

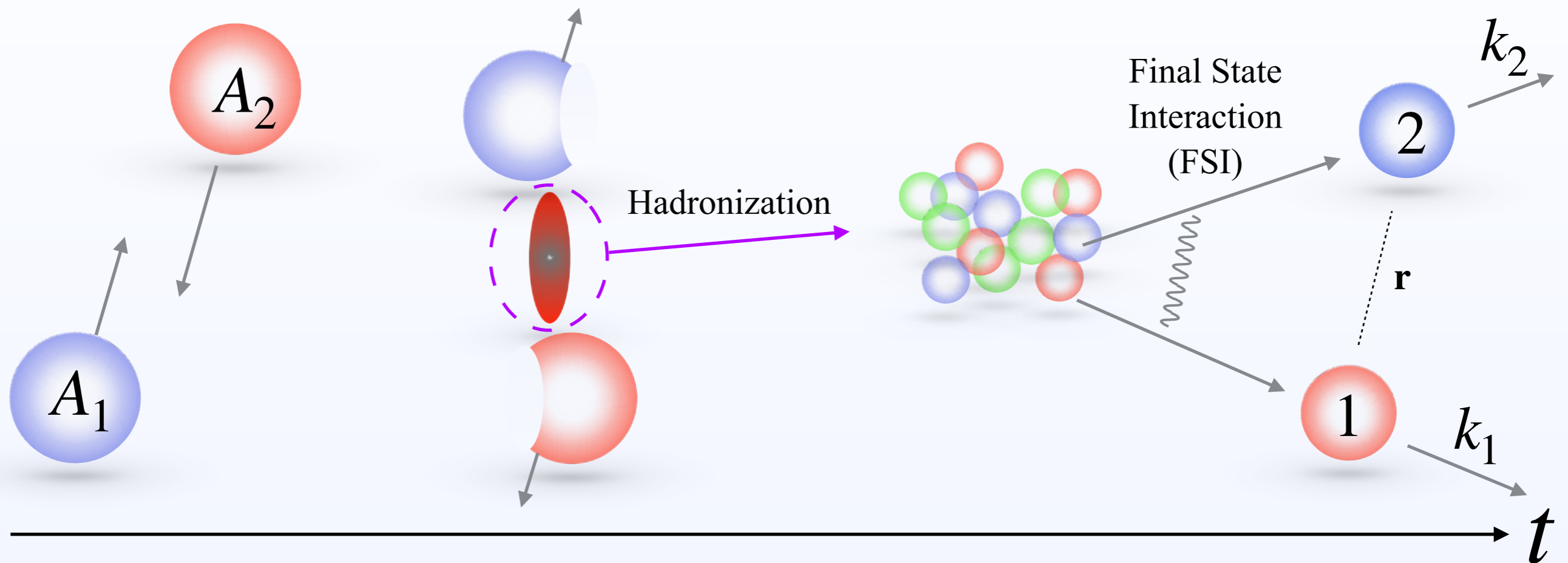
Yuki Kamiya  
HISKP, Bonn Univ.

# ハドロン運動量相関を用いたチャームハドロン 相互作用の研究

ハドロン分光に迫る反応と構造の物理  
@ online 2022/12/6

# Hadron correlation in high energy nuclear collision

- High energy nuclear collision and FSI

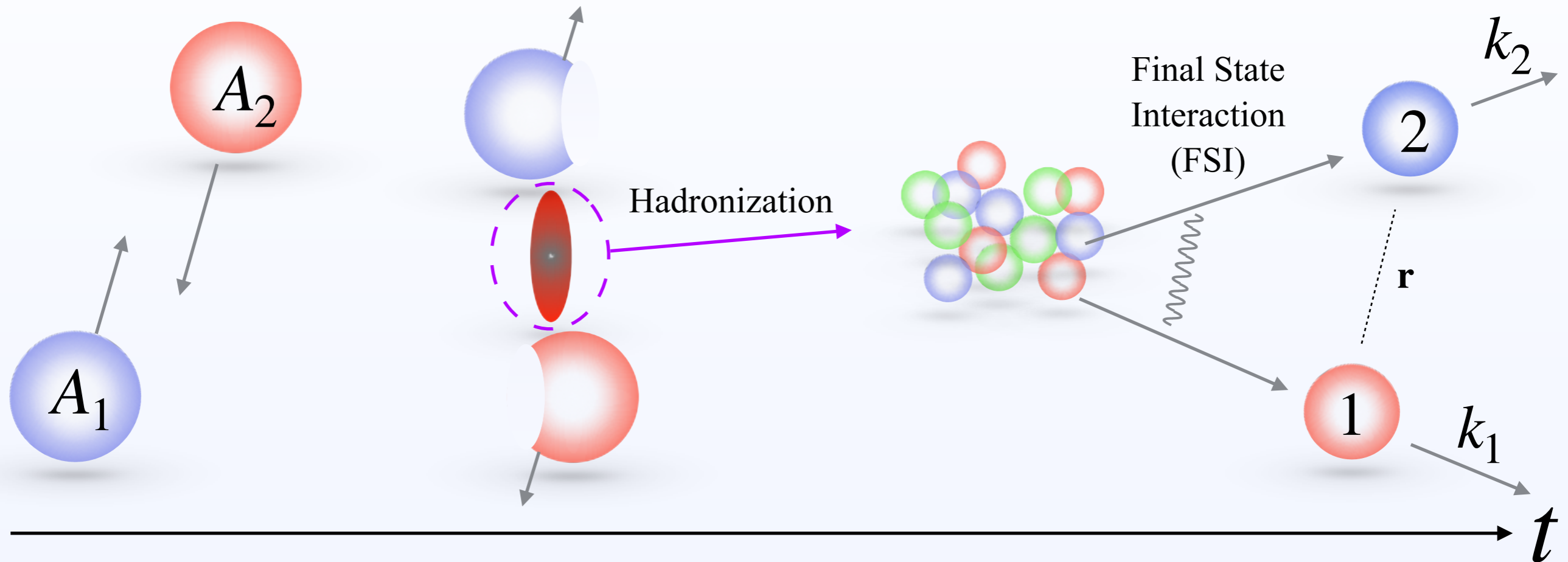


- Hadron-hadron correlation

$$C_{12}(k_1, k_2) = \frac{N_{12}(k_1, k_2)}{N_1(k_1)N_2(k_2)}$$
$$= \begin{cases} 1 & \text{(w/o correlation)} \\ \text{Others (w/ correlation)} \end{cases}$$

# Hadron correlation in high energy nuclear collision

- High energy nuclear collision and FSI



- Hadron-hadron correlation

- Koonin-Pratt formula : S.E. Koonin, PLB 70 (1977)  
S. Pratt et. al. PRC 42 (1990)

$$C(\mathbf{q}) \simeq \int d^3\mathbf{r} S(\mathbf{r}) |\varphi^{(-)}(\mathbf{q}, \mathbf{r})|^2$$

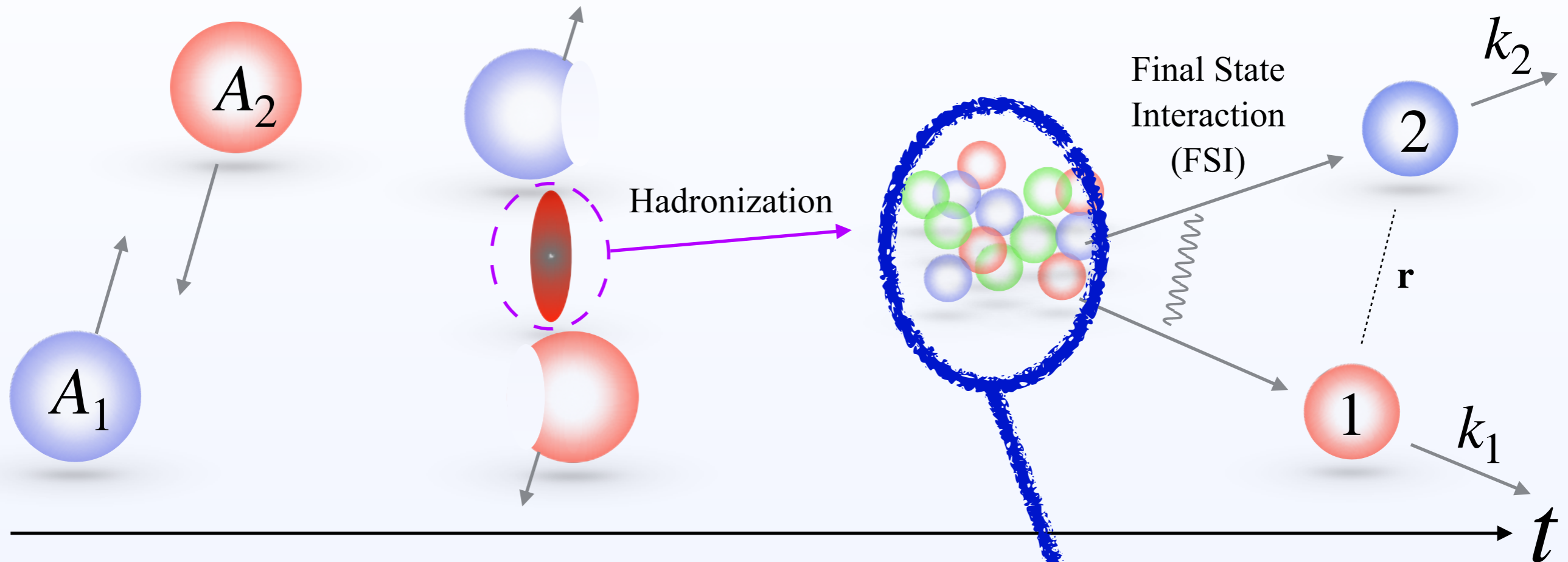
$$\mathbf{q} = (m_2\mathbf{k}_1 - m_1\mathbf{k}_2)/(m_1 + m_2)$$

$S(\mathbf{r})$  : Source function

$\varphi^{(-)}(\mathbf{q}, \mathbf{r})$  : Relative wave function

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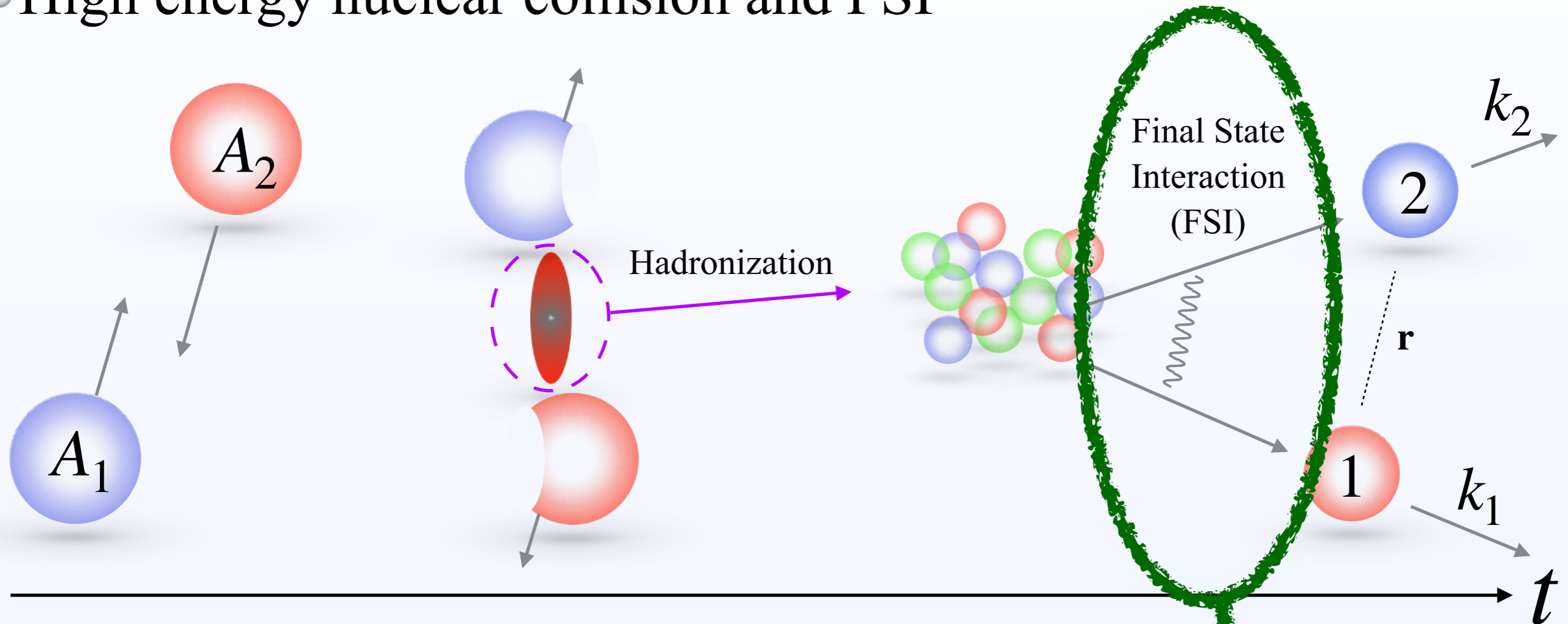
$S(\mathbf{r})$  : Source function

$\varphi^{(-)}(\mathbf{q}, \mathbf{r})$  : Relative wave function

- Depends on ...
- Collision detail ( $A_i$ , energy, centrality)
- Including information of...
  - size of hadron source,
  - momentum dependence, weight...

# Hadron correlation in high energy nuclear collision

- High energy nuclear collision and FSI



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- Depends on ...

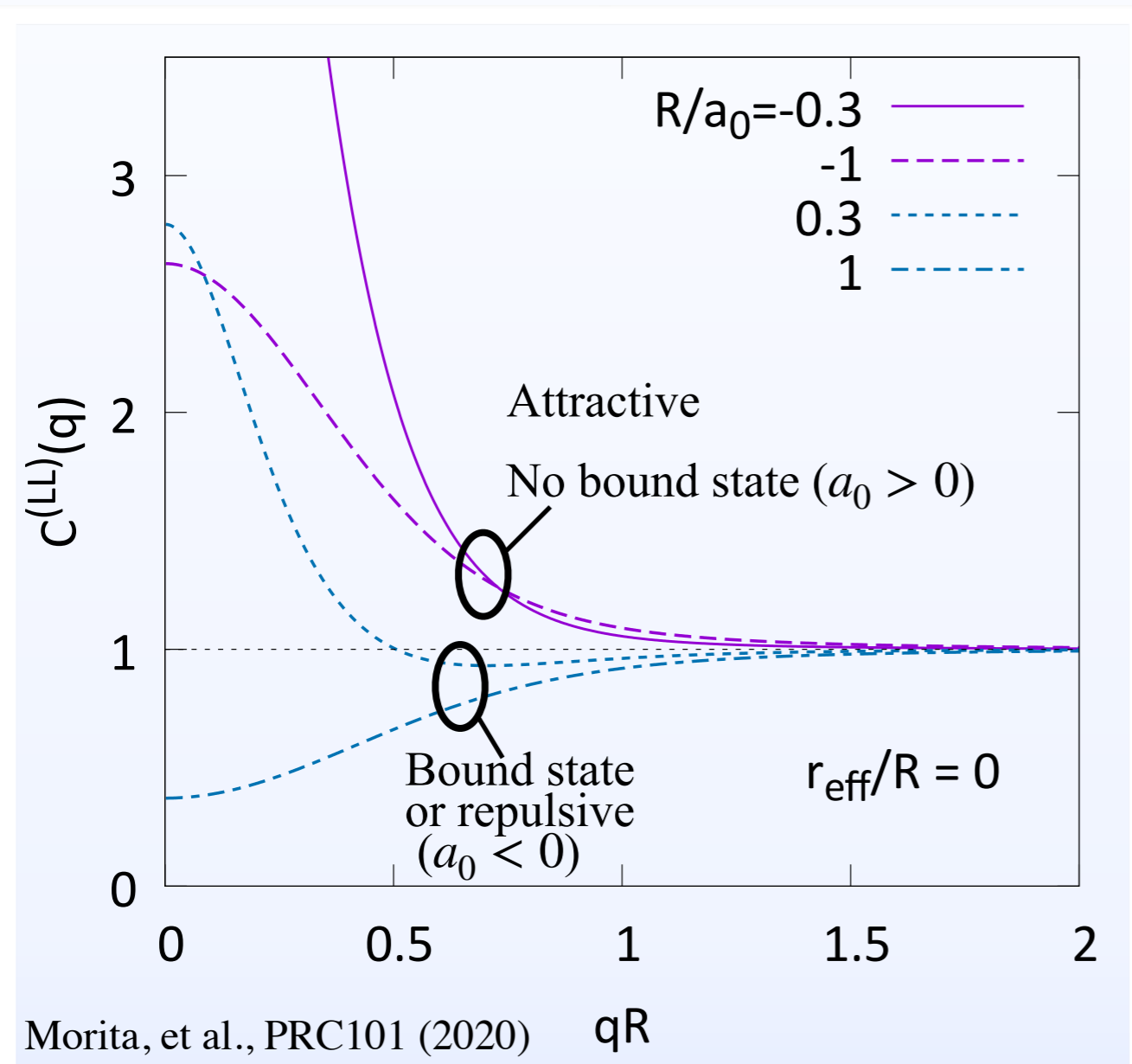
Interaction (strong and Coulomb)

quantum statistics (Fermion, boson)

# Hadron correlation in high energy nuclear collision

- Line shapes of  $C(q)$ : relation to interaction

$$C(\mathbf{q}) \simeq \int d^3\mathbf{r} S(\mathbf{r}) |\varphi^{(-)}(\mathbf{q}, \mathbf{r})|^2$$



- Scattering length  $a_0$  and source size  $R$  determines the suppression/enhancement of line shape \*  $a_0 = \mathcal{F}(q = 0)$

- Repulsive int. ( $a_0 < 0$ , small  $|a_0|$ )

➔ Suppressed  $C(q)$

- Attractive int. w/ bound state ( $a_0 < 0$ , large  $|a_0|$ )

➔ Suppressed  $C(q)$  for Large  $R$   
Enhanced  $C(q)$  for small  $R$

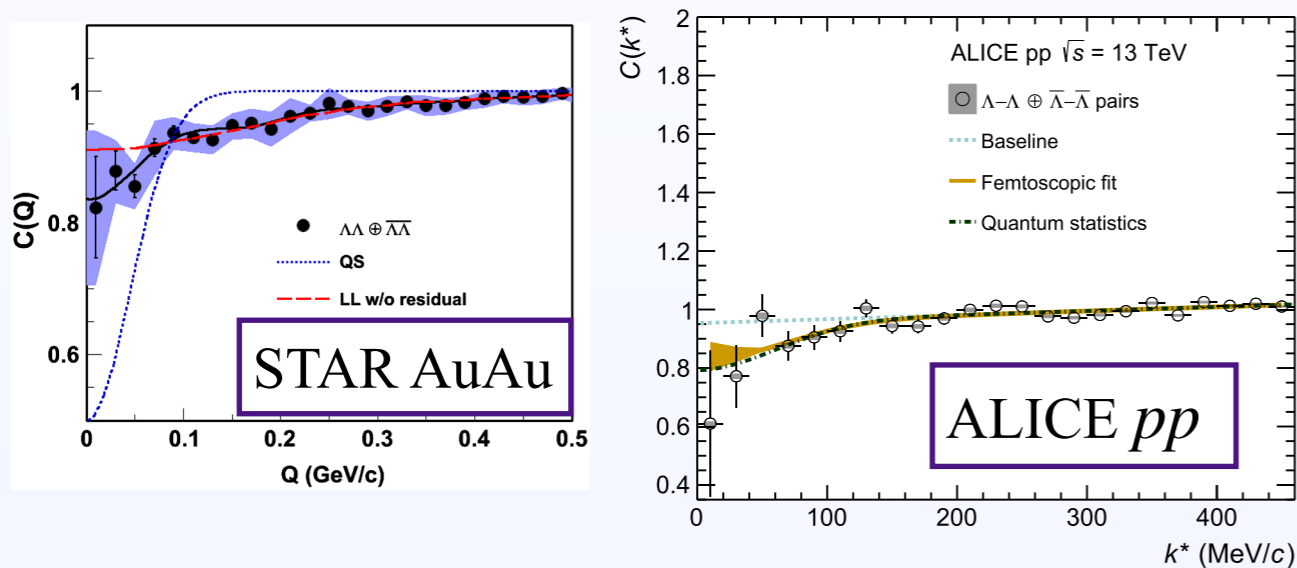
- Attractive int. w/o bound state ( $a_0 > 0$ )

➔ Enhanced  $C(q)$

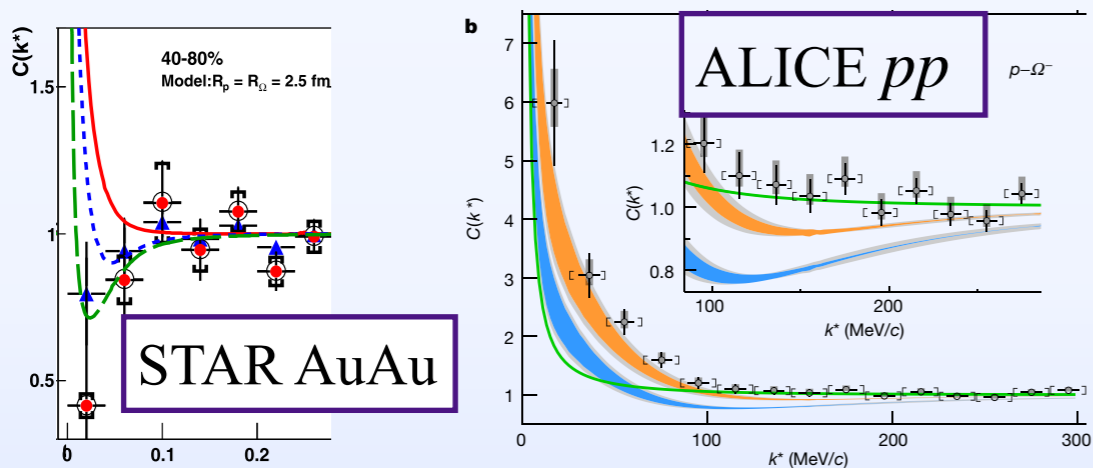
# Hadron correlation in high energy nuclear collision

## Experimental data in various sectors

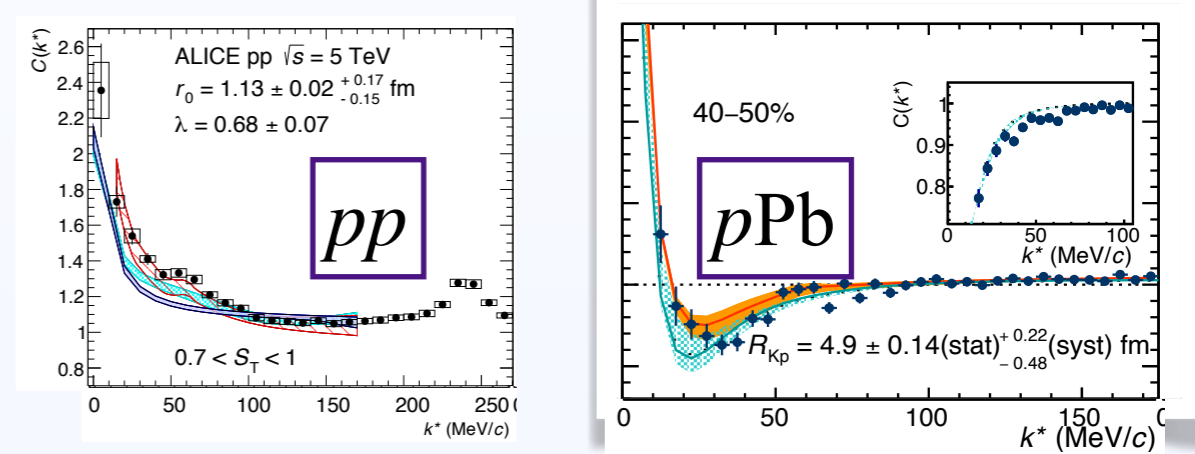
- $\Lambda\Lambda$ 
  - STAR AuAu: PRL 114,022301(2015)
  - ALICE pp: PLB 797 (2019) 134822
  - PbPb: PRC99, 024001 (2019)



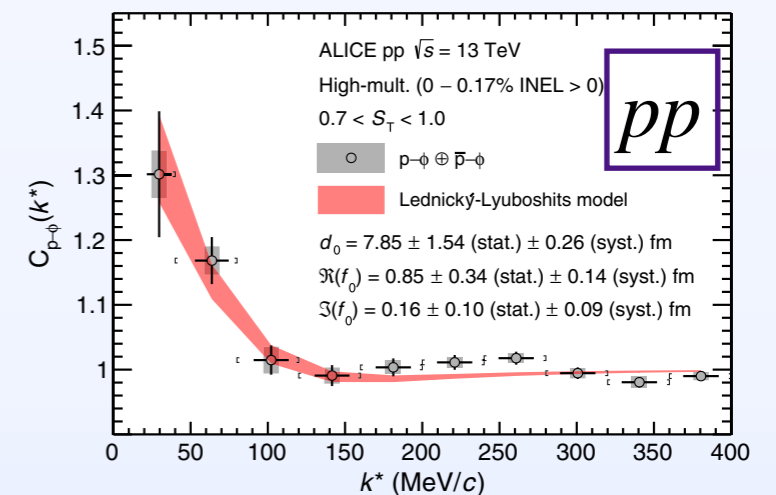
- $p\Omega$ 
  - STAR AuAu: PLB 790, 490 (2019)
  - ALICE pp: Nature 588 (2020) 232



- $K^\pm p$ 
  - ALICE pp: PRL 124 (2020) 9, 092301
  - PbPb: PLB 822 (2021) 136708
  - STAR AuAu: NPA 982 (2019) 359



- $p\phi$ 
  - ALICE pp: PRL 127 (2021) 17, 172301



- How to construct correlation model from theory;  $\mathcal{F}(q) \rightarrow C(q)$

- Using effective potential

- Construct the eff. potential by reproducing the amplitude  $\mathcal{F}$  (or threshold parameters  $(a_0, r_e)$ )

- Solving the Schrödinger eq.  $\longrightarrow \varphi$

- Using half offshell  $T$ -matrix  $T_l(q, k; E)$  Haidenbauer, Nuclear Physics A 981 (2019) 1–16

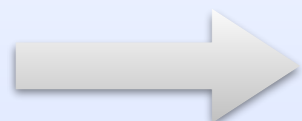
- $T_l(q, k; E) \longrightarrow \varphi$

$$\tilde{\psi}(k, r) = j_l(kr) + \frac{1}{\pi} \int j_l(qr) dq q^2 \frac{1}{E - E_1(q) - E_2(q) + i\epsilon} T_l(q, k; E)$$

- Using Lednicky-Lyuboshitz formula

- Direct relation between  $C(q)$  and  $\mathcal{F}(q)$

- Detail  $\longrightarrow$  Next slide



Comparison of model predictions and correlation data



# Hadron correlation in high energy nuclear collision

- How to extract interaction from Correlation data;  $C(q) \rightarrow \mathcal{F}(q)$

- Lednicky-Lyuboshitz (LL) formula R. Lednicky, et al. Sov. J. Nucl. Phys. 35(1982).


- Approximate  $\varphi$  by asymptotic wave func.(s-wave only)

$$C(\mathbf{q}) \simeq \int d^3\mathbf{r} S(\mathbf{r}) |\varphi^{(-)}(\mathbf{q}, \mathbf{r})|^2$$

$$\varphi^{(-)}(\mathbf{q}, \mathbf{r}) \xrightarrow{r \rightarrow \infty} \exp(-i\mathbf{q} \cdot \mathbf{r}) + \frac{\mathcal{F}(-q)}{r} \exp(-iqr)$$

- Use effective range expansion for amplitude  $\mathcal{F}$

$$\mathcal{F}(q) = \left[ \frac{1}{a_0} + \frac{r_e}{2} q^2 - iq \right]^{-1}$$


$$C(q) = 1 + \left[ \frac{|\mathcal{F}(q)|^2}{2R^2} F_3\left(\frac{r_{\text{eff}}}{R}\right) + \frac{2\text{Re } \mathcal{F}(q)}{\sqrt{\pi}R} F_1(2qR) - \frac{\text{Im } \mathcal{F}(q)}{R} F_2(2qR) \right]$$

- Fit the data with formula

- Direct relation between  $C(q)$  and  $\mathcal{F}(q)$
- Difficult to introduce the detailed interaction e.g. coupled-channel
- Coulomb int. can be only introduced with Gamow factor (too crude for  $C(q)$ )

- How to extract interaction from Correlation data;  $C(q) \rightarrow \mathcal{F}(q)$

- Potential method

$$C(q) \rightarrow V(r) \rightarrow \mathcal{F}(q)$$

- Parametrize the potential

e.g.  $V(r) = V_0 \exp(-mr)$   $\xrightarrow{H\varphi = E\varphi}$   $\varphi \xrightarrow{C(\mathbf{q}) = \int d^3\mathbf{r} S(\mathbf{r}) |\varphi^{(-)}(\mathbf{q}, \mathbf{r})|^2}$   $C(q)$

- Determine the parameters by fitting the data
- Calculate the amplitude or threshold parameters ( $a_0, r_e$ ) from  $V(r)$ 
  - More fitting costs (needs to solve Schrödinger eq. for every change of parameters.)
  - Easy to introduce coupled-channel effect
  - Coulomb effect can be precisely calculated by adding Coulomb pot. in  $H$ .



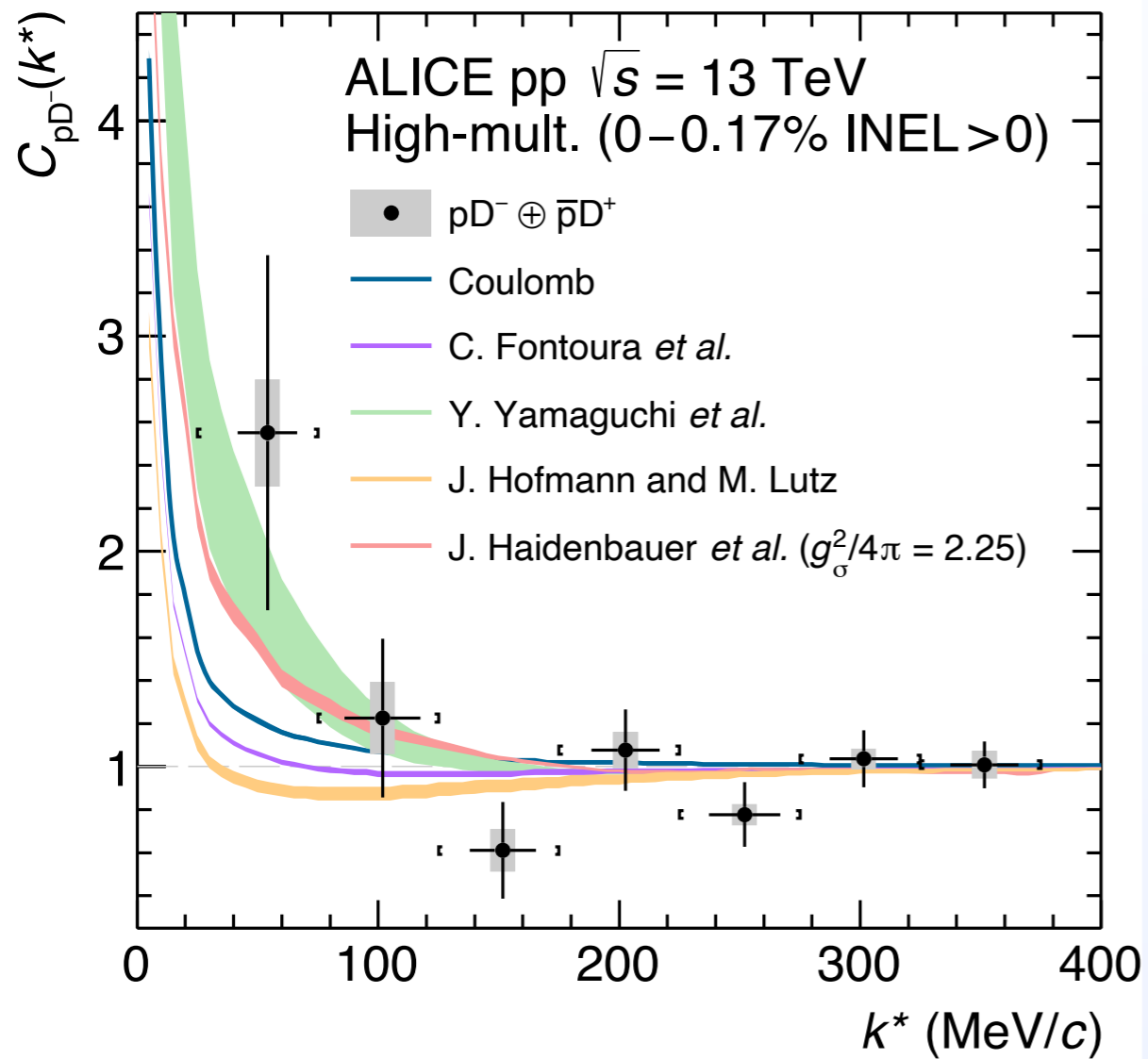
Amplitude can be directly determined from correlation data

# $\bar{D}N$ interaction and $D^-p$ correlation function

- $\bar{D}(\bar{c}l)N$  interaction ( $C = -1$ )

- $D^-p$  correlation function ALICE PRD 106 (2022) 5, 052010

$f_0 \equiv \mathcal{F}(E = E_{\text{th}})$   
 + : attractive w/o bound  
 - : repulsive  
 or attractive w/ bound



- Model scattering lengths  $f_0$

Model	$f_0 (I=0)$	$f_0 (I=1)$	$n_\sigma$
Coulomb			(1.1–1.5)
Haidenbauer et al. [21]			
– $g_\sigma^2/4\pi = 1$	0.14	–0.28	(1.2–1.5)
– $g_\sigma^2/4\pi = 2.25$	0.67	0.04	(0.8–1.3)
Hofmann and Lutz [22]	–0.16	–0.26	(1.3–1.6)
Yamaguchi et al. [24]	–4.38	–0.07	(0.6–1.1)
Fontoura et al. [23]	0.16	–0.25	(1.1–1.5)

- pure Coulomb case is compatible with data
- Better agreement with strongly attractive interaction models for  $I = 0$ .
- pion exchange model of Yamaguchi et al. predicting 2 MeV bound state gives the lowest  $n_\sigma$

\* Background including miss PID is subtracted

# $\bar{D}N$ interaction and $D^-p$ correlation function

## • Constraint on $I = 0$ scattering length $f_0$

ALICE PRD 106 (2022) 5, 052010

- Analysis with one range Gaussian potential

$$V(r) = V_0 \exp(-m^2 r^2)$$

- $m \leftarrow \rho$  exchange ( $m = m_\rho$ )

- Assume negligible  $I = 1$  int.

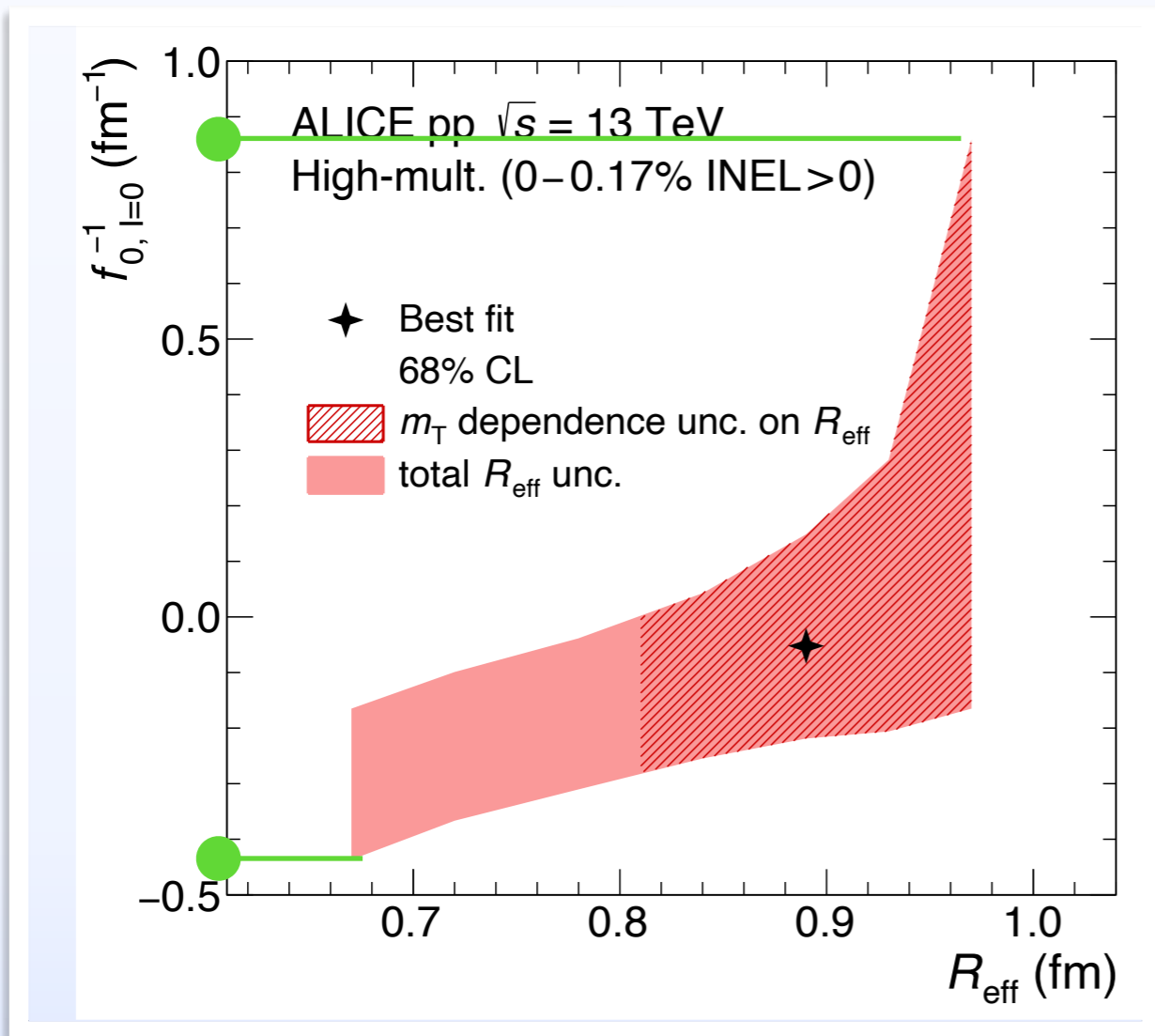
$$f_0 \equiv \mathcal{F}(E = E_{\text{th}})$$

+ : attractive w/o bound

- : repulsive

or attractive w/ bound

- Constraint on  $f_{0, I=0}$

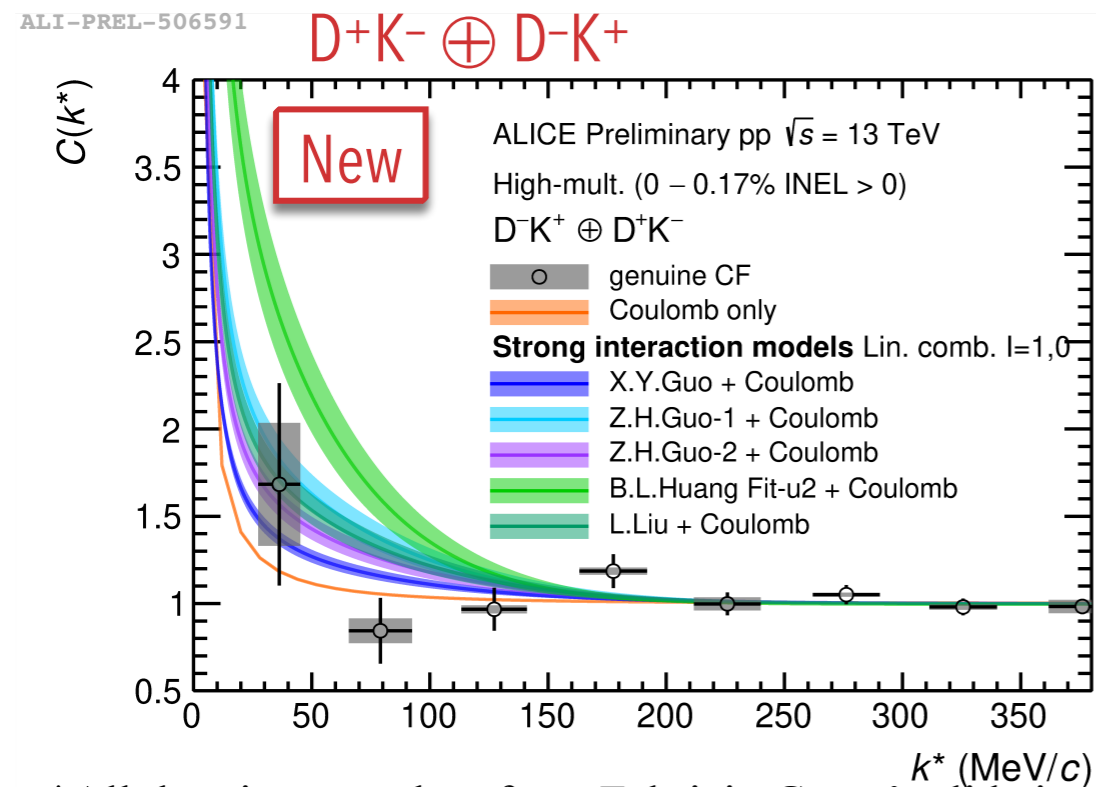
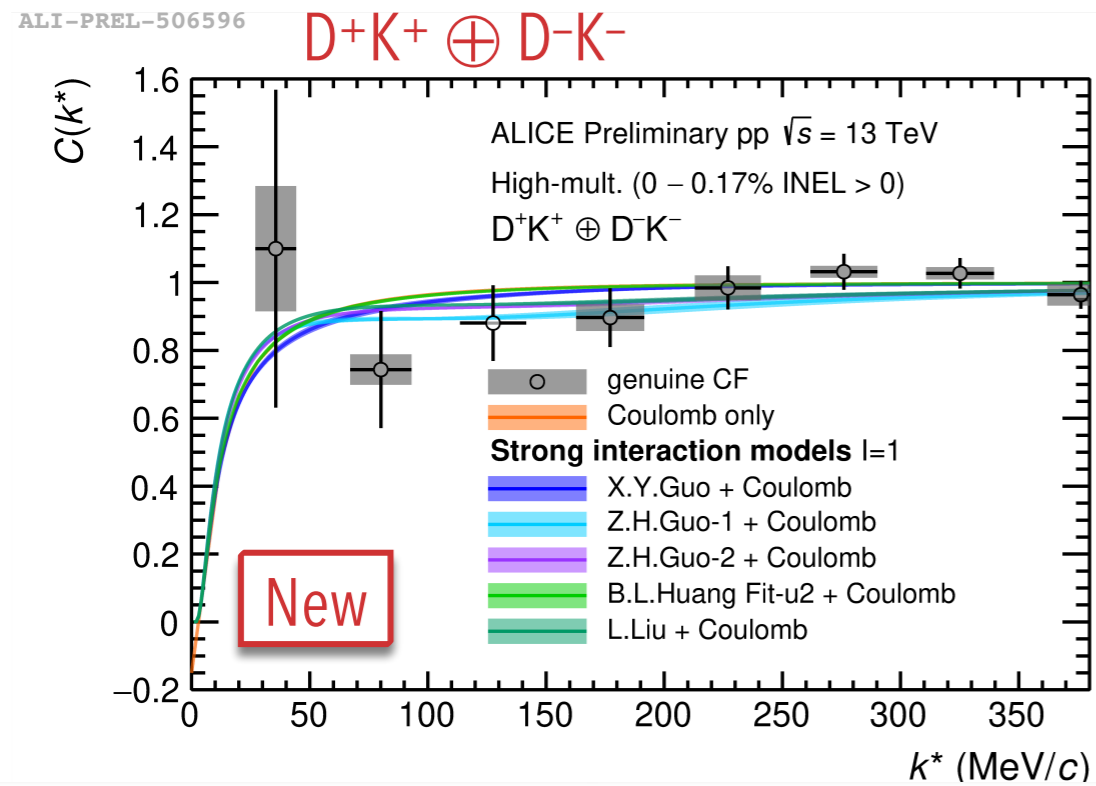


- $1\sigma$  constraint  $\rightarrow f_{0, I=0}^{-1} \in [-0.4, 0.9] \text{ fm}^{-1}$ :

- strongly attractive with or without bound state

- \* Most models predicts repulsive int. for  $I = 1$   
 $\rightarrow I = 0$  may have more attraction in reality.

# $DK$ and $D\bar{K}$ interaction



\*All the pictures taken from Fabrizio Grosa's slide in

## $DK(\bar{K})$ , $D\pi$ models:

- All based on chiral Lagrangian
- Using the lattice data in  $DK/D\pi$  sector

L. Liu et al, Phys. Rev. D87 (2013) 014508

X.-Y. Guo et al, Phys. Rev. D 98 (2018) 014510

B.-L. Huang et al, Phys. Rev. D 105 (2022) 036016

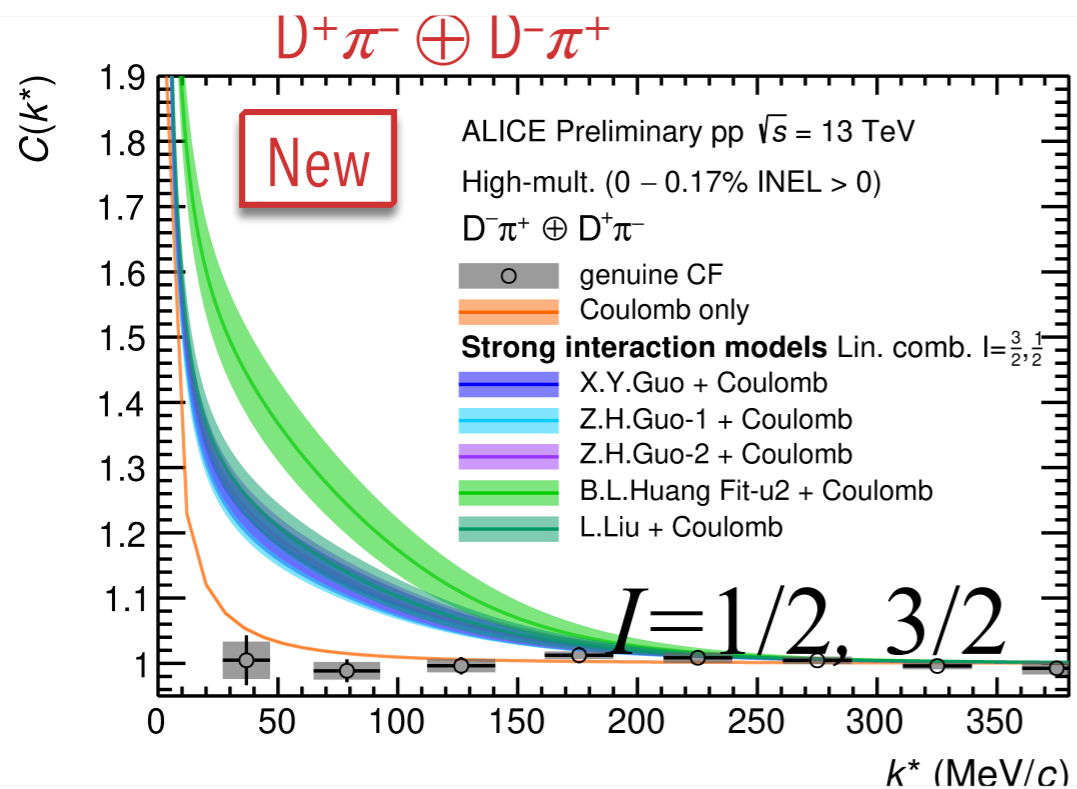
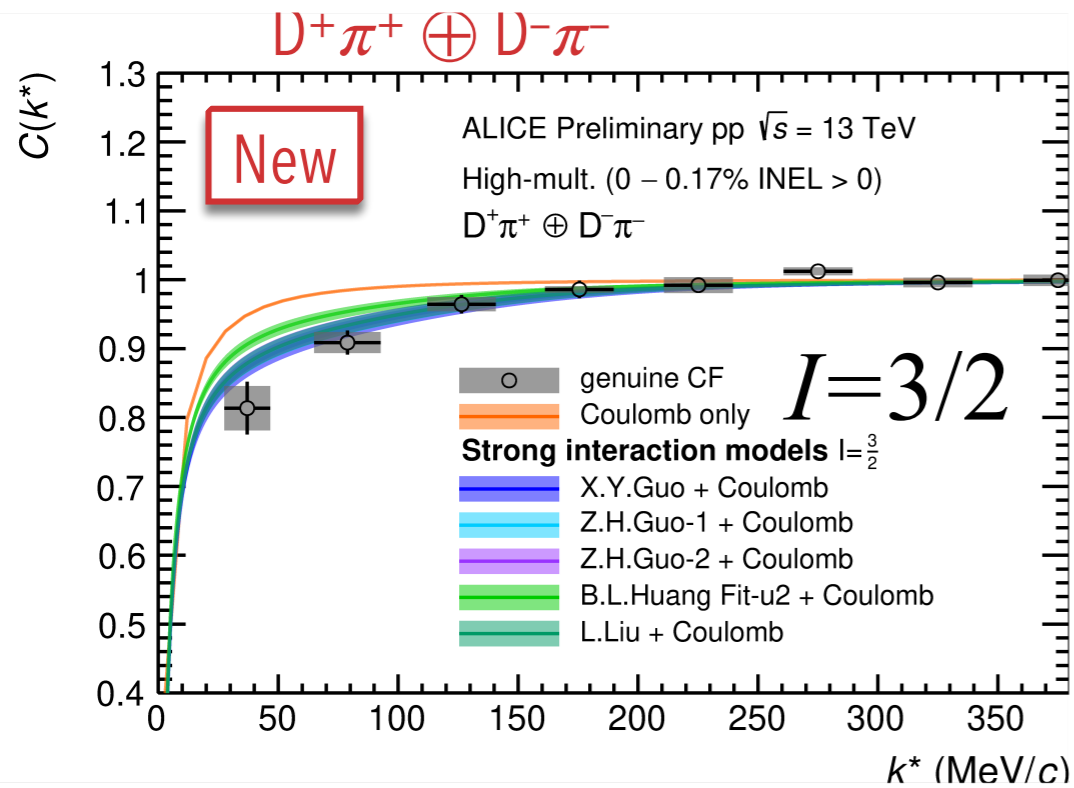
Z.-H. Guo et al Eur. Phys. J. C 79 (2019) 13

model	$DK(I=1)$	$D\bar{K}(I=1)$	$D\bar{K}(I=0)$
<b>X.Y. Guo</b>	-0.05	-0.22	0.46
<b>Z.H. Guo 1</b>	$0.06+i0.30$	-0.18	0.96
<b>Z.H. Guo 2</b>	$0.05+i0.17$	-0.19	0.68
<b>B.L.Huang</b>	-0.01	-0.24	1.81
<b>L.Liu</b>	$0.07+i0.17$	-0.20	0.84

## $DK(\bar{K})$ correlation data

- No significant signals from the pure Coulomb case
- In good agreement with Chiral models

# $D\pi$ interaction



## $D\pi$ interaction

- L. Liu et al, Phys. Rev. D87 (2013) 014508
- X.-Y. Guo et al, Phys. Rev. D 98 (2018) 014510
- B.-L. Huang et al, Phys. Rev. D 105 (2022) 036011
- Z.-H. Guo et al Eur. Phys. J. C 79 (2019) 13

model	$D\pi(I = 3/2)$	$D\pi(I = 1/2)$
<b>X.Y. Guo</b>	-0.11	0.33
<b>Z.H. Guo 1</b>	-0.101	0.31
<b>Z.H. Guo 2</b>	-0.099	0.34
<b>B.L. Huang</b>	-0.06	0.61
<b>L.Liu</b>	-0.10	0.37

## $D\pi$ correlation data

- $D^+\pi^+$  pair ( $I = 3/2$ ) is in good agreement with chiral models
- $D^+\pi^-$  pair does not show no enhancement from pure Coulomb case while all the chiral models predict enhanced CF

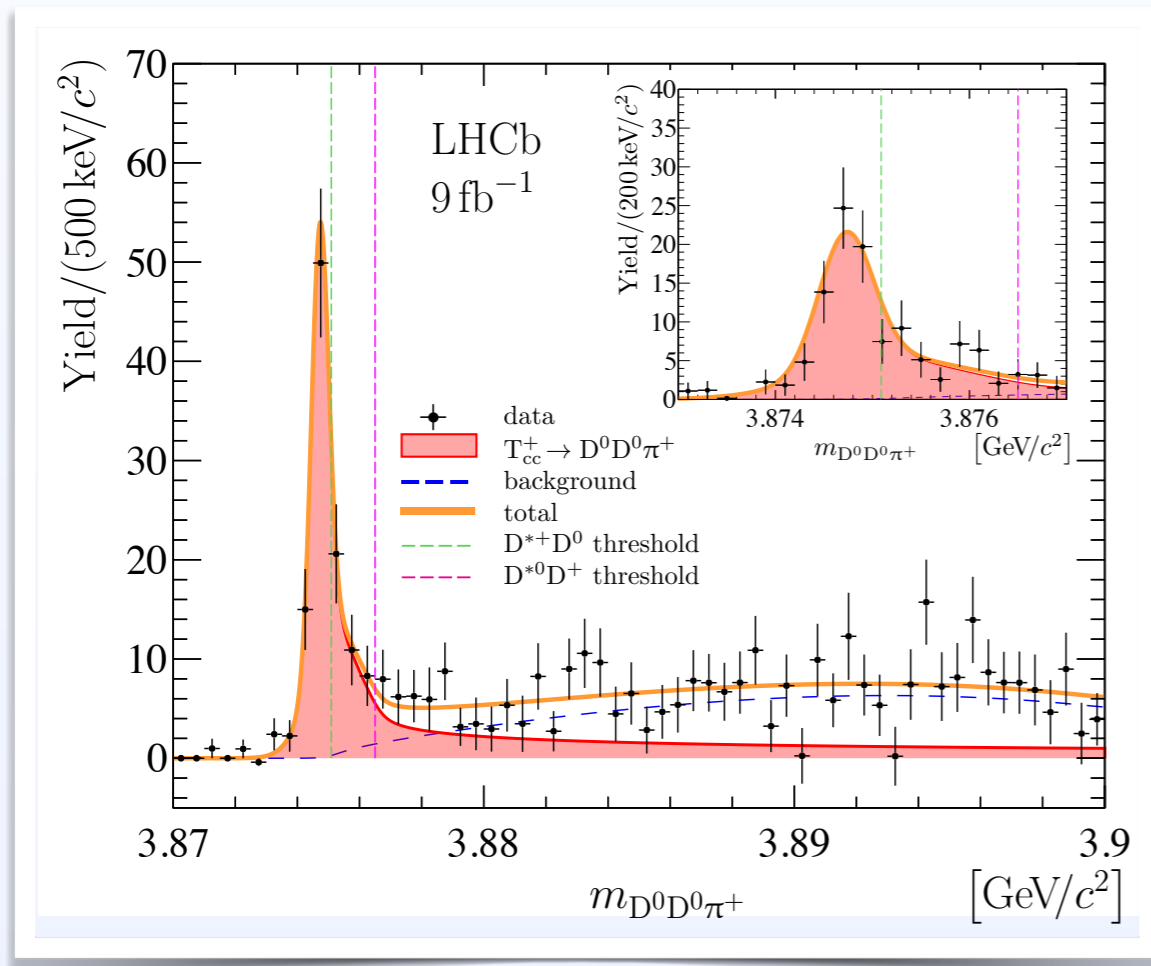
\*All the pictures taken from Fabrizio Grosa's slide in Quark Matter 2022

# $DD^*$ and $D\bar{D}^*$ int. from femtoscopy

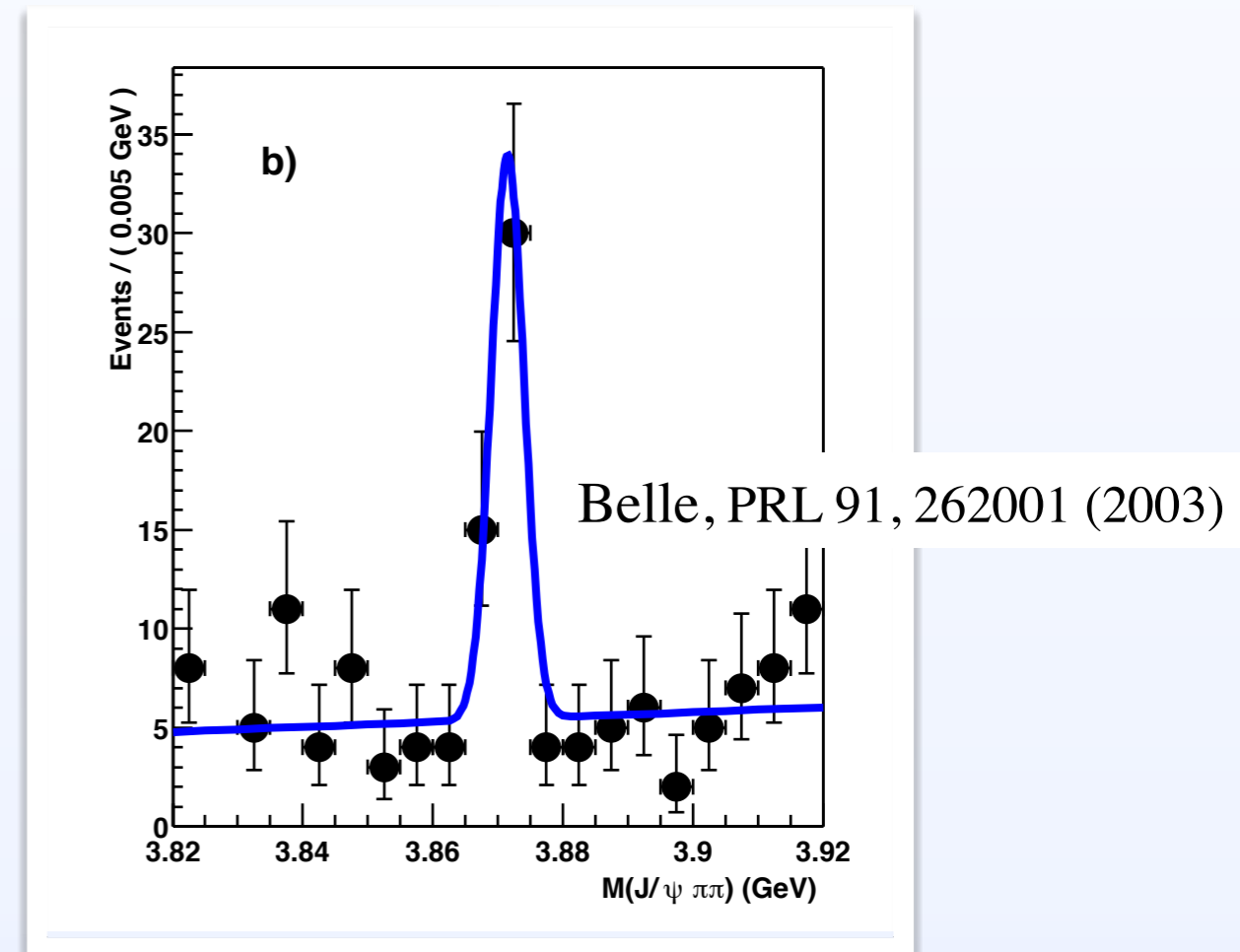
- $T_{cc}$

- Observed in  $D^0D^0\pi$  spectrum

LHCb, Nature Com. 13 (2022) 1



- $X(3872)$  or  $\chi_{c1}$ 
  - Firstly observed in  $\pi\pi J/\Psi$  spectrum  
Belle, PRL 91, 262001 (2003)
  - Confirmed by Babar: PRD71, 071003 (2003)  
CDF: PRL 93 072001 (2004)  
D0: PRL 93 162002 (2004)

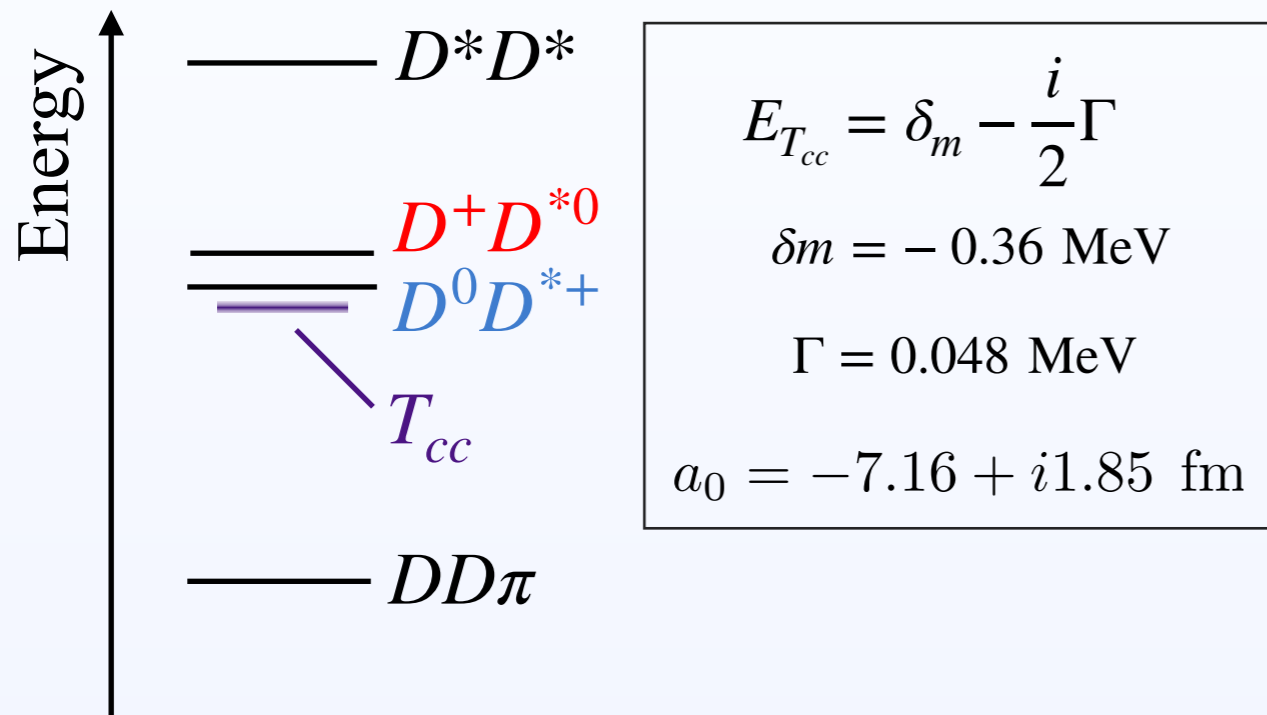


# $DD^*$ and $D\bar{D}^*$ int. from femtoscopy

## • $DD^*$ and $D\bar{D}^*$ sector

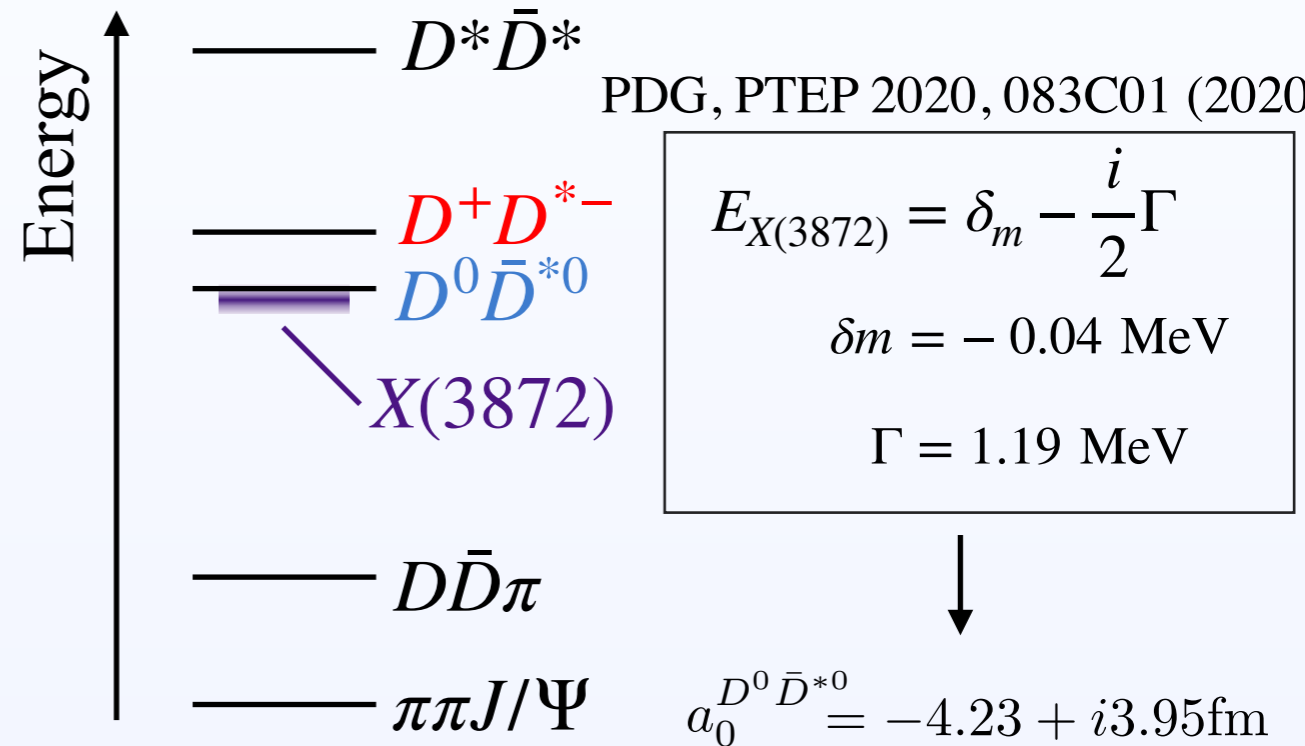
$C = 2$

LHCb, Nature Com. 13 (2022) 1



$C = 0$

PDG, PTEP 2020, 083C01 (2020).



$$a_0 \equiv \mathcal{F}(E = E_{\text{th}})$$

+ : attractive w/o bound

- : repulsive

or attractive w/ bound

- $T_{cc}/X(3872)$  lies nearby  $DD^*/D\bar{D}^*$

==> meson-meson molecule?

==> Strong attractive interaction

- Gaussian potential

$$V(r) = V_0 \exp(-m^2 r^2)$$

- $m \leftarrow \pi$  exchange ( $m = m_\pi$ )

- $V_0 \leftarrow$  scattering lengths

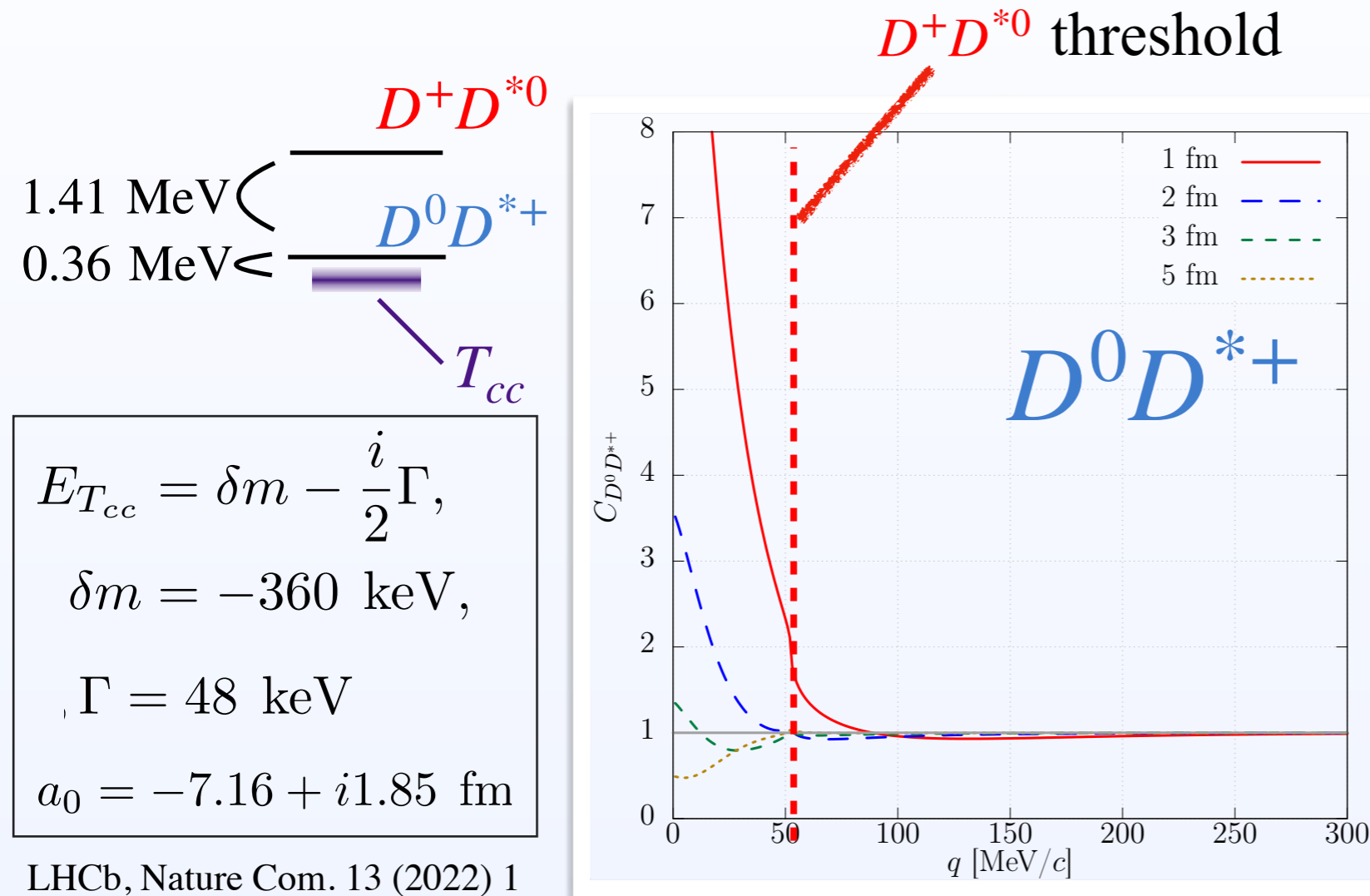
- Assume dominant contribution from exotic channel ( $I = 0$ )

- Coupled-channel of two isospin channels



# $DD^*$ and $D\bar{D}^*$ int. from femtoscopy

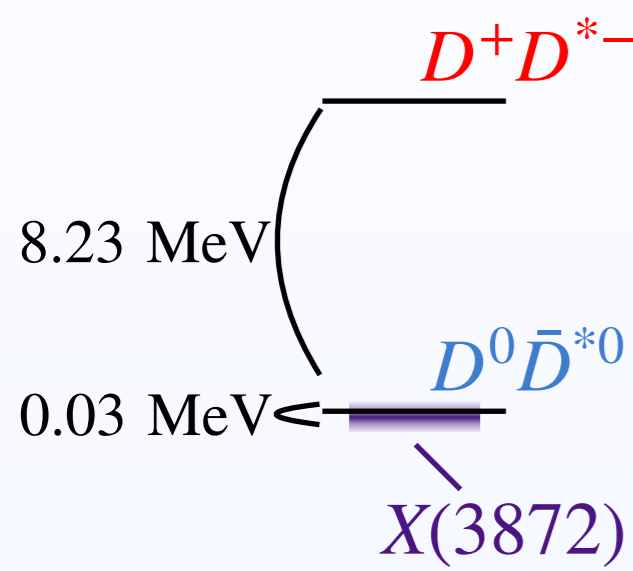
- $DD^*$  correlation and  $T_{cc}$  state



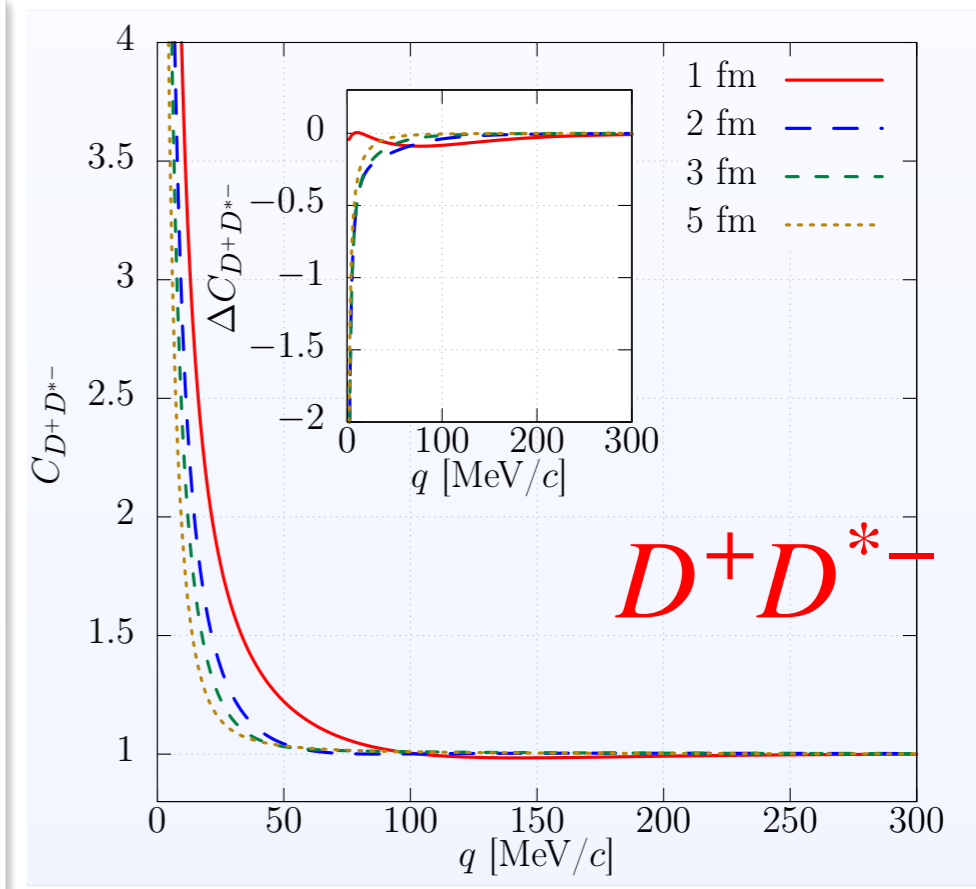
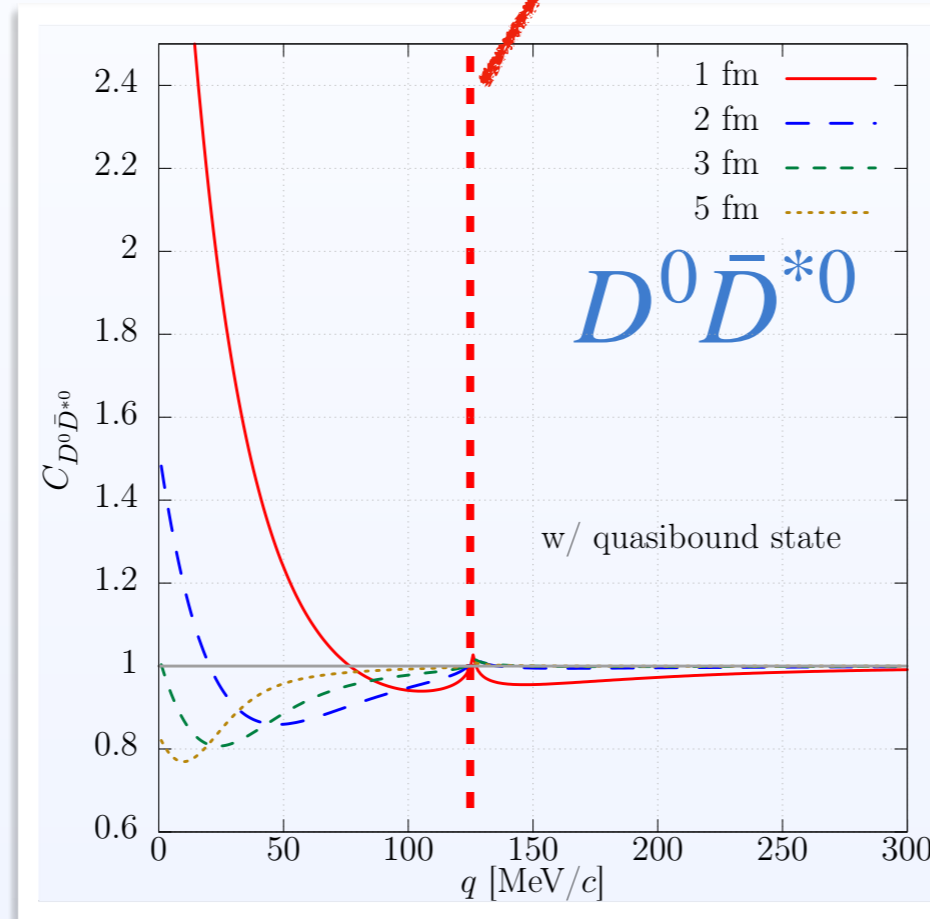
- Bound state like behavior for both pairs
- Stronger source size dep. for  $D^0D^{*+}$
- $D^+D^{*0}$  cusp is not prominent

# $DD^*$ and $D\bar{D}^*$ int. from femtoscopy

- $D\bar{D}^*$  correlation and  $X(3872)$  state



$D^+D^{*-}$  threshold



PDG, PTEP 2020, 083C01 (2020)

$$E_{X(3872)} = \delta_m - \frac{i}{2}\Gamma$$

$$\delta m = -0.04 \text{ MeV}$$

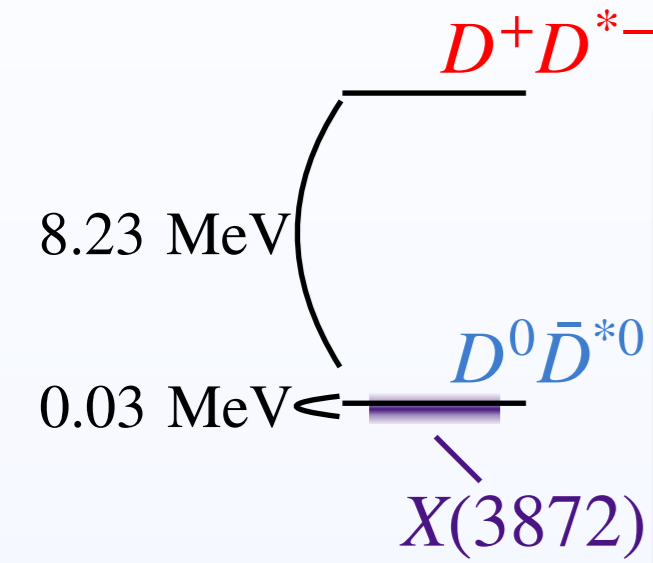
$$\Gamma = 1.19 \text{ MeV}$$

$$a_0^{D^0\bar{D}^{*0}} = -4.23 + i3.95 \text{ fm}$$

- $D^0D^{*+}$  : Strong source size dep.
- $D^+D^{*-}$  : Small effect of the strong int. (Coulomb int dominance)
- Moderate  $D^+D^{*+}$  cusp

# $DD^*$ and $D\bar{D}^*$ int. from femtoscopy

- $D\bar{D}^*$  correlation and  $X(3872)$  state



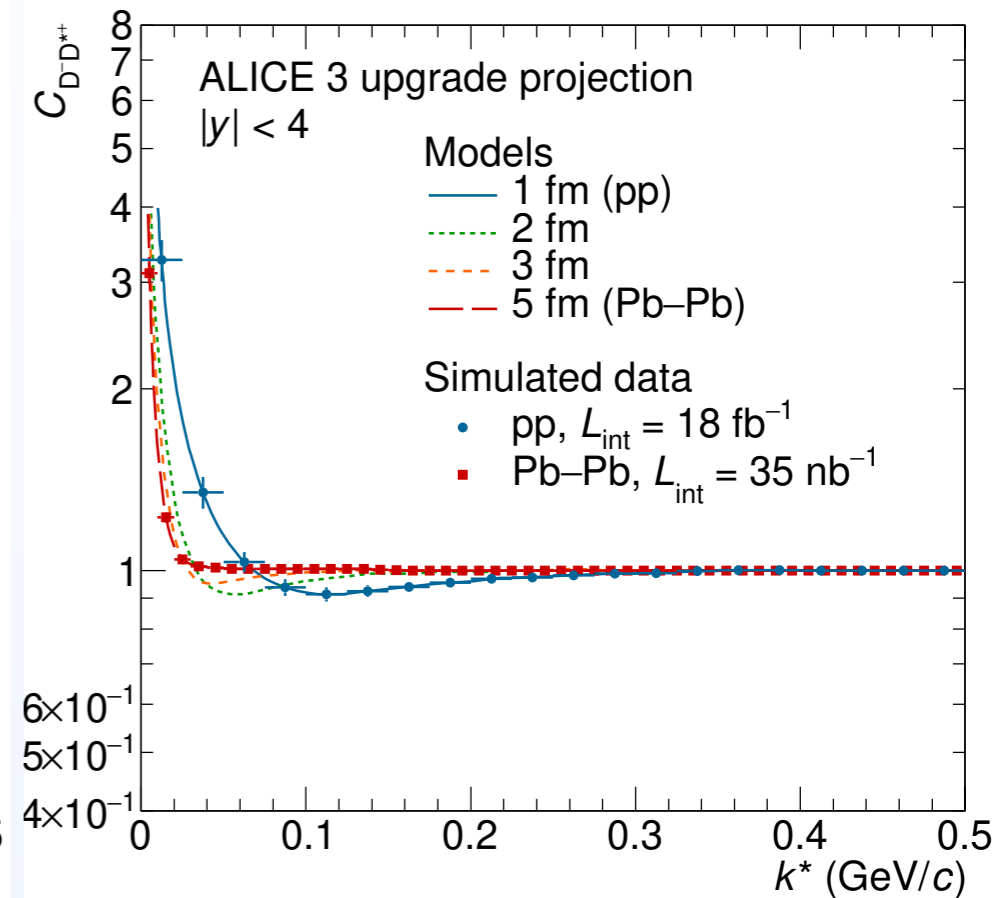
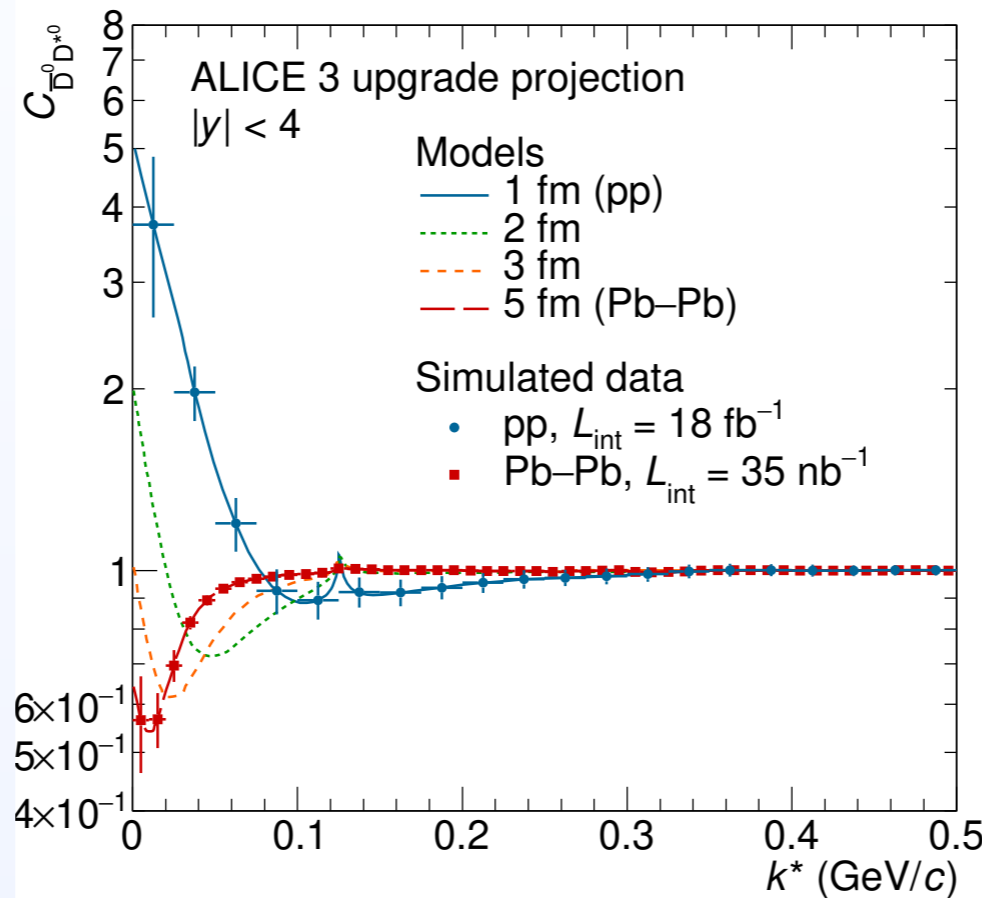
PDG, PTEP 2020, 083C01 (2020)

$$E_{X(3872)} = \delta_m - \frac{i}{2}\Gamma$$

$$\delta m = -0.04 \text{ MeV}$$

$$\Gamma = 1.19 \text{ MeV}$$

$$a_0^{D^0\bar{D}^{*0}} = -4.23 + i3.95 \text{ fm}$$



ALICE collab., CERN-LHCC-2022-009 (2022).

- $D^0D^{*+}$  : Strong source size dep.
- $D^+D^{*-}$  : Small effect of the strong int. (Coulomb int dominance)
- Moderate  $D^+D^{*+}$  cusp

# $X(3872)$ with various assumptions

- $X(3872)$  as cusp

$$a_0 \equiv \mathcal{F}(E = E_{\text{th}})$$

+ : attractive w/o bound

- : repulsive

or attractive w/ bound

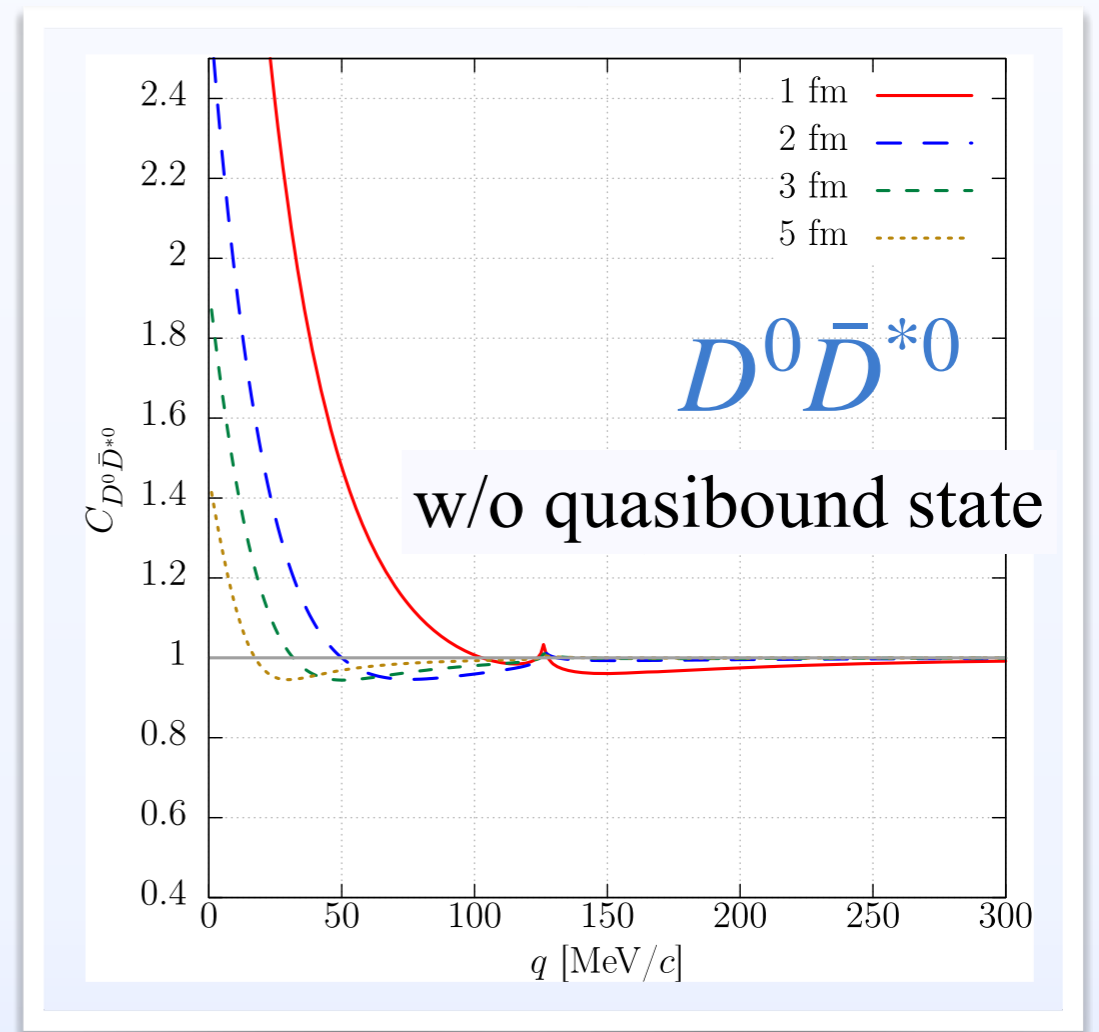
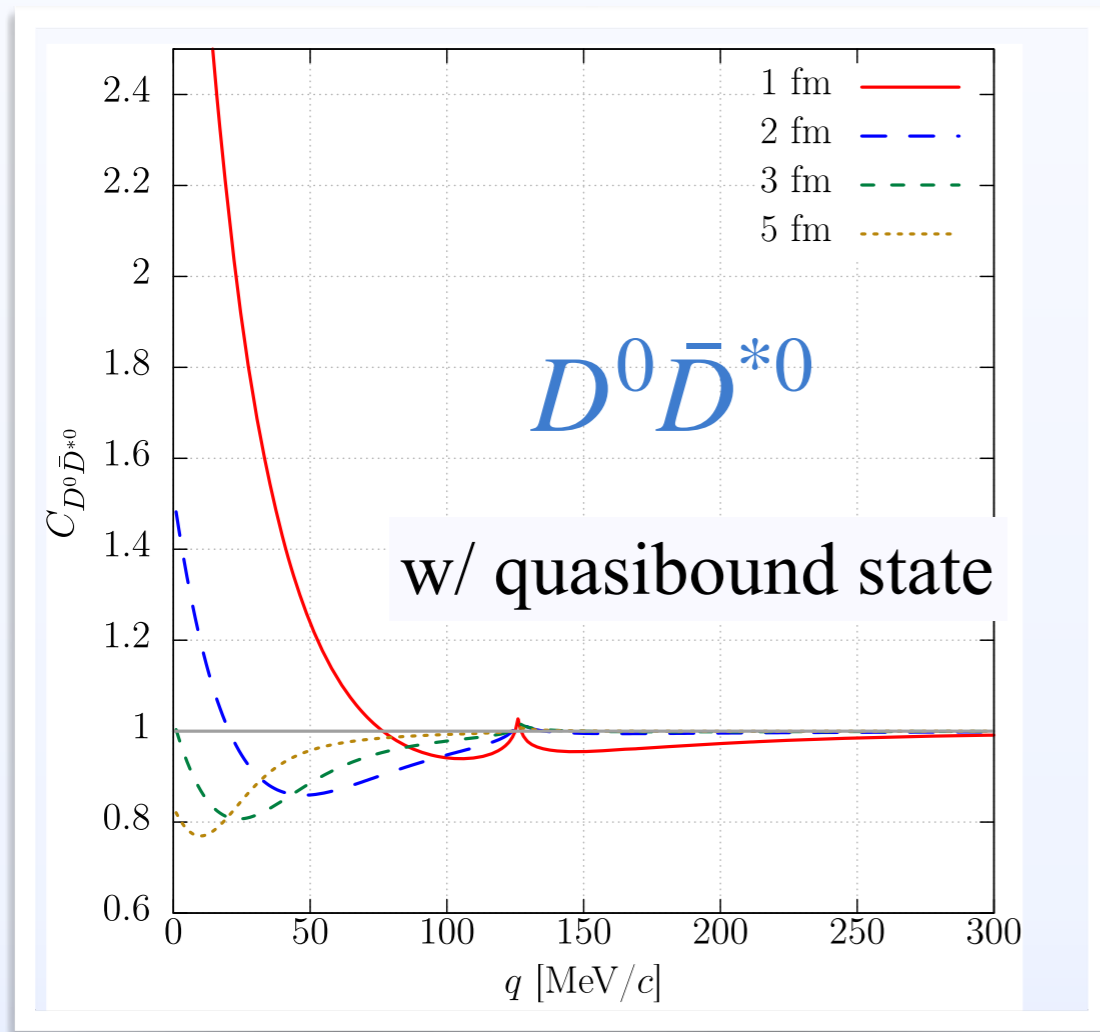
- Empirical scattering length

$$a_0^{D^0 \bar{D}^{*0}} = -4.23 + i3.95 \text{ fm}$$

weaken interaction

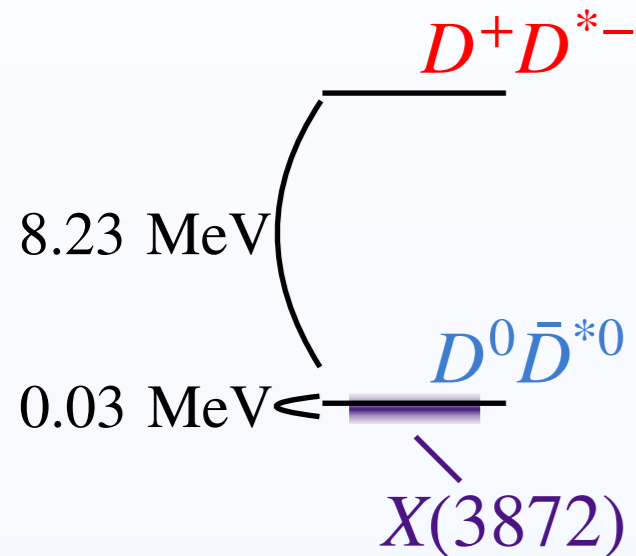


$$a_0^{D^0 \bar{D}^{*0}} = 2.30 + i4.00 \text{ fm}$$



# X(3872) with various assumptions

- X(3872) with short range force



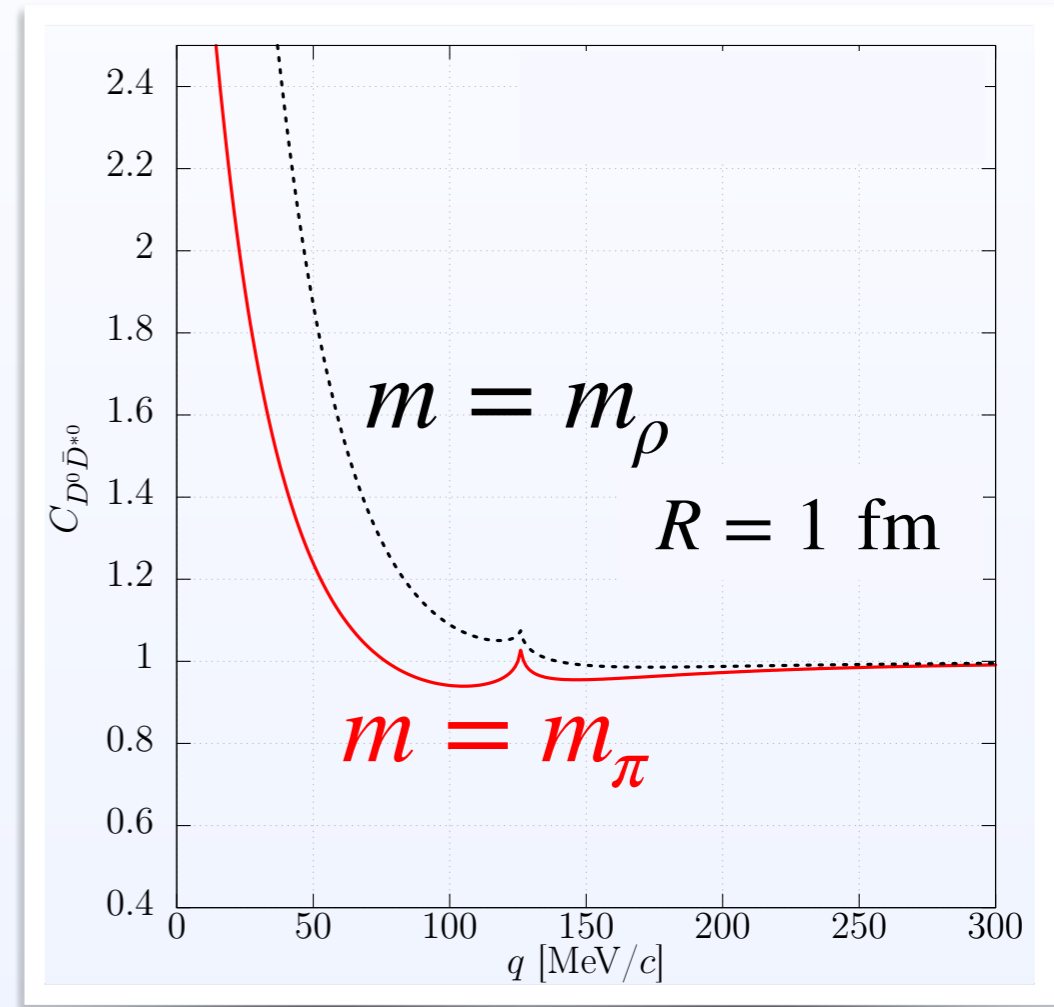
- Potential shape dependence

$$V(r) = V_0 \exp(-m^2 r^2)$$

Two potentials fitted to same scattering length

$$a_0^{D^0\bar{D}^{*0}} = -4.23 + i3.95 \text{ fm}$$

- Long range pot. :  $m = m_\pi$
- Short range pot. :  $m = m_\rho$



PDG, PTEP 2020, 083C01 (2020)

$$E_{X(3872)} = \delta_m - \frac{i}{2}\Gamma$$

$$\delta m = -0.04 \text{ MeV}$$

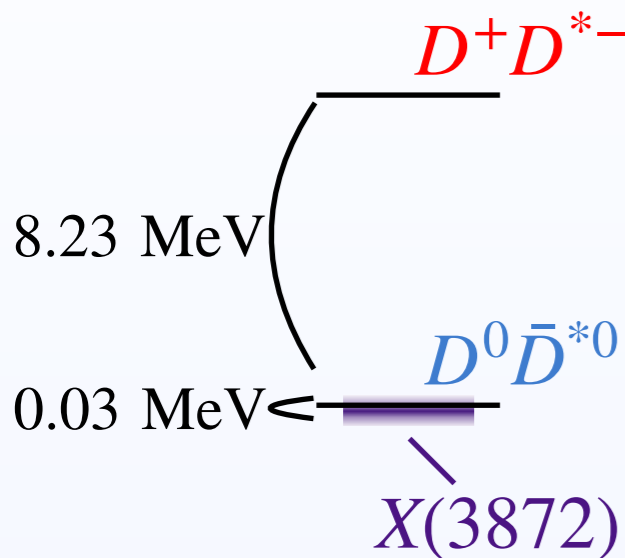
$$\Gamma = 1.19 \text{ MeV}$$



Change of the interaction range gives moderate enhancement

# X(3872) with various assumptions

- X(3872) with  $I = 1$  interaction



- $\pi$  exchange interaction
- $\propto I \cdot I' \implies V_{I=0}/V_{I=1} = -3$
- Fit  $a_0^{D^0\bar{D}^{*0}}$  for each case

repulsive  $V_{I=1}$

$$V_{I=1} = -\text{Re } V_{I=0}/3$$

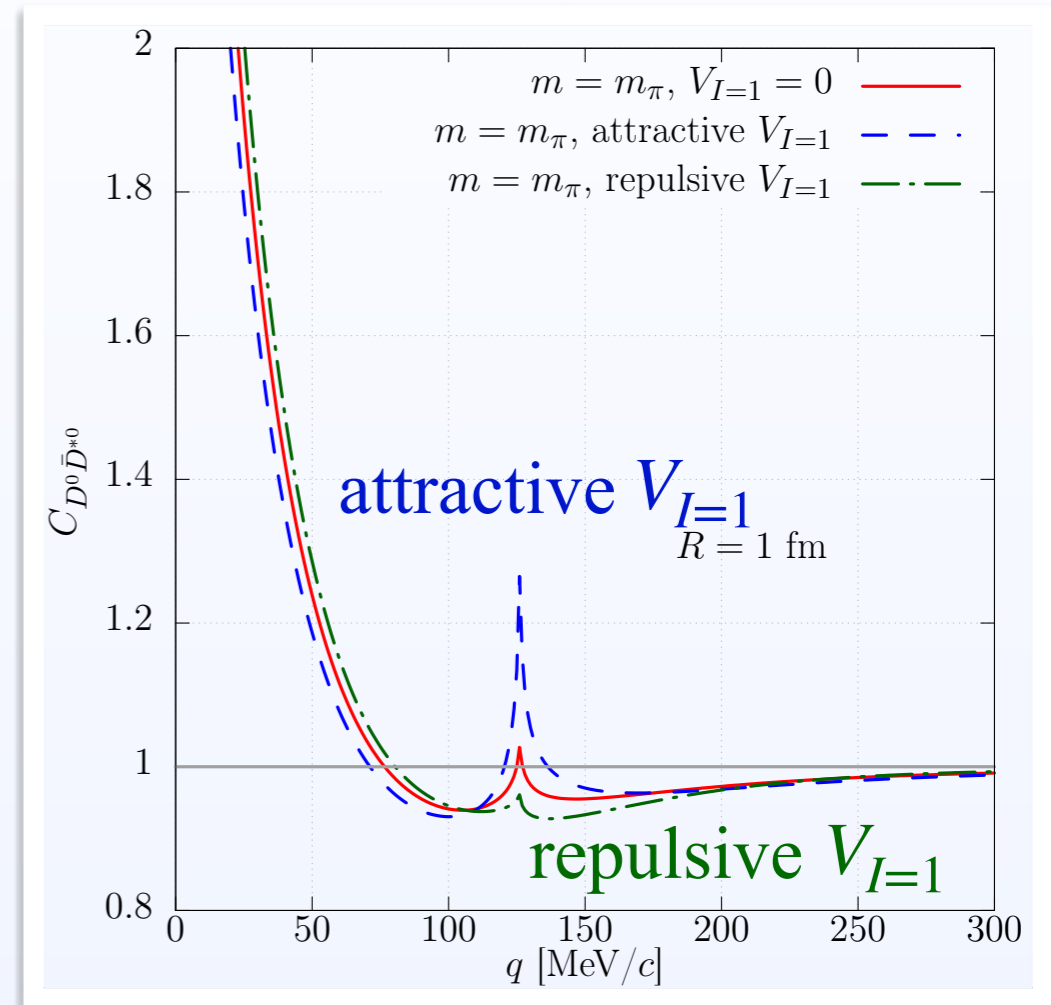
$\implies$  Weaker cusp

attractive  $V_{I=1}$

$$V_{I=1} = \text{Re } V_{I=0}/3$$

$\implies$  prominent cusp

\* Due to the additional virtual pole around  $D^+D^{*-}$  threshold



PDG, PTEP 2020, 083C01 (2020)

$$E_{X(3872)} = \delta_m - \frac{i}{2}\Gamma$$

$$\delta m = -0.04 \text{ MeV}$$

$$\Gamma = 1.19 \text{ MeV}$$



The strength of the cusp depends on the detailed isospin structure of the interaction.

# Summary

- Femtoscopic correlation function in high energy nuclear collisions is a powerful tool to investigate the nature of bound state.
  - Comparison to model prediction
  - Direct extraction from  $C(q)$  data
- $D^-p$   
Non-interacting model can explain data but strong attractive interaction reduce the standard deviation.
- $DK(\bar{K})$  : Coulomb int. dominant and consistent with chiral models  
 $D\pi$  : Opposite-charge pair shows the discrepancy from chiral models
- $DD^*/D\bar{D}^*$   
The lower isospin partner channels are expected to show the strong source size dependence due to the near threshold  $T_{cc}/X(3872)$  states.

*Thank you for your attention!*