PEKING UNIVERSITY

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$N-\phi$ interaction from lattice QCD

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Dec. 07, 2022

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Phys. Rev. D 106, 074507 (2022) [arXiv: 2205.10544]

Contents

$N\phi$ interaction

- Fundamental inputs for studying

R. S. Hayano and T. Hatsuda, Rev. Mod. Phys. **82**, 2949 (2010)

  - meson properties in nuclear matter
  - chiral symmetry and its restoration

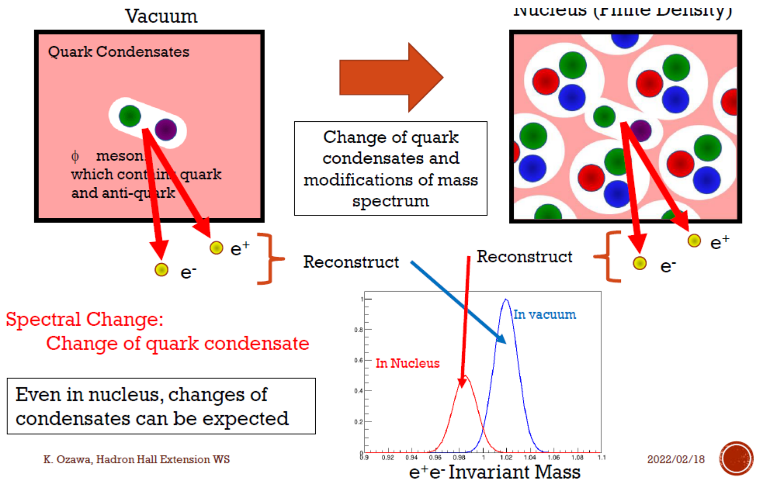
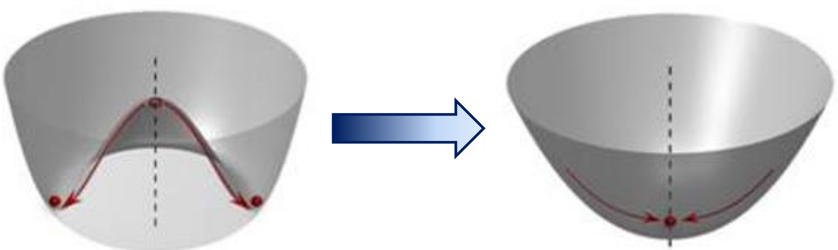


Fig. taken from K. Ozawa



- Theoretical studies

P. Gubler and D. Satow, Prog. Part. Nucl. Phys. 106, 1 (2019); T. Hatsuda and S. H. Lee, Phys. Rev. C 46, R34 (1992)
J. Castella and G. Krein, Phys. Rev. D 98, 014029 (2018)

  - ϕ in-medium property $\leftrightarrow \bar{s}s$ condensate
  - open questions on $N\phi$: microscopic origin, the role of dynamical π

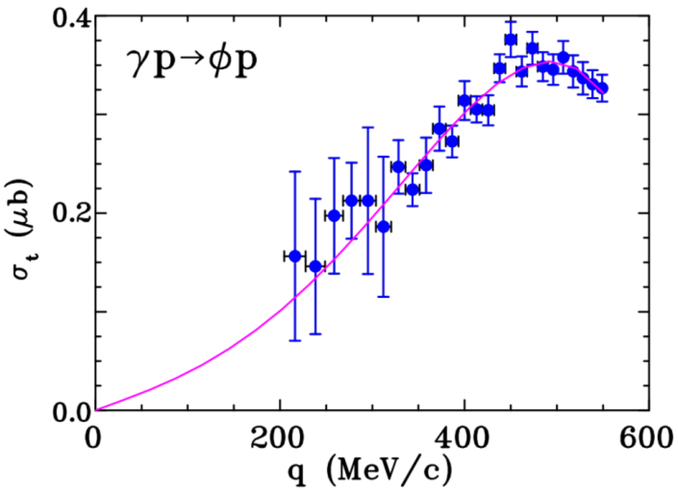
Experiments on $N\phi$ interaction

- Production and absorption experiments
  - Spring8-LEPS
  - KEK-PS E325
  - JLab-CLAS
  - COSY-ANKE

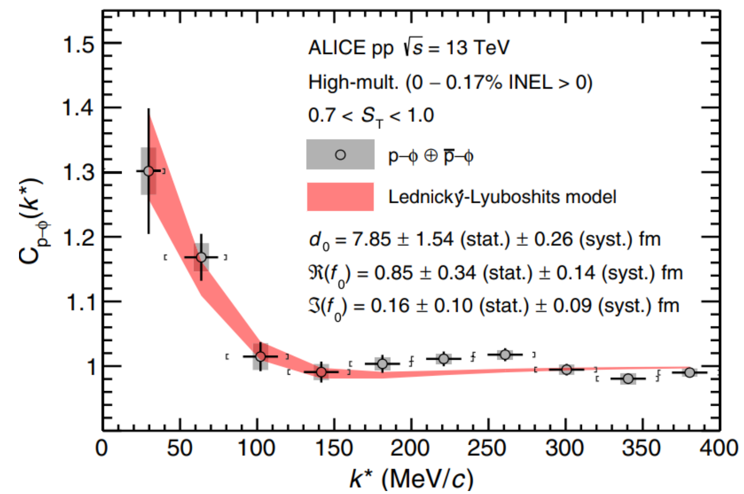
T. Ishikawa *et al.*, Phys. Lett. B **608**, 215 (2005)
R. Muto *et al.* (KEK-PS-E325 Coll.), Phys. Rev. Lett. **98**, 042501 (2007)
M. Wood *et al.* (CLAS Coll.), Phys. Rev. Lett. **105**, 112301 (2010)
A. Polyanskiy et al., Phys. Lett. **B** 695, 74 (2011)

- Recent measurements on scattering length

I. Strakovsky, L. Pentchev, and A. Titov, Phys. Rev. C 101, 045201 (2020)



ALICE Coll, Phys. Rev. Lett. 127, 172301 (2021)



Reference	Spin-averaged a_0 [fm]
Strakovsky *et al.* 2020	$\pm 0.063(10)$
ALICE 2021	$-0.85(34)_{\text{stat.}}(14)_{\text{syst.}}$

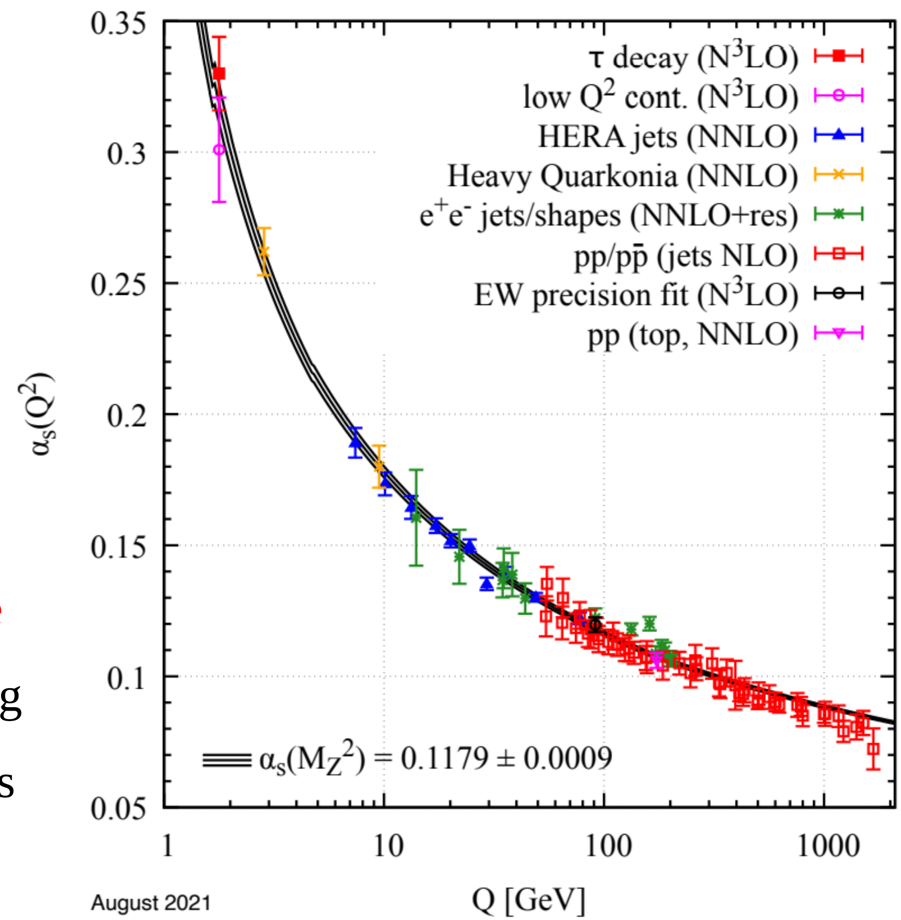
- A first principle study on $N\phi$ interaction is highly needed

QCD at different scales

➢ Strong interaction strength strongly depends on energy scale (Q)

- High Q (> few GeV) region, perturbative
  - ✓ asymptotic freedom
  - ✓ weakly interacting quarks and gluons
  - ✓ small coupling
  - ✓ pen and paper

- Low Q (< 1 GeV) region, non perturbative
  - ✓ chiral symmetry spontaneous breaking
  - ✓ strongly interacting quarks and gluons
  - ✓ large coupling
  - ✓ lattice QCD

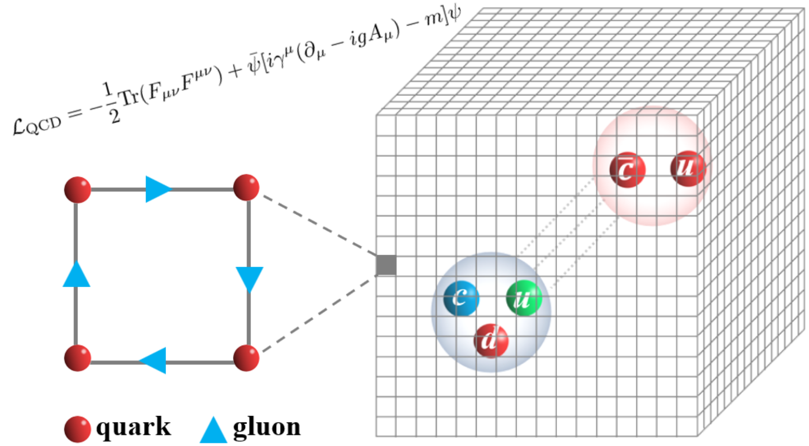


PDG, https://pdg.lbl.gov/

LQCD

- Lattice discretization

K.G. Wilson
(1974)



  - quarks: lattice site, $q(x)$
  - gluons: link between lattice sites, $U_\mu(x) = e^{iagA_\mu(x)}$

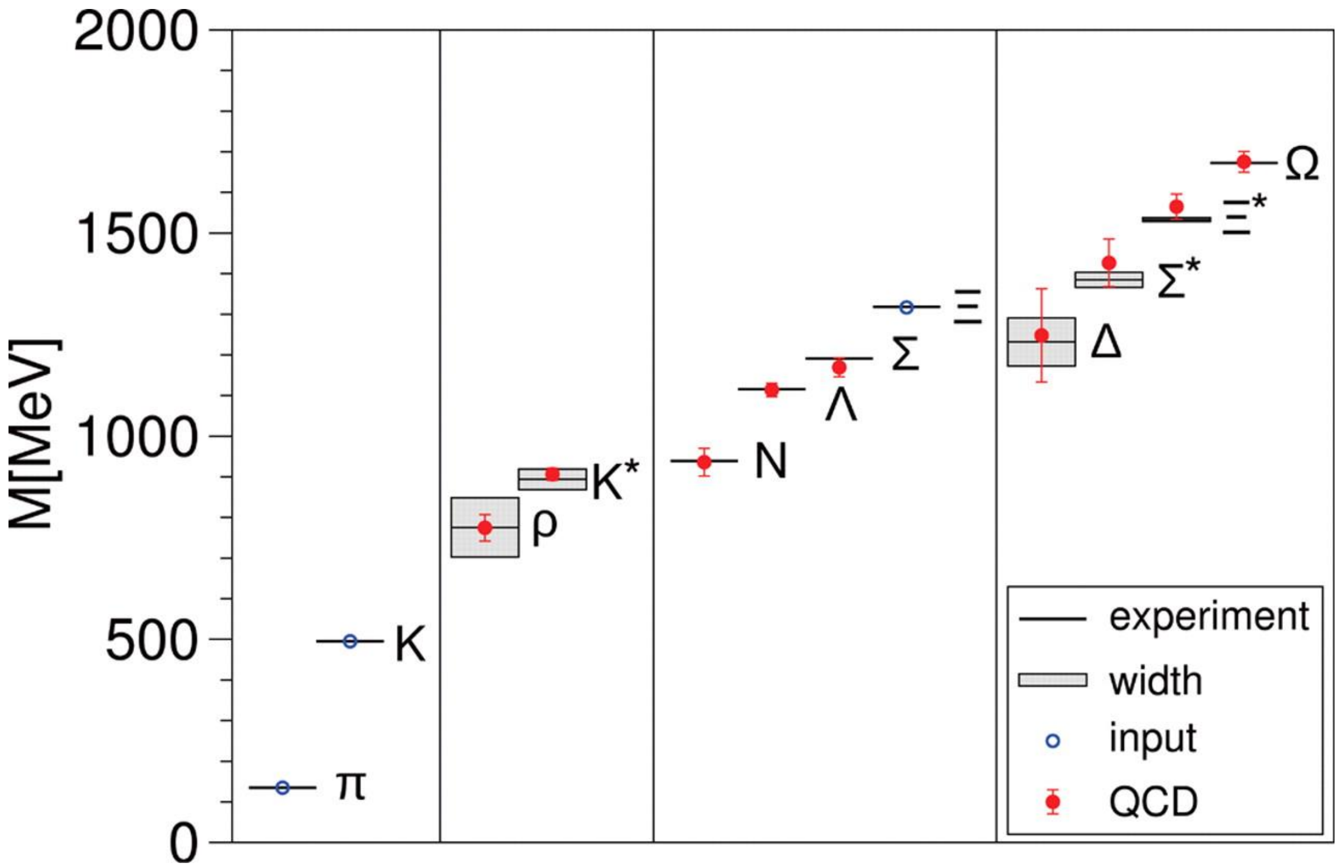
- Euclidean path integral with QCD action S_E

$$\langle \hat{O} \rangle = \frac{1}{Z_E} \int [\mathcal{D}\bar{q}][\mathcal{D}q][\mathcal{D}U] O e^{-S_E}$$

  - dof: $\sim 100^4 \times 8 \sim 10^9$

- Monte Carlo simulation: gauge configuration with probability e^{-S_E}

Single hadron mass from LQCD



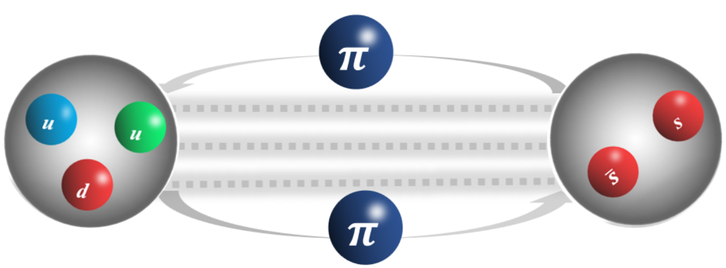
S. Durr *et al.* [BMW Coll.] Science 322, 1224 (2008)

- Successful for single hadron system

Lattice calculation of $N\phi$ interaction

➢ In this talk: First realistic LQCD results on $N\phi$ system

- interaction potential from lattice QCD
- interaction mechanism and the role of dynamical pion
- scattering properties
- detailed systematic error analysis



PHYSICAL REVIEW D **106,** 074507 (2022)

Attractive N-ϕ interaction and two-pion tail from lattice QCD near physical point

Yan Lyu,[1,2,*] Takumi Doi,[2,†] Tetsuo Hatsuda,[2,‡] Yoichi Ikeda,[3,§] Jie Meng,[1,4,∥] Kenji Sasaki,[3,¶] and Takuya Sugiura[2,**]

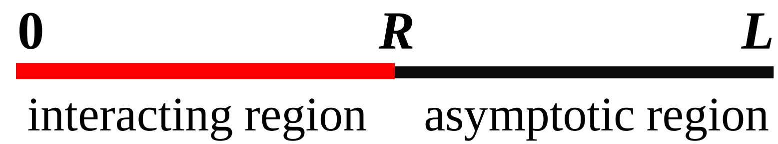
Finite volume method

M. Lüscher, Nucl. Phys. B 354, 531 (1991)

- QM in 1d (PBC $\psi(0) = \psi(L) = 0$)
  - Free theory

$$\psi(x) \sim \sin(kx), \quad k = \frac{n\pi}{L}$$

  - Interacting theory

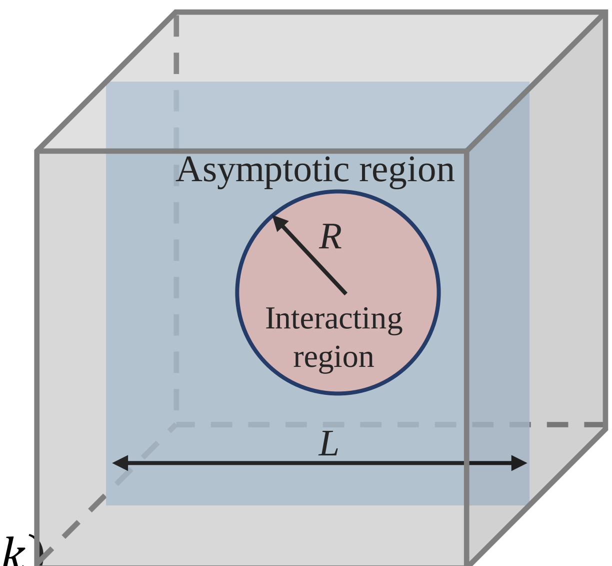


$$\psi(x > R) \sim \sin(kx + \delta(k)), \quad k + \delta(k) = \frac{n\pi}{L}$$

- Lüscher's formula (3d box)

$$k \cot \delta_0(k) = \frac{2}{\sqrt{\pi}L} Z_{00}(1, q^2)$$

$$Z_{00}(s, q^2) = \frac{1}{4\sqrt{\pi}} \sum_{\vec{n}\in\mathbb{Z}^3} \frac{1}{(\vec{n}^2 - q^2)^s}, \quad q = \frac{kL}{2\pi}$$



  - FV momentum k from LQCD → scattering phase shift $\delta(k)$

HAL QCD method

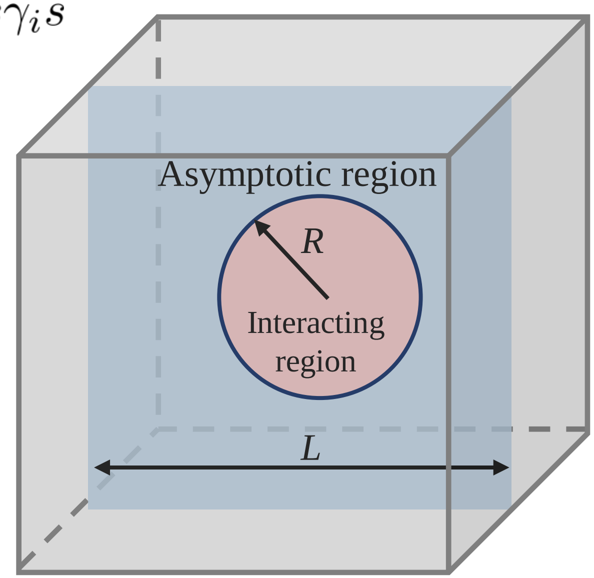
N. Ishii, S. Aoki and T. Hatsuda, Phys. Rev. Lett. **99,** 022001 (2007)

- Nambu-Bethe-Salpeter (NBS) amplitude

$$\psi^k(\boldsymbol{r},t) = \langle 0|N(\boldsymbol{r},0)\phi(\boldsymbol{0},0)|N(\boldsymbol{k})\phi(-\boldsymbol{k});E\rangle$$

$N_\alpha = (u^T C\gamma_5 d)u_\alpha$ $\qquad$ $\phi_i = \bar{s}\gamma_i s$

- asymptotic region
  - ✓ Helmholtz eq. $(\nabla^2 + k^2)\psi(r) \simeq 0$
  - ✓ Scattering phase shifts $\psi^k(r) \simeq A\dfrac{\sin(kr+\delta(k))}{kr}$



- "interacting" region

$$(\nabla^2 + k^2)\psi^k(\boldsymbol{r}) = m \times \int d^3r' U(\boldsymbol{r},\boldsymbol{r}')\psi^k(\boldsymbol{r}')$$

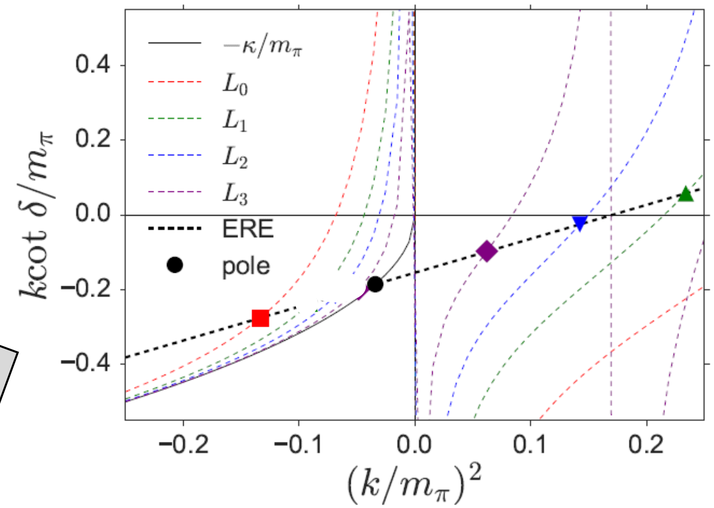
  - ✓ non-local energy-independent kernel $U(\boldsymbol{r},\boldsymbol{r}')$
  - ✓ derivative expansion $U(\boldsymbol{r},\boldsymbol{r}') = V(\boldsymbol{r})\delta(\boldsymbol{r}-\boldsymbol{r}') + \sum V_i(\boldsymbol{r})\nabla^i\delta(\boldsymbol{r}-\boldsymbol{r}')$

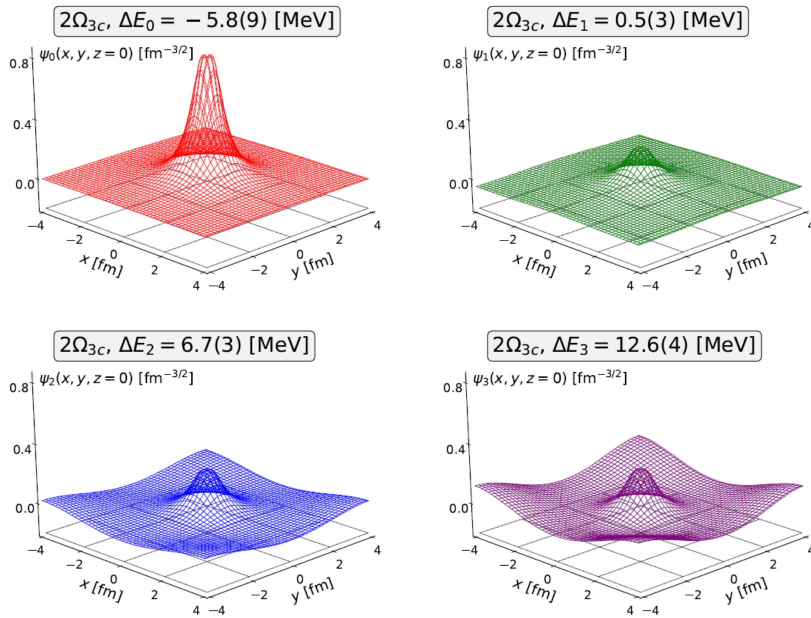
Check on convergence: Iritani *et al.*, JHEP 03, 007 (2019); YL *et al.*, PRD 105, 074512 (2022)

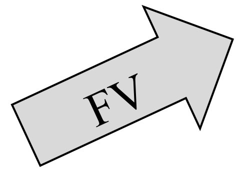
FV v.s. HAL

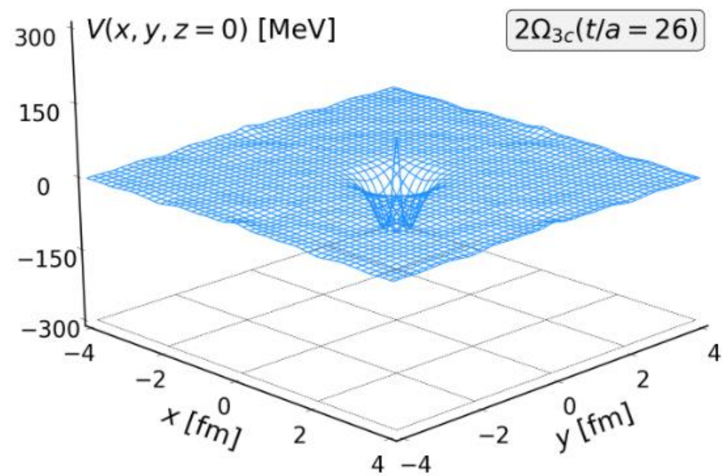
- If elastic states are available
  - FV: $\Delta E_{0,1,2,3\ldots} \rightarrow$ phase shifts
  - HAL: $\psi_{0,1,2,3\ldots} \rightarrow$ potential

phase shift









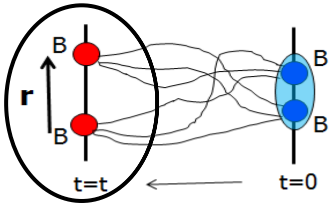
eigenstates

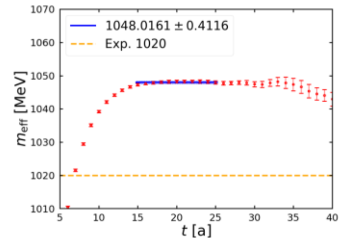
potential

Challenge in real calculation

- Ground state saturation

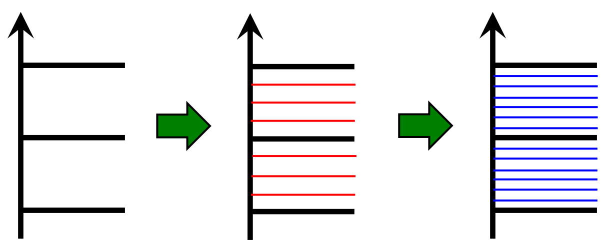
$$R(\boldsymbol{r},t) = \langle \hat{O}(\boldsymbol{r},t)\hat{O}^\dagger(0)\rangle = \sum_n \langle 0|\hat{O}(\boldsymbol{r},t)|n\rangle \langle n|\hat{O}^\dagger(0)|0\rangle$$
$$= \sum_n a_n \psi_n(\boldsymbol{r}) e^{-E_n t} \xrightarrow{t \gg \frac{1}{E_1 - E_0}} a_0 \psi_0(\boldsymbol{r}) e^{-E_0 t}$$





- More and more dense states for large box and/or heavy system

$$t \gg \frac{1}{E_1 - E_0} \sim \mu \left(\frac{L}{2\pi}\right)^2$$



- Signal-to-noise problem for baryonic system

$$\frac{\text{Signal}}{\text{Noise}} = \frac{\langle N(t)\bar{N}\rangle}{\sqrt{\langle |N(t)\bar{N}|^2\rangle}} = \frac{e^{-m_N t}}{e^{-3/2 m_\pi t}} = e^{-(m_N - 3/2 m_\pi)t}$$

  - signal exponentially decrease as t increasing

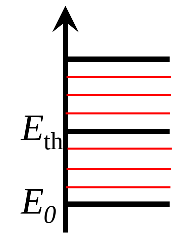
G. P. Lepage, in From Actions to Answers: Proceedings of the TASI 1989

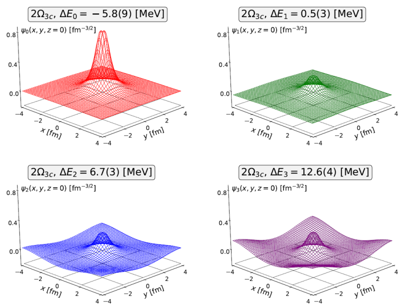
Time-dependent HAL QCD method

N. Ishii, *et al.* (HAL QCD Collaboration), Phys. Lett. B **712,** 437 (2012)

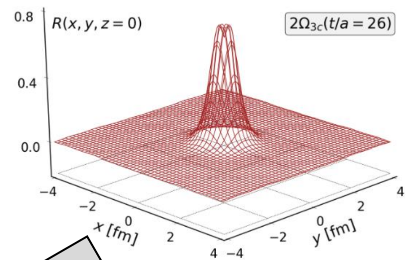
- Define R-correlator as a summation over all elastic states

$$R(\boldsymbol{r},t) = \sum_n a_n \psi_n(\boldsymbol{r}) e^{-E_n t} \xrightarrow{t \gg \frac{1}{E_{\mathrm{th}}-E_0}} \sum_{n<N_{\mathrm{th}}} a_n \psi_n(\boldsymbol{r}) e^{-E_n t}$$

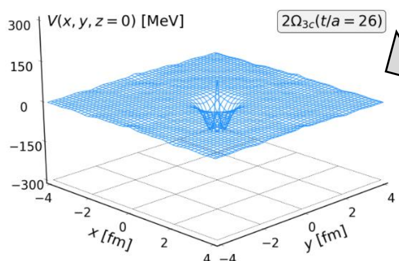




eigenstates



Rcorr.



potential

- Master equation

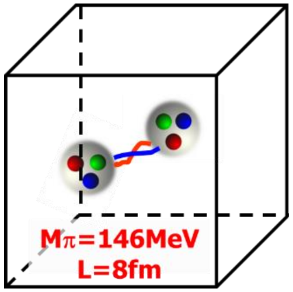
$$\sim k^2/(2\mu)$$

$$\left(\frac{1+3\delta^2}{8\mu}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu}\right) R(\boldsymbol{r},t) = \int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}') R(\boldsymbol{r}',t)$$

- $\delta = \frac{m_N - m_\phi}{m_N + m_\phi}$; $\mu = \frac{m_N m_\phi}{m_N + m_\phi}$

Lattice setup

- (2+1)-flavor configuration near the physical point
  - Iwasaki gauge action and $O(a)$-improved Wilson quark action



$L \times T$	a [fm]	La [fm]	m_π [MeV]	m_K [MeV]
$96^3 \times 96$	0.0846(7)	8.1	146	525

K.-I. Ishikawa, *et al.* (PACS Collaboration), *Proc. Sci.*, LATTICE2015 (2016) 075

- Relevant hadron masses

Hadron	Lattice Mass [MeV]	Physical Mass[MeV]
N	**954.0(2.9)**	938.9
ϕ	**1048.0(4)**	1019.5

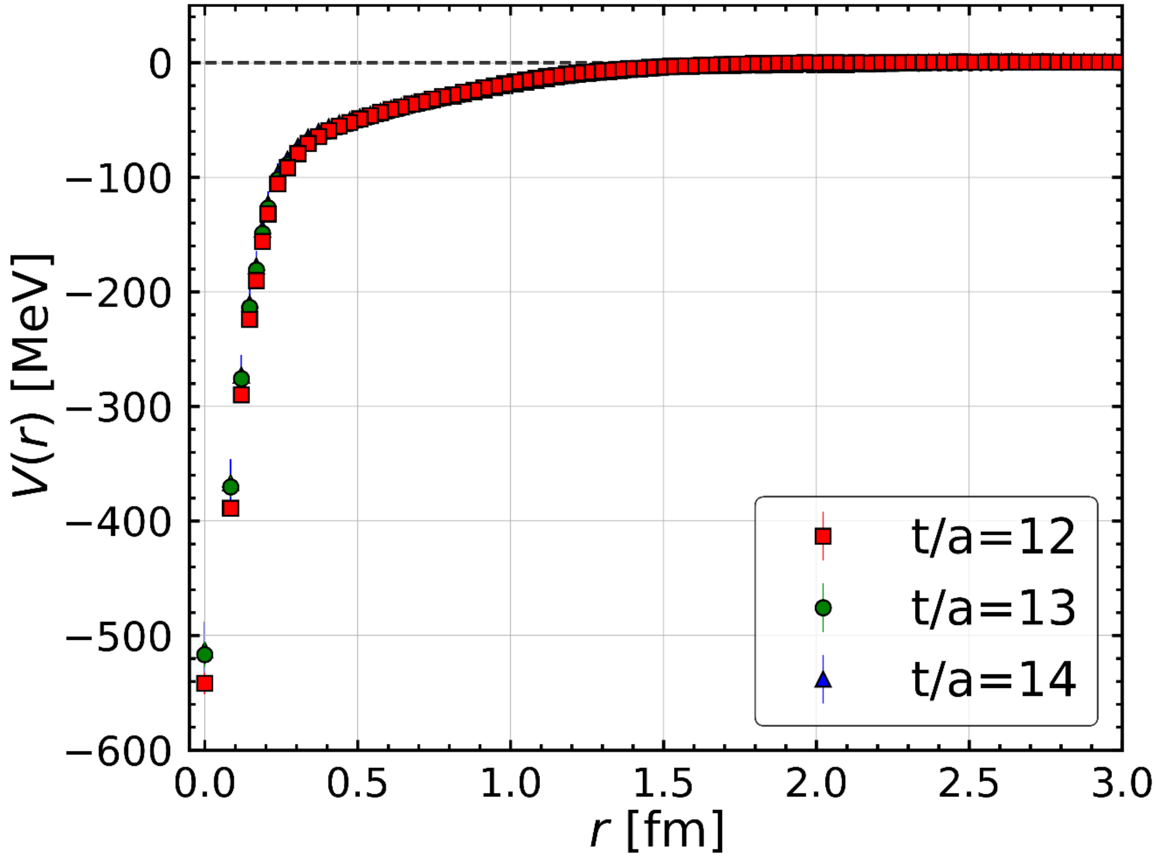
  - Note that $\phi \to K\bar{K}$ is prohibited due to $m_\phi < 2m_K$

- Statistics

$$200_{\text{conf.}} \times 80_{\text{src}} \times 4_{\text{rot.}} \times 2_{\text{b.f.}} = 128000$$

Potential

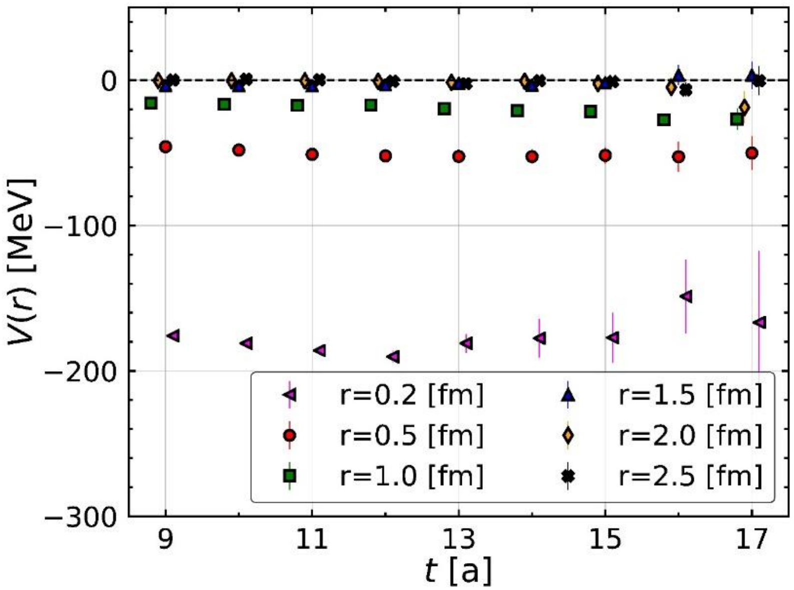
- $N\phi(\,^4S_{3/2})$ potential at Euclidean time 12, 13 and 14



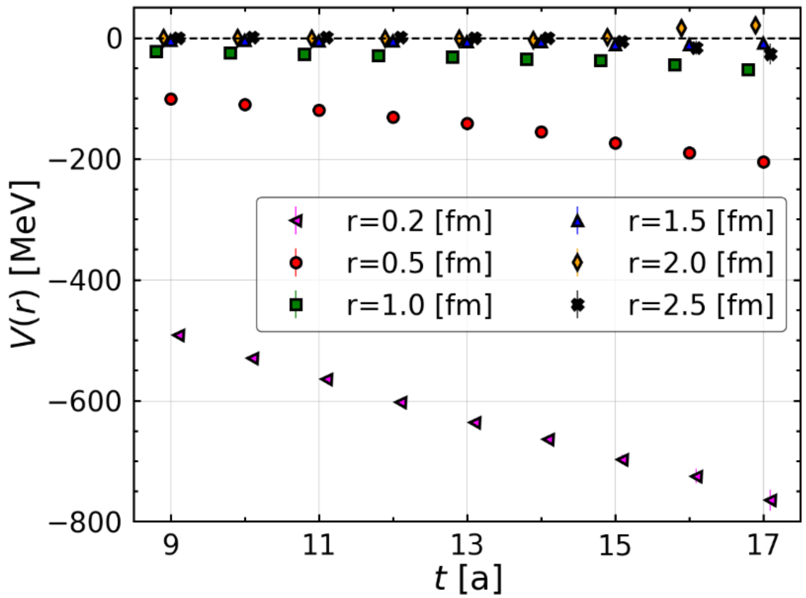
- attractive core, Pauli exclusion does not operate due to no common quarks
- long-ranged attractive tail, hints of pion dynamics
- weak t dependence

Potential in wider range of t

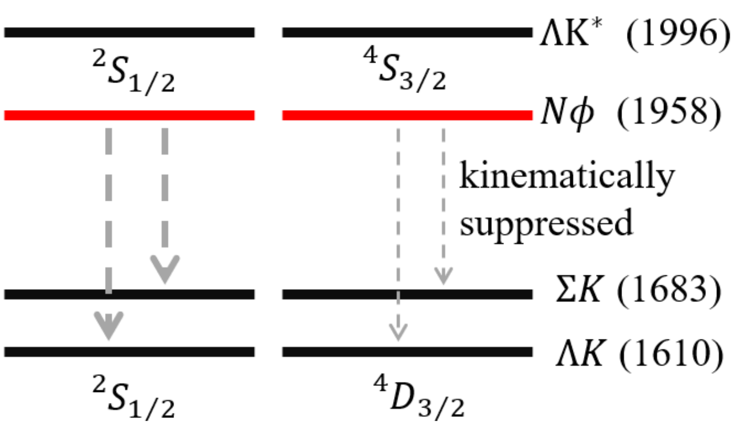
- $^4S_{3/2}$



- $^2S_{1/2}$

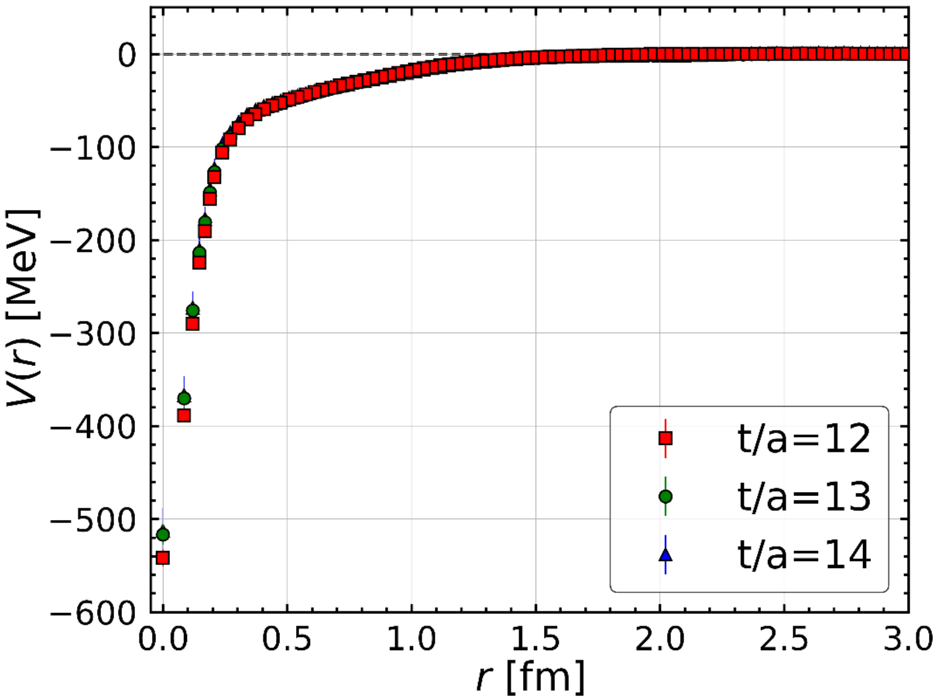


- correlation function is dominated by the elastic states in $^4S_{3/2}$
- this is in sharp contrast to $^2S_{1/2}$ case, where potential show clear t-dependence

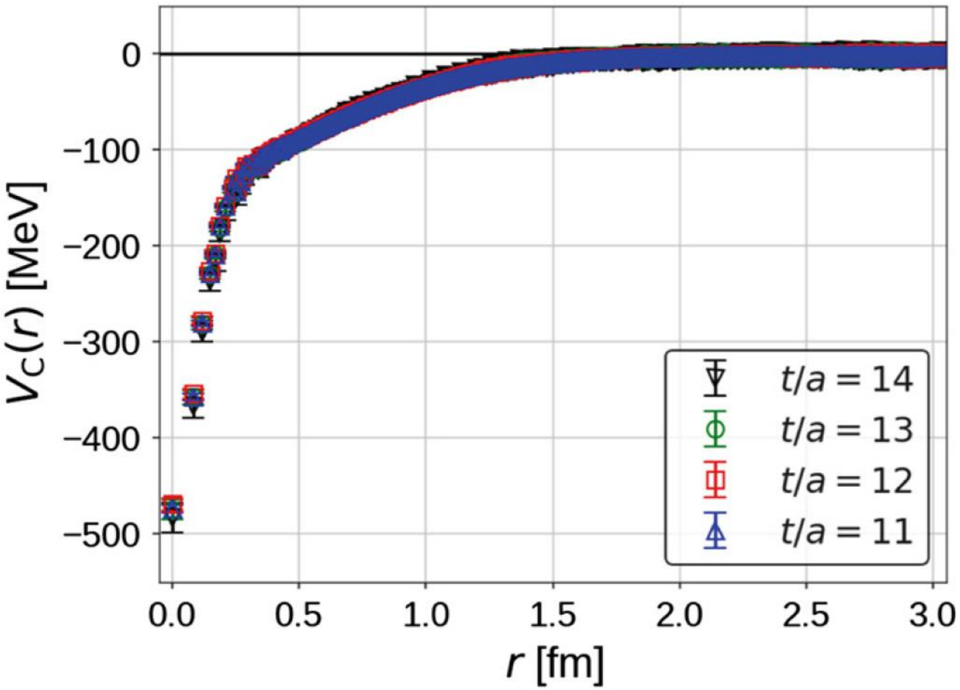


$N\phi$ v.s. $N\Omega$

- $N\phi(\,^4S_{3/2})$ potential



- $N\Omega(\,^5S_2)$ potential



T. Iritani, *et al.* (HAL QCD Coll.), Phys. Lett. B 792, 284 (2019)

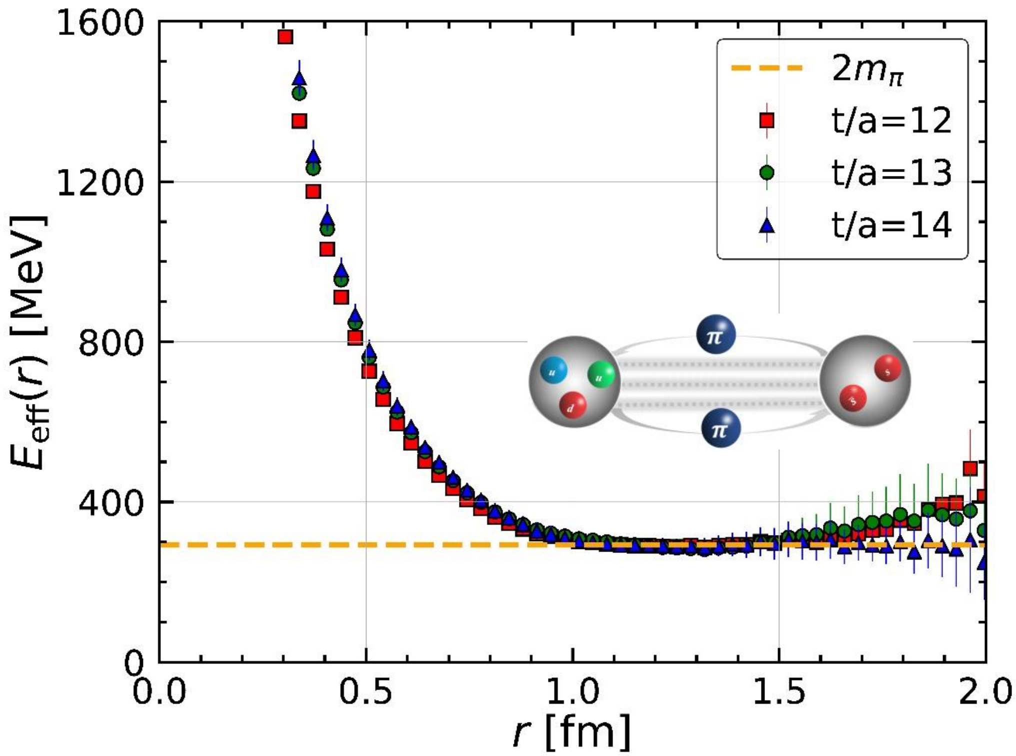
- universality of pion dynamics (?)

Two-pion-exchange tail

- Interaction between nucleon and heavy quarkonium at large distances

J. Castella and G. Krein, Phys. Rev. D **98**, 014029 (2018);H. Fujii and D. Kharzeev, Phys. Rev. D **60**, 114039 (1999)
G. Bhanot and M. E. Peskin, Nucl. Phys. B **156**, 391(1979)

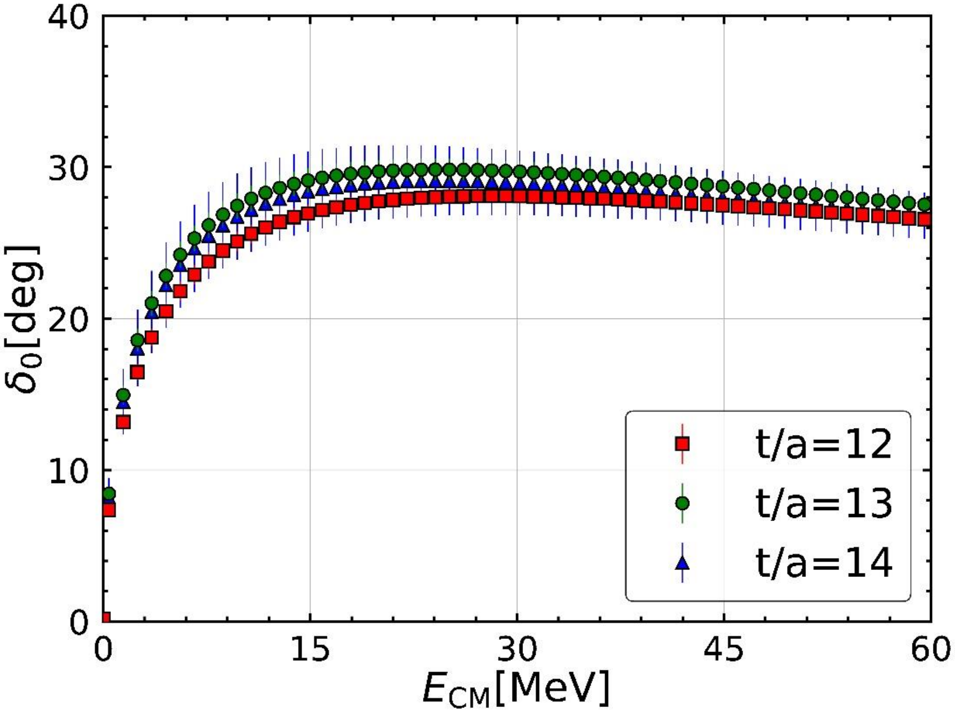
$$V(r) = -\alpha \frac{e^{-2m_\pi r}}{r^2} \quad \Rightarrow \quad E_{\text{eff}}(r) = -\frac{\ln\left[-V(r)r^2/\alpha\right]}{r}$$



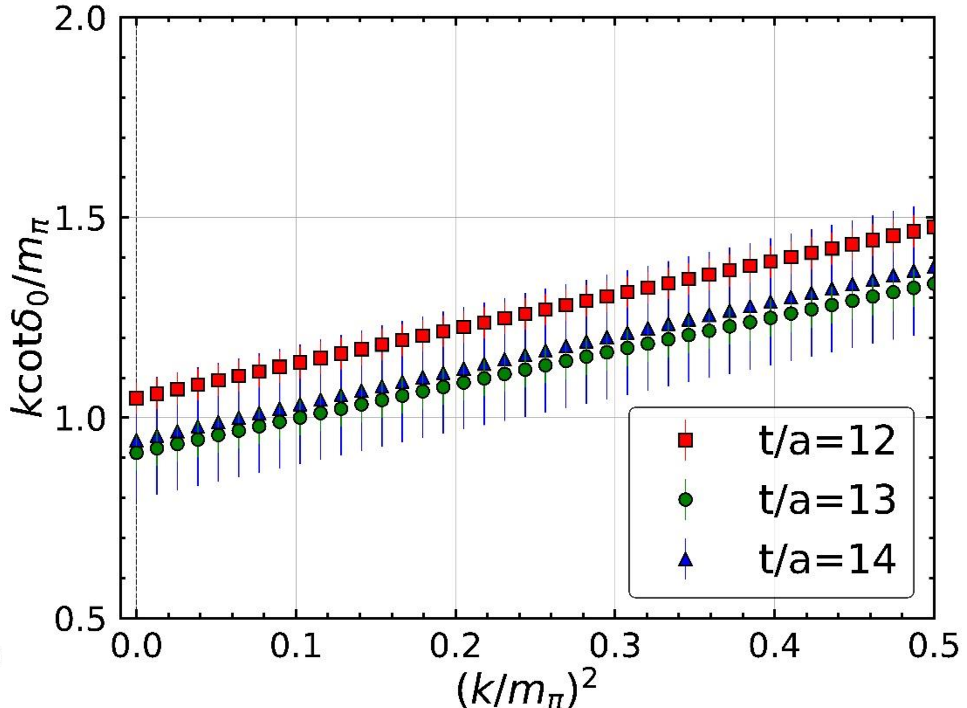
- long-range ($r > 1$ fm) potential is indeed dominated by the TPE

Physical observables

- Scattering phase shifts



- Effective range expansion



$$k \cot \delta_0 = -\frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} k^2 + O(k^2)$$

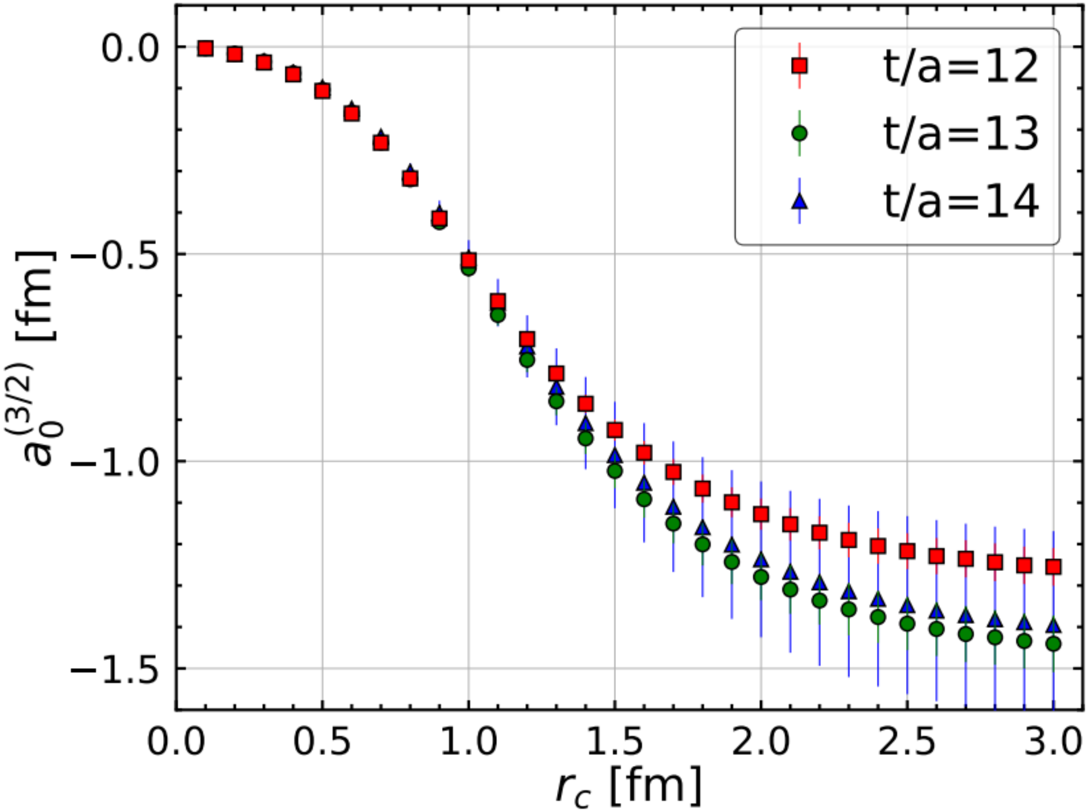
m_π [MeV]	$a_0^{(3/2)}$ [fm]	$r_{\text{eff}}^{(3/2)}$ [fm]
146.4	$-1.43(23)_{\text{stat.}} \left(^{+36}_{-06}\right)_{\text{syst.}}$	$2.36(10)_{\text{stat.}} \left(^{+02}_{-48}\right)_{\text{syst.}}$

consistent w/ ALICE Exp.

Long-range potential's contribution to scattering length

- Scattering length from potential with long-range cutoff

$$V(r;r_c) = \begin{cases} V(r), & r \le r_c \\ 0, & r > r_c \end{cases}$$



- long-range potential significantly contributes to scattering length

Other systematic errors

- Slightly heavy quark masses
  - TPE becomes slightly long-ranged with weaker strength $m_\pi(\text{Lat.}) \rightarrow m_\pi(\text{Phy.})$
  - hadron masses decrease $m_{\phi,N}(\text{Lat.}) \rightarrow m_{\phi,N}(\text{Phy.})$
  - towards physical point
    physical $m_{\phi,N}$, change m_π to 138.0 MeV for TPE tail

m_π [MeV]	$a_0^{(3/2)}$ [fm]	$r_{\text{eff}}^{(3/2)}$ [fm]
146.4	$-1.43(23)_{\text{stat.}} \left(^{+36}_{-06}\right)_{\text{syst.}}$	$2.36(10)_{\text{stat.}} \left(^{+02}_{-48}\right)_{\text{syst.}}$
138.0	$\simeq -1.25$	$\simeq 2.49$

- Finite volume
  - expected to be as small as $\exp(-2m_\pi L/2) \simeq 0.3\%$

- Finite cutoff
  - non-perturbative $O(a)$-improvement for u, d, s quark, $O\left(\left(a\Lambda_{\text{QCD}}\right)^2\right) \sim O(1)\%$
  - even completely cut $V(r)$ at $r < 0.1$ fm $\rightarrow$ 2%

Summary and outlook

- **Summary:** $N\phi(\,^4S_{3/2})$ interaction is studied from LQCD for the first time w/ nearly physical quark masses and large volume
  - Extract potential from spacetime correlation function
  - Potential: attractive; TPE tail
  - Scattering properties

- **Outlook:**
  - $N\phi(\,^2S_{1/2})$ interaction from a coupled-channel analysis $\Lambda K - \Sigma K - p\phi$
  - Physical point simulations
  - N- $\bar{c}c$ interaction

Thanks for your attention