クォーク共鳴群法によるバリオン間短距離ポテンシャルの計算

Calculation of short range potential between baryons in the quark resonance group method

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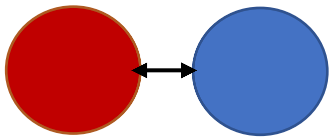
- **Introduction**
- Formulation
- Numerical Results
- Summary

Nuclear force determined by the experiments

- **Long range part**

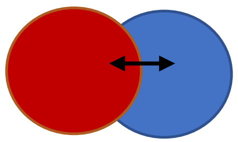
  attractive



➡ **Meson exchange**

- **Short range part**

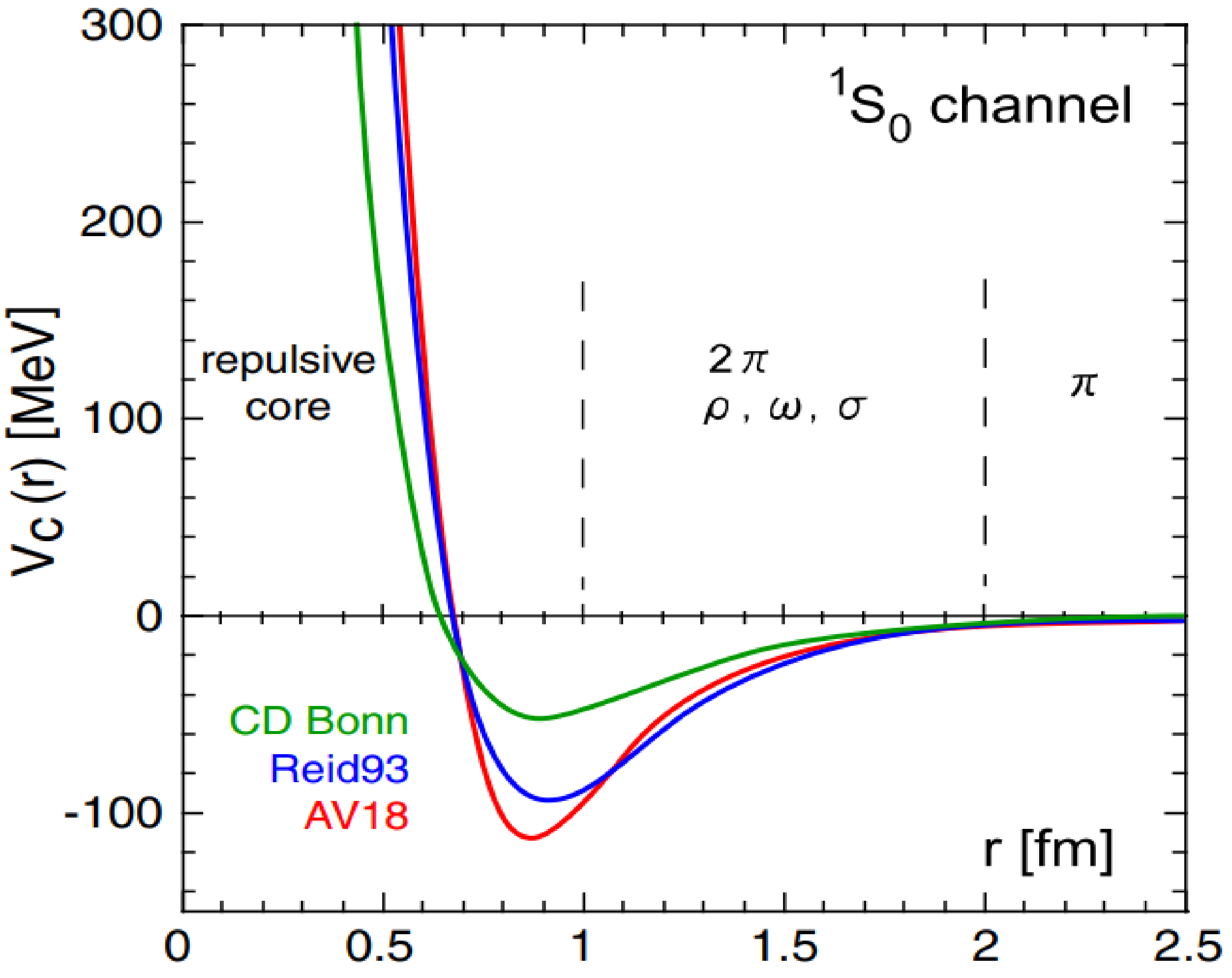
  repulsive



➡ **Pauli exclusion principle**

Spin-spin interaction

between quarks

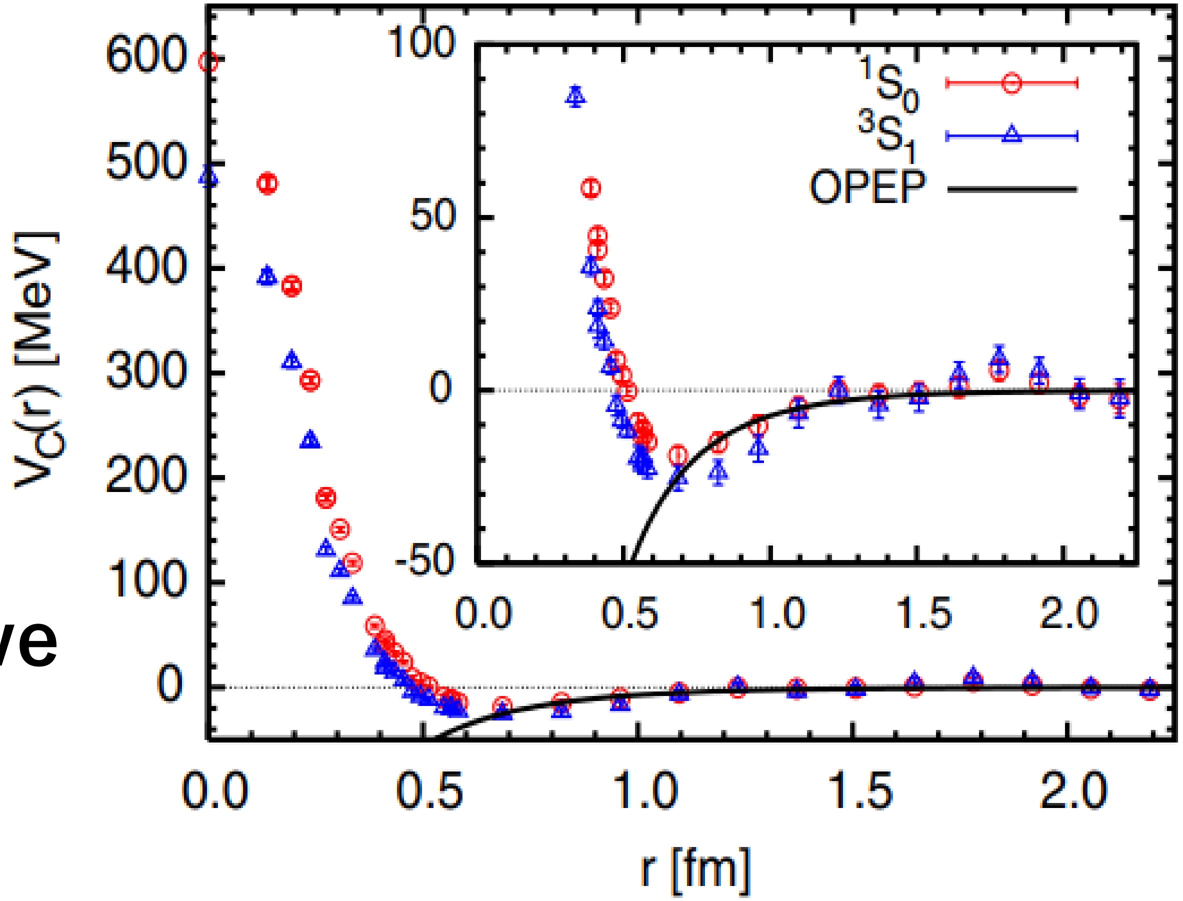


N. Ishii,S. Aoki,T. Hatsuda” PRL 99, 022001 (2007)

Lattice QCD simulation

Recently, nuclear force has been studied in lattice QCD simulation.

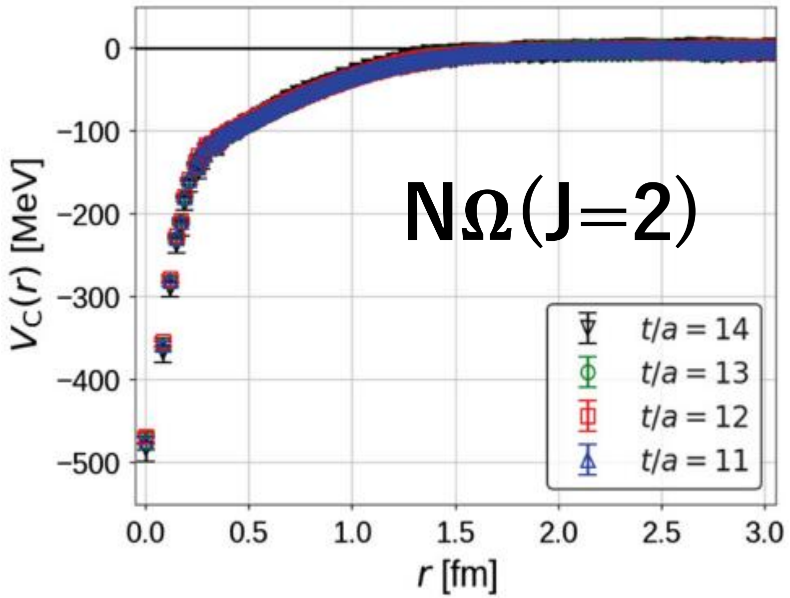
In particular, the HAL QCD method successfully reproduced the repulsive core of the nuclear force.



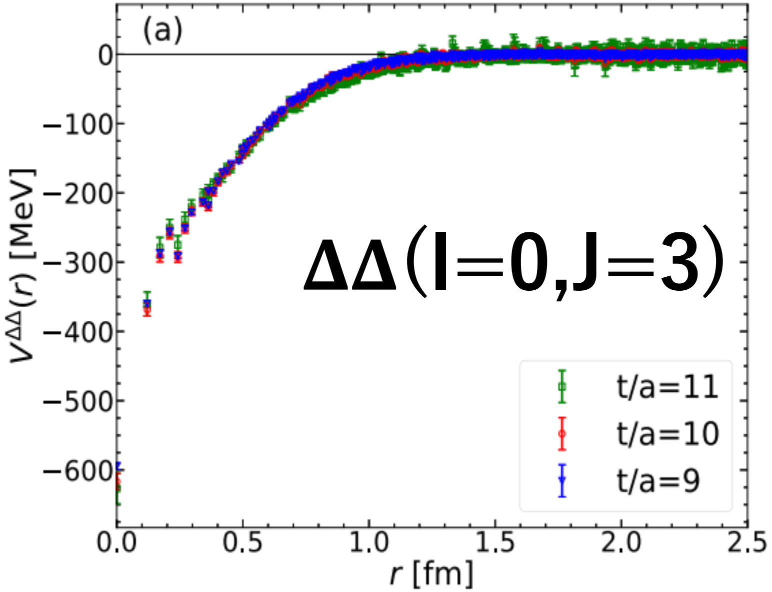
N. Ishii,S. Aoki,T. Hatsuda" PRL 99, 022001 (2007)

Lattice QCD simulation

Studies of various two-baryon systems including decuplets.



T. Iritani et al., Physics Letters B 792,284 (2017)



S. Gongyo et al.,a Physics Letters B 811, 135935 (2020)

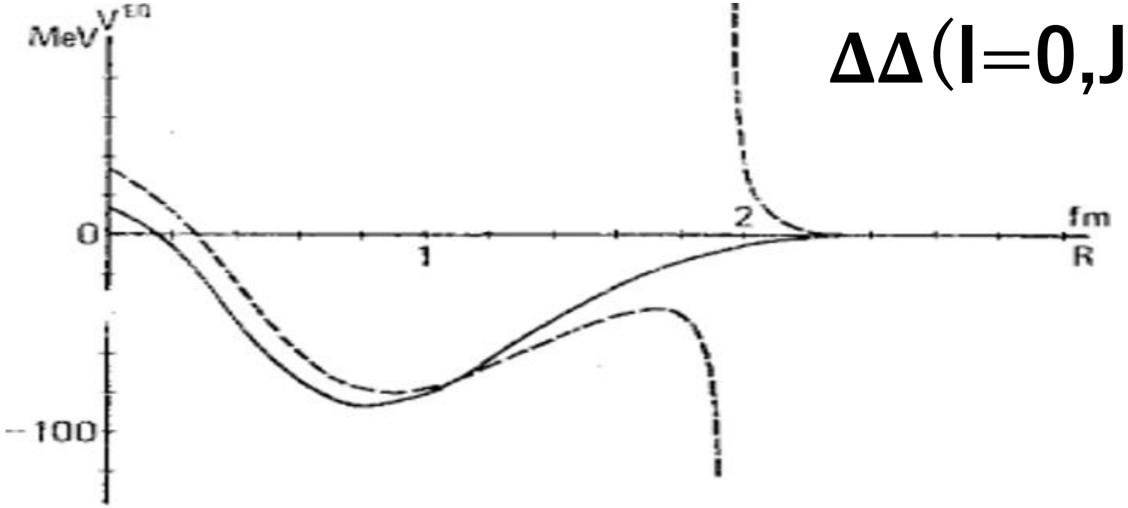
The lattice QCD simulations gives us hints of the quark dynamics inside baryons, which is reflected in the short-range potentials.

Quark model calculation

To confirm the quark dynamics, the constituent quark model will be suitable.

Potential of ΔΔ(I=0,J=3)

Previous calculations in quark models



M. Oka,K. Yazaki, Progress of Theoretical Physics,Vol 66,572(1981)

we want to update this systematically so as to include decuplets as well as octets.

Research Goals

Based on the quark model,
calculate short range local potential
by the resonance group method .

- **More precise calculation in quark model**
- **Non local potential ➡ Local potential**
- **Decuplet as well as octet**

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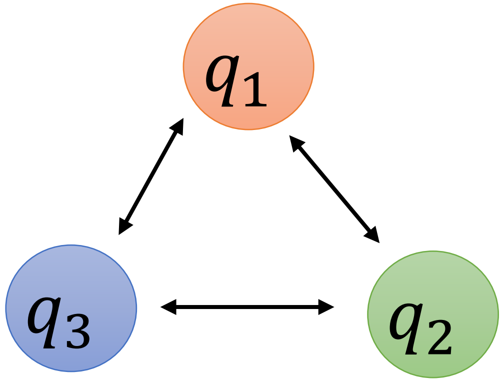
Strategy



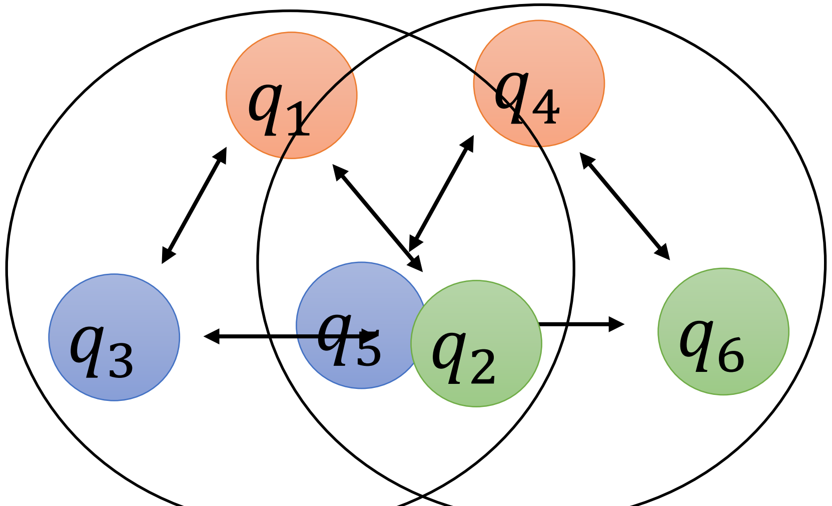
(1) Calculate the mass and wave function of one baryon.

Obtained precise wave function by **Gaussian expansion method** 。

Determine the parameters that reproduce the baryon masses well.

(2) Calculate the potential between two baryons by using the one-baryon wave function.

Two baryons in **the resonating group method**



How to calculate one baryon

Gaussian expansion method

The wave function is described by a set of Gaussian functions.

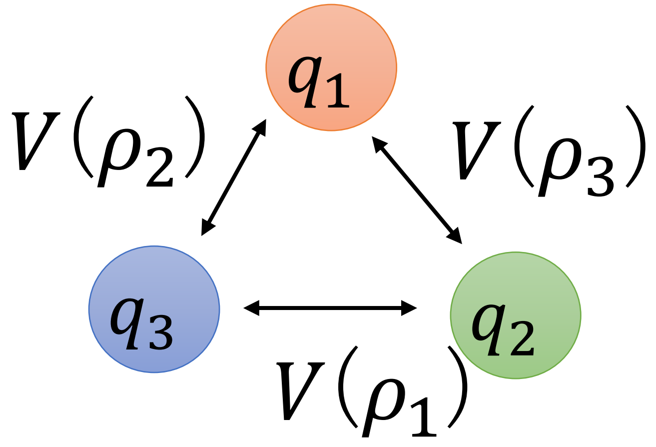
$$\Psi(\lambda,\rho)=\sum_{c=1}^{3}\sum_{n,n'=1}^{N}C_{c,n,n'}\exp(-\frac{\lambda_c^2}{r_n^2}-\frac{\rho_c^2}{r_{n'}^2})$$

Interactions between quarks in one baryon

Color electric interaction

Confining force

Color magnetic interaction as a spin-spin force



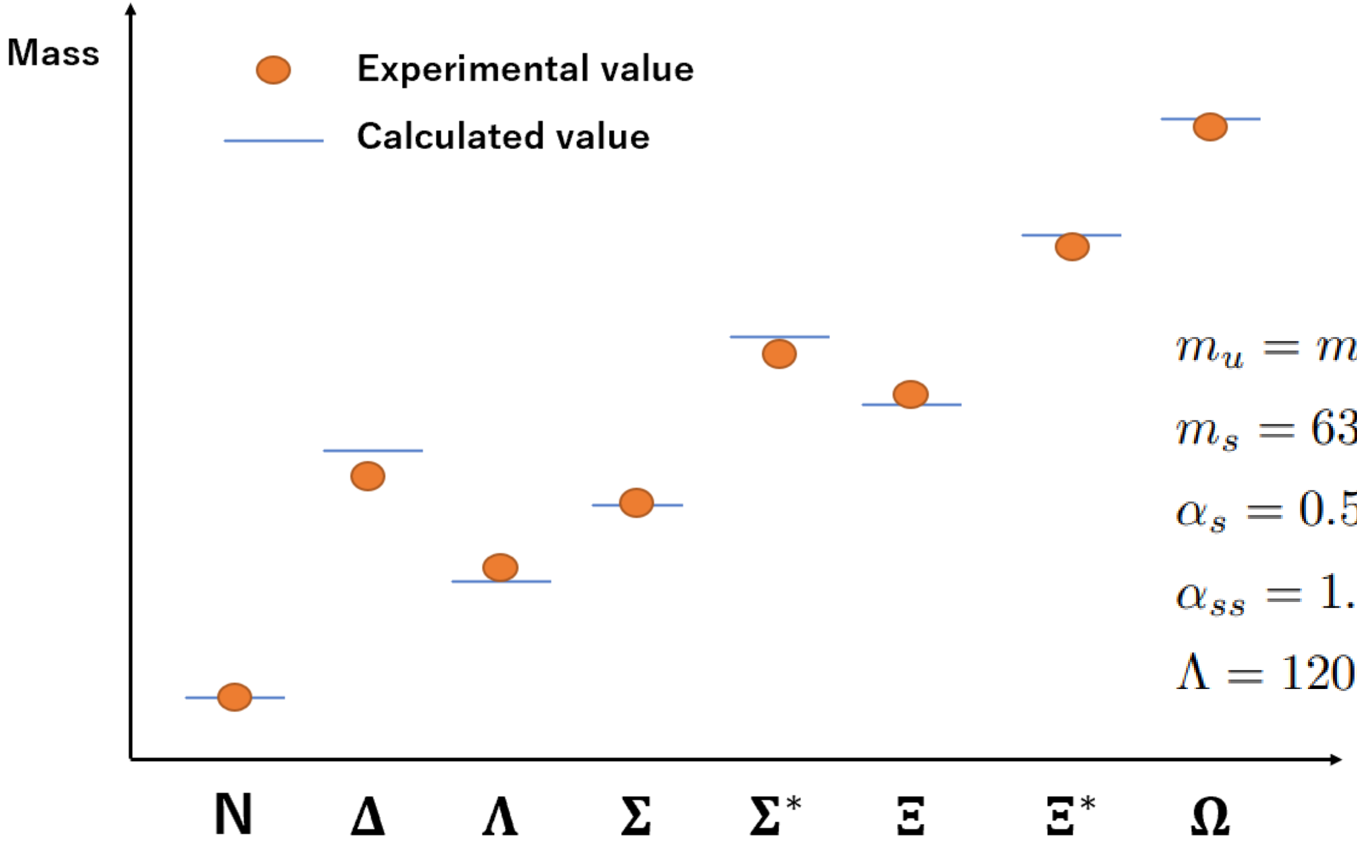
$$V(\rho)=-\frac{2\alpha_s}{3\rho}+\frac{k\rho}{2}+\frac{4\pi\alpha_{ss}}{9}\frac{\boldsymbol{\sigma_q}\cdot\boldsymbol{\sigma_q}}{m_q m_q}\frac{\Lambda^2}{4\pi r}e^{-\Lambda\rho}$$

Deltafunction like

Parameters

$$V(\rho) = -\frac{2\alpha_s}{3\rho} + \frac{k\rho}{2} + \frac{4\pi\alpha_{ss}}{9}\frac{\boldsymbol{\sigma_q}\cdot\boldsymbol{\sigma_q}}{m_q m_q}\frac{\Lambda^2}{4\pi r}e^{-\Lambda\rho}$$

baryons	Mass[MeV](calc / Exp)
$N(u,u,d:\frac{1}{2})$	939/939
$\Delta(u,u,u:\frac{3}{2})$	1260/1232
$\Lambda(u,d,s:\frac{1}{2})$	1103/1116
$\Sigma(u,u,s:\frac{1}{2})$	1187/1193
$\Sigma^*(u,u,s:\frac{3}{2})$	1406/1385
$\Xi(u,s,s:\frac{1}{2})$	1303/1318
$\Xi^*(u,s,s:\frac{3}{2})$	1545/1533
$\Omega(s,s,s:\frac{3}{2})$	1677/1672



$m_u = m_d = 363\text{MeV}$

$m_s = 632\text{MeV}$

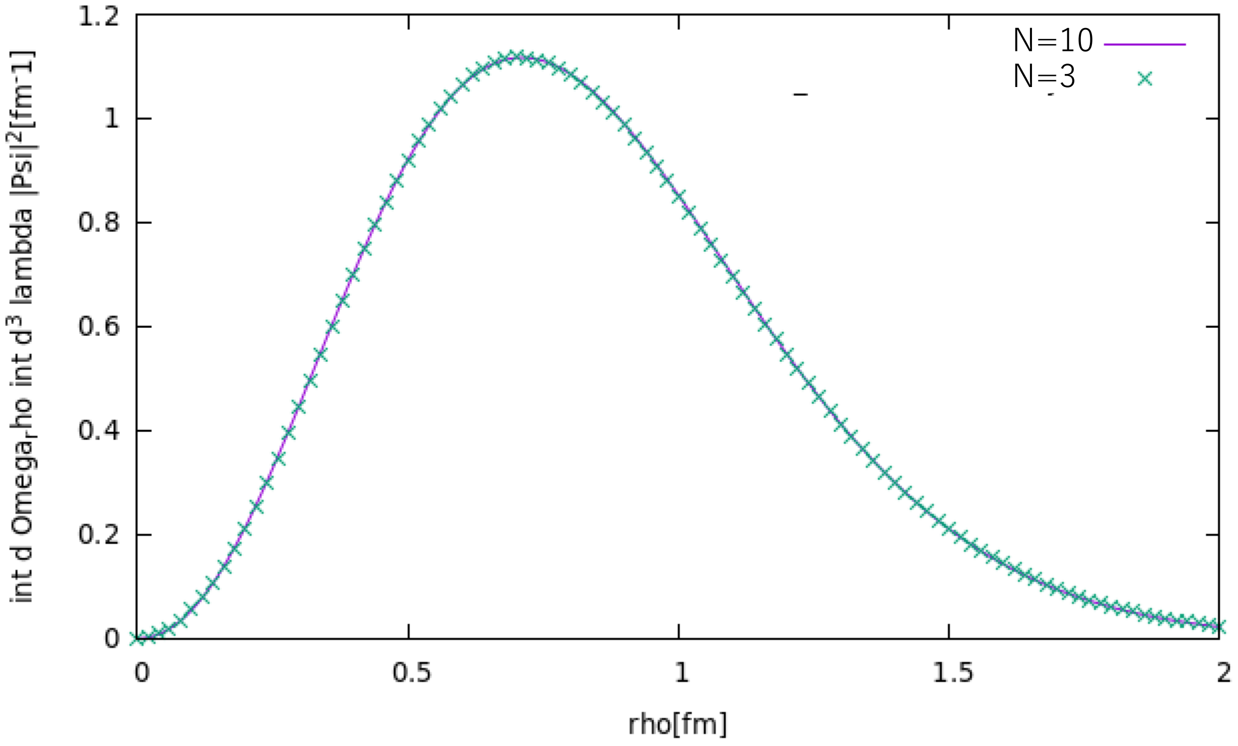
$\alpha_s = 0.5$

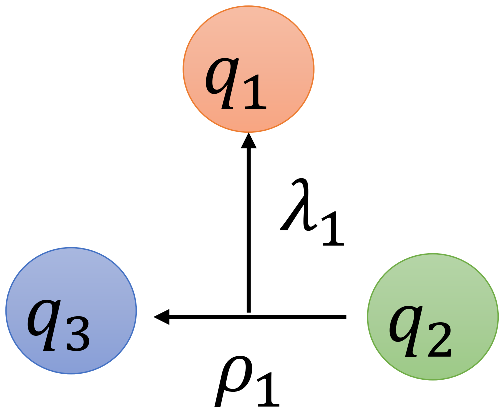
$\alpha_{ss} = 1.0$

$\Lambda = 1200\text{MeV}$

Wave function

u-u distribution in Δ^{++}





$$\Psi(\lambda,\rho)=\sum_{c=1}^{3}\sum_{n,n'=1}^{N}C_{c,n,n'}\exp\left(-\frac{\lambda_c^2}{r_n^2}-\frac{\rho_c^2}{r_{n'}^2}\right)$$

Distribution $\int d\Omega_\rho \int d^3\lambda|\Psi|^2$

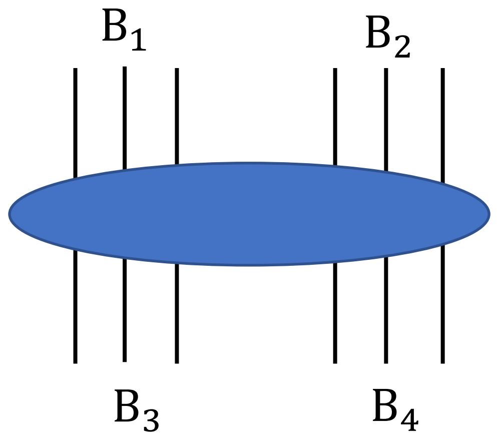
The wave function of N = 3 is good enough to calculate the distribution of quarks.

Interaction between baryons

- **Pauli exclusion principle**
- **Color magnetic interaction**

← significant in short range baryon-baryon interaction.

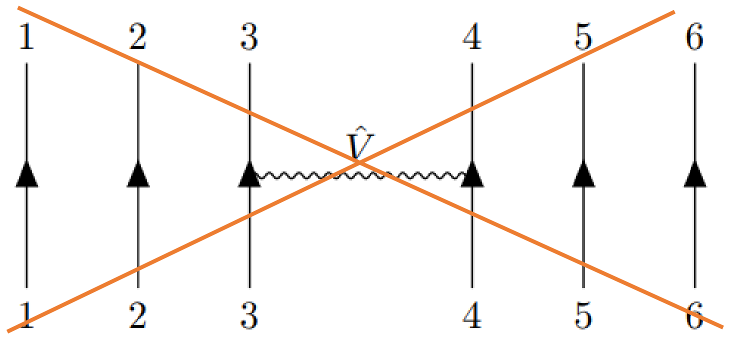
The most general diagram of the two-baryon system.



➡ **In the present study we focus on the color magnetic interaction.**

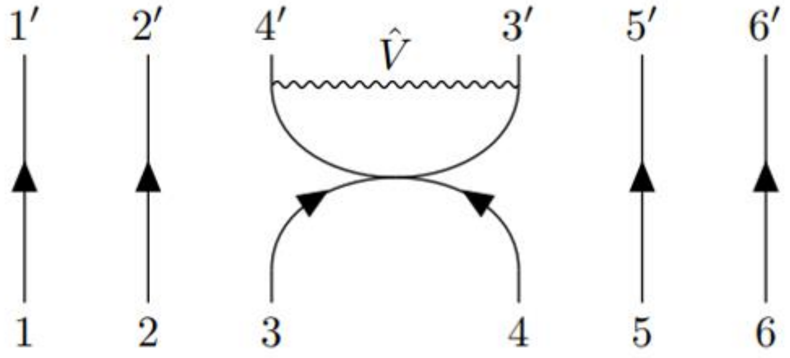
Color magnetic interaction

Owing to the color symmetry,
gluon cannot mediate between color singlets.



Therefore, when gluon mediates,
the quark exchange also takes place at the same time.

Here we show the results
with the following diagram.



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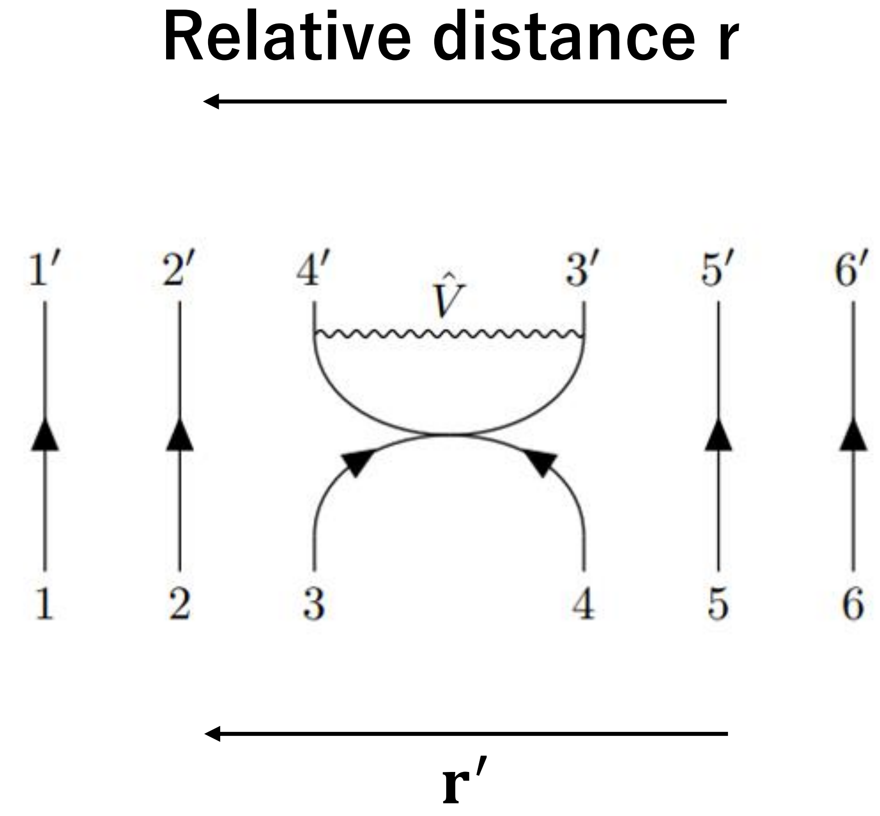
Local potential

ΔΔ(I=3,J=0),ΔΔ(I=0,J=3)

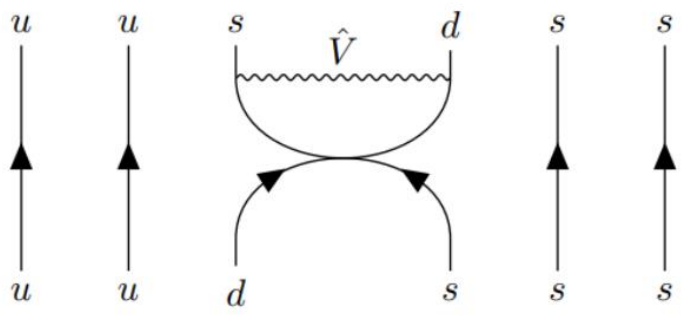
In general, non-local $\mathbf{r} \neq \mathbf{r}'$.

The interaction range of color magnetic interaction is short,
so it can be regarded as $\mathbf{r} = \mathbf{r}'$.

➡local potential



Local potential



pΩ(J=2)

They don't have quarks of same flavor, so $\boldsymbol{p\Omega}$ have to interact via other channels.
Then, non-local potential would be essential.

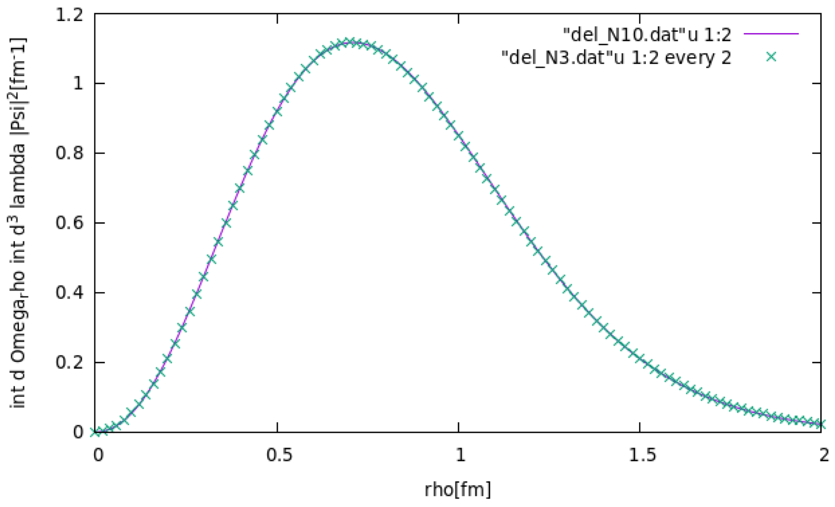
$$N\Omega \Rightarrow \Lambda\Xi^{*0} \text{ - } \Sigma^0\Xi^0 \text{ - } \Sigma^+\Xi^{*-} \text{ - } \Sigma^{*0}\Xi^0 \text{ - } \Sigma^{*+}\Xi^- \text{ - } \Sigma^{*0}\Xi^{*0} \text{ - } \Sigma^{*+}\Xi^{*-} \Rightarrow N\Omega$$

Our strategy:

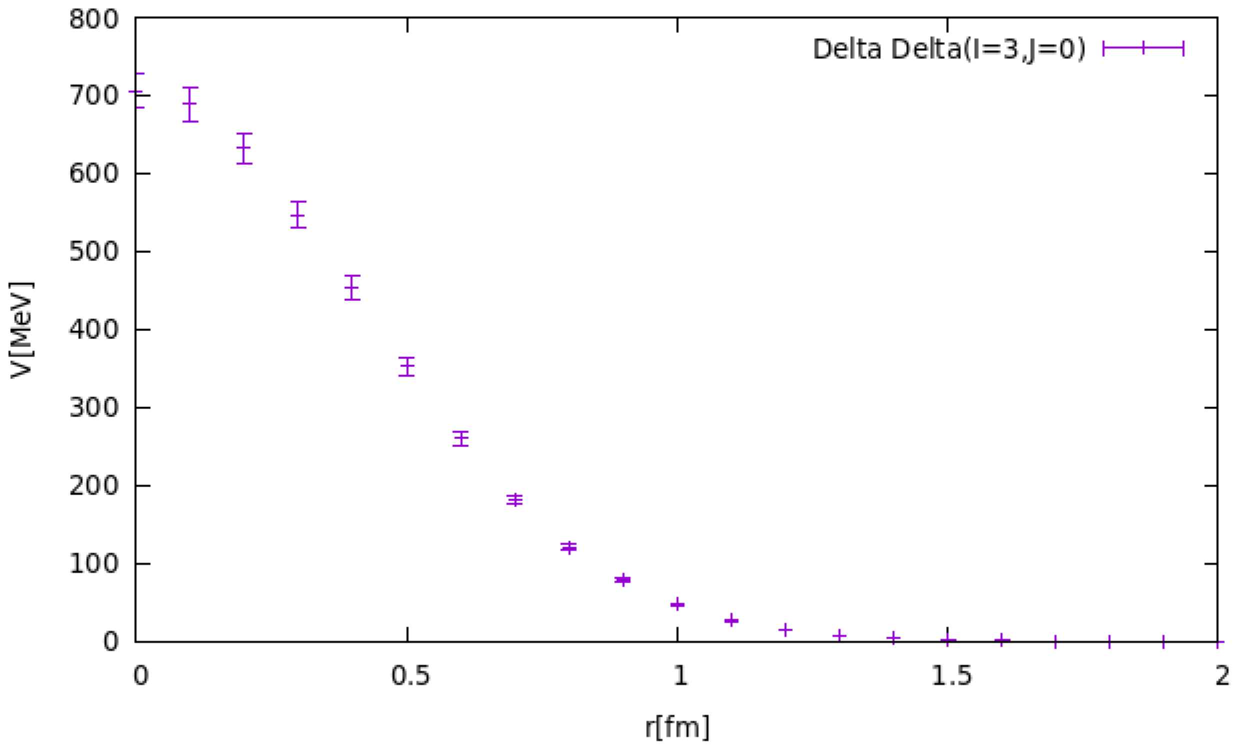
- Calculate $\boldsymbol{p\Omega}$ relative wave function $\chi(\boldsymbol{r})$ at $\boldsymbol{E} = \boldsymbol{m_p} + \boldsymbol{m_\Omega}$
- Calculate equivalent local potential via $\mathbf{V}(\mathbf{r}) = \frac{1}{2\mu\chi(r)}\nabla^2\chi(\boldsymbol{r})$

Numerical results

ΔΔ(I=3,J=0)



- We obtain local potential of the range 1fm.

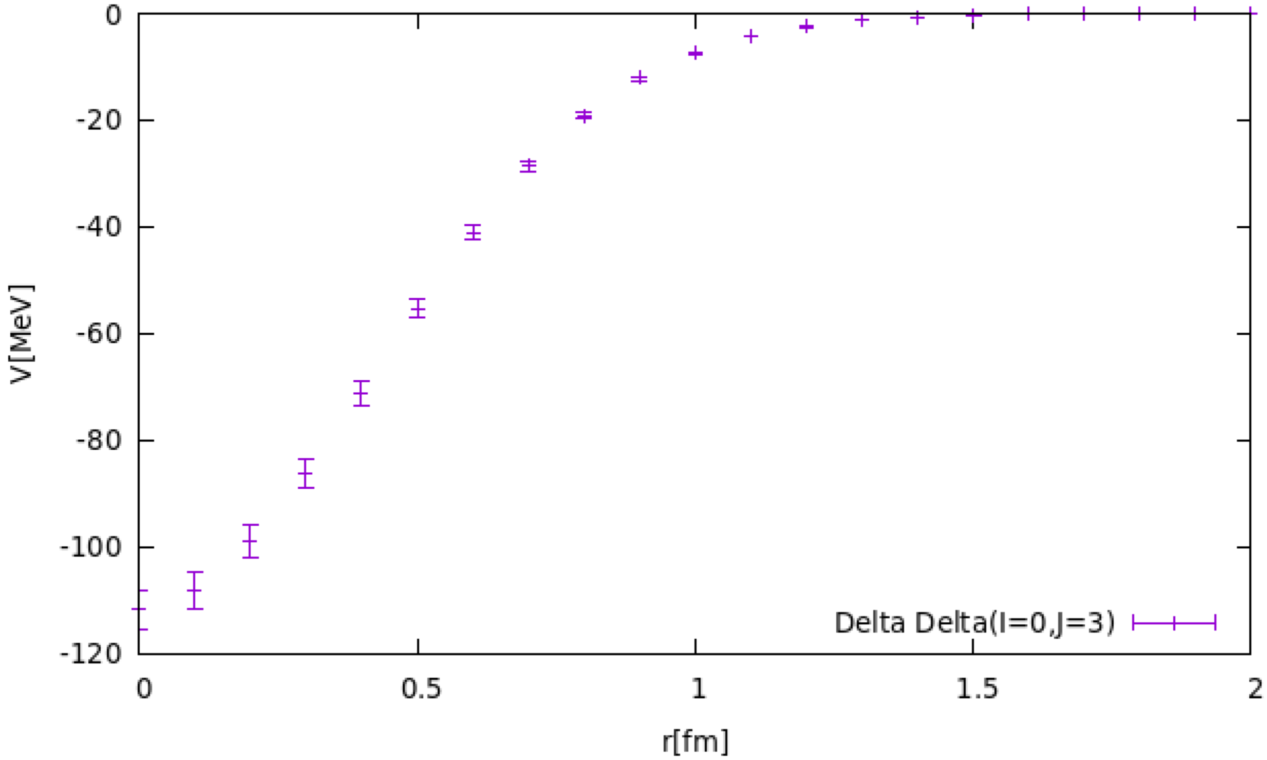


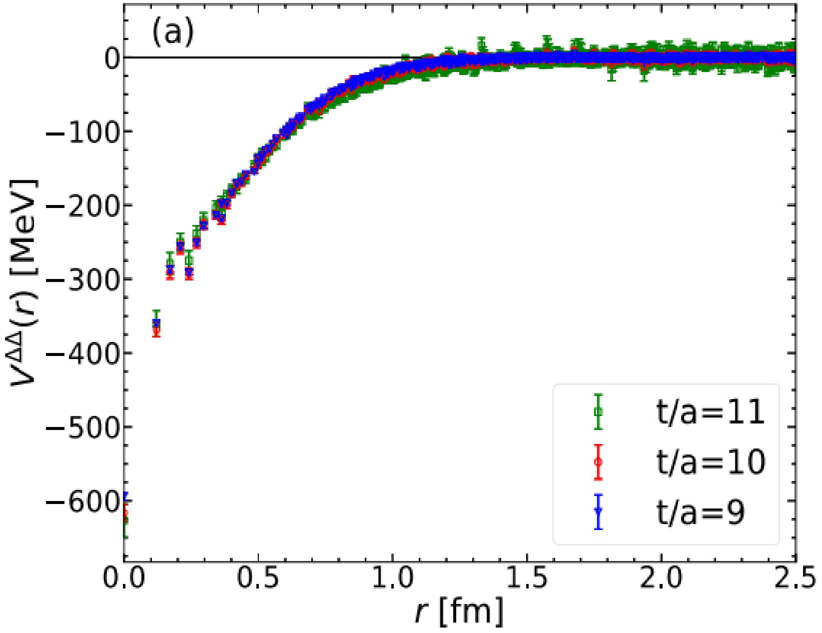
- Color magnetic interaction brings repulsive core in the I=3, J=0 channel.

Numerical results

ΔΔ(I=0,J=3)

- **Color magnetic interaction brings attriactive core in the I=0, J=3 channel.**
- **Compared with the Gongyo et al., the attraction in I=0, J=3 is weak.**
  ➡Other contribution is necessary.

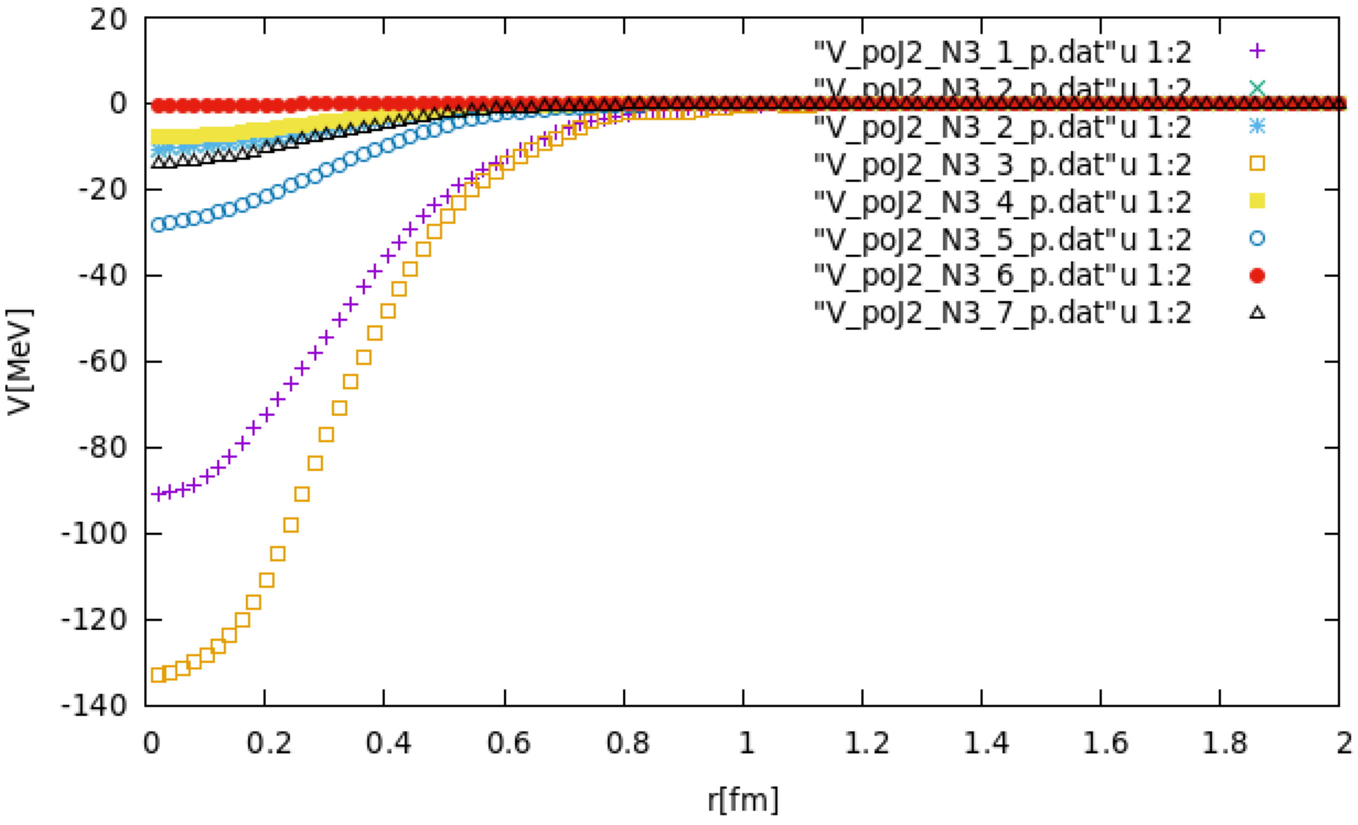




S. Gongyo et al.,a Physics Letters B 811, 135935 (2020)

Numerical results

NΩ(J=2)



(1) $N\Omega \Rightarrow \Lambda\Xi^{*0}$ - $\Rightarrow N\Omega$

(2) $N\Omega \Rightarrow \Sigma^{0}\Xi^{0}$ - $\Rightarrow N\Omega$

(3) $N\Omega \Rightarrow \Sigma^{+}\Xi^{*-}$ - $\Rightarrow N\Omega$

(4) $N\Omega \Rightarrow \Sigma^{*0}\Xi^{0}$ - $\Rightarrow N\Omega$

(5) $N\Omega \Rightarrow \Sigma^{*+}\Xi^{-}$ - $\Rightarrow N\Omega$

(6) $N\Omega \Rightarrow \Sigma^{*0}\Xi^{*0}$ - $\Rightarrow N\Omega$

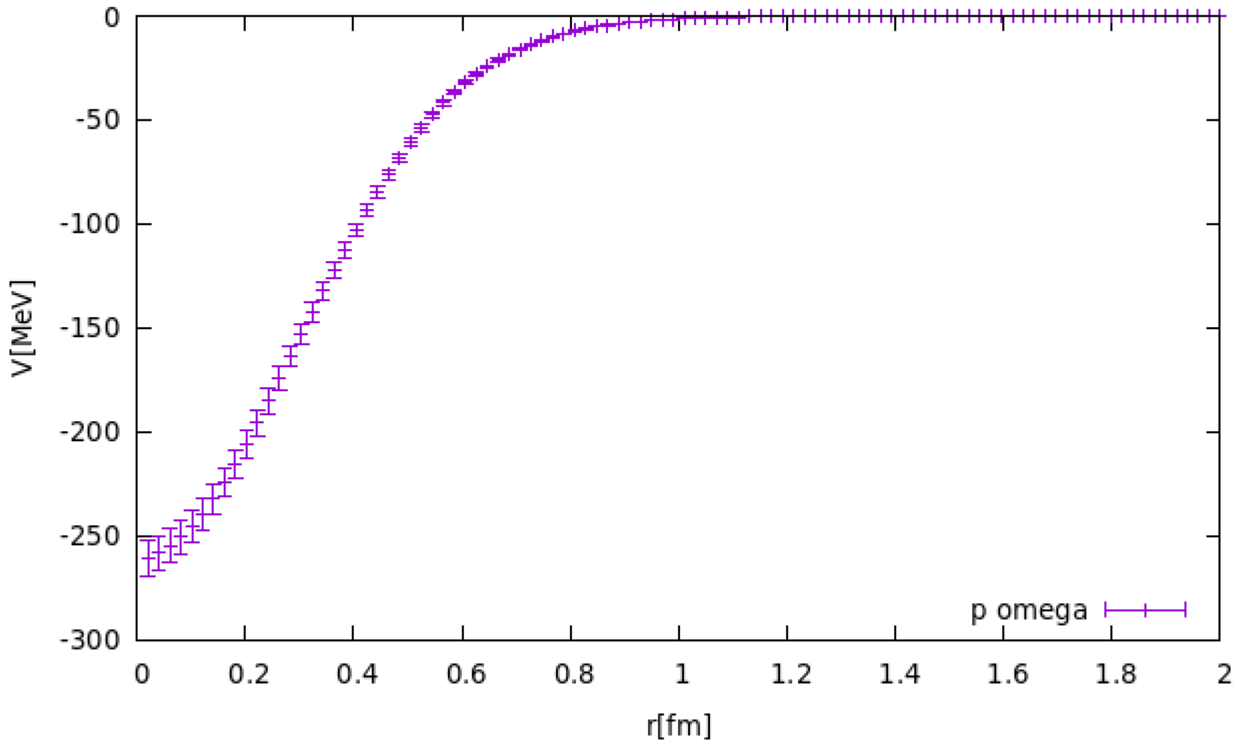
(7) $N\Omega \Rightarrow \Sigma^{*+}\Xi^{*-} \Rightarrow N\Omega$

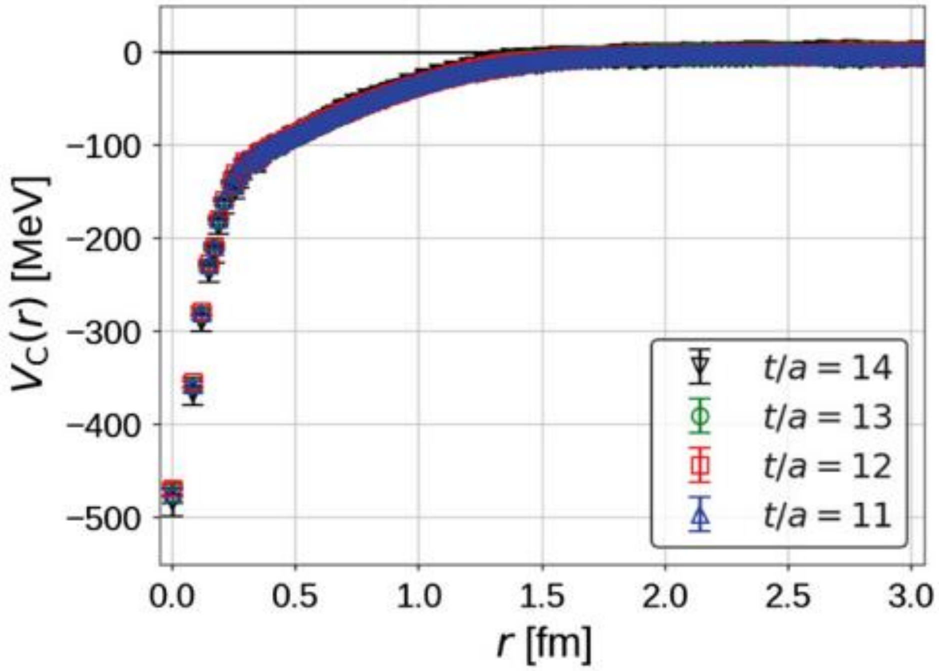
(3) Works as the most attractive

Numerical results

NΩ(J=2) $N\Omega \Rightarrow \Lambda\Xi^{*0}$ - $\Sigma^0\Xi^0$ - $\Sigma^+\Xi^{*-}$ - $\Sigma^{*0}\Xi^0$ - $\Sigma^{*+}\Xi^-$ - $\Sigma^{*0}\Xi^{*0}$ - $\Sigma^{*+}\Xi^{*-}$ $\Rightarrow N\Omega$

Compared with the Iritani et al., the attraction is weak.





T. Iritani et al., Physics Letters B 792,284 (2017)

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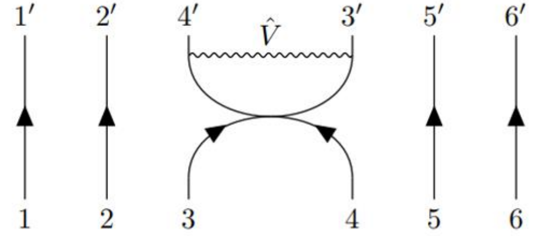
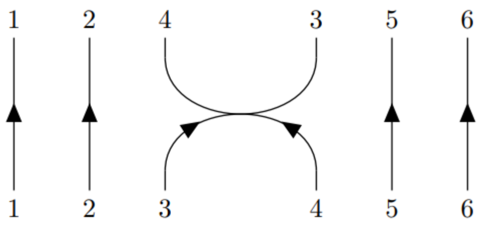
Summary

- We calculated baryon-baryon potential of $\Delta\Delta$(I=3,J=0), $\Delta\Delta$(I=0,J=3) and NΩ(J=2) in the resonating group method by considering the color magnetic interaction in this diagram.

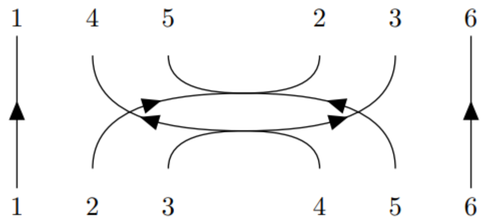


- The color magnetic interaction brings attraction in the $\Delta\Delta$(I=0,J=3), while the attraction in the NΩ(J=2) comes from the coupled channels.



- **Prospects for the future**

Evaluate the most general two-baryon diagram, which contains, for example, these diagrams.



How to calculate potential between baryons

- **The resonating group method**

$\hat{a}^\dagger \cdots$ creation operater

$\hat{a} \cdots$ annihilation operater

Baryon‘s Wave Function $|\mathrm{B}\rangle$ and potential operators $\hat{\mathrm{V}}$ Define as follows .

$$|B\rangle = \sum \mathcal{W} \int d^3r_1 d^3r_2 d^3r_3 \psi(\boldsymbol{r_1})\psi(\boldsymbol{r_2})\psi(\boldsymbol{r_3})\hat{a}^\dagger_{f_1,s_1,c_1}(\boldsymbol{r_1})\hat{a}^\dagger_{f_2,s_2,c_2}(\boldsymbol{r_2})\hat{a}^\dagger_{f_3,s_3,c_3}(\boldsymbol{r_3})|0\rangle$$

$$\hat{V} = \frac{1}{2}\int d^3r_A d^3r_A \frac{-2\pi\alpha_{ss}}{3m_A m_B}\delta^3(\boldsymbol{r_A}-\boldsymbol{r_B}) \quad \hat{a}^\dagger_{f'_A,s'_A,c'_A}(\boldsymbol{r_A})\frac{\lambda_{c_A,c_{A'}}}{2}\sigma_{s_A,s_{A'}}\hat{a}_{f_A,s_A,c_A}(\boldsymbol{r_A})\hat{a}^\dagger_{f'_B,s'_B,c'_B}(\boldsymbol{r_B})\frac{\lambda_{c_B,c_{B'}}}{2}\sigma_{s_B,s_{B'}}\hat{a}_{f_B,s_B,c_B}(\boldsymbol{r_B})$$

Then, the expected value of the potential is as follows.

$$V = \langle BB|\hat{V}|BB\rangle$$

ex） exchange 4 with 3.

$$V_{34} = \langle BB|\hat{V}|BB\rangle \Rightarrow \langle 0|\hat{a}_6\hat{a}_5\hat{a}_4\hat{a}_3\hat{a}_2\hat{a}_1(\hat{a}^\dagger_{A'}\frac{\lambda_{c_A,c_{A'}}}{2}\sigma_{s_A,s_{A'}}\hat{a}_A)(\hat{a}^\dagger_{B'}\frac{\lambda_{c_B,c_{B'}}}{2}\sigma_{s_B,s_{B'}}\hat{a}_B)\hat{a}^\dagger_{1'}\hat{a}^\dagger_{2'}\hat{a}^\dagger_{3'}\hat{a}^\dagger_{4'}\hat{a}^\dagger_{5'}\hat{a}^\dagger_{6'}|0\rangle$$

$$= \langle 0|\hat{a}_4\hat{a}_3(\hat{a}^\dagger_{A'}\frac{\lambda_{c_A,c_{A'}}}{2}\sigma_{s_A,s_{A'}}\hat{a}_A)(\hat{a}^\dagger_{B'}\frac{\lambda_{c_B,c_{B'}}}{2}\sigma_{s_B,s_{B'}}\hat{a}_B)\hat{a}^\dagger_3\hat{a}^\dagger_4|0\rangle\delta_{1-1'}\delta_{2-2'}\delta_{5-5'}\delta_{6-6'}$$

Select interactions.