Fully-Charmed Tetraquark and Quark Confinement

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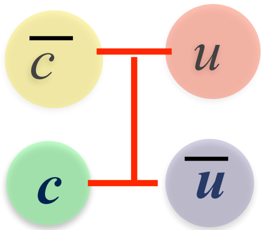
ELPH Workshop ハドロン分光に迫る反応と構造の物理 (2012-12-06)

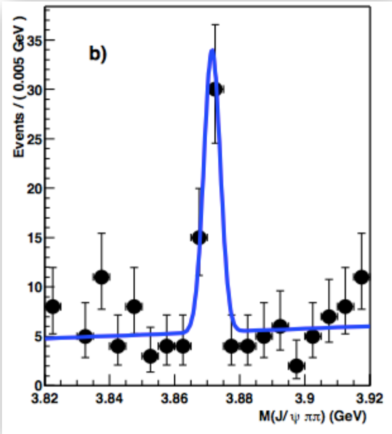
Exotic Multiquark Hadrons

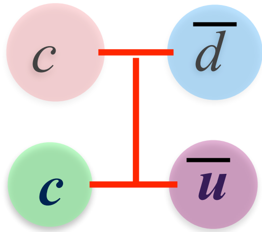
- Hadrons that do not fit the simple quark model picture

X(3872) ***Belle***

PRL 91 (2003)

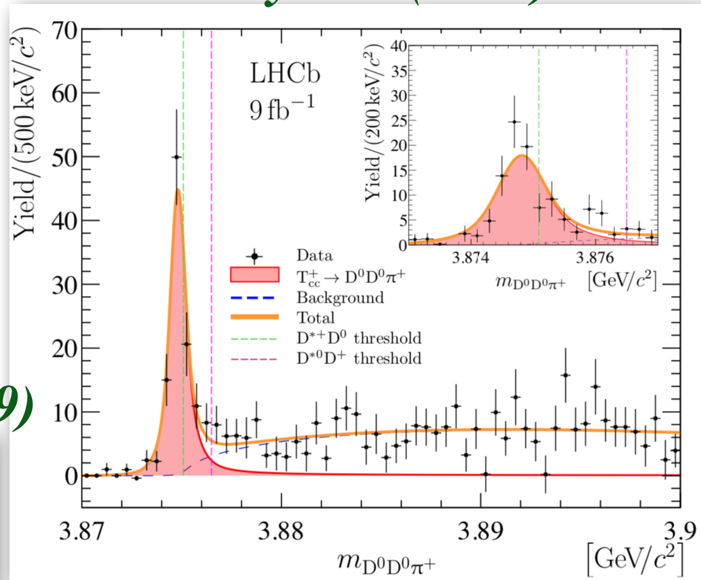






T_{cc} ***LHCb***

Nature Phys. 18 (2022)751

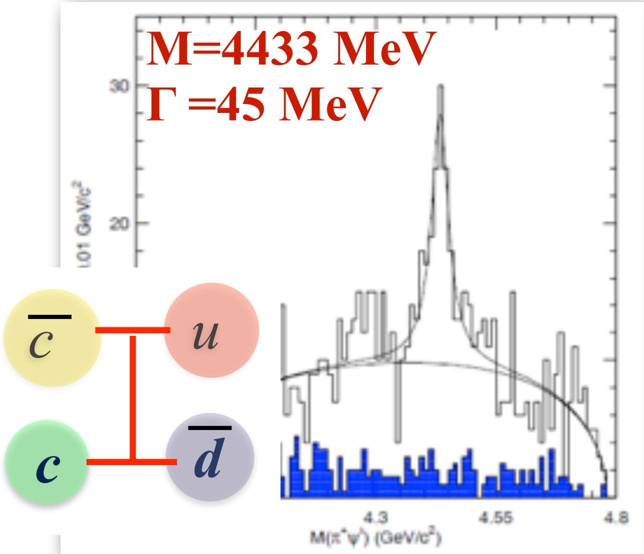


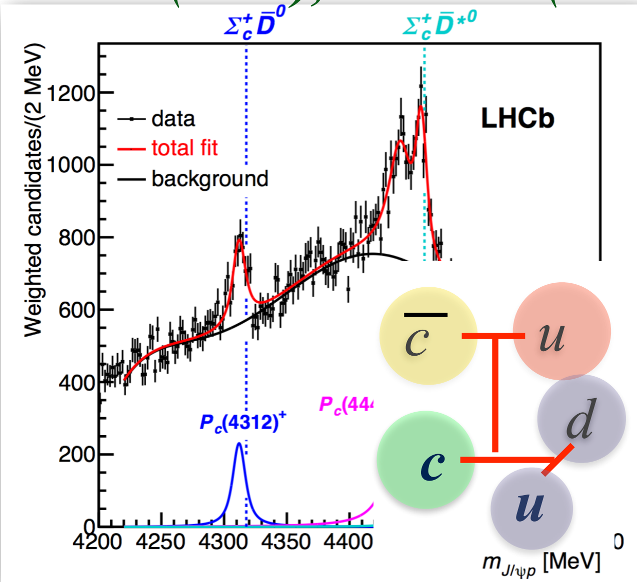
Z_c^+(4430) ***Belle***

PRL 100 (2008) 142001

P$_c$ (4312) (4440) (4457) ***LHCb***

PRL 115 (2015), PRL 122 (2019)

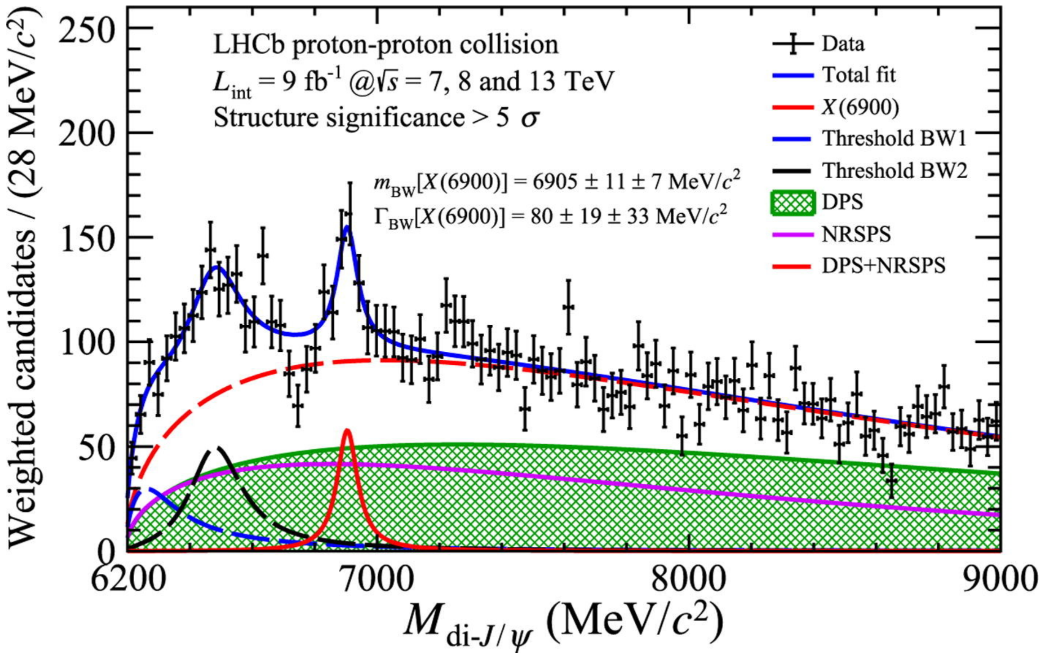




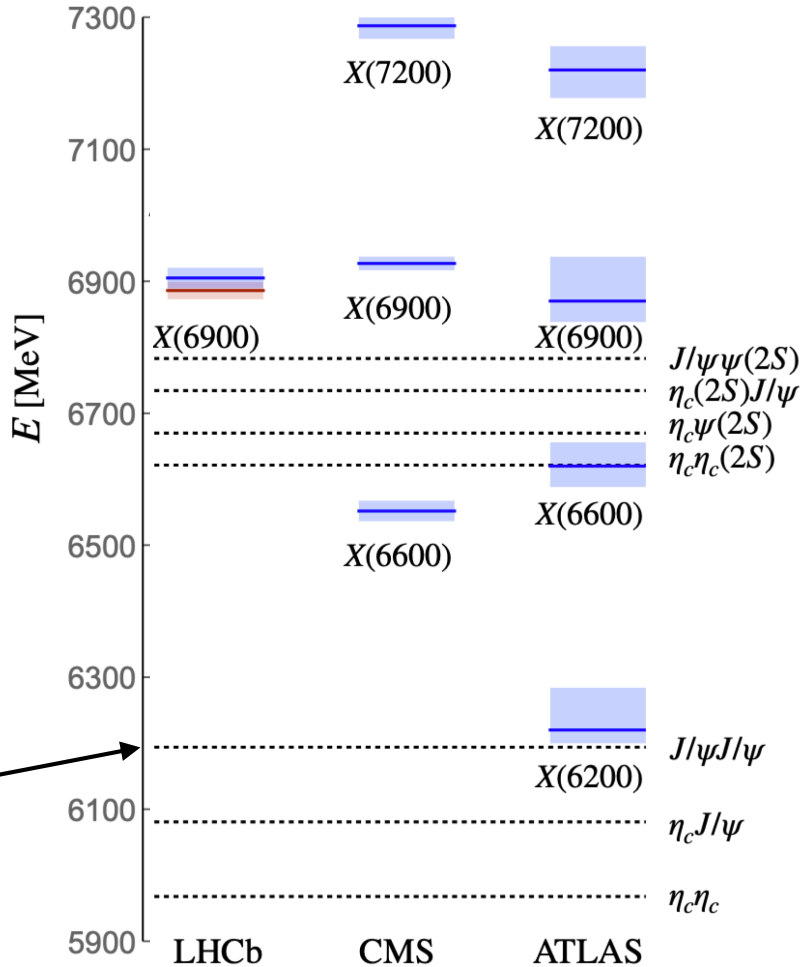
Exotic Multiquark Hadrons

- ***Fully charmed tetra-quark resonances*** $X(cc\bar{c}\bar{c}) \rightarrow (c\bar{c})(c\bar{c})$

$$m_{BW}[X(6900)] = 6905 \pm 11 \pm 7\ \mathrm{MeV}/c^2$$
$$\Gamma_{BW}[X(6900)] = 80 \pm 19 \pm 33\ \mathrm{MeV}/c^2$$



J/ψ-J/ψ resonances observed at LHC (quantum numbers unknown)



Invariant mass of two *J/ψ*'s threshold @ 6.194 GeV

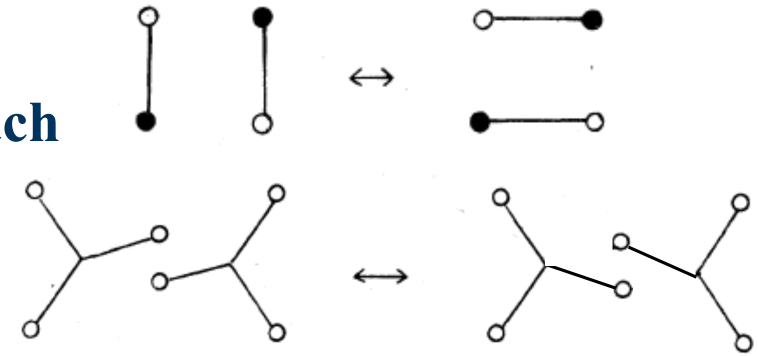
Exotic Multiquark Hadrons

- **Why are the multi-quark hadrons exciting and important?**
- **What do they add to our understanding of QCD?**
  Color configuration in multi-quark (N≧4) systems is not unique.
  Corresponding confinement mechanism is not well specified.
  The tetra-quark, the simplest multi-quark, will clarify the quark color dynamics and will tell us how quarks are confined.
- **We propose**
  String-like confinement potential by extending the color configuration space of the "quark model".
  Contributions of compact multi-quark states vs two-meson molecular states can be separated.

String flip-flop model

- “Reconnection of strings and quark matter”,
  H. Miyazawa, Phys. Rev. D20, 2953 (1979)

  “String Flip-Flop” model
  based on the Born-Oppenheimer approach



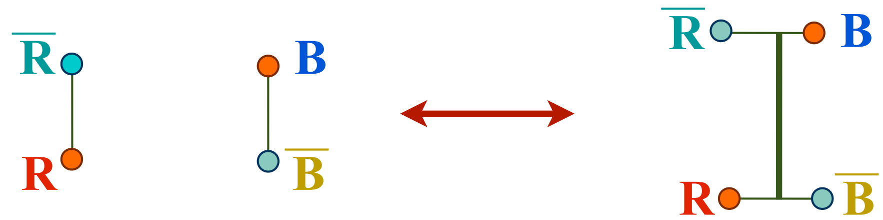
$$V_{\text{string}} = \sigma \times \underset{\text{links}}{\text{Min}} \sum r_{\text{link}}$$

- Similar “string-type” confinement potential models for multi-quark systems were discussed by

  O.W. Greenberg, J. Hietarinta, Phys. Lett. B 86, 309 (1979)
  N. Isgur, J. E. Paton, Phys. Lett. B 124, 247 (1983)
  M. Oka, Phys. Rev. D 31, 2274 (1985).
  F. Lenz, et al., Annals Phys. 170, 65 (1986).
  Y. Koike et al., Nucl. Phys. A449, 635 (1986), PTP S137, 21 (2000).
  G.A. Miller, Phys. Rev D37, 2431 (1998).
  J. Vijande, A. Valcarce, J.M. Richard, Phys. Rev. D 85, 014019 (2012).

String flip-flop model

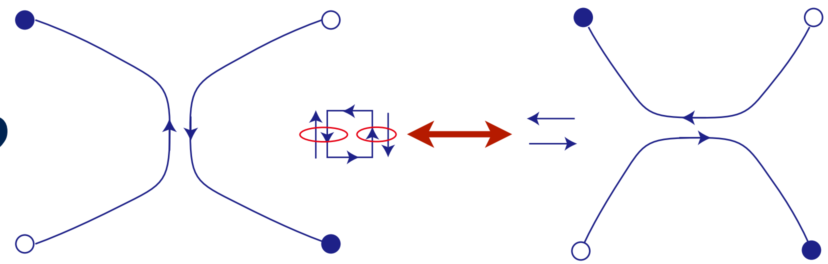
- **The string FF model works for the U(1) charge, but in the color SU(3) theory, hidden-color channels are automatically mixed.**



Hidden Color channel

- **In QCD, the transition between two color configurations is dynamically generated, and the HC channel is regarded as an independent configuration.**

Strong coupling expansion of QCD



- **In the quark model, only 2 independent color states are allowed.**

$$\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} = 2 \times \mathbf{1} \oplus 4 \times \mathbf{8} \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}$$

Quark model - color configurations

- Color singlet $Q_1\ Q_2\ Q_3^{bar}\ Q_4^{bar}$ system is described by

  Two singlets (mesons) states:

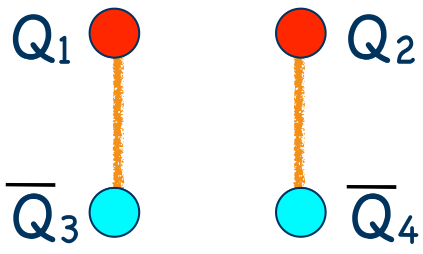
  $|1\rangle = |(Q_1\bar{Q}_3)_{\mathbf{1}}(Q_2\bar{Q}_4)_{\mathbf{1}}\rangle \quad |1'\rangle = |(Q_1\bar{Q}_4)_{\mathbf{1}}(Q_2\bar{Q}_3)_{\mathbf{1}}\rangle$

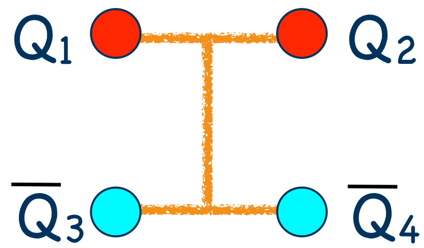
  Singlet + hidden color states:

  $|1\rangle = |(Q_1\bar{Q}_3)_{\mathbf{1}}(Q_2\bar{Q}_4)_{\mathbf{1}}\rangle \quad |8\rangle = |(Q_1\bar{Q}_3)_{\mathbf{8}}(Q_2\bar{Q}_3)_{\mathbf{8}}\rangle$

  Diquarks with color 3^{bar} and 6:

  $|3\rangle = |(Q_1Q_2)_{\bar{\mathbf{3}}}(\bar{Q}_3\bar{Q}_4)_{\mathbf{3}}\rangle \quad |6\rangle = |(Q_1Q_2)_{\mathbf{6}}(\bar{Q}_3\bar{Q}_4)_{\bar{\mathbf{6}}}\rangle$

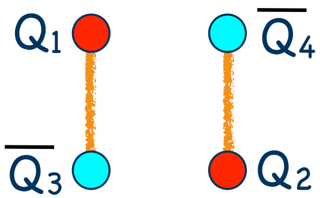


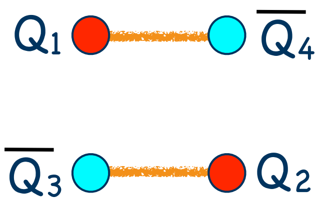


- ***These bases are all equivalent.***

$$|1\rangle = \sqrt{\frac{1}{3}}|3\rangle + \sqrt{\frac{2}{3}}|6\rangle \qquad |8\rangle = -\sqrt{\frac{2}{3}}|3\rangle + \sqrt{\frac{1}{3}}|6\rangle$$

- **Two-meson states are not orthogonal.**

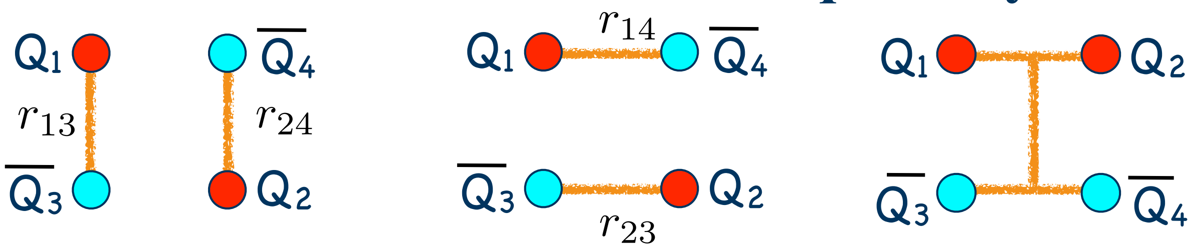
$|1\rangle =$ 

$|1'\rangle =$ 

$$\langle 1|1'\rangle = \frac{1}{3}$$

Novel string-like potential

- **Three color basis states in the tetraquark system**



$$|\mathbf{1}\rangle\rangle \equiv |(Q_1 \to \bar{Q}_3)_1(Q_2 \to \bar{Q}_4)_1\rangle$$
$$|\mathbf{1}'\rangle\rangle \equiv |(Q_1 \to \bar{Q}_4)_1(Q_2 \to \bar{Q}_3)_1\rangle$$
$$|\mathbf{hc}\rangle\rangle \equiv |(Q_1 \leftrightarrow Q_2)_{\bar{3}} \leftarrow (\bar{Q}_3 \leftrightarrow \bar{Q}_4)_3\rangle$$

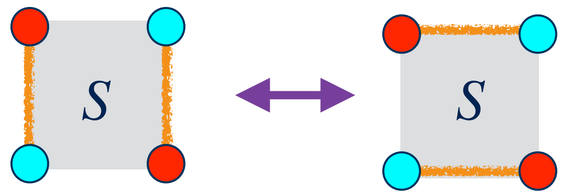
orthogonal bases

- **The string confinement potential**

$$\langle\langle\mathbf{1}|V_{\mathrm{ST}}|\mathbf{1}\rangle\rangle = \sigma(r_{13} + r_{24}),$$
$$\langle\langle\mathbf{1}'|V_{\mathrm{ST}}|\mathbf{1}'\rangle\rangle = \sigma(r_{14} + r_{23}).$$

σ: string tension

transition by quantum tunneling
filled the area by gauge field



$$\langle\langle\mathbf{1}|V_{\mathrm{ST}}|\mathbf{1}'\rangle\rangle = \kappa e^{-\sigma S}$$

S: Minimal surface area

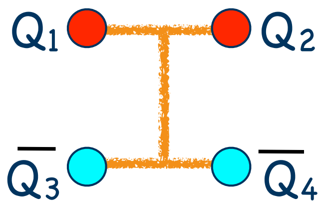
Y. Koike, O. Morimatsu, K. Yazaki, PTP S137, 21 (2000)

Novel string-like potential

- Confinement in $|\mathbf{hc}\rangle\rangle$ channel

$$\langle\langle \mathbf{hc}|V_{\rm ST}|\mathbf{hc}\rangle\rangle = \sigma\left[\frac{r_{13}+r_{24}+r_{14}+r_{23}}{4}+\frac{r_{12}+r_{34}}{2}\right]$$

$$\langle\langle \mathbf{1}|V_{\rm ST}|\mathbf{hc}\rangle\rangle = \langle\langle \mathbf{1}'|V_{\rm ST}|\mathbf{hc}\rangle\rangle = \pm\kappa' \exp(-\sigma S)$$



$$\kappa' = \sqrt{8}\kappa$$

- 3-channel confinement potential

$$V_{\rm ST} = \begin{pmatrix} \sigma(r_{13}+r_{24}) & \kappa e^{-\sigma S} & \kappa' e^{-\sigma S} \\ \kappa e^{-\sigma S} & \sigma(r_{14}+r_{23}) & -\kappa' e^{-\sigma S} \\ \kappa' e^{-\sigma S} & -\kappa' e^{-\sigma S} & \frac{\sigma}{4}[r_{13}+r_{24}+r_{14}+r_{23}+2(r_{12}+r_{34})] \end{pmatrix} \begin{matrix} |\mathbf{1}\rangle\rangle \\ |\mathbf{1}'\rangle\rangle \\ |\mathbf{hc}\rangle\rangle \end{matrix}$$

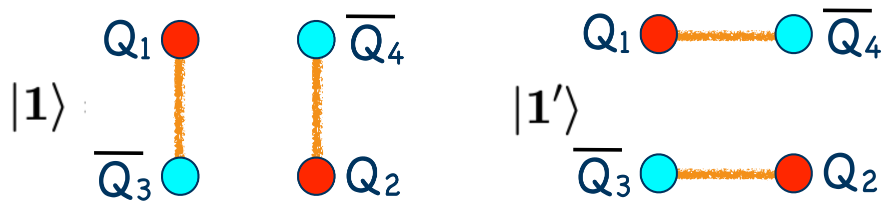
Local 4-body potential

Quark model confinement potential

- **Quark model with conventional confinement**

$$V = -\frac{3}{4}\sigma \sum_{i<j} (T_i \cdot T_j) r_{ij}$$

$$T_i(Q) = \frac{\lambda_i}{2} \qquad T_i(\bar{Q}) = -\frac{\lambda_i^*}{2}$$



$$V_{\rm QM} = \begin{pmatrix} \sigma(r_{13}+r_{24}) & (\sigma/3)(r_{13}+r_{24}+r_{14}+r_{23}-r_{12}-r_{34}) \\ (\sigma/3)(r_{13}+r_{24}+r_{14}+r_{23}-r_{12}-r_{34}) & \sigma(r_{14}+r_{23}) \end{pmatrix} \begin{matrix} |\mathbf{1}\rangle \\ |\mathbf{1}'\rangle \end{matrix}$$

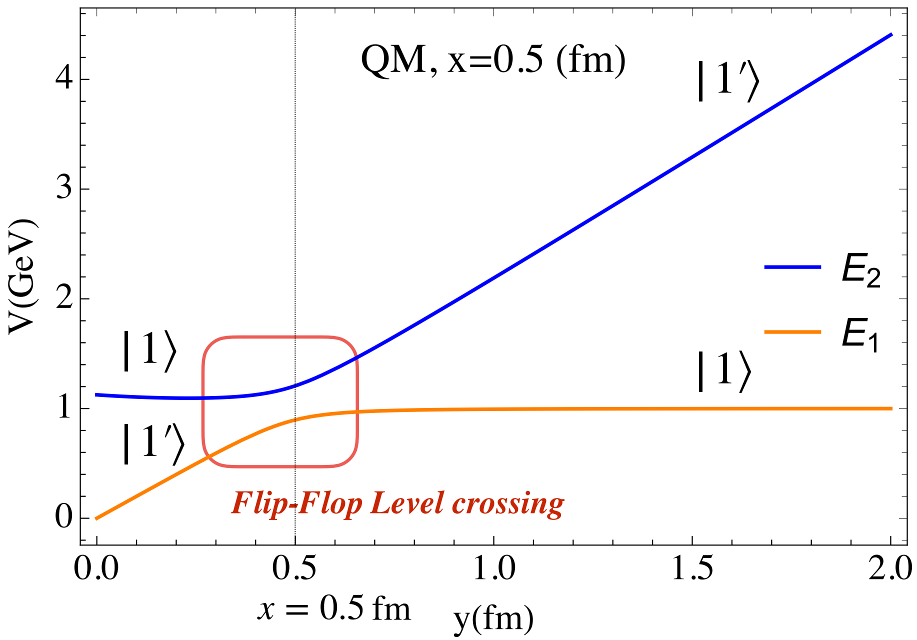
$\Rightarrow$ ***induces long-range color van der Waals force via color-octet P-wave polarized intermediate states***

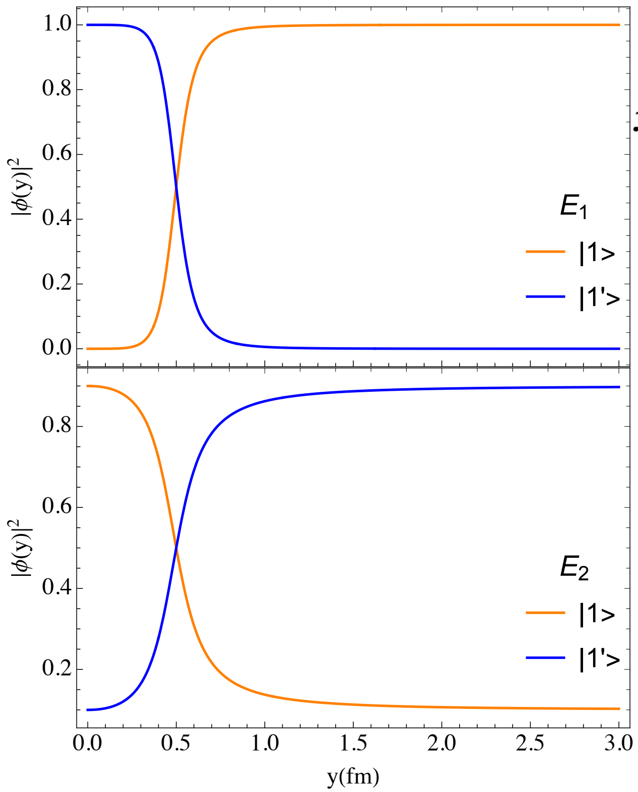
$$V_{\rm vdW} = \frac{|\langle \mathbf{8}|V|\mathbf{1}\rangle|^2}{\Delta E} \propto -\frac{1}{R^3}$$

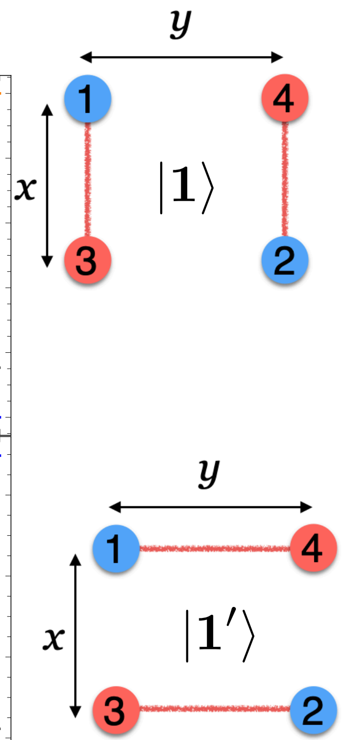
T. Appelquist, W. Fischler, Phys. Lett. B77, 405 (1978)

Born-Oppenheimer potential

- **For the QM conventional confinement,**
  ***y*-dependence of the adiabatic potential for a fixed *x*:**



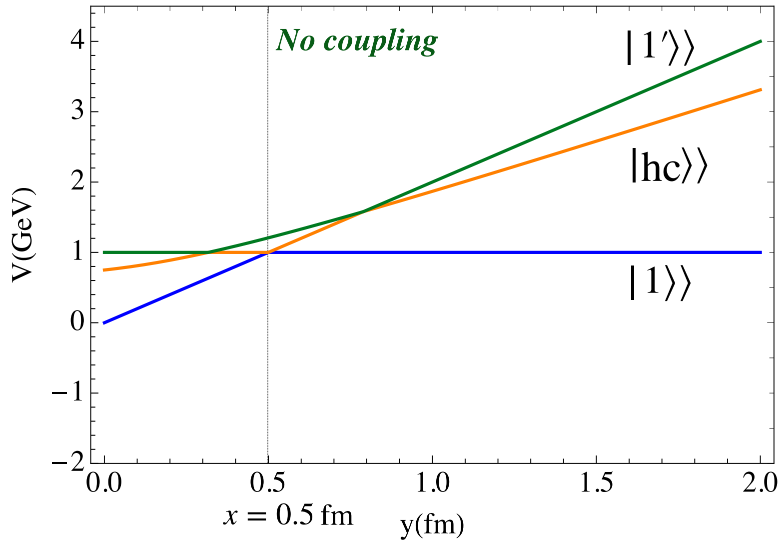


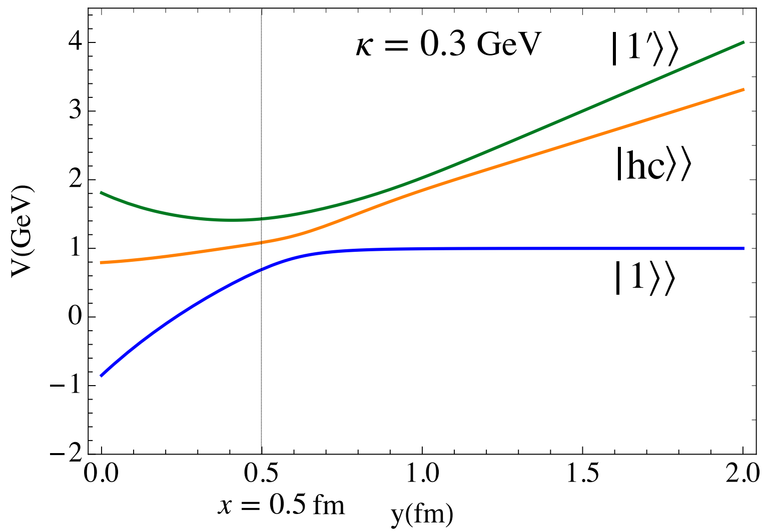


$$V = \begin{pmatrix} 2\sigma x & (2/3)\sigma(x+y-\sqrt{x^2+y^2}) \\ (2/3)\sigma(x+y-\sqrt{x^2+y^2}) & 2\sigma y \end{pmatrix}$$

Born-Oppenheimer potential

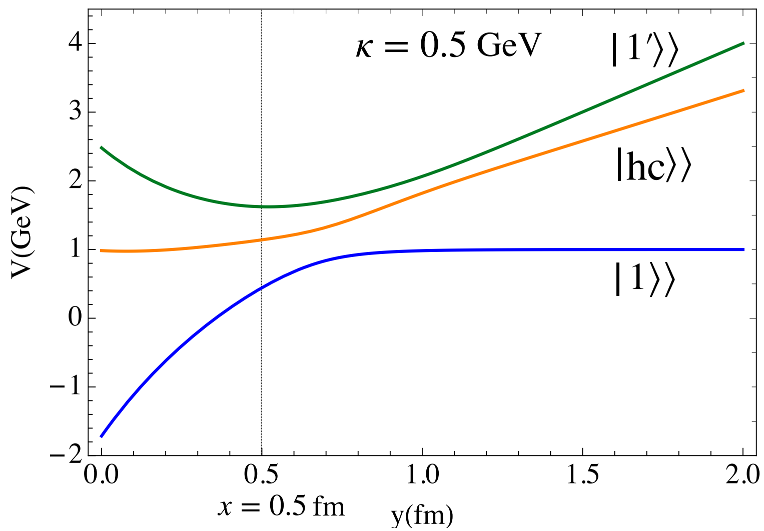
Novel String-like confinement: adiabatic potential at fixed x





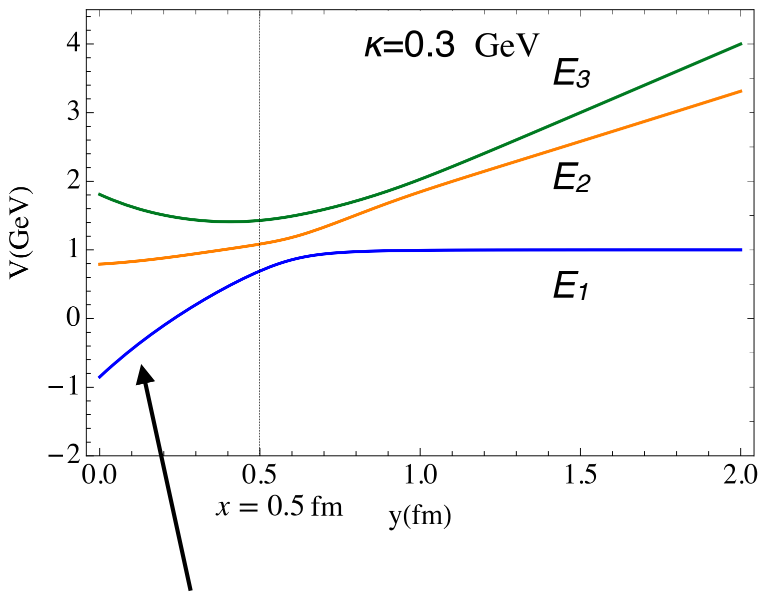
$$V_{\mathrm{ST}}(x,y) = \begin{pmatrix} 2\sigma x & \kappa e^{-\sigma x y} & \kappa' e^{-\sigma x y} \\ \kappa e^{-\sigma x y} & 2\sigma y & -\kappa' e^{-\sigma x y} \\ \kappa' e^{-\sigma x y} & -\kappa' e^{-\sigma x y} & \sigma\left(\frac{x+y}{2} + \sqrt{x^2+y^2}\right) \end{pmatrix}$$

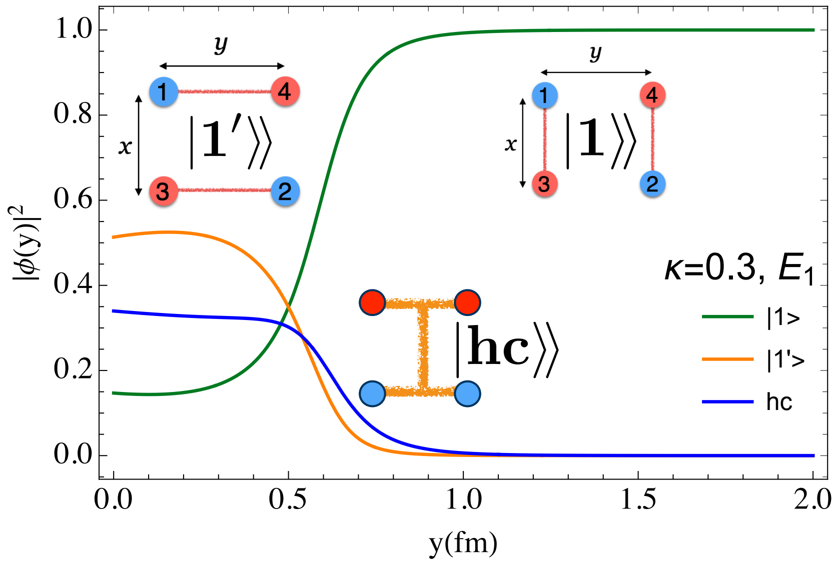
$$\kappa' = \sqrt{8}\kappa$$



Born-Oppenheimer potential

- **Novel String-like confinement: adiabatic potential at fixed *x***



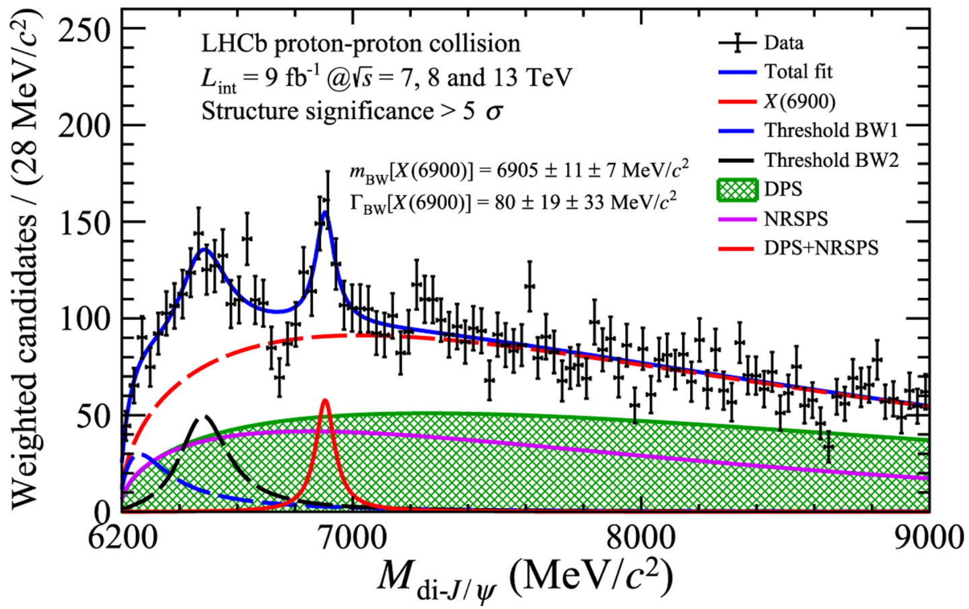


- **There appears a strong attraction at short distances with a large mixing of the hidden color (hc) state. The compact tetra-quark configurations become important at short distances.**
- **The range of the free parameter *κ***

$$0 \le \kappa' = \sqrt{8}\kappa \le 2\sigma a \sim 2\sqrt{\sigma} \sim 1\text{GeV} \longrightarrow \kappa \le 0.3\text{GeV}$$

Tetraquark states

- Next step is to compute the spectrum of realistic tetra-quark systems with different choices of confinement and compare them with experiment and/or lattice QCD results.
  (1) Fully heavy tetraquark, $QQ\bar{Q}\bar{Q}$
  (2) Doubly heavy tetraquark, $T_{QQ} = QQ\overline{qq}$
- As an example, X(6900), a candidate of *fully charmed tetra-quark*



$$m_{BW}[X(6900)] = 6905 \pm 11 \pm 7\ \text{MeV}/c^2$$
$$\Gamma_{BW}[X(6900)] = 80 \pm 19 \pm 33\ \text{MeV}/c^2$$

J/ψ-J/ψ resonances observed at LHCb (quantum numbers unknown) → candidates of $cc\bar{c}\bar{c}$ resonant states

Tetra-Charm states

- **With one-gluon exchange interaction**
  Coulomb + spin-spin interaction
- **Parameters to fit to the charmonium masses**
  → reproduce the two meson thresholds

$$V_{\text{cen}}(r_{ij}) = V_{\text{Coul}} + V_{\text{lin}} + V_{\text{hyp}} = \frac{\alpha_s}{r_{ij}} - \frac{3}{4} b r_{ij} - \frac{8\pi\alpha_s}{3m_i m_j}\left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma^2 r_{ij}^2} \mathbf{s}_i \cdot \mathbf{s}_j$$

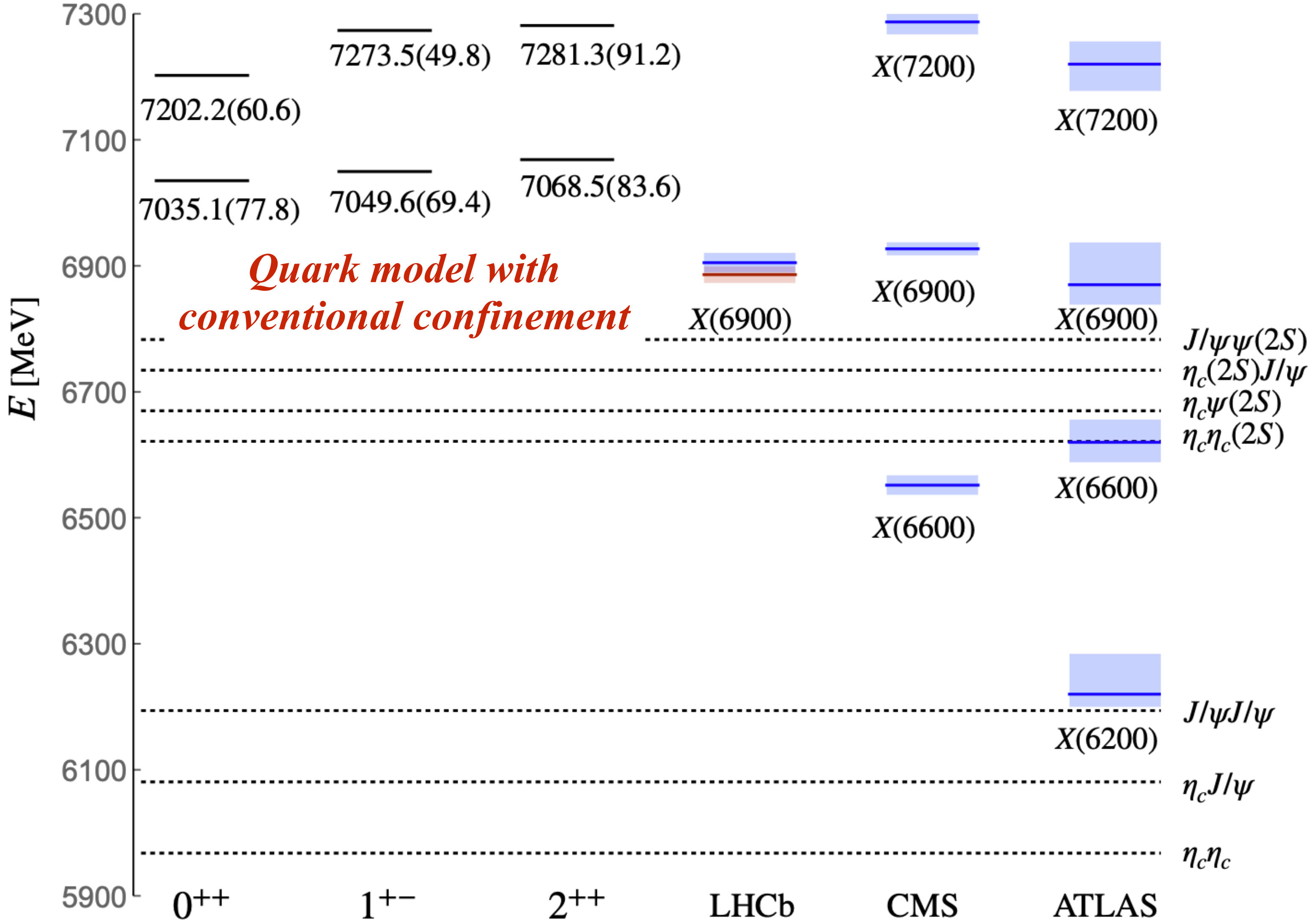
$$b = \sigma(\text{conf}) = 0.1425\ \text{GeV}^2 = 0.722\ \text{GeV/fm}$$

		Mass spectrum (MeV)			
Parameter		$^{2S+1}L_J$	Meson	EXP	THE
α_s	0.5461	$^1S_0$	η_c	2983.9	2984
b [GeV^2]	0.1425	$^3S_1$	J/ψ	3096.9	3092
m_c [GeV]	1.4794	$^3P_0$	χ_{c0}	3414.7	3426
σ [GeV]	1.0946	$^3P_1$	χ_{c1}	3510.7	3506
		$^1P_1$	$h_c(1P)$	3525.4	3516
		$^3P_2$	χ_{c2}	3556.2	3556
		$^1S_0$	$\eta_c(2S)$	3637.5	3634
		$^3S_1$	$\psi(2S)$	3686.1	3675
		$^3S_1$	$\psi(3S)$	4039.0	4076
		$^3S_1$	$\psi(4S)$	4421.0	4412

- **The conventional confinement cannot reproduce the tetra-charm spectrum.**
- **Replace the linear confinement by the *string-like confinement* to calculate the tetra-charm states.**

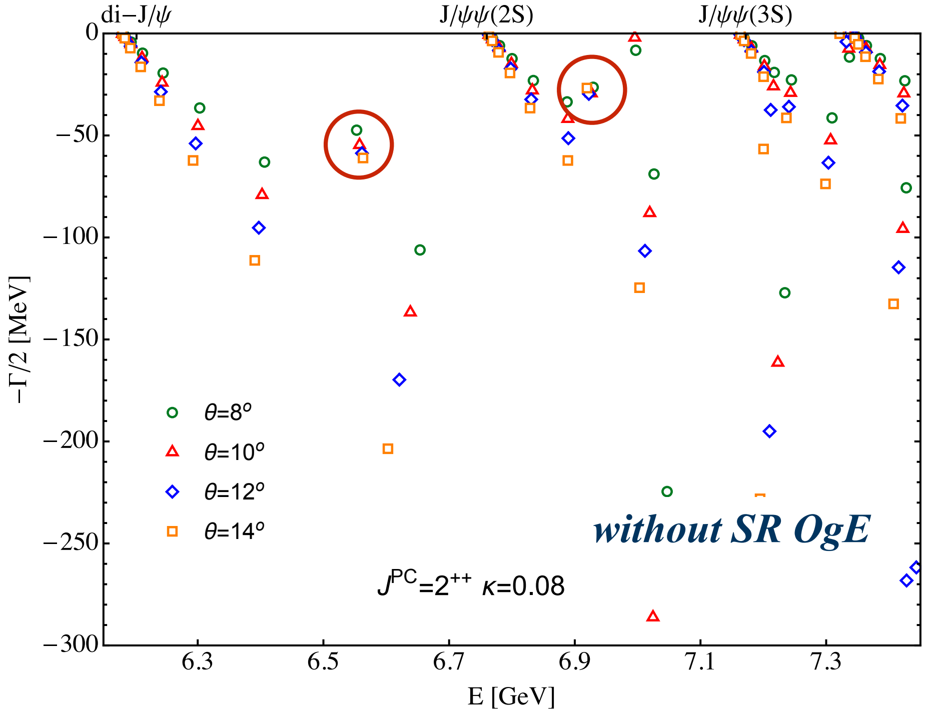
Tetra-Charm states

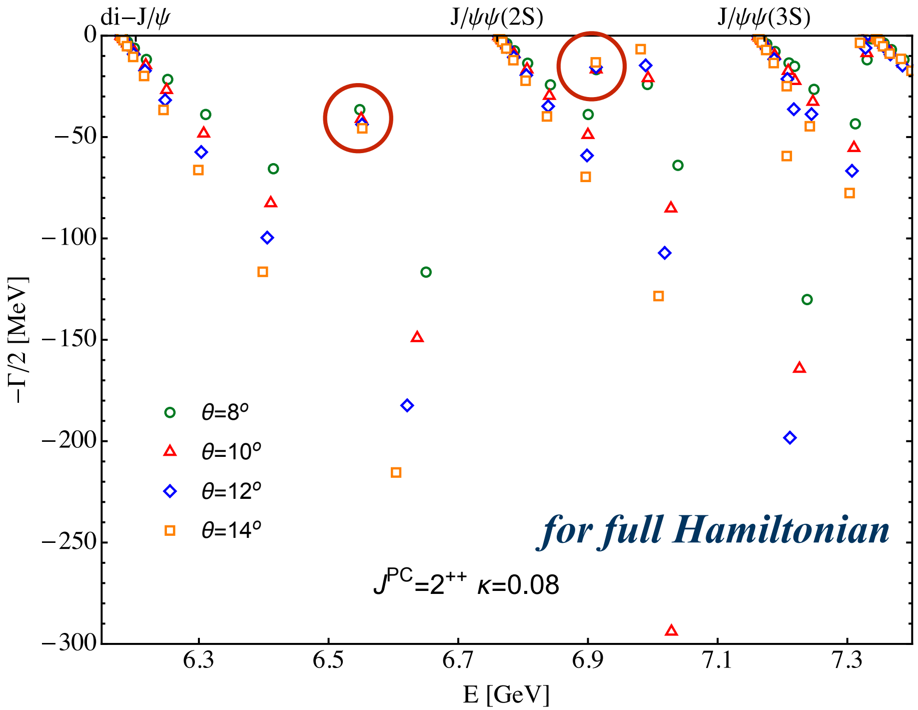
Fully charmed tetraquarks



Tetra-Charms with novel confinement

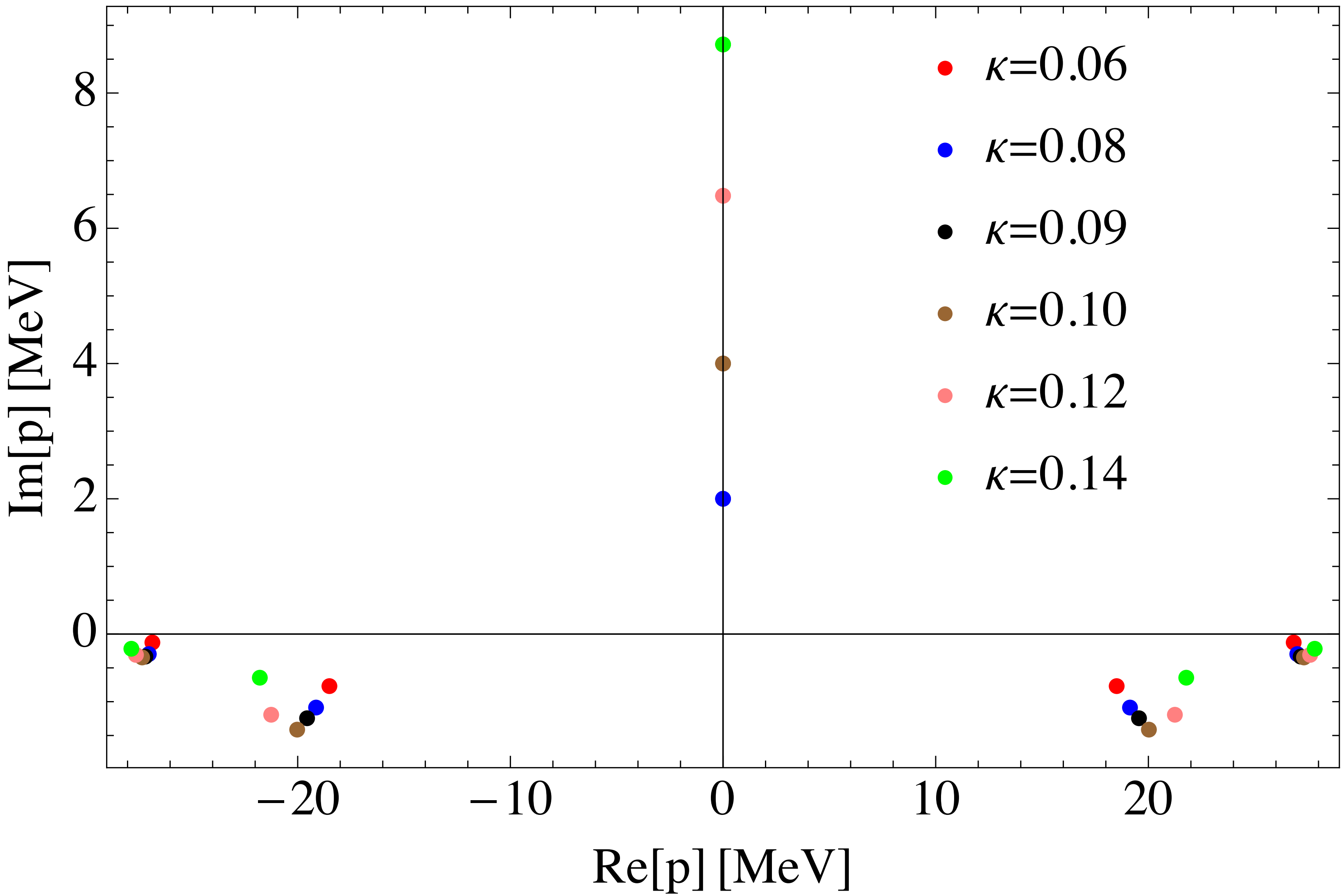
Complex scaling method for the spin-aligned 2^{++} states



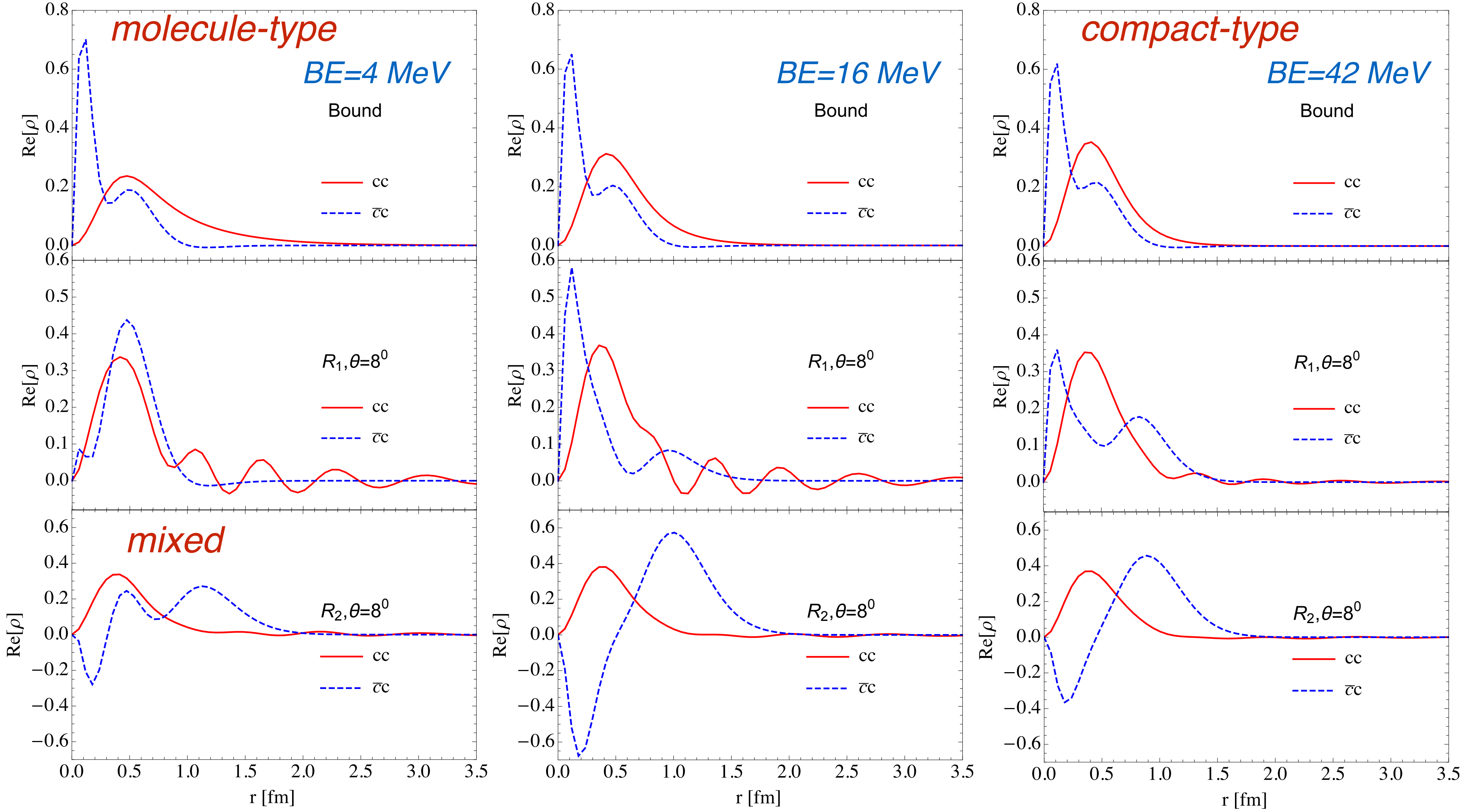


2^{++} Bound and resonance states (*without SR OgE potential)

κ (GeV)	BS* (MeV)	BS (MeV)	BE(MeV)	1st Res	2nd Res
0.08	6180	6180	4	6550 (41)	6913 (17)
0.10	6168	6166	16	6582 (54)	6929 (19)
0.12	6142	6140	42	6631 (51)	6944 (16)

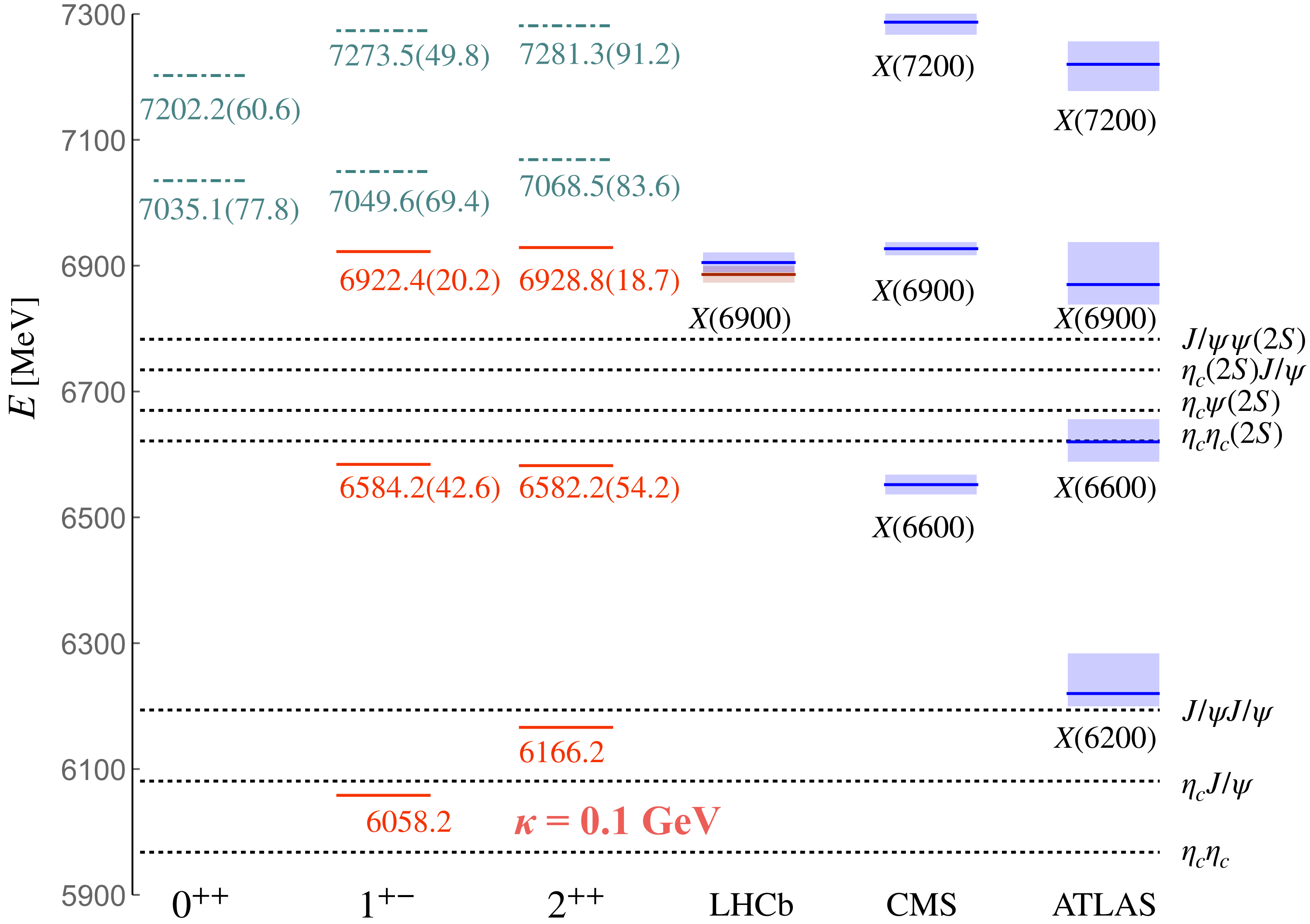


Im[p] [MeV]
Re[p] [MeV]
κ=0.06
κ=0.08
κ=0.09
κ=0.10
κ=0.12
κ=0.14



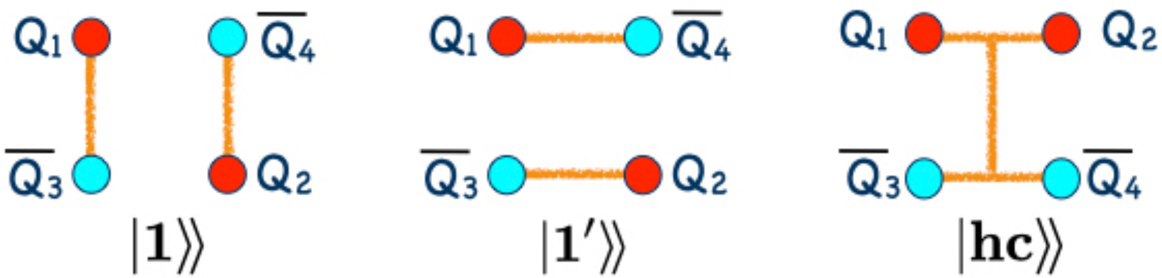
$\kappa = 0.10$ $\kappa = 0.12$ $\kappa = 0.14$

Tetra-Charms with novel confinement



Summary

- Confinement in the tetra (multi)-quark system is not trivial nor well established from the quark model viewpoints.
- In QCD string picture, two choices ($|1\rangle\rangle$, $|1'\rangle\rangle$) of quark-antiquark combinations are regarded as independent color configurations. Furthermore, the hidden-color state ($|hc\rangle\rangle$) can be introduced as an independent component.



- A new color confinement force is proposed and can be used for the quark model of multiquark systems. Mixings of $|hc\rangle\rangle$ state induce a strong attraction among the quarks.
- A bound state below the two-charmonia threshold appears due to the attraction. Furthermore, the complex scaling method provides us with two or more resonances that are consistent with experiment.