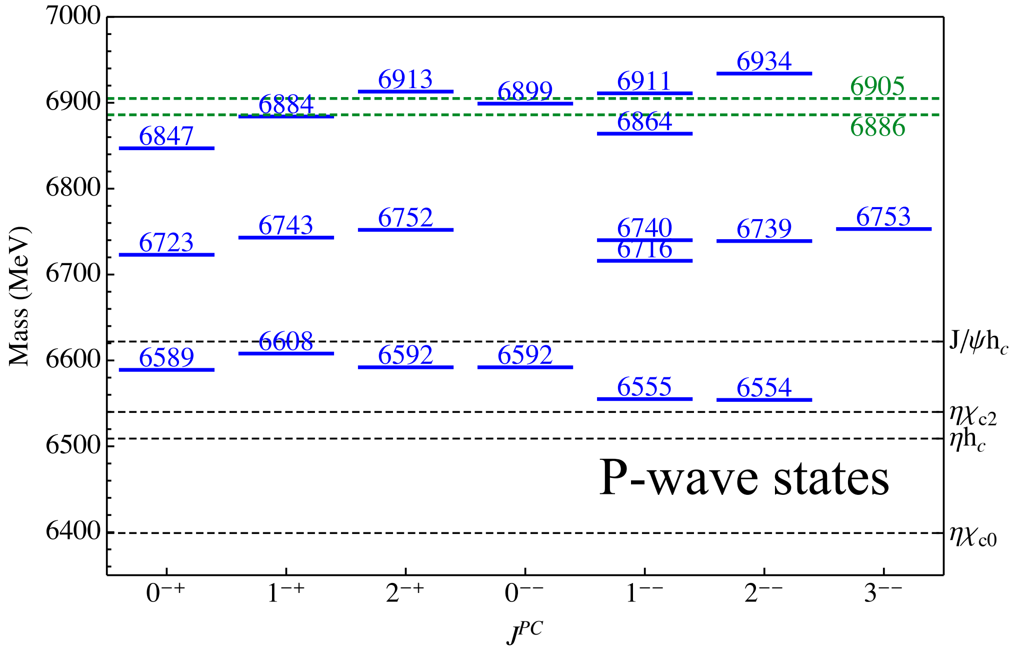


J^{PC}	Decay modes
0^{-+}	$\boldsymbol{J/\psi J/\psi}$(P-wave), $\eta_c\chi_{c0}$, $J/\psi h_c$, $J/\psi\psi(2S)$(P-wave)
1^{-+}	$\boldsymbol{J/\psi J/\psi}$(P-wave) $J/\psi h_c$, $J/\psi\psi(2S)$(P-wave)
2^{-+}	$\boldsymbol{J/\psi J/\psi}$(P-wave), $\eta_c\chi_{c2}$, $J/\psi h_c$, $J/\psi\psi(2S)$(P-wave)
0^{--}	$\eta_c J/\psi$(P-wave), $J/\psi\chi_{c1}$, $\eta_c\psi(2S)$(P-wave)
1^{--}	$\eta_c J/\psi$(P-wave), $\eta_c h_c$, $J/\psi\chi_{c0}$, $J/\psi\chi_{c1}$, $J/\psi\chi_{c2}$, $\eta_c\psi'$(P-wave)
2^{--}	$\eta_c J/\psi$(P-wave), $J/\psi\chi_{c1}$, $J/\psi\chi_{c2}$, $\eta_c\psi'$(P-wave), $h_c\chi_{c0}$(P-wave)
3^{--}	$J/\psi\chi_{c2}$

- New narrow $T_{cc\bar{c}\bar{c}}$ tetraquark: especially $J^{PC} = 0^{--}$ or 1^{-+}.
- P-wave decay channels dominated: small decay widths.
- $X(6900)$: Narrow P-wave state with $J^{PC} = 1^{-+}$ or 2^{-+}.

Questions





- No stable bound states exist in the quark models.
- The lowest fully charmed tetraquark state : **in mass region (6. 5, 6. 7, 6. 9) GeV**
- $X(6900)$: wide S-wave states $J^{PC}=0^{++}$ or 2^{++} or narrow P-wave states with $J^{PC}=1^{-+}$ or 2^{-+}.
- **Redundancy states :**

✓ The finite number of the bases → discrete eigenvalues of the scattering states

✓ Multiquark states with large decay widths → hard to observe

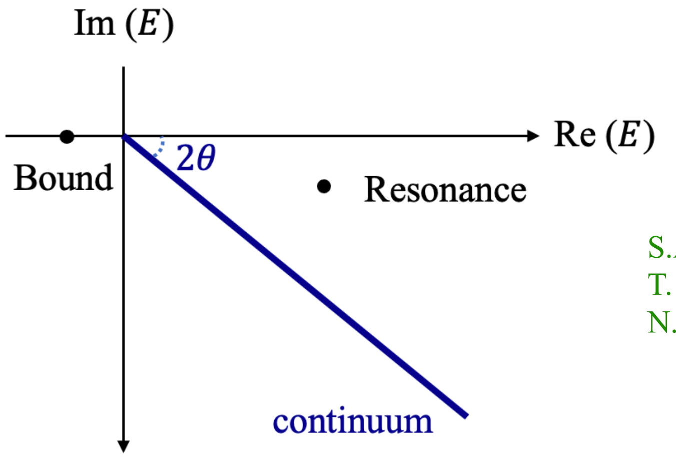
Phys. Rev. D. 100, 096013

Phys. Rev. D. 104, 036016

Complex scaling method (CSM): $T_{cc\bar{c}\bar{c}}$

- **Complex scaling method:** identifying resonances

$$\mathbf{r} \to \mathbf{r}e^{i\theta}, \quad \mathbf{k} \to \mathbf{k}e^{-i\theta}$$

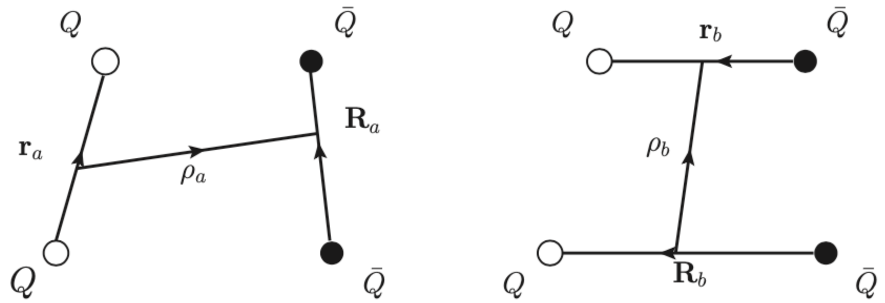


With the increasing θ, bound and resonant states will stay stable while scattering states (continuum states) will rotate with 2θ.

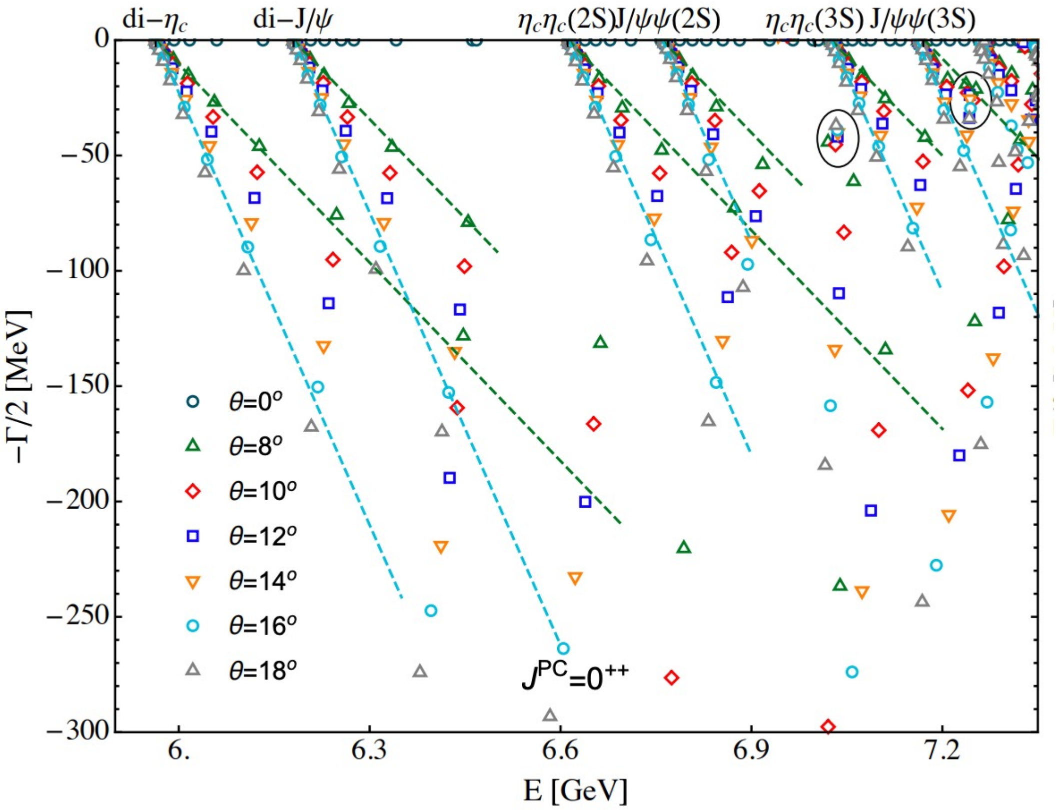
S.Aoyama et al. PTP. 116, 1 (2006).
T. Myo et al. PPNP. 79, 1 (2014)
N. Moiseyev, Physics reports 302, 212 (1998)

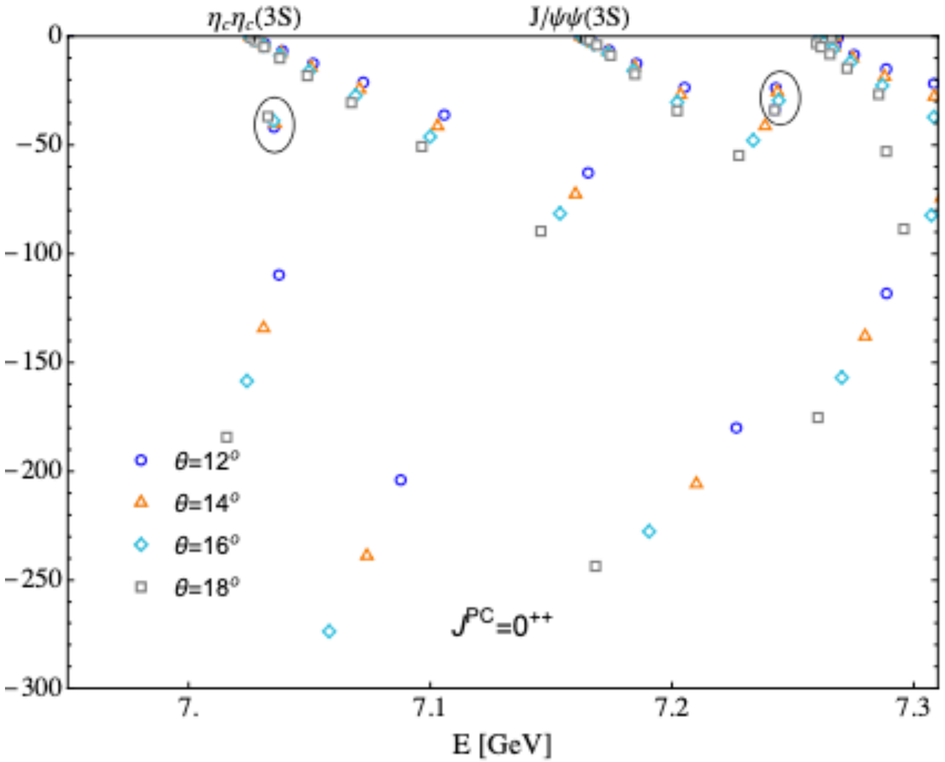
- Complex scaling method for tetraquark state:

(Diquark-antidiquark; Coupled-channel effect; …; Meson-meson scattering channels (di-J/ψ, $J/\psi\ \psi$…))



$T_{cc\bar{c}\bar{c}}$





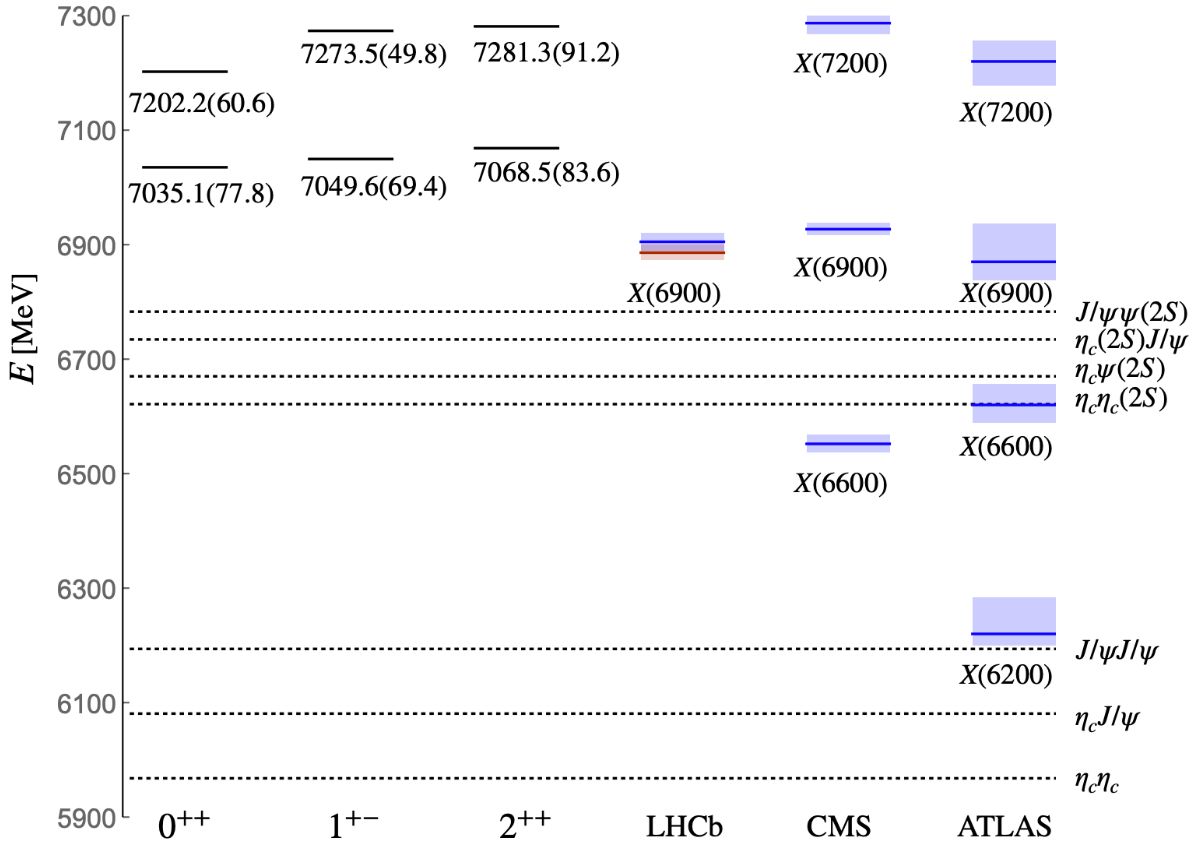
arXiv: 2008.07292

- 1st pole: good convergency

- 2nd pole: quite close to the threshold lines with a scaling angle in the (8, 10) degree

✓ Higher states are more difficult to describe.

$T_{cc\bar{c}\bar{c}}$



J^{PC}	1st	2nd
0^{++}	$7035.1 - i\ 38.9$	$7202.2 - i\ 30.3$
1^{+-}	$7049.6 - i\ 34.7$	$7273.5 - i\ 24.9$
2^{++}	$7068.5 - i\ 41.8$	$7281.3 - i\ 45.6$

- ***1st pole VS X(6900):***

✓ 100 MeV higher mass & consistent decay width

- ***2nd pole: a candidate for X(7200).***

- ***Absence of the lower $X(6600)$ state.***

✓ a wide resonance asymptote will oscillate very strongly in the complex plane.

- The confinement mechanism~ br.

→ Oka san's talk

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Thank you for your attention!