Analysis of $D^{(*)}D^{(*)}$ and $\bar{D}^{(*)}\Xi_{cc}^{(*)}$ molecular state by one boson exchange model based on heavy quark symmetry

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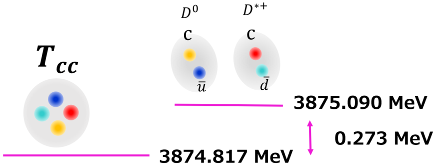
$T^+_{cc}(3874)\ I(J^P) = 0(1^+)$

- composed of $cc\bar{u}\bar{d}$
- just below the shreshold of $D^{*+}(2010)D^0(1864)$

$$m_{T^+_{cc}} - (m_{D^{*+}} + m^0_D) = -273 \pm 63\,[\mathrm{KeV}]$$

LHCb collaboration, Nature Physics volume 18, pages751-754(2022)



$\rightarrow$ mass difference is small $\sim 0.273\,\mathrm{MeV}$

$\rightarrow$ naturally assumed that T_{cc} is a loosely bound state of $D^0 D^{*+}$

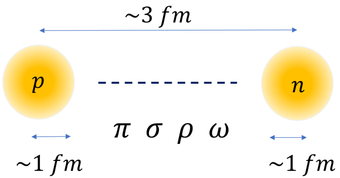
$\rightarrow$ analogy of deuteron (a loosely bound state of proton & neutron)

OBE model

Deuteron : a loosely bound state of proton & neutron

the properties are reproduced by one boson exchange potential (OBEP)

- $B_{in} \simeq 2.2\mathrm{MeV}$
- $P_D \simeq 5\ \%$
- $\sqrt{\langle r^2 \rangle} \simeq 4\,\mathrm{fm}$



Apply one boson exchange model to a molecule state of $D^{(*)}D^{(*)}$

Outline

1. Analysis of $D^{(*)}D^{(*)}$
   - assume that T_{cc}^{+} is $D^{(*)}D^{(*)}$ molecule state
   - study bound states of $D^{(*)}D^{(*)}$ molecule by OBEM
   - fit a free parameter Λ to the experimental data of T_{cc}^{+}
2. Analysis of $\bar{D}^{(*)}\Xi_{cc}^{(*)}$
   - construct doubly heavy baryon lagrangian based on heavy quark anti-diquark symmetry
   - study bound states of $\bar{D}^{(*)}\Xi_{cc}^{(*)}$ by OBEM

Heavy quark spin symmetry

Heavy quark field

$$Q(x) = e^{-im_Q v\cdot x}[e^{im_Q v\cdot x}\frac{1+\not{v}}{2}Q(x) + e^{im_Q v\cdot x}\frac{1-\not{v}}{2}Q(x)]$$

$$= e^{-im_Q v\cdot x}[Q_v(x) + \chi_v(x)]$$

erase the degree of freedom of χ_v

$$\mathcal{L}_{heavy} = \bar{Q}(x)(i\not{D} - m_Q)Q(x)$$

$$= \bar{Q}_v(x) v \cdot iDQ_V(x)$$

$$+ \bar{Q}_V(x)\frac{(i\not{D}_\perp)^2)}{2m_Q}Q_v(x) - g_s\bar{Q}_v(x)\frac{\sigma^{\mu\nu}G^{\mu\nu}}{4m_Q}Q_v(x) + \mathcal{O}(\frac{1}{m_Q^2})$$

HQSS

Heavy quark spin interaction is completely suppressed in the HQ limit

A. V. Manohar and M. B. Wise, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 10, 1 (2000)

Heavy quark anti-diquark symmetry

Q in $Q\bar{q}$ and $\bar{Q}\bar{Q}$ in $\bar{Q}\bar{Q}\bar{q}$ have the same color representaion 3

$$Q\bar{q} \sim 3 \otimes \bar{3}$$

$$\bar{Q}\bar{Q}\bar{q} \sim \bar{3} \otimes \bar{3} \otimes \bar{3} = (3 \oplus \bar{6}) \otimes \bar{3}$$

$\bar{Q}\bar{Q}$ is a sizable object

$\rightarrow$ point-like particle in the HQ limit

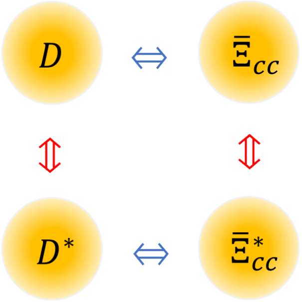
HADS

Heavy quark Q can be identified with heavy anti-diquark $\bar{Q}\bar{Q}$ in HQ limit

Mart1n J. Savage et al., Phys.Lett. B248, 177 (1990)

application of HQS

- Heavy quark spin interaction is completely suppressed (HQSS)
- Heavy quark Q can be identified with anti-heavy diquark $\bar{Q}\bar{Q}$ (HADS)



Heavy meson doublet field

doublet field H of P and P^*

$$H = \Lambda_+ \left[P^{*\mu}\gamma_\mu - \gamma^5 P \right]$$

$$\bar{H} = \gamma^0 H^\dagger \gamma^0 = [P^{*\dagger\mu}\gamma_\mu - \gamma^5 P^\dagger]\Lambda_+$$

$$\Lambda_+ = \frac{1+\not{v}}{2},\ \Lambda_- = \frac{1-\not{v}}{2}$$

A. V. Manohar and M. B. Wise, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 10, 1 (2000)

- $H_a \to D(\Lambda) H_a D^{-1}(\Lambda)$ (Lorentz)
- $H_a \to H_b h^\dagger_{ba}(x)$ (chiral & hidden local)
- $H_a \to S_v H_a$ (heavy quark spin)
- $H_a \to \gamma^0 H_a \gamma^0$ (parity)

Construct a invariant lagrangian under these symmetry

interaction lagrangians

pseudo scalar & scalar & vector coupling

$$\mathcal{L}_{HH\mathcal{A}} = g \,\mathrm{Tr}\left[H\gamma_\mu\gamma_5 A^\mu \bar{H}\right]$$

$$\mathcal{L}_{HH\sigma} = g_\sigma \,\mathrm{Tr}\left[H\sigma\bar{H}\right]$$

$$\mathcal{L}_{HHV} = -\beta \,\mathrm{Tr}\left[Hv_\mu(V^\mu - \rho^\mu)\bar{H}\right] + \lambda \,\mathrm{Tr}\left[H\sigma_{\mu\nu}F^{\mu\nu}(\rho)\bar{H}\right]$$

R. Casalbuoni et al., N, Phys. Rep. 281, 145 (1997)

$$A^\mu \equiv \frac{i}{2}\left(\xi^\dagger\partial^\mu\xi - \xi\partial^\mu\xi^\dagger\right), V^\mu \equiv \frac{i}{2}\left(\xi^\dagger\partial^\mu\xi + \xi\partial^\mu\xi^\dagger\right), \xi = e^{\frac{i\hat{\mathcal{M}}}{\sqrt{2}f_\pi}}$$

$$F_{\mu\nu}(\rho) \equiv \partial_\mu\rho_\nu - \partial_\nu\rho_\mu - [\rho_\mu, \rho_\nu]$$

$$\rho_\mu = i\frac{g_v}{\sqrt{2}}\hat{\rho}_\mu$$

Coupling constant

Coupling constant	(MeV)
g	0.59
g_σ	0.76
g_V	5.9
β	0.9
λ	$0.56\,\mathrm{GeV}^{-1}$

Ning Li et al. PHYSICAL REVIEW D 88, 114008 (2013)

- $D^* \to D\pi$: $g \simeq 0.59$ S. Ahmed et al., CLEO Collaboration, Phys. Rev. Lett. 87, 251801 (2001)
- σ couples to only light quark : $g_\sigma = \frac{g_{\sigma NN}}{3} \simeq 0.76$

  X. Liu, Y.-R. Liu, W.-Z. Deng, and S.-L. Zhu, Phys. Rev. D77, 094015 (2008)
- V.M.D : $g_V \simeq 5.9, \beta \simeq 0.9$ R. Casalbuoni et al., Phys. Lett. B 299, 139 (1993)
- charming penguin diagrams in B meson decay processes : $\lambda \simeq 0.56$

  C. Isola et al., Phys. Rev. D68, 114001 (2003)

Possible channels

DD^* molecule with $I(J^P) = 0(1^+)$

- $m_{D^*}(2008) - m_D(1867) \simeq 140\mathrm{MeV} > m_\pi(138)$
- Tensor force mixes $(^3S_1)$ and $(^3D_1)$

$$\begin{pmatrix} [DD^*]_-(^3S_1) \\ [DD^*]_-(^3D_1) \\ D^*D^*(^3S_1) \\ D^*D^*(^3D_1) \end{pmatrix} \quad [DD^*]_- = \frac{DD^* - D^*D}{\sqrt{2}}$$

OBEP for $D^{(*)}D^{(*)} - D^{(*)}D^{(*)}$

$$V_\pi(r) = C_\pi(\frac{g}{2f_\pi})^2\frac{1}{3}\left[(\mathcal{O}_1\cdot\mathcal{O}_2)C(r,m_\pi,\Lambda) + S_{\mathcal{O}_1,\mathcal{O}_2}T(r,m_\pi,\Lambda)\right]\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$$

$$V_\sigma(r) = -\left(\frac{g_\sigma}{m_\sigma}\right)^2 C(r,m_\sigma,\Lambda)$$

$$V_v(r) = C_v(\lambda g_V)^2\frac{1}{3}\left[2\mathcal{O}_1\cdot\mathcal{O}_2 C(r,m_v,\Lambda) - S_{\mathcal{O}_1,\mathcal{O}_2}T(r,m_\rho,\Lambda)\right]\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$$

$$V_v(r) = \left(\frac{\beta g_V}{2m_V}\right) C(r,m_v,\Lambda)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$$

Λ is a free parameter dependent of a hadron size

Form factor

include the finite size effect of hadron by the cutoff in the momentum integral → Pauli-Villars regularization

form factor

$$F[q] = \frac{\Lambda^2 - m_{ex}^2}{\Lambda^2 - q^2}, \quad \Lambda : \text{a free parameter}$$

- $F[q] \xrightarrow{\boldsymbol{q}\to 0} 1, \quad F[q] \xrightarrow{\boldsymbol{q}\to\infty} 0$
- attach $F[q]$ at each vertex
- set the same Λ in all $F[q]$

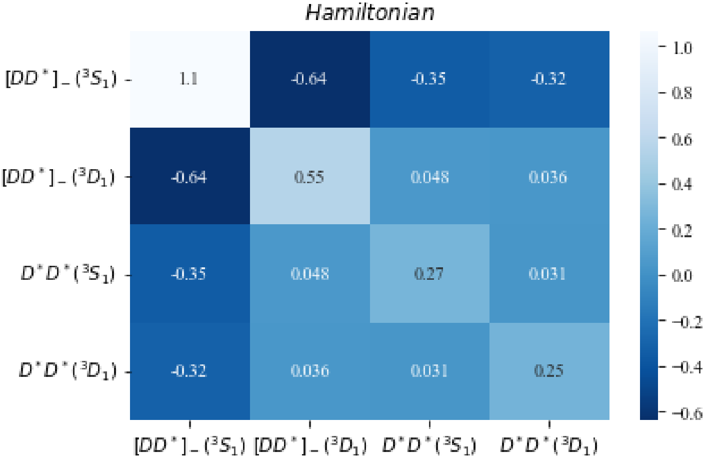
fit Λ to fit the experimental data of $T_{cc}\,(B_{in} \simeq 0.273\,[\mathrm{MeV}])$

Numerical results

OBE	(MeV)		
Λ	1220	**1234**	1240
E_{bin}	0.15	**0.27**	0.34
$P_{[DD^*]_-}(^3S_1)$	0.9927	**0.9916**	0.9904
$P_{[DD^*]_-}(^3D_1)$	0.0058	**0.0065**	0.0072
$P_{D^*D^*}(^3S_1)$	0.0009	**0.0012**	0.0016
$P_{D^*D^*}(^3D_1)$	0.0004	**0.0005**	0.00069
$\sqrt{\langle r^2 \rangle}$	8.4	**6.9**	5.8

No bound state by only one pion exchange at more than $\Lambda = 1800\mathrm{MeV}$

$\langle H \rangle$



non-diagonal part plays a role in making a bound state

$\Rightarrow$ **tensor force mixing** $S - D$ wave & **transition** $[DD^*]_-$ - D^*D^* help to bind

Analysis of $\bar{D}^{(*)}\Xi_{cc}^{(*)}$ molecular states

- analysis of $D^{(*)}D^{(*)}$ molecular state
  - **tensor force mixing** $D^{(*)}D^{(*)}(^3S_1) - D^{(*)}D^{(*)}(^3D_1)$
  - **transition of** $[DD^*]_-$ & D^*D^*

$\rightarrow$ important to make a bound state

- sis of $\bar{D}^{(*)}\Xi_{cc}^{(*)}$ molecular state

$$\bar{D}^{(*)}\bar{D}^{(*)}(S, D) \Rightarrow \bar{D}^{(*)}\Xi_{cc}^{(*)}(S, D)$$

$$(\bar{Q}q)(\bar{Q}q) \Rightarrow (QQq)(\bar{Q}q)$$

Anti-heavy meson doublet field

doublet field $\tilde{H}$ of $\tilde{P}$ and $\tilde{P}^*$

$$\tilde{H}_a = \left[\tilde{P}_a^{*\mu}\gamma_\mu - \tilde{P}_a\gamma_5\right]\Lambda_-$$

$$\bar{\tilde{H}}_a \equiv \gamma_0 \bar{\tilde{H}}_a^\dagger \gamma_0 = \Lambda_- \left[\tilde{P}_a^{*\dagger}\gamma_\mu + \tilde{P}_a^\dagger \gamma_5\right]$$

pseudo scalar & scalar & vector coupling

$$\mathcal{L}_{\tilde{H}\tilde{H}A} = g\,\mathrm{Tr}\left[\bar{\tilde{H}}_b\gamma_\mu\gamma_5 A_{ba}^\mu \tilde{H}_a\right]$$

$$\mathcal{L}_{\tilde{H}\tilde{H}\sigma} = -g_\sigma \mathrm{Tr}\bar{\tilde{H}}_a \sigma \tilde{H}_a$$

$$\mathcal{L}_{\tilde{H}\tilde{H}\beta} = -i\beta\, Tr\left[\bar{\tilde{H}}_b v_\mu (V_{ba}^\mu - \rho_{ba}^\mu)\tilde{H}_a\right]$$

$$\mathcal{L}_{\tilde{H}\tilde{H}\lambda} = i\lambda\, Tr\left[\bar{\tilde{H}}_a \sigma_{\mu\nu} F_{ba}^{\mu\nu} \tilde{H}_a\right]$$

Doubly heavy baryon doublet field

QQq in the ground state

$$\Psi_{QQ}(\text{antisym}) = \text{space(sym)} \times \text{color(antisym)} \times \text{spin(sym)}$$

$$spin_{QQ} \otimes spin_q = 1 \otimes \frac{1}{2} = \frac{1}{2} \oplus \frac{3}{2}$$

$\rightarrow QQq$ with spin $\frac{1}{2}$ and $\frac{3}{2}$ are degenerate in HQ limit

doublet field Ψ_{QQ} of $\mathcal{B}_{QQ}$ and $\mathcal{B}^*_{QQ}$

$$\psi^\mu_{QQ} = \mathcal{B}^{*\mu}_{QQ} + \sqrt{\frac{1}{3}}(\gamma^\mu + v^\mu)\gamma^5 \mathcal{B}_{QQ}$$

$$\bar{\psi}^\mu_{QQ} = \bar{\mathcal{B}}^{*\mu}_{QQ} - \sqrt{\frac{1}{3}}\bar{\mathcal{B}}_{QQ}\gamma^5(\gamma^\mu + v^\mu)$$

Lu Meng et al., PHYSICAL REVIEW D 100, 014006 (2019)

$\mathcal{L}_{int}$ of ψ_{QQ}

pseudo scalar & scalar & vector coupling

$$\mathcal{L}_{\psi_{QQ}\psi_{QQ}A} = \hat{g}\bar{\psi}^{\mu}_{QQ}A^{\nu}\gamma_{\nu}\gamma^{5}\psi_{QQ\mu}$$

$$\mathcal{L}_{\psi_{QQ}\psi_{QQ}\sigma} = -\hat{g_{\sigma}}\bar{\psi}^{\mu}_{QQ}\sigma\psi_{QQ\mu}$$

$$\mathcal{L}_{\psi_{QQ}\psi_{QQ}\beta} = i\hat{\beta}\bar{\psi}^{\mu}_{QQ}v_{\nu}\rho^{\nu}\psi_{QQ\mu}$$

$$\mathcal{L}_{\psi_{QQ}\psi_{QQ}\lambda} = i\hat{\lambda}\bar{\psi}^{\mu}_{QQ}\sigma_{\alpha\beta}F^{\alpha\beta}\psi_{QQ\mu}$$

identification of $\bar{D}$ with Ξ_{cc} from HADS

$$g = \hat{g}$$
$$g_{\sigma} = \hat{g}_{\sigma}$$
$$\beta = \hat{\beta}$$
$$\lambda = \hat{\lambda}$$

Jie Hu et al., PHYSICAL REVIEW D 73, 054003 (2006)

OBEP for $\bar{D}^{(*)}\Xi_{cc}^{(*)}$

$$V_\pi(r) = C_\pi(\frac{g}{2f_\pi})^2\frac{1}{3}\left[(\mathcal{O}_1\cdot\mathcal{O}_2)C(r,m_\pi,\Lambda) + S_{\mathcal{O}_1,\mathcal{O}_2}T(r,m_\pi,\Lambda)\right]\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$$

$$V_\sigma(r) = -\frac{1}{2}\left(\frac{g_\sigma}{m_\sigma}\right)^2 C(r,m_\sigma,\Lambda)$$

$$V_v(r) = C_v(\lambda g_V)^2\,\frac{1}{3}\,\left[2\mathcal{O}_1\cdot\mathcal{O}_2 C(r,m_v,\Lambda) - S_{\mathcal{O}_1,\mathcal{O}_2}T(r,m_\rho,\Lambda)\right]\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$$

$$+\frac{1}{2}\left(\frac{\beta g_V}{2m_V}\right)C(r,m_v,\Lambda)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$$

Now set $\Lambda = 1234\,[\mathrm{MeV}]$

Possible channels

- $J = \frac{1}{2}$

$$\begin{pmatrix} \bar{D}\Xi_{cc}(^2S_{\frac{1}{2}}) \\ \bar{D}\Xi^*_{cc}(^4D_{\frac{1}{2}}) \\ \bar{D}^*\Xi_{cc}(^2S_{\frac{1}{2}}, ^4D_{\frac{1}{2}}) \\ \bar{D}^*\Xi^*_{cc}(^2S_{\frac{1}{2}}, ^4D_{\frac{1}{2}}, ^6D_{\frac{1}{2}}) \end{pmatrix}$$

- $J = \frac{3}{2}$

$$\begin{pmatrix} \bar{D}\Xi_{cc}(^2D_{\frac{3}{2}}) \\ \bar{D}\Xi^*_{cc}(^4S_{\frac{3}{2}}, ^4D_{\frac{3}{2}}) \\ \bar{D}^*\Xi_{cc}(^2D_{\frac{3}{2}}, ^4S_{\frac{3}{2}}, ^4D_{\frac{3}{2}}) \\ \bar{D}^*\Xi^*_{cc}(^2D_{\frac{3}{2}}, ^4S_{\frac{3}{2}}, ^4D_{\frac{3}{2}}, ^6D_{\frac{3}{2}}) \end{pmatrix}$$

Possible channels

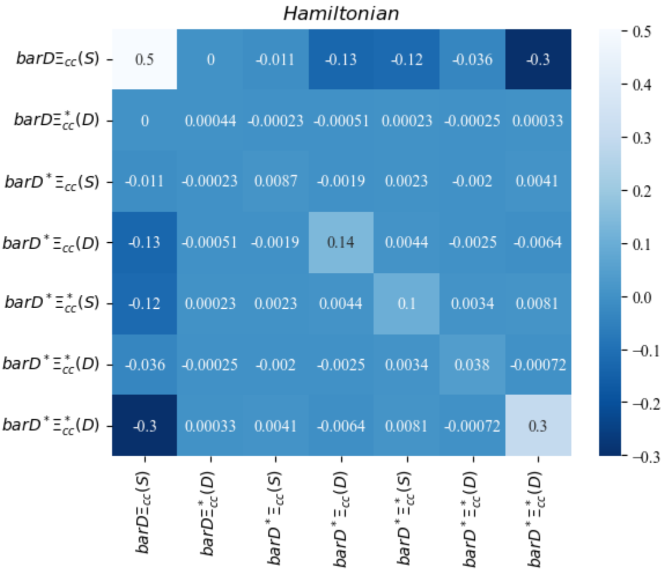
- $J = \frac{5}{2}$

$$\begin{pmatrix} \bar{D}\Xi_{cc}(^2D_{\frac{5}{2}}) \\ \bar{D}\Xi^*_{cc}(^4D_{\frac{5}{2}}) \\ \bar{D}^*\Xi_{cc}(^2D_{\frac{5}{2}},^4D_{\frac{5}{2}}) \\ \bar{D}^*\Xi^*_{cc}(^2D_{\frac{5}{2}},^6S_{\frac{5}{2}},^6D_{\frac{5}{2}}) \end{pmatrix}$$

Numerical results of $J = \frac{1}{2}$ case

OBE	[MeV]			
Λ	1234	1400	1450	1500
E_{bin}	$\times$	$\times$	0.096	0.479
$P_{\bar{D}\Xi_{cc}}(^2S_{\frac{1}{2}})$	$\times$	$\times$	0.998	0.997
$P_{\bar{D}\Xi_{cc}^*}(^4D_{\frac{1}{2}})$	$\times$	$\times$	$8.8e-07$	$2.0e-6$
$P_{\bar{D}^*\Xi_{cc}}(^2S_{\frac{1}{2}})$	$\times$	$\times$	$4.4e-05$	$1.2e-4$
$P_{\bar{D}^*\Xi_{cc}}(^4D_{\frac{1}{2}})$	$\times$	$\times$	$2.7e-04$	$2.7e-4$
$P_{\bar{D}^*\Xi_{cc}^*}(^2S_{\frac{1}{2}})$	$\times$	$\times$	$6.9e-05$	$6.4e-4$
$P_{\bar{D}^*\Xi_{cc}^*}(^4D_{\frac{1}{2}})$	$\times$	$\times$	$6.9e-05$	$1.6e-4$
$P_{\bar{D}^*\Xi_{cc}^*}(^6D_{\frac{1}{2}})$	$\times$	$\times$	$5.4e-04$	$1.2e-3$
$\sqrt{\langle r^2 \rangle}$	$\times$	$\times$	9.2	4.4

$\langle H \rangle$



Summary

1. Analysis of $D^{(*)}D$

- assume $T_{cc}(3875)$ is a molecule of DD^*
- reproduce the experimental data at $\Lambda \simeq 1234\,\mathrm{MeV}$
- tensor force mixing $D^{(*)}D^{(*)}(S) - D^{(*)}D^{(*)}(D)$ and transition effect of $[DD^*]_- - D^*D^*$ are important to bind

2. Prediction of $\bar{D}^{(*)}\Xi_{cc}^{(*)}$

- determine the coupling constants in $\mathcal{L}_{\psi_{QQ}}$ based on HADS
- No bound state at $\Lambda \simeq 1234\,[\mathrm{MeV}]$
- only $J = \frac{1}{2}$ state is bound at $\Lambda \simeq 1450\,[\mathrm{MeV}]$

Future works

Consider **HQ limit** ($m_Q \to \infty$) through my study

but,

$$m_{D^*}(2008) - m_D(1867) \simeq 140\mathrm{MeV} > m_\pi(138)$$

- include the next leading order of $\mathcal{O}(1/m_Q)$ **in HQSS**

  S. Yasui and K. Sudoh Phys. Rev. C 89, 015201(2014)

$$\bar{D}^{(*)} \sim \Xi_{cc}^{(*)}$$

- include the next leading order of $\mathcal{O}(1/m_Q v)$ **in HADS**

  take into account QQ sizable effect $\to$ take Λ smaller ($\Lambda \propto \frac{1}{r}$)