K^+N elastic scattering for estimation of in-medium quark condensate with strange

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ELPH workshop C033

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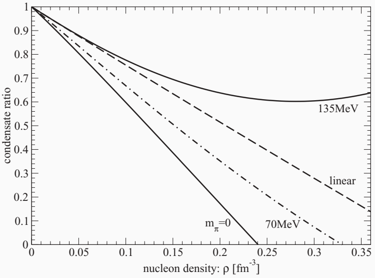
Partial restoration of ChSB in-medium

- Chiral symmetry (ChS) is spontaneously broken by physical states (ChSB)
  - Quark condensate $\Rightarrow \langle \bar{\psi}\psi \rangle \neq 0$
- ChS is considered to be **partially restored** even at finite density like nuclei
  - $|\langle \bar{\psi}\psi \rangle|$ is expected to decreases in nuclear medium

$\rightarrow$ want to investigate the behavior of in-medium quark condensate from **observables**

In-medium quark condensate $\langle \bar{u}u + \bar{d}d \rangle^*$



taken from N. Kaiser *et al.*, PRC 77, 025204 (2008)

- up to linear-density:

$$\frac{\langle \bar{u}u + \bar{d}d \rangle^*}{\langle \bar{u}u + \bar{d}d \rangle_0} = 1 - \frac{\sigma_{\pi N}}{F_\pi^2 M_\pi^2}\rho$$

E.G. Drukarev, E.M. Levin, Nucl. Phys. A 511, 679 (1990)

- $c_1 = -\dfrac{\sigma_{\pi N}}{4M_\pi^2} = -0.59\ \mathrm{GeV}^{-1}$ with $\sigma_{\pi N} = 45\ \mathrm{MeV}$ determined by πN scattering

J. Gasser, H. Leutwyler, and M. E. Sainio, Phys. Lett. B253, 252 (1991)

- 30 % reduction at ρ_0 (normal nuclear density 0.17 fm^{-3})

- **Chiral symmetry is 30% restored at normal nuclear density**
- Theoretical prediction can be seen in $\pi-$nucleus system experimentally

pionic atom:
-K. Suzuki *et al.*, PRL 92, 072302 (2004);
-E. E. Kolomeitsev *et al.*, PRL 90, 092501 (2003);
-Jido, Hatsuda, Kunihiro, PLB 670, 109 (2008)

πA scattering:
-E. Friedman *et al.*, PRL 93, 122302 (2004); PRC 72, 034609 (2005)

In-medium condensate with strange quarks

Systematic point of view, we are interested in
in-medium quark condensate with strange $\langle \bar{u}u + \bar{s}s \rangle^*$:

1. represent $\langle \bar{u}u + \bar{s}s \rangle^*$ in terms of correlation function Π^{ab}
2. expand Π^{ab} based on low-density theorem then obtain $\langle \bar{u}u + \bar{s}s \rangle^*$ in terms of T_{KN} in soft-limit $q \to 0$
3. calculate T_{KN} with ChPT and the LECs are determined from the experiments
4. evaluate $\langle \bar{u}u + \bar{s}s \rangle^*$ using the obtained T_{KN}

Correlation function approach for in-medium condensate

Correlation function approach (Jido, Hatsuda, Kunihiro, PLB 670 (2008), 109, Goda, Jido, PRC 88 (2013), 0652049, Hübsch, Jido, PRC 104 (2021), 015202.)

Using **chiral Ward identity**, $\langle\bar{\psi}\psi\rangle^*$ is represented in terms of two correlation functions in soft-limit $q \to 0$

$$i\delta^{ab}\left\langle\bar{u}u+\bar{s}s\right\rangle^* = \frac{m+m_s}{2}\Pi^{ab}(0) + i\lim_{q\to 0} q^\mu \Pi^{ab}_{5\mu}(q) \text{ with } a,b=4,5,$$

$$\Pi^{ab}(q) = \mathrm{F.T.}\left\langle\Omega\right| \mathrm{T}[P^a(x)P^b(0)]\left|\Omega\right\rangle$$

$$\Pi^{ab}_{5\mu}(q) = \mathrm{F.T.}\left\langle\Omega\right| \mathrm{T}[A^a_\mu(x)P^b(0)]\left|\Omega\right\rangle$$

$P^a(x) = \bar{q}i\gamma_5\lambda^a q$: pseudoscalar field:
$A^a_\mu(x) = \bar{q}\gamma_\mu\gamma_5\frac{\lambda^a}{2}q$: axial-vector current
$|\Omega\rangle$: nuclear-matter ground state

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$\langle\bar{\psi}\psi\rangle^*$ is obtained by evaluating Π^{ab} with soft-limit $q \to 0$
($q^\mu\Pi^{ab}_{5\mu}(q)$ vanishes in soft-limit because of no zero modes.)

Correlation function approach for in-medium condensate

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Using **chiral Ward identity**, $\langle\bar{\psi}\psi\rangle^*$ is represented in terms of two correlation functions in soft-limit $q \to 0$

$$i\delta^{ab}\left\langle \bar{u}u + \bar{s}s \right\rangle^* = \frac{m+m_s}{2}\Pi^{ab}(0) + \cancel{i\lim_{q\to 0} q^\mu \Pi^{ab}_{5\mu}(q)} \text{ with } a,b = 4,5,$$

$$\Pi^{ab}(q) = \text{F.T.} \left\langle \Omega \right| \mathrm{T}[P^a(x)P^b(0)] \left| \Omega \right\rangle$$

$$\Pi^{ab}_{5\mu}(q) = \text{F.T.} \left\langle \Omega \right| \mathrm{T}[A^a_\mu(x)P^b(0)] \left| \Omega \right\rangle$$

$$\begin{aligned}
\left\langle \bar{u}u + \bar{s}s \right\rangle^* &= -i\frac{m+m_s}{2}\Pi^{4+i5,4-i5}(q=0;\rho) \\
&= -i\frac{m+m_s}{2}\left\langle \left(\frac{P^4(0)+iP^5(0)}{\sqrt{2}}\right)\left(\frac{P^4(0)-iP^5(0)}{\sqrt{2}}\right)\right\rangle^* \\
&\sim -i\frac{m+m_s}{2}\left\langle \Omega \right| K^-(q)K^+(q) \left| \Omega \right\rangle
\end{aligned}$$

Low-density expansion of $\Pi^{4+i5,4-i5}$

Low-density theorem (E.G. Drukarev, E.M. Levin, Nucl. Phys. A 511, 679 (1990))

$$\langle\Omega|\,\mathcal{O}\,|\Omega\rangle = \langle 0|\,\mathcal{O}\,|0\rangle + \rho\,\langle N|\,\mathcal{O}\,|N\rangle + O(\rho^{n>1})$$

Applying to Π^{ab}:

$$\Pi^{4+i5,4-i5}(q;\rho) = G_K\,\langle 0|\,K^-(q)K^+(q)\,|0\rangle + \rho G_K\,\langle N|\,K^-(q)K^+(q)\,|N\rangle + O(\rho^{n>1})$$

$$G_K \equiv \langle 0|\,\frac{P^4 - iP^5}{\sqrt{2}}\,|K^+\rangle$$

and using the reduction formula (S. Weinberg, Phys. Rev. Lett. 17, 616 (1966)):

$$\langle N|\,K^-(q)K^+(q)\,|N\rangle = \frac{i}{q^2 - M_K^2}\,\frac{1}{q^2 - M_K^2}\left(-\frac{T_{KN}(q)}{2M_N}\right),$$

we obtain

$$\frac{\langle\bar{u}u + \bar{s}s\rangle^*}{\langle\bar{u}u + \bar{s}s\rangle_0} = \left(1 + \frac{\rho}{M_K^2}\,\frac{T_{KN}(q=0)}{2M_N}\right).$$

Construction of T_{KN}

In order to evaluate $\langle \bar{u}u + \bar{s}s \rangle^*$, T_{KN} is needed:

$$\frac{\langle \bar{u}u + \bar{s}s \rangle^*}{\langle \bar{u}u + \bar{s}s \rangle_0} = \left(1 + \frac{\rho}{M_K^2} \frac{T_{KN}(q=0)}{2M_N}\right).$$

1. construct K^+N scattering amplitude by ChPT:
   - ChPT is suitable for taking soft-limit
   - T_{KN} includes the low-energy constants
   - In order to improve **extrapolation to strange quark sector**, add some NNLO in addition to Leading Order + NLO, 12 LECs

Construction of T_{KN}: LO

Isospin combinations:

$$
\begin{aligned}
T_{K^+p\to K^+p} &= T_{KN}^{I=1} \\
T_{K^+n\to K^+n} &= \frac{1}{2}(T_{KN}^{I=1} + T_{KN}^{I=0}) \\
T_{K^+n\to K^0p} &= \frac{1}{2}(T_{KN}^{I=1} - T_{KN}^{I=0})
\end{aligned}
$$

calculate $T_{KN}^{I=0,1}$

- Leading order of SU(3) chiral Lagrangian:

$$
\mathcal{L}_{MB}^{(1)} = \mathrm{Tr}\{\bar{B}(i\not{D} - M_0)B)\} - \frac{D}{2}\,\mathrm{Tr}\{\bar{B}\gamma^\mu\gamma^5\{u_\mu, B\}\} - \frac{F}{2}\,\mathrm{Tr}\{\bar{B}\gamma^\mu\gamma^5[u_\mu, B]\}
$$

$$
D = 0.80,\ F = 0.46
$$

Construction of T_{KN}: NLO

- NLO of SU(3) chiral Lagrangian: Aoki, Jido, PTEP2019,013D01(19)

$$
\begin{aligned}
\mathcal{L}^{(2)}_{MB} &= b_D \,\mathrm{Tr}\big\{\bar{B}\{\chi_+, B\}\big\} + b_F \,\mathrm{Tr}\big\{\bar{B}[\chi_+, B]\big\} + b_0 \,\mathrm{Tr}\big\{\bar{B}B\big\}\,\mathrm{Tr}\{\chi_+\} \\
&+ d_1 \mathrm{Tr}\left(\bar{B}\{u_\mu, [u^\mu, B]\}\right) + d_2 \mathrm{Tr}\left(\bar{B}[u_\mu, [u^\mu, B]]\right) + d_3 \mathrm{Tr}\left(\bar{B}u_\mu\right)\mathrm{Tr}(u^\mu B) \\
&+ d_4 \mathrm{Tr}\left(\bar{B}B\right)\mathrm{Tr}(u^\mu u_\mu) \\
&- \frac{g_1}{8M_N^2}\mathrm{Tr}\left(\bar{B}\{u_\mu, [u_\nu, \{D^\mu, D^\nu\}B]\}\right) - \frac{g_2}{8M_N^2}\mathrm{Tr}\left(\bar{B}[u_\mu, [u_\nu, \{D^\mu, D^\nu\}B]]\right) \\
&- \frac{g_3}{8M_N^2}\mathrm{Tr}(\bar{B}u_\mu)\mathrm{Tr}(u_\nu, \{D^\mu, D^\nu\}B) - \frac{g_4}{8M_N^2}\mathrm{Tr}(\bar{B}\{D^\mu, D^\nu\}B)\mathrm{Tr}(u_\mu u_\nu) \\
&- \frac{h_1}{4}\mathrm{Tr}\left(\bar{B}[\gamma^\mu, \gamma^\nu]B u_\mu u_\nu\right) - \frac{h_2}{4}\mathrm{Tr}\left(\bar{B}[\gamma^\mu, \gamma^\nu]u_\mu [u_\nu, B]\right) \\
&- \frac{h_3}{4}\mathrm{Tr}\left(\bar{B}[\gamma^\mu, \gamma^\nu]u_\mu \{u_\nu, B\}\right) - \frac{h_4}{4}\mathrm{Tr}(\bar{B}[\gamma^\mu, \gamma^\nu]u_\mu)\mathrm{Tr}(u_\nu B) + \mathrm{h.c.}
\end{aligned}
$$

$$
\begin{aligned}
b^{I=0} &= b_0 - b_F, & b^{I=1} &= b_0 + b_D, \\
d^{I=0} &= 2d_1 + d_3 - 2d_4, & d^{I=1} &= -2d_2 - d_3 - 2d_4, \\
g^{I=0} &= 2g_1 + g_3 - 2g_4, & g^{I=1} &= -2g_2 - g_3 - 2g_4, \\
h^{I=0} &= h_1 + h_2 + h_3 + h_4, & h^{I=1} &= h_1 - h_2 - h_3 - h_4.
\end{aligned}
$$

Construction of T_{KN}: NNLO

- In order to improve **extrapolation to strange quark sector**, we introduce some terms which contain strange quark mass from the next-to-next-to-leading order (NNLO): Oller et el. JHEP09 (2006) 079

$$
\begin{aligned}
\mathcal{L}_{MB}^{(3)} = {} & v_D \,\mathrm{Tr}\big(\bar{B}\{\chi_-, \gamma_5 B\}\big) + v_F \,\mathrm{Tr}\big(\bar{B}[\chi_-, \gamma_5 B]\big) \\
& + w_1 \,\mathrm{Tr}\big(\bar{B}\gamma_\mu B[\chi_-, u^\mu]\big) + w_2 \,\mathrm{Tr}\big(\bar{B}[\chi_-, u^\mu]\gamma_\mu B\big) \\
& + w_3 \big[\mathrm{Tr}\big(\bar{B}u^\mu\big)\,\mathrm{Tr}(\chi_-\gamma_\mu B) - \mathrm{Tr}\big(\bar{B}\chi_-\big)\,\mathrm{Tr}(u^\mu\gamma_\mu B)\big]
\end{aligned}
$$

$$
\begin{aligned}
v_- &= v_D - v_F, & v_+ &= v_D + 3v_F, \\
w^{I=0} &= w_1 - w_2 + w_3, & w^{I=1} &= w_1 + w_2 - w_3
\end{aligned}
$$

Strange quark mass is contained in χ_-

Construction of T_{KN}: in-medium condensate

KN scattering amplitudes:

$$T^I_{KN} = T^I_{\mathrm{WT}} + T^I_{\mathrm{Born}} + T^I_{\mathrm{NLO}} + T^I_{\mathrm{NNLO}}$$

Taking isospin-average and soft-limit, we have

$$\frac{\langle \bar{u}u + \bar{s}s \rangle^*}{\langle \bar{u}u + \bar{s}s \rangle_0} = 1 + \frac{(3b^{I=1} + b^{I=0})}{F_K^2}\rho$$

Construction of T_{KN}: Fitting

In order to evaluate $\langle \bar{u}u + \bar{s}s \rangle^*$, we need to obtain T_{KN}:

2. fit LECs to the experimental data:
    - K^+p elastic diff. cross sections, $P_{\rm lab} = 145 \text{ to } 726\ \mathrm{MeV}/c$
    - $K^+n \to K^0p$ diff. cross sections, $P_{\rm lab} = 434 \text{ to } 780\ \mathrm{MeV}/c$
    - $I = 1,\ 0$ total cross sections

   Two choices for $I = 0$ total cross section data:
    - Carroll et al. 1973
    - Bowen et al. 1970, 1973 $\leftarrow$ Broad resonance could exist

   Aoki, Jido, PTEP2019,013D01(19)

Construction of T_{KN}: $I = 0$ broad resonance state

- **Broad resonance** with $I = 0, S = +1$ around $P_{\rm lab} = 600$ MeV has been reported in Aoki, Jido, PTEP2019,013D01(19)

	Resonance (J^P)	mass [MeV]	width [MeV]
Solution 1	P_{01} ($\frac{1}{2}^+$)	1617	305
Solution 2	P_{03} ($\frac{3}{2}^+$)	1678	463

- The resonance may affect $I = 0$ K^+N scattering

$\rightarrow$ fit LECs with

FIT 1: Carroll et al. (1973) for $I = 0$, no resonance
FIT 2: Bowen et al. (1970) for $I = 0$, no resonance
FIT 3: Bowen et al. (1970) for $I = 0$ and P_{01} resonance
FIT 4: Bowen et al. (1970) for $I = 0$ and P_{03} resonance

Fitted LECs

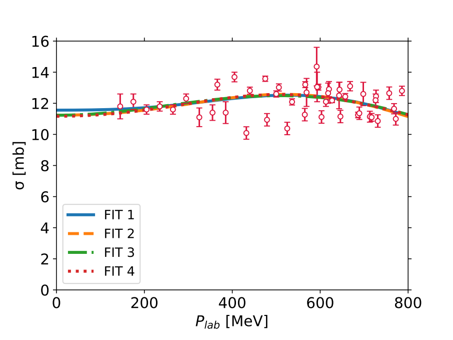
Fitting	FIT 1 (Carroll 1973)	FIT 2 (Bowen 1970)	FIT 3 (Bowen 1970 with P_{01})	FIT 4 (Bowen 1970 with P_{03})
$b^{I=1}$	-1.07 ± 0.11	-1.13 ± 0.10	-0.11 ± 0.12	-1.08 ± 0.11
$d^{I=1}$	-2.05 ± 0.20	-2.08 ± 0.17	0.19 ± 0.19	-1.97 ± 0.17
$g^{I=1}$	-0.82 ± 0.22	-0.90 ± 0.18	-0.80 ± 0.20	-1.01 ± 0.19
$h^{I=1}$	3.70 ± 0.50	4.20 ± 0.60	0.90 ± 0.54	4.20 ± 0.60
$w^{I=1}$	-0.76 ± 0.11	-1.01 ± 0.10	-0.36 ± 0.10	-1.05 ± 0.10
$b^{I=0}$	-3.66 ± 0.30	1.40 ± 0.40	2.40 ± 0.48	2.30 ± 0.40
$d^{I=0}$	-9.20 ± 0.40	-0.30 ± 0.40	-1.40 ± 0.58	-0.60 ± 0.50
$g^{I=0}$	1.50 ± 0.50	6.10 ± 0.70	8.30 ± 0.95	8.10 ± 0.80
$h^{I=0}$	16.30 ± 0.70	-3.90 ± 0.80	-1.60 ± 0.96	-4.90 ± 0.80
$w^{I=0}$	-0.57 ± 0.29	4.19 ± 0.35	4.90 ± 0.46	5.00 ± 0.40
v_-	42.90 ± 1.70	12.70 ± 1.70	5.00 ± 0.19	10.1 ± 1.70
v_+	-7.60 ± 0.90	4.60 ± 0.90	-0.36 ± 0.93	4.70 ± 0.90
$\chi^2_{\rm dof}$	2.41	2.75	2.95	2.96

LECs except for w^I: GeV^{-1}, w^I: GeV^{-2}

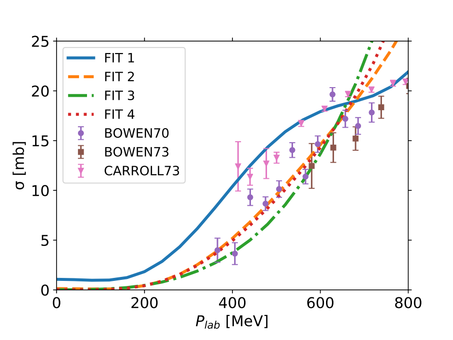
$I = 1$ LECs are better determined than $I = 0$ LECs.

Total cross section

$I = 1$ total cross section



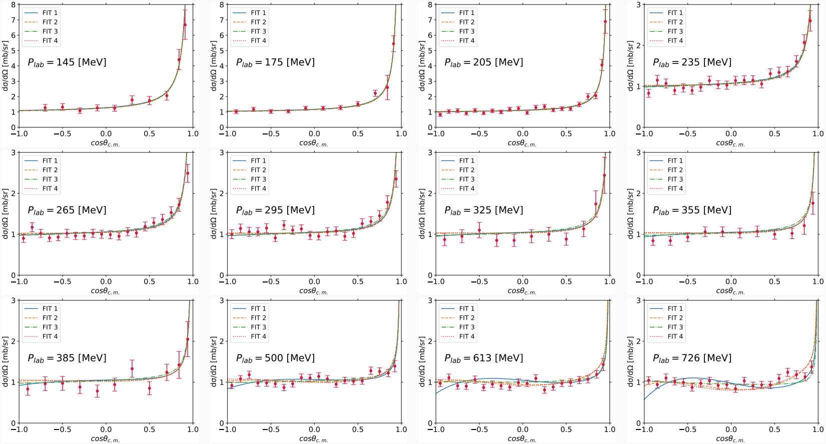
$I = 0$ total cross section



$\surd$ $I = 1$ total cross section is reproduced very well

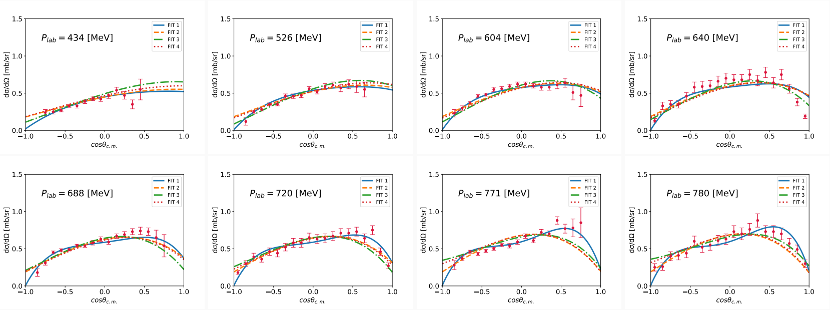
$\surd$ $I = 0$ total cross section is also reproduced well

K^+p differential cross sections



$\surd$ K^+p differential cross sections are reproduced very well

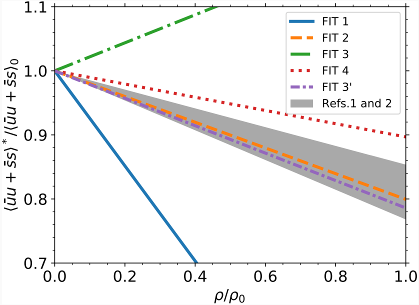
$K^+ n \to K^0 p$ charge exchange



$\surd$ $K^+ n \to K^0 p$ differential cross sections are reproduced well

- The obtained LECs $\to$ in-medium $\langle \bar{u}u + \bar{s}s \rangle$

Behavior of in-medium $\langle \bar{u}u + \bar{s}s \rangle$



$$\frac{\langle \bar{u}u + \bar{s}s \rangle^*}{\langle \bar{u}u + \bar{s}s \rangle_0} = 1 + \frac{(3b^{I=1} + b^{I=0})}{F_K^2}\rho$$

- Gray area: taken from
  L.S. Geng, Frontiers of Physics 8, 328 (2013),
  B. Kubis and U. G. Meißner, Eur. Phys. J. C18, 747 (2001)
- FIT 1: Carroll et al. (1973) for $I = 0$
- FIT 2: Bowen et al. (1970) for $I = 0$
- FIT 3: Bowen with the P_{01} resonance
- FIT 4: Bowen with the P_{03} resonance

- Whether $\langle \bar{u}u + \bar{s}s \rangle^*$ increases or decreases, and to what degree depends on the existence of resonances and the choice of experimental data for $I = 0$

Summary

- We derive $\langle \bar{u}u + \bar{s}s \rangle^*$ using Correlation function approach and Low-density theorem
- We estimate LECs from K^+N elastic scattering to evaluate $\langle \bar{u}u + \bar{s}s \rangle^*$
  - improve extrapolation to strange sector
  - consider effect of broad resonance state with $S = +1, I = 0$ around $P_{\mathrm{lab}} = 600\ \mathrm{MeV}$
  - obtain the LECs good to reproduce the data
- Whether $\langle \bar{u}u + \bar{s}s \rangle^*$ increases or decreases and to what degree depends on the existence of resonances and the choice of data for $I = 0$
  - $I = 1\ K^+N$ scattering data: $P_{\mathrm{lab}} = 145 - 786\ \mathrm{MeV}$
  - $I = 0\ K^+N$ scattering data: $P_{\mathrm{lab}} = 366 - 794\ \mathrm{MeV}$

  $\rightarrow$ **Need lower energy data** with low ambiguity of $I = 0\ K^+N$ scattering in order to avoid the effect of the resonance and the choice of $I = 0$ total cross section

K^+n differential cross sections

K^+n elastic scattering are not reproduced

not used in fitting