Structure of X(3872) with hadronic potentials coupled to quarks

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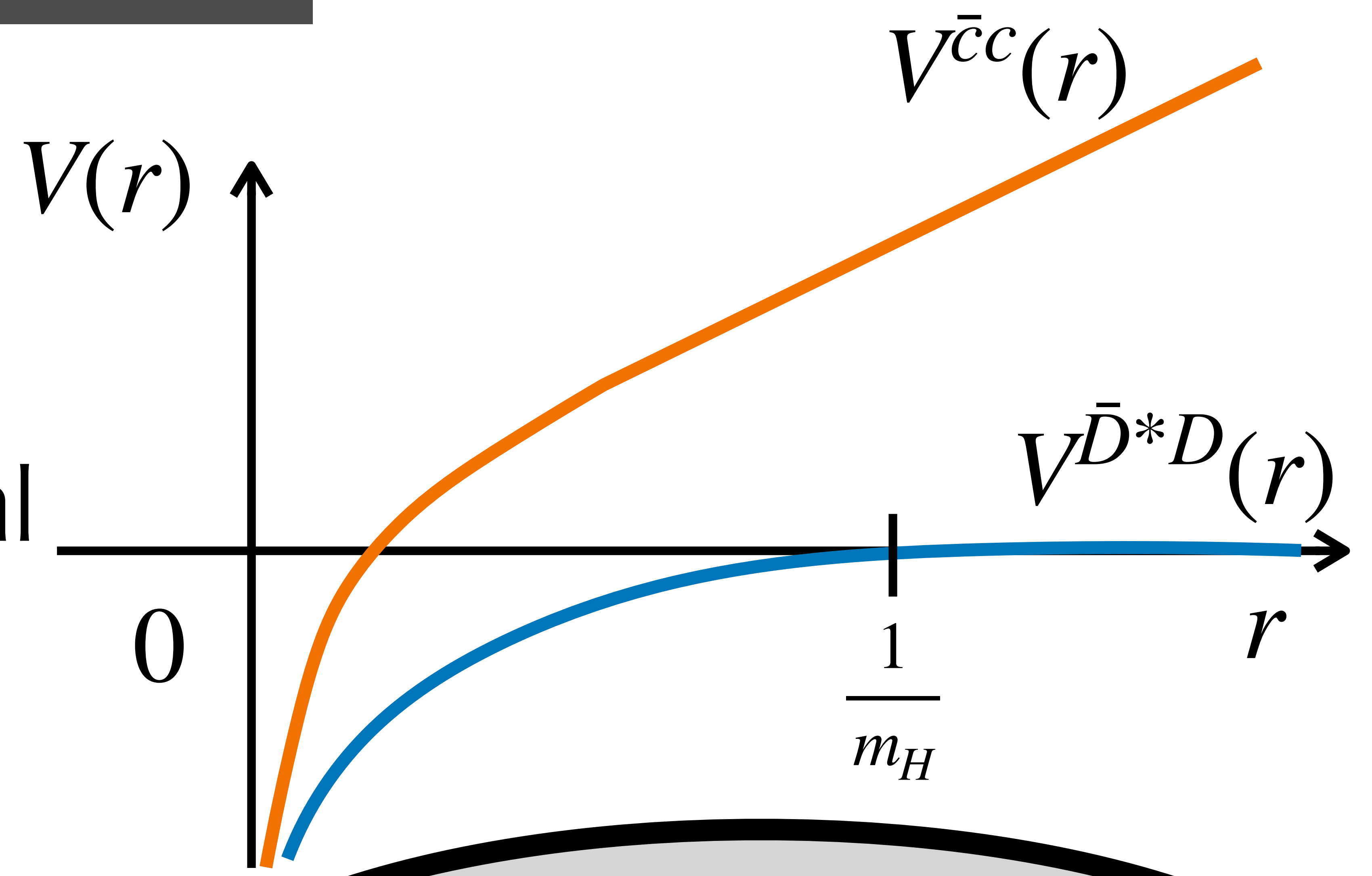
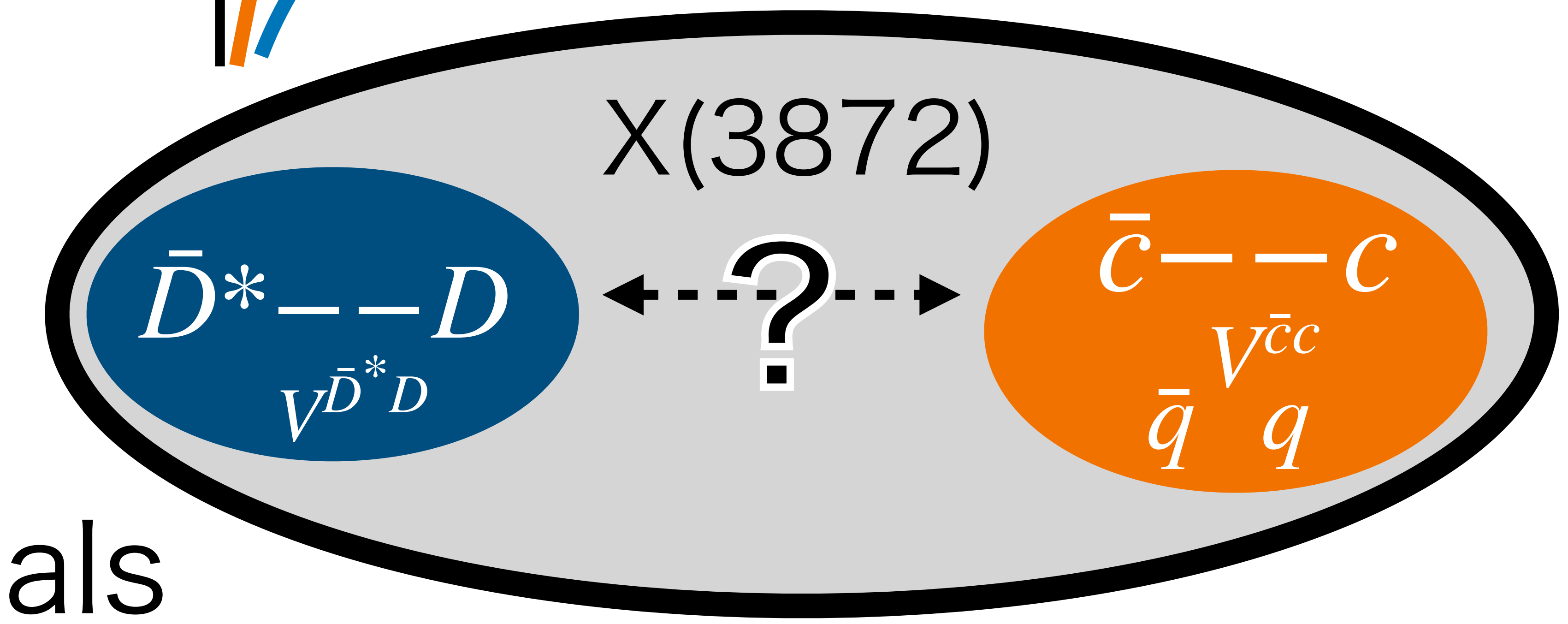
Introduction

$$V^{\bar{c}c}(r) = -\frac{A}{r} + \sigma r + V_0 \xrightarrow[r\to\infty]{} \infty$$: Confinement potential

$$V^{\bar{D}^*D}(r) = K_{\bar{D}^*D}\frac{\exp[-m_H r]}{r} + \cdots \xrightarrow[r\to\infty]{} 0$$: Scattering potential





Problem

- $V^{\bar{c}c}(r)$ and $V^{\bar{D}^*D}(r)$ are calculated independently

This Study

- **Channel couplings** between $\bar{c}c$ and $\bar{D}^*D$ potentials
- Obtain the **effective potential** $V_{\text{eff}}^{\bar{D}^*D}$ as a local potential
- **Convert non-local** effective potential $V_{\text{eff}}^{\bar{D}^*D}$ to **local** ones
- Apply $V_{\text{eff}}^{\bar{D}^*D}$ to the model of **X(3872)**

Formulation : $V_{\mathrm{eff}}^{\bar{D}^*D}$

- Hamiltonian H with coupled-channel between $\bar{c}c$ and $\bar{D}^*D$

$$H = \begin{pmatrix} T^{\bar{c}c} & 0 \\ 0 & T^{\bar{D}D} + \Delta \end{pmatrix} + \begin{pmatrix} V^{\bar{c}c} & V^t \\ V^t & V^{\bar{D}D} \end{pmatrix}$$

$T^{\bar{c}c}, T^{\bar{D}D}$: kinetic energy
Δ : threshold energy
V^t : transition potential

- **non-local**
- Energy dependent

- Eliminate $\bar{c}c$ to obtain effective potential $V_{\mathrm{eff}}^{\bar{D}^*D}(\boldsymbol{r}, \boldsymbol{r}', E)$[1,2]

$$V_{\mathrm{eff}}^{\bar{D}^*D}(\boldsymbol{r}, \boldsymbol{r}', E) = V^{\bar{D}^*D}(\boldsymbol{r})\delta(\boldsymbol{r}' - \boldsymbol{r}) + \sum_n \frac{\langle \boldsymbol{r}'_{\bar{D}^*D} | V^t | \phi_n \rangle \langle \phi_n | V^t | \boldsymbol{r}_{\bar{D}^*D} \rangle}{E - E_n}$$

When

- $V^{\bar{D}^*D}(\boldsymbol{r}) = 0$
- $\langle \phi_n | V^t | \boldsymbol{r}_{\bar{D}^*D} \rangle = g_0 e^{-\mu r}/r$
- only take $n = 0$

$$= \frac{g_0^2}{E - E_0} \frac{e^{-\mu r'}}{r'} \frac{e^{-\mu r}}{r}$$

g_0 : coupling constant,
E_0 : binding energy of $\bar{c}c$
μ : cut-off constant

[1] H. Feshbach, Ann. Phys. **5**, 357 (1958); ibid., **19**, 287 (1962)
[2] I. Terashima, T. Hyodo arXiv:2208.14075 [hep-ph]

Formulation : Conversion to local

① Formal derivative expansion

Express non-local potential in terms of derivative of local potential
by Taylor expansion directly

$$V_{\text{eff}}^{\bar{D}^*D}(\boldsymbol{r},\boldsymbol{r}',E) = \frac{g_0^2}{E-E_0}\frac{e^{-\mu r}}{r}\frac{e^{-\mu r'}}{r'} \xrightarrow{\text{derivative expansion}} \boxed{V^{\text{formal}}(r,E) = \frac{4\pi g_0^2}{\mu^2(E-E_0)}\frac{e^{-\mu r}}{r} + O(\nabla)}$$

② HAL QCD method [S.Aoki and K.Yazaki, PTEP **2022**, no.3, 033B04 (2022)]

Construct from wave function $\psi_k(r)$ obtained from non-local

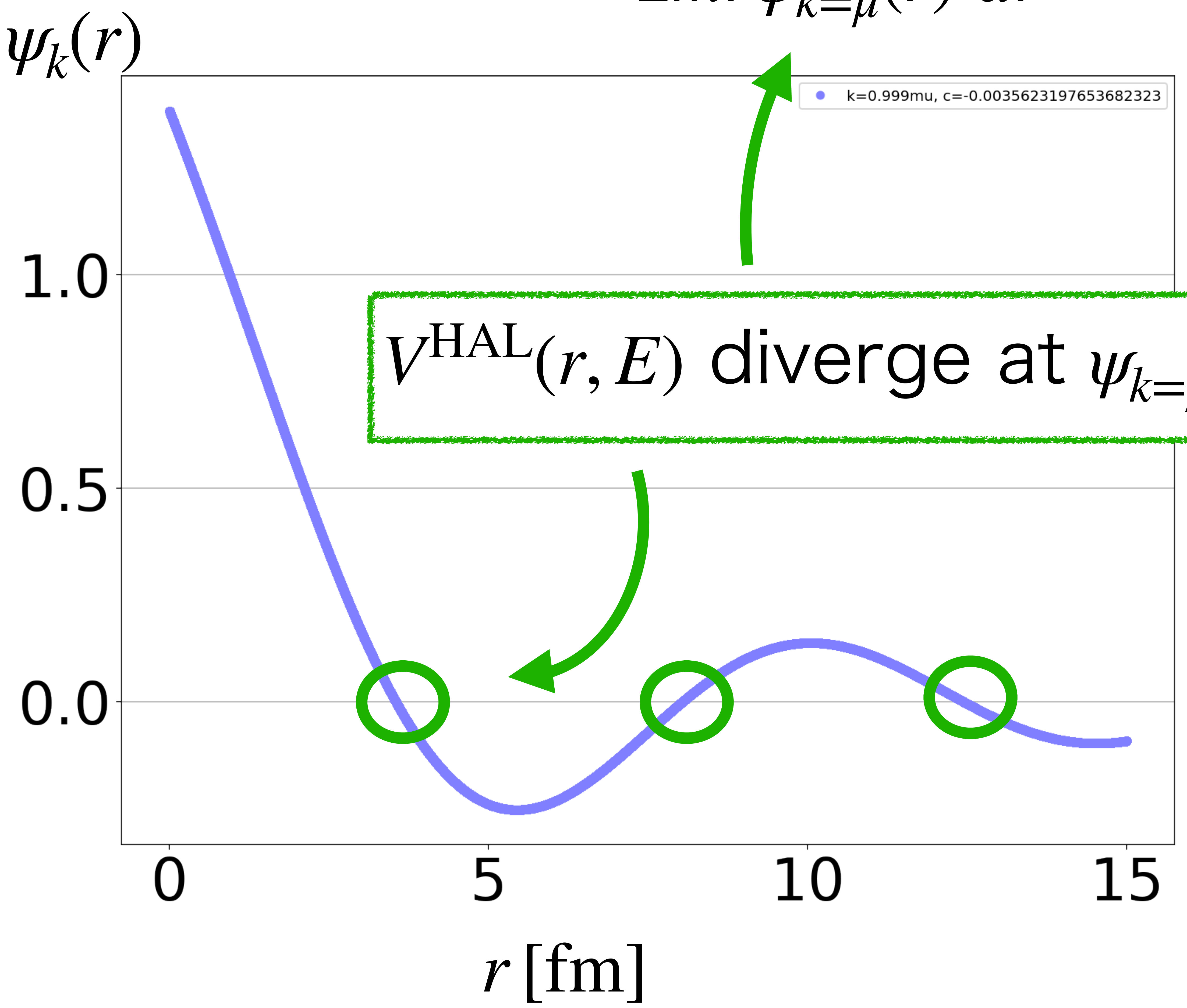
potentials with momentum $k = \sqrt{2mE}$

$$\boxed{V^{\text{HAL}}(r,E) = E + \frac{1}{2mr\psi_k(r)}\frac{d^2}{dr^2}\left[r\psi_k(r)\right] + O(\nabla^2)}$$

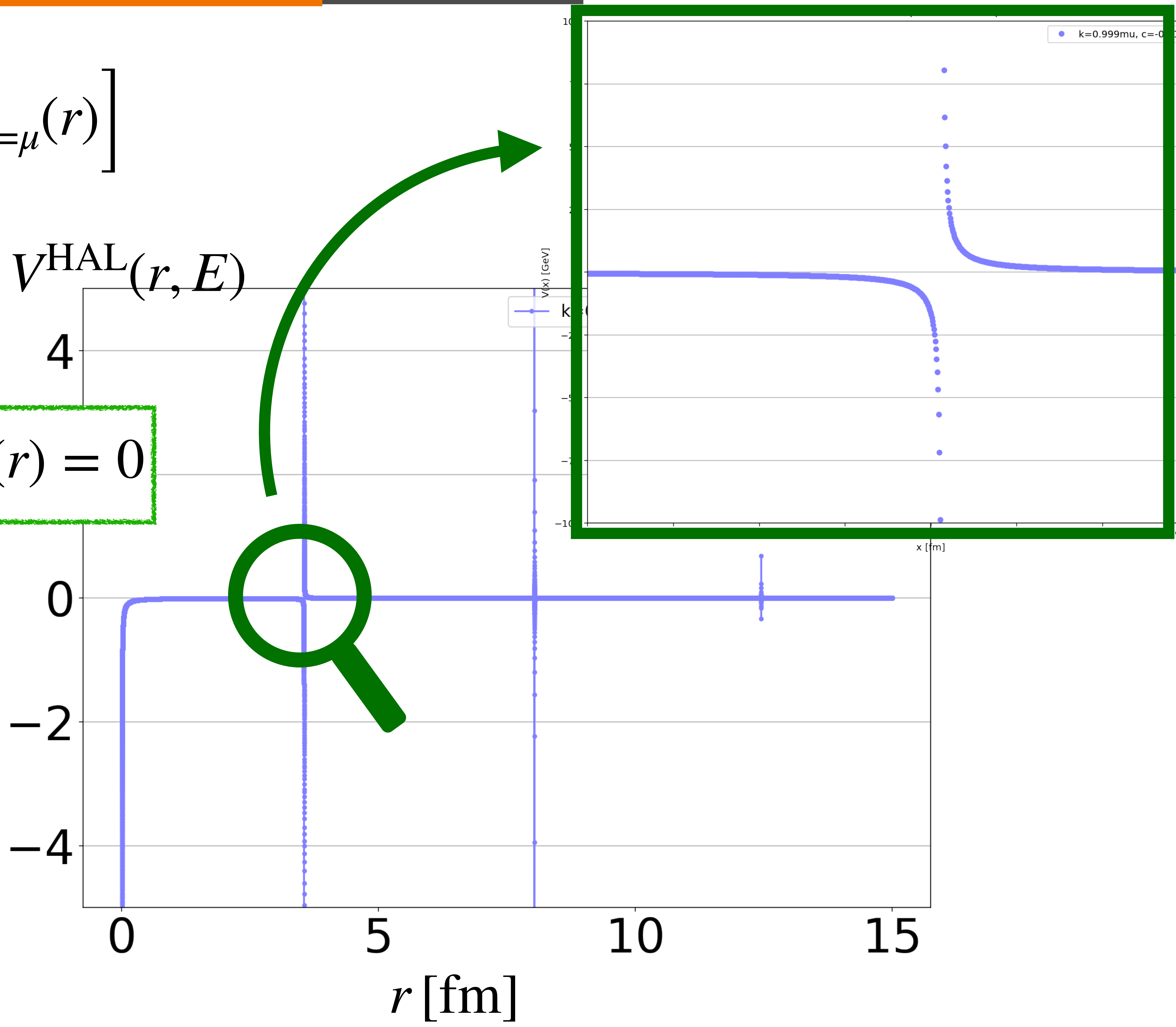
- $\psi_k(r)$ can be solved analytically by virtue of Yukawa potential

Note : Analytics nature of HAL QCD method

$$V^{\mathrm{HAL}}(r,E) = \frac{1}{2mr\psi_{k=\mu}(r)}\frac{d^2}{dr^2}\left[r\psi_{k=\mu}(r)\right]$$



$V^{\mathrm{HAL}}(r,E)$ diverge at $\psi_{k=\mu}(r)=0$



Result : Energy dependance of HAL QCD method

Change E to investigate energy dependance of $V^{\mathrm{HAL}}(r, E)$

The greater E, the smaller r of diverges occur

- Because frequency $\dfrac{\sqrt{2mE}}{\hbar}$ get higher when E increase

Formulation of X(3872)

$$V_{\mathrm{eff}}^{\bar{D}^*D}(\boldsymbol{r},\boldsymbol{r}',E) = \frac{g_0^2}{E-E_0}v(\boldsymbol{r})v(\boldsymbol{r}')$$

Construct a model of X(3872)

- NOT consider
  - channel of $D^{*\mp}D^{\pm}$
  - decays to $J/\psi\ \pi^+\ \pi^-\ (\pi^0)$

- g_0 is determined to reproduce mass of X(3872)[4]
- cut-off μ takes as energy of π[4]

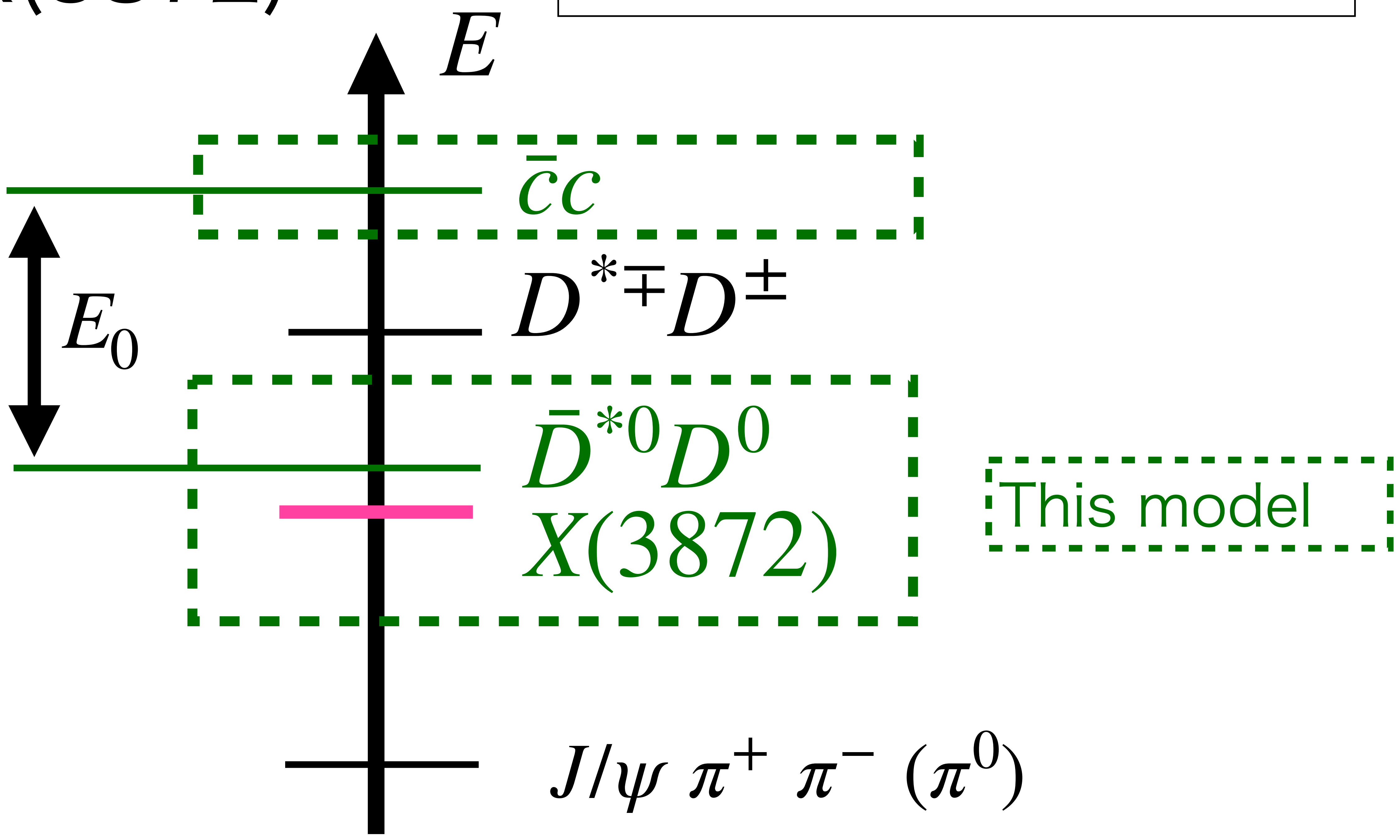
lightest meson

- Binding energy E_0 of $\bar{c}c$

$$E_0 = m_{c\bar{c}} - m_{D^0} - m_{\bar{D}^{*0}}$$

[3] [4] [4]

- Reduced mass $m = \dfrac{m_{D^0} + m_{\bar{D}^{*0}}}{m_{D^0}m_{\bar{D}^{*0}}}$



[3] S. Godfrey and N. Isgur, Phys. Rev. D, **32**, 189 (1985), [4] PDG Live

Result : ①formal derivative expansion in X(3872)

$$V^{\text{formal}}(r,E) = \frac{4\pi g_0^2}{\mu^2(E-E_0)}\frac{e^{-\mu r}}{r} + O(\nabla)$$

- Local approximation by formal derivative expansion
  $V_{\text{eff}}^{\bar{D}^*D}(\boldsymbol{r},\boldsymbol{r}',E) \to V_0^{\text{formal}}(r,E)$

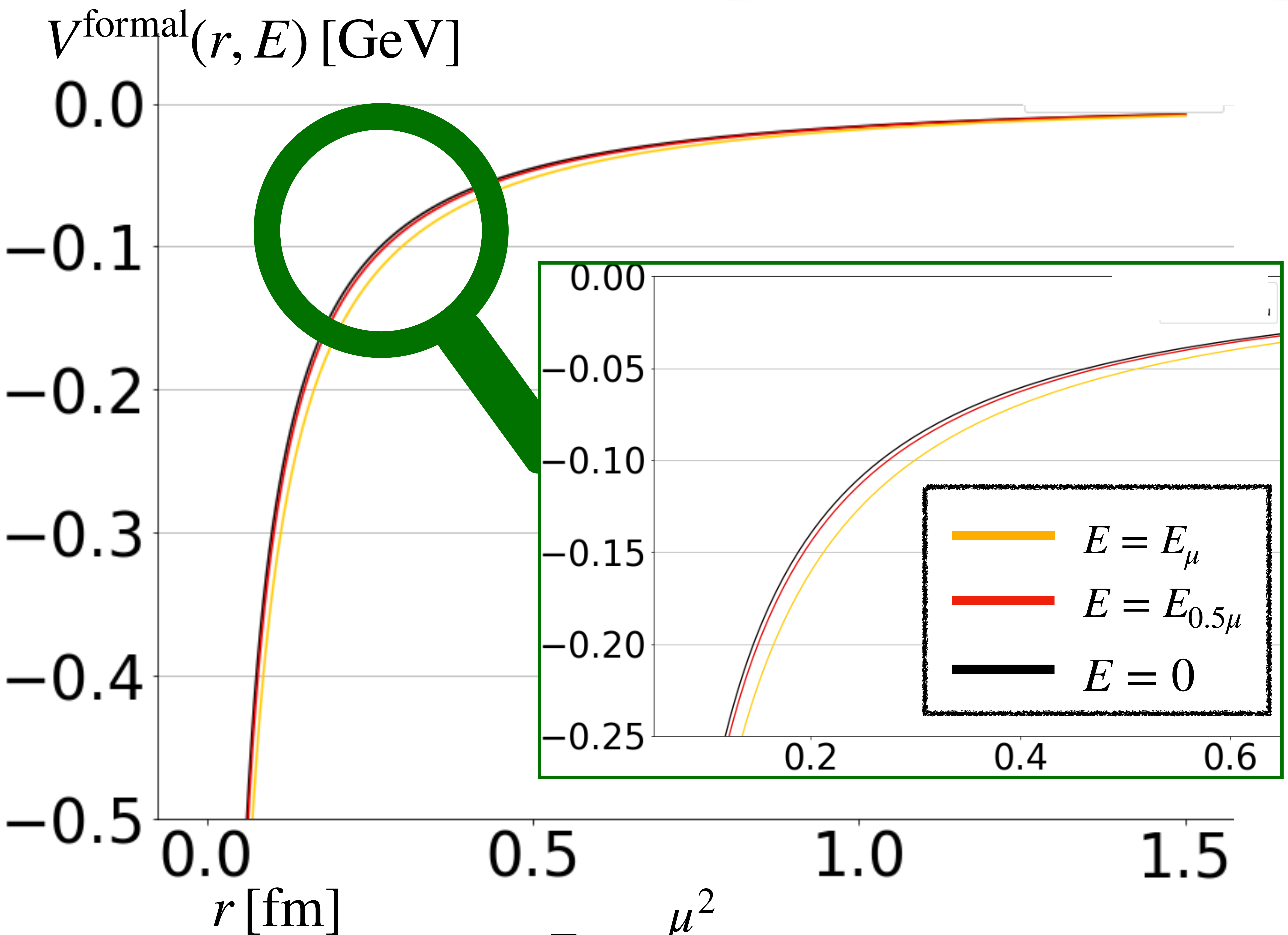
$\boxed{\text{Less } E \text{ dependance}}$

Reason:

$$V^{\text{formal}}(r,E) = \frac{4\pi g_0^2}{\mu^2(E-E_0)}\frac{e^{-\mu r}}{r}$$

$$E \ll E_0 = 0.078\,\text{GeV}$$

$(E_\mu \simeq 0.01$ GeV$)$



$E_\mu = \dfrac{\mu^2}{2m}$ $\quad\mu$: cut-off (0.14GeV)

Result : ②HAL QCD method in X(3872)

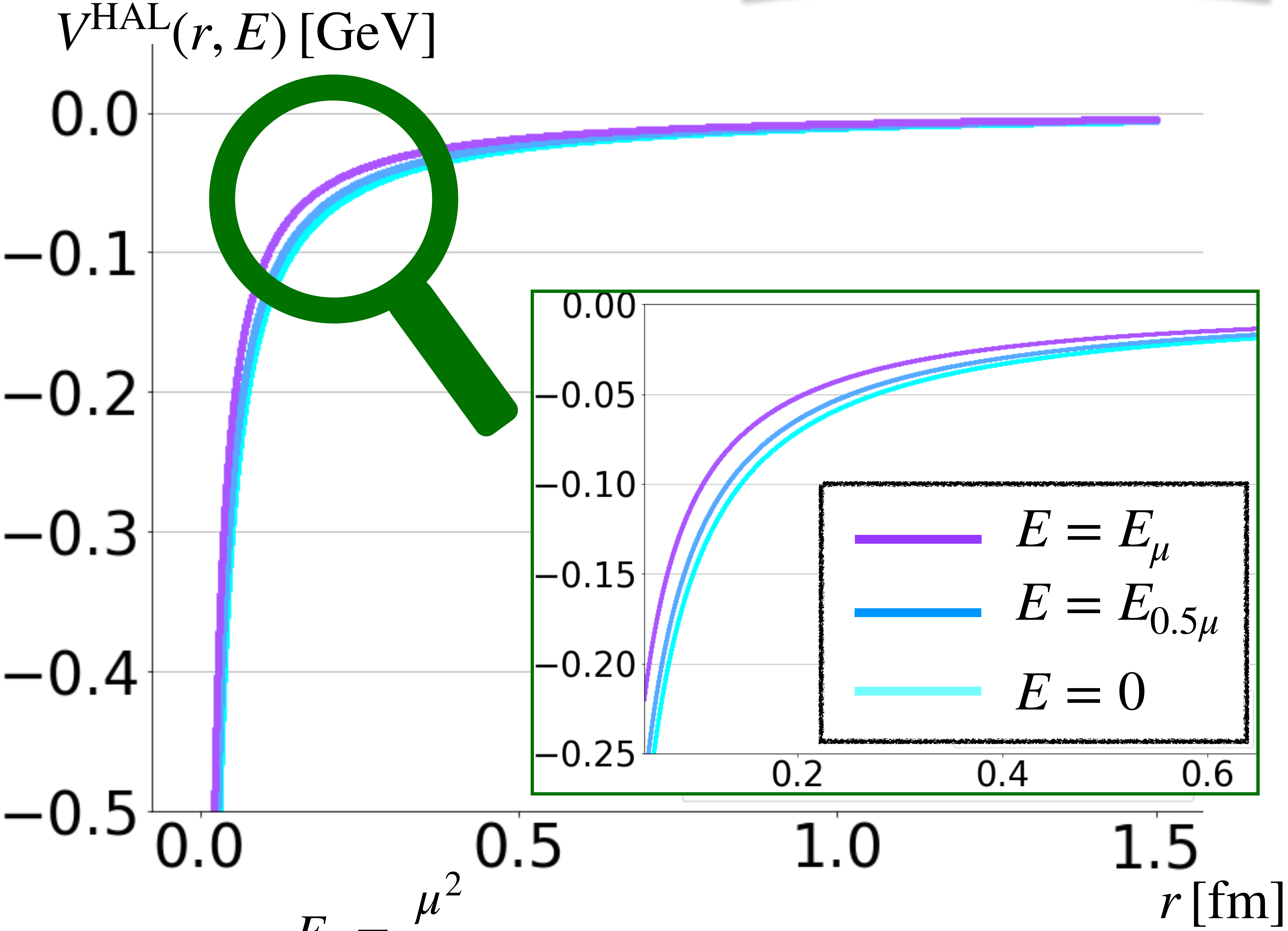
$$V^{\mathrm{HAL}}(r,E) = \frac{1}{2mr\psi_k(r)}\frac{d^2}{dr^2}[r\psi_k(r)] + O(\nabla^2)$$

- Local approximation by HAL QCD method

$V_{\mathrm{eff}}^{\bar{D}^*D}(\boldsymbol{r},\boldsymbol{r}',E) \to V_0^{\mathrm{HAL}}(r,E)$

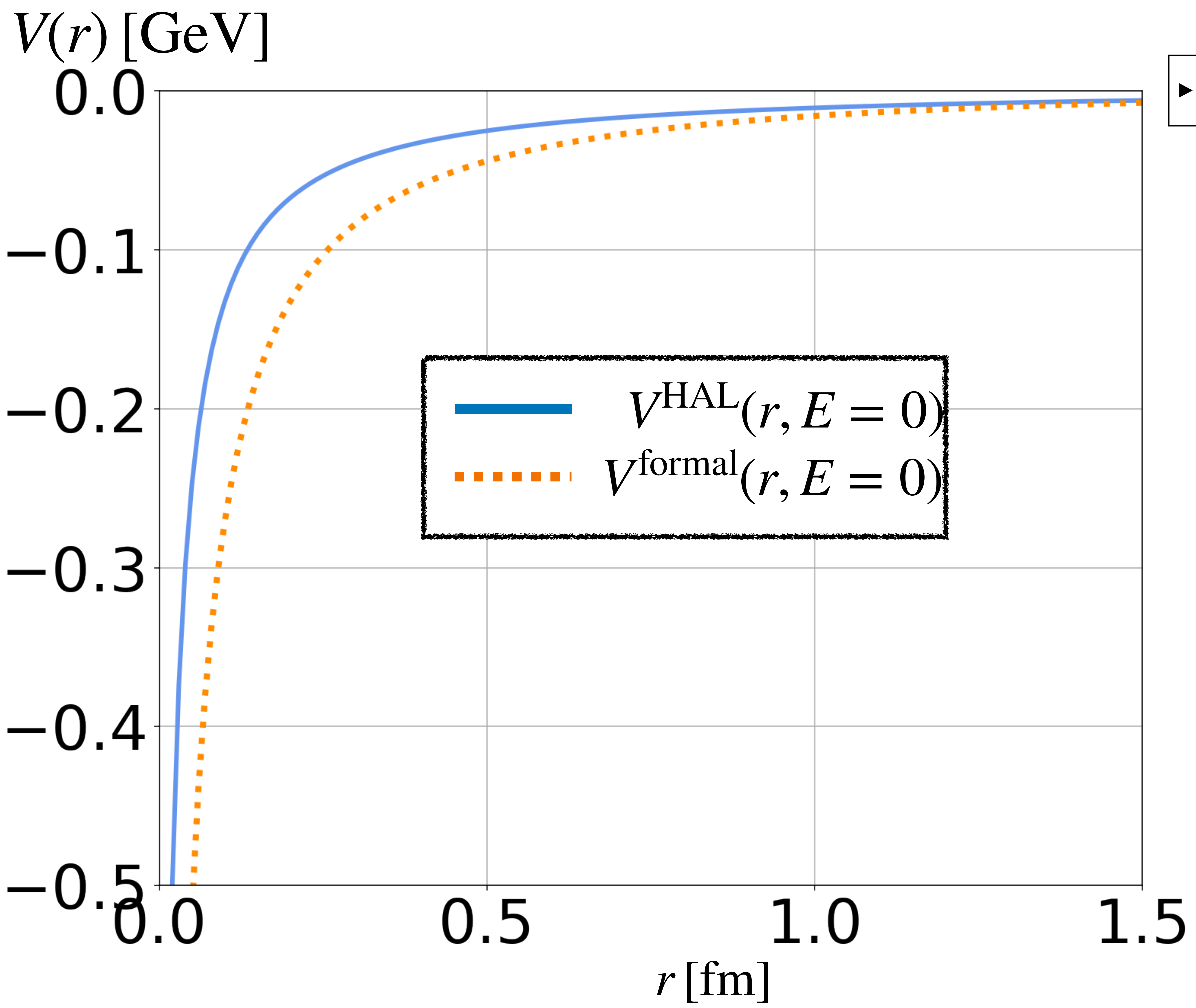
→ Less E dependance

→ Same as ①formal derivative



$E_\mu = \dfrac{\mu^2}{2m}$ μ :cut-off (0.14GeV)

Result : compare in $X(3872)$



- Comparing approximated potentials

Potentials are different

- How about physical quantities from these potentials?

Check it including k-dependent

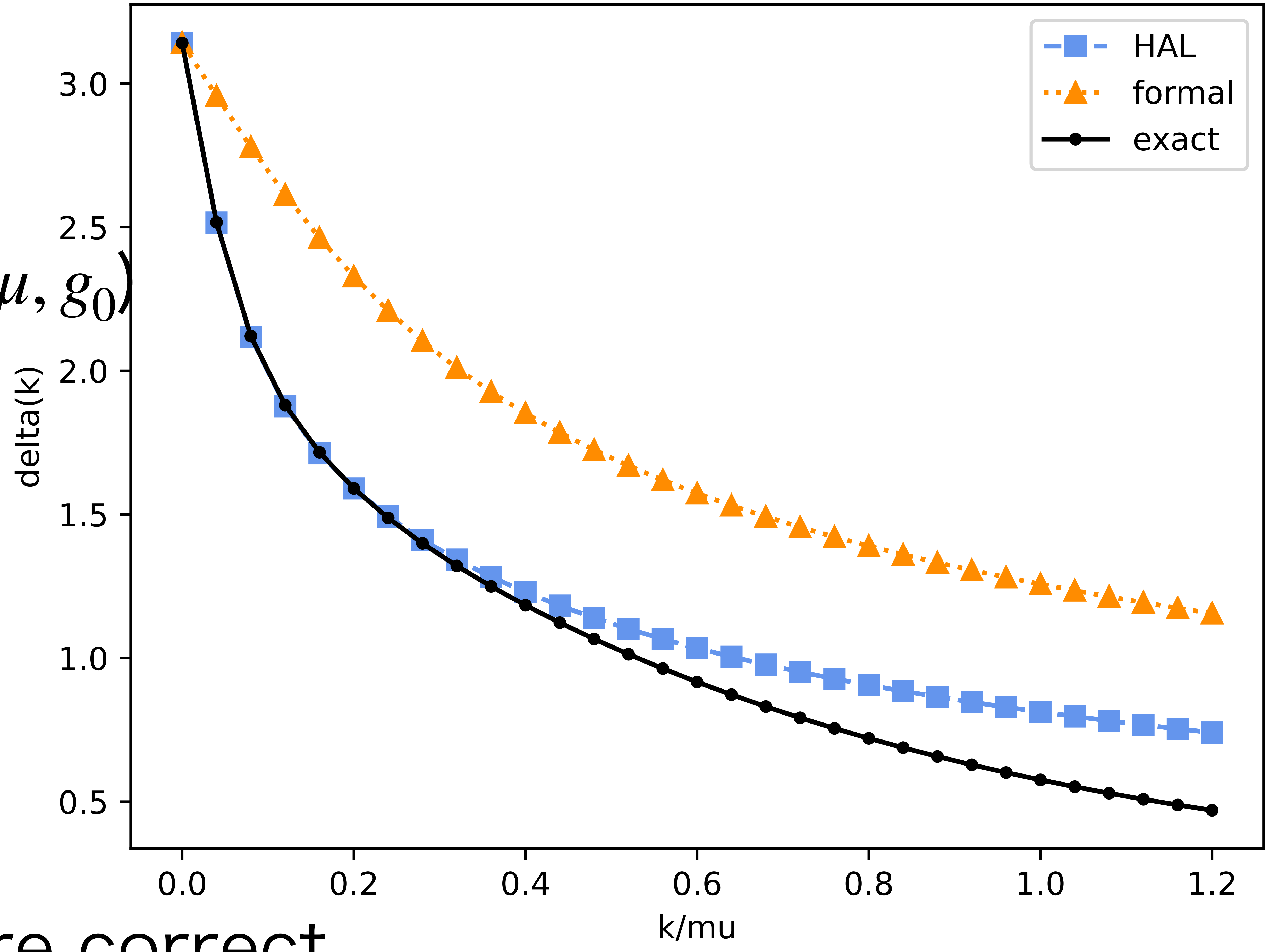
Result : Phase sift δ

Compare phase shift δ (k)

from $V_0^{\mathrm{formal}}(r, E=0)$ and $V^{\mathrm{HAL}}(r, E=0)$,

with exact $\delta(k)$ from model conditions (μ, g_0)

δ from HAL QCD method is more consistent with exact δ, especially for small k



➡ Scattering length from V^{HAL} is more correct

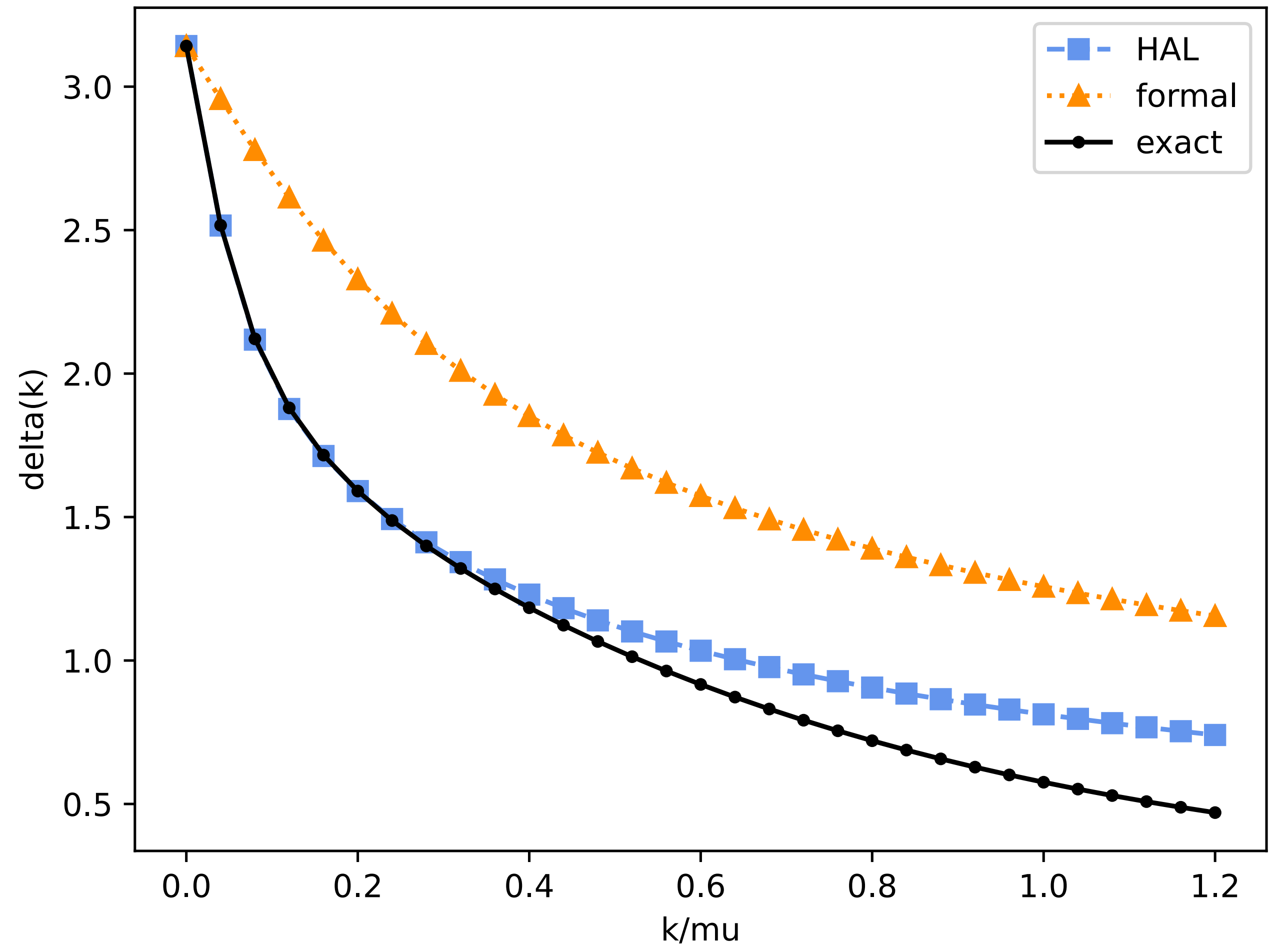
Summary

$$V_{\text{eff}}^{\bar{D}^*D}(\boldsymbol{r},\boldsymbol{r}',E) = \frac{g_0^2}{E-E_0} v(\boldsymbol{r}) v(\boldsymbol{r}')$$

- Consider **channel coupling** of $X(3872)$ between $V^{\bar{c}c}(r)$ and $V^{\bar{D}^*D}(r)$
- Convert $V_{\text{eff}}^{\bar{D}D}(E)$ obtained by eliminating $c\bar{c}$ channel, non-local to local
- compare of two conversion methods ①formal derivative expansion V_0^{formal} and ②HAL QCD method V_0^{HAL}

➡

- V^{formal} and V^{HAL} are different
- Phase shift δ from V^{HAL} is more consistent with exact δ



Future outlook

- Comparison of what physical quantities (e.g., scattering amplitude) are obtained for each conversion
- Add $D^{*\mp}D^{\pm}$ channel [M. Takizawa, PTEP **2013**, 093 D 01 (2013)]