Near-threshold hadron scattering using effective field theory

Tokyo Metropolitan University

Katsuyoshi Sone
Tetsuo Hyodo

Background

Exotic hadrons ⇨ $T_{cc}, X(3872), f_0(980), a_0, P_c, Z_c$

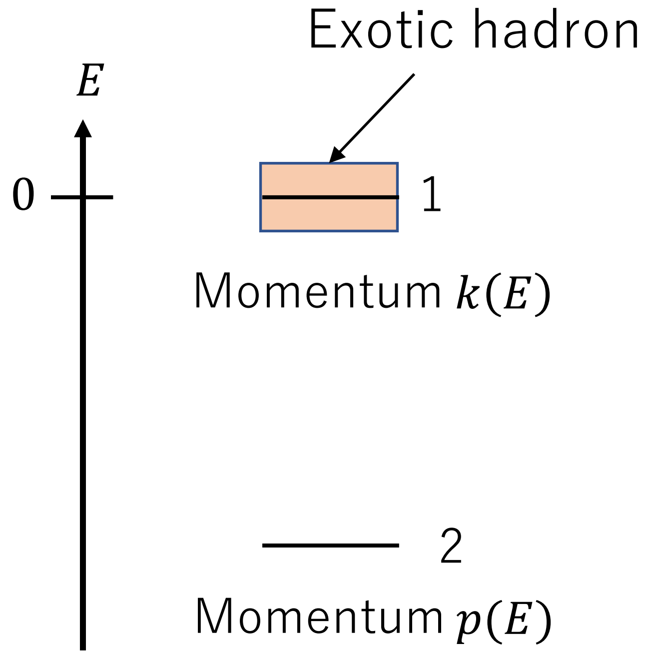
Internal structure ⇔ Scattering lengths a and effective range r

For near-threshold exotic hadrons, channel couplings are important.

Unstable exotic hadron near the threshold of channel 1

⇨ Flatté amplitude has been used[1].

Scattering lengths a_F and effective range r_F have been determined by the Flatté amplitude[2].



a and r in more general framework?

[1] R.Aaij et al. [LHCb], Phys. Rev. D 102, no.9, 092005 (2020)

[2] V. Baru et al. Eur. Phys. J. A, 23, 523-533 (2005)

Flatté amplitude

The Flatté amplitude

$$f^F = h(E)\begin{pmatrix} g_1^2 & g_1 g_2 \\ g_1 g_2 & g_2^2 \end{pmatrix}$$

The Flatté parameters

g_1^2, g_2^2 : Real coupling constants

E_{BW} : Bare energy

The Flatté amplitude has the threshold effect.

$$h(E) = -\frac{1}{2}\frac{1}{E - E_{BW} + i\,g_2^2 p(E)/2 + i g_1^2\,k(E)/2}$$

f_{11}^F, f_{22}^F can be written as the effective range expansion in k.

$$f_{11}^F, f_{22}^F \propto \left(-\frac{1}{a_F} + \frac{1}{2}r_F k^2 - ik + O(k^4)\right)^{-1}$$

a_F : Scattering lengths

r_F : Effective range

a_F and r_F

We consider near the threshold 1(region Ⅱ and Ⅲ).

1→1 scattering does not occur in region Ⅱ.

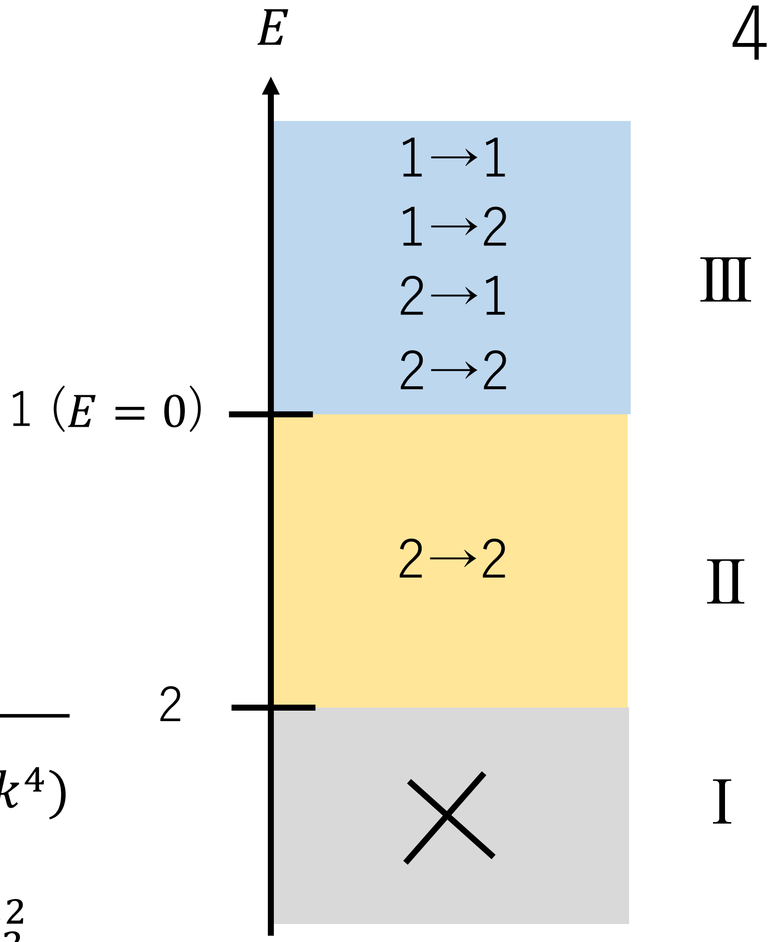
2→2 scattering occurs in both region Ⅱ and Ⅲ.

⇨ a_F, r_F are determined from f_{22}^F (ex. [3][1] for $X(3872)$).

$$f_{22}^F = \frac{\frac{g_2^2}{g_1^2}}{\left(\frac{2E_{BW}}{g_1^2} - i\frac{g_2^2}{g_1^2}p_0\right) - \left(\frac{2}{m_k g_1^2} + i\frac{g_2^2}{2p_0 g_1^2}\right)k^2 - ik + O(k^4)}$$

$$a_F = -\frac{g_1^2}{2E_{BW} - i g_2^2 p_0} \qquad r_F = -\frac{4}{m_k g_1^2} + i\frac{g_2^2}{p_0 g_1^2}$$

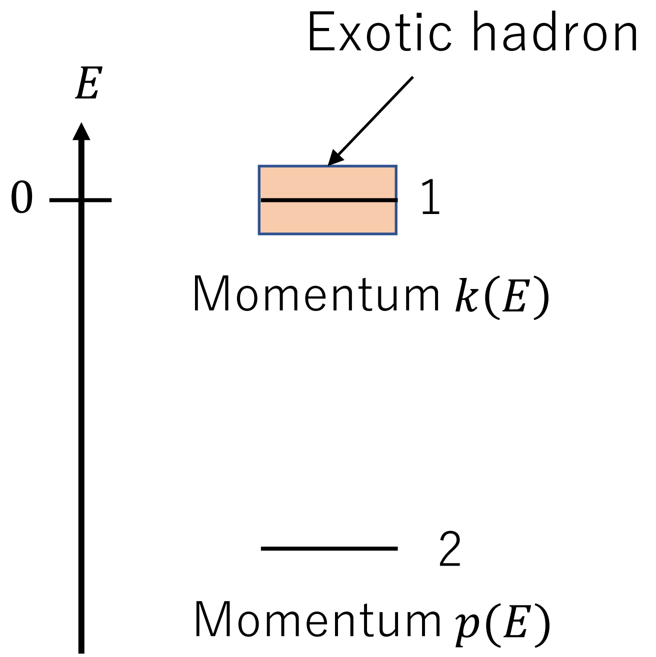
Scattering length Effective range



[3] A. Esposito et al., Phys. Rev. D 105 (2022) 3, L031503
[1] R. Aaij et al. [LHCb], Phys. Rev. D102, no.9, 092005 (2020)

General form

We consider the two-channel scattering.



- Conservation of probability

⇨ Optical theorem with channel couplings

⇩

$$f^{-1} = \begin{pmatrix} M_{11}(E) - ik & M_{12}(E) \\ M_{21}(E) & M_{22}(E) - ip \end{pmatrix}$$

M_{nm}: Analytic functions of E
k, p: Momentum

Flatté amplitude : $\det(f^F)=0$

⇨ **Flatté amplitude does not satisfy the optical theorem.**

EFT amplitude

As more general framework, we consider the effective field theory(EFT).

The effective field theory(EFT)
- Nonrelativistic
- Contact interaction
- Two channels

The scattering amplitude derived from EFT[5]

$$f^{EFT}(E) = \left\{\frac{1}{a_{12}^2} - \left(\frac{1}{a_{22}} + ip(E)\right)\left(\frac{1}{a_{11}} + ik(E)\right)\right\}^{-1} \begin{pmatrix} \left(\frac{1}{a_{22}} + ip(E)\right) & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & \left(\frac{1}{a_{11}} + ik(E)\right) \end{pmatrix}$$

a_{11}, a_{12}, a_{22} : Three EFT parameters

f^{EFT} satisfies the optical theorem with channel couplings.

[5]T.D.Cohen et al. , Phys. Lett. B 588 (2004) 57-66

f_{11} component

Effective range expansion for f_{11}^{EFT}

$$f_{11}^{EFT} = \frac{\frac{1}{a_{22}} + ip}{\frac{1}{a_{12}^2} - \left(\frac{1}{a_{22}} + ip\right)\left(\frac{1}{a_{11}} + ik\right)}$$

$$= \frac{1}{\left(\frac{1}{\frac{a_{12}^2}{a_{22}} + ip_0 a_{12}^2} - \frac{1}{a_{11}}\right) - \frac{i}{2\left(\frac{a_{12}}{a_{22}} + ip_0 a_{12}\right)^2 p_0} k^2 + O(k^4) - ik}$$

$$a_{EFT} = \frac{a_{11} a_{12}^2 (1 + ip_0 a_{22})}{a_{12}^2 (1 + ip_0 a_{22}) - a_{11} a_{22}}$$

$$r_{EFT} = -\frac{i}{p_0}\left\{\frac{a_{22}}{a_{12}(1 + ip_0 a_{22})}\right\}^2$$

f_{11}^{EFT} can be written as the effective range expansion in k.

f_{22} component

Effective range expansion for f_{22}^{EFT}

$$f_{22}^{EFT}=\frac{\frac{1}{a_{11}}+ik}{\frac{1}{a_{12}^2}-\left(\frac{1}{a_{22}}+ip\right)\left(\frac{1}{a_{11}}+ik\right)}$$

$$=\frac{\left(\frac{a_{12}}{a_{11}}\right)^2}{\left(\frac{1}{a_{11}}-\frac{a_{12}^2}{a_{11}^2a_{22}}-i\frac{p_0a_{12}^2}{a_{11}^2}\right)-\left(a_{11}+i\frac{a_{12}^2}{2p_0a_{11}^2}\right)k^2-ik+O(k^3)}$$

f_{22}^{EFT} cannot be written as the effective range expansion in k.

⇨ a_{EFT} and r_{EFT} cannot be determined by f_{22}^{EFT}.

The correct scattering length and effective range must be determined by f_{11}.

The validity of a_F and r_F determined in f_{22}^F in Flatté amplitude

Analytic comparison

We compare a_F, r_F with a_{EFT}, r_{EFT}, with EFT parameters.

Matching of f_{22} at small k.

$$f_{22}^{F} = \frac{\frac{g_2^2}{g_1^2}}{\left(\frac{2E_{BW}}{g_1^2} - i\frac{g_2^2}{g_1^2}p_0\right) - \left(\frac{2}{m_1 g_1^2} + i\frac{g_2^2}{2p_0 g_1^2}\right)k^2 - ik + O(k^4)} \qquad f_{22}^{EFT} = \frac{\left(\frac{a_{12}}{a_{11}}\right)^2}{\left(\frac{1}{a_{11}} - \frac{a_{12}^2}{a_{11}^2 a_{22}} - i\frac{p_0 a_{12}^2}{a_{11}^2}\right) - \left(a_{11} + i\frac{a_{12}^2}{2p_0 a_{11}^2}\right)k^2 - ik + O(k^3)}$$

$$\Downarrow$$

$$g_1^2 = \frac{2}{m_1 a_{11}} \qquad g_2^2 = \frac{2a_{12}^2}{m_1 a_{11}^3} \qquad E_{BW} = \frac{1}{m_1}\left(\frac{1}{a_{11}^2} - \frac{a_{12}^2}{a_{11}^3 a_{22}}\right)$$

a_F and r_F can be expressed by the EFT parameter a_{11}, a_{12}, a_{22}.

$$\begin{aligned} a_F(g_1^2, g_2^2, E_{BW}) \\ r_F(g_1^2, g_2^2, E_{BW}) \end{aligned} \quad \Rightarrow \quad \begin{aligned} a_F(a_{11}, a_{12}, a_{22}) \\ r_F(a_{11}, a_{12}, a_{22}) \end{aligned}$$

Analytic comparison

a_{EFT} and r_{EFT} in EFT amplitude

$$a_{EFT} = \frac{a_{11}a_{12}^2(1 + ip_0a_{22})}{a_{12}^2(1 + ip_0a_{22}) - a_{11}a_{22}} \qquad r_{EFT} = -\frac{i}{p_0}\left\{\frac{a_{22}}{a_{12}(1 + ip_0a_{22})}\right\}^2$$

p_0 : channel 2 momentum at $E = 0$

a_F and r_F in Flatté amplitude with EFT parameters a_{11}, a_{12}, a_{22}

$$a_F = \frac{a_{11}^2a_{12}}{a_{12}^2(1 + ip_0a_{22}) - a_{11}a_{22}} \qquad r_F = -2a_{11} - i\frac{a_{12}^2}{p_0a_{11}^2}$$

a_F and r_F are analytically different from a_{EFT} and r_{EFT}.

Numerical comparison

Application to the $\pi\pi$-$K\bar{K}$ system with $f_0(980)$ for quantitative comparison

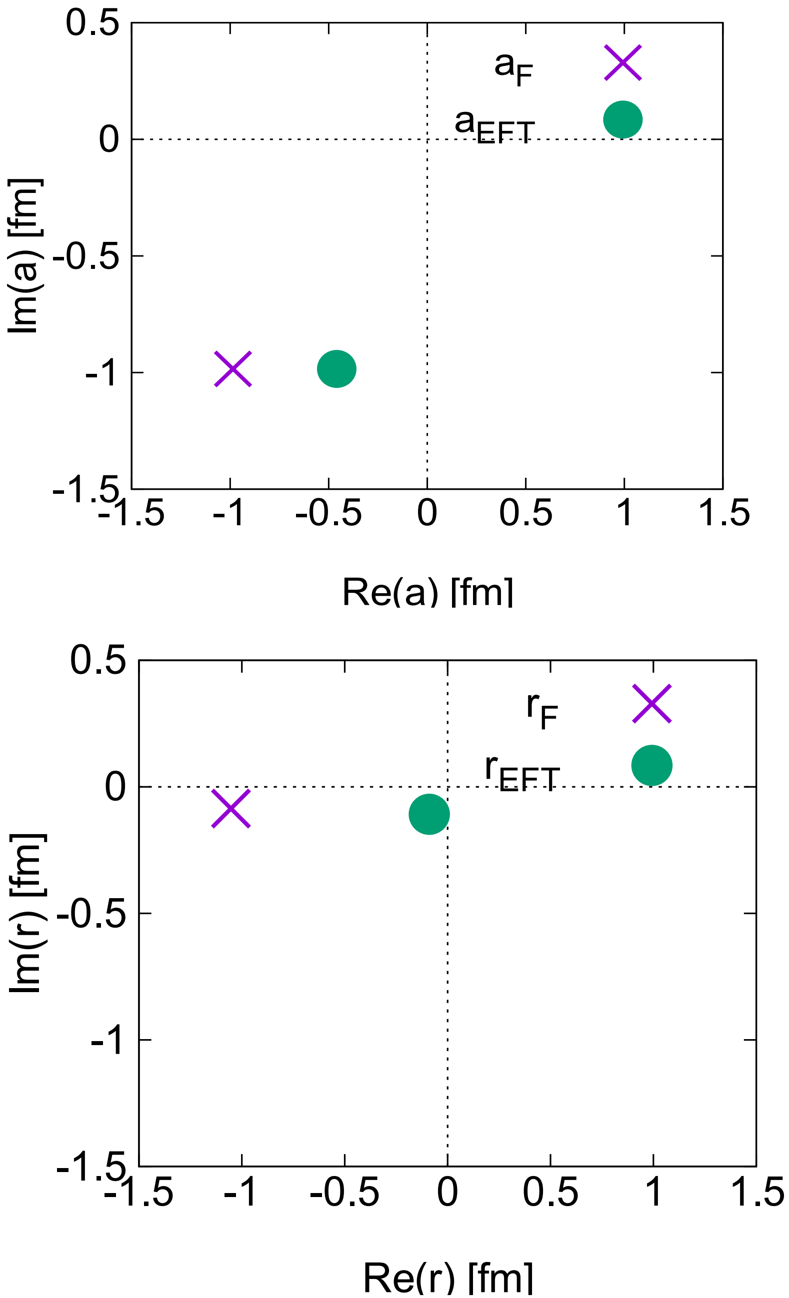
EFT parameters a_{11}, a_{12}, a_{22} corresponding to Ref.[6]

$$a_{11} = 0.53 \text{ [fm]},\ a_{12} = 0.24 \text{ [fm]},\ a_{22} = 0.15 \text{ [fm]}$$

⇩

$a_F = -0.98 - 0.98i$[fm]	$r_F = -1.05 - 0.08i$[fm]
$a_{EFT} = -0.45 - 0.98i$[fm]	$r_{EFT} = -0.09 - 0.10i$[fm]

a_F and r_F are quantitatively different from a_{EFT} and r_{EFT} in the physical system.



[6] R.R. Akhametshin et al., Phys. Lett B 462, 380 (1999)

Pole position

We compare the pole potion of f^{EFT} with that of f^{F}.

The EFT scattering amplitude

$$f^{EFT}(E)=\underline{\left\{\frac{1}{a_{12}^2}-\left(\frac{1}{a_{22}}+ip(E)\right)\left(\frac{1}{a_{11}}+ik(E)\right)\right\}}^{-1}\begin{pmatrix}\left(\frac{1}{a_{22}}+ip(E)\right) & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & \left(\frac{1}{a_{11}}+ik(E)\right)\end{pmatrix}$$

$$=0$$

a_{11}, a_{12}, a_{22}; EFT parameters

The Flatté scattering amplitude

$$f^{F}(E)=\underline{\left\{2E_{BW}-E-ig_1^2k(E)-ig_2^2p(E)\right\}}^{-1}\begin{pmatrix} g_1^2 & g_1g_2 \\ g_1g_2 & g_2^2\end{pmatrix}$$

$$=0$$

E_{BW}, g_1^2, g_2^2; Flatté parameters

$k(E)$; channel 1 momentum

$p(E)$; channel 2 momentum

Pole position

$\pi\pi$-$K\bar{K}$ system with $f_0(980)$

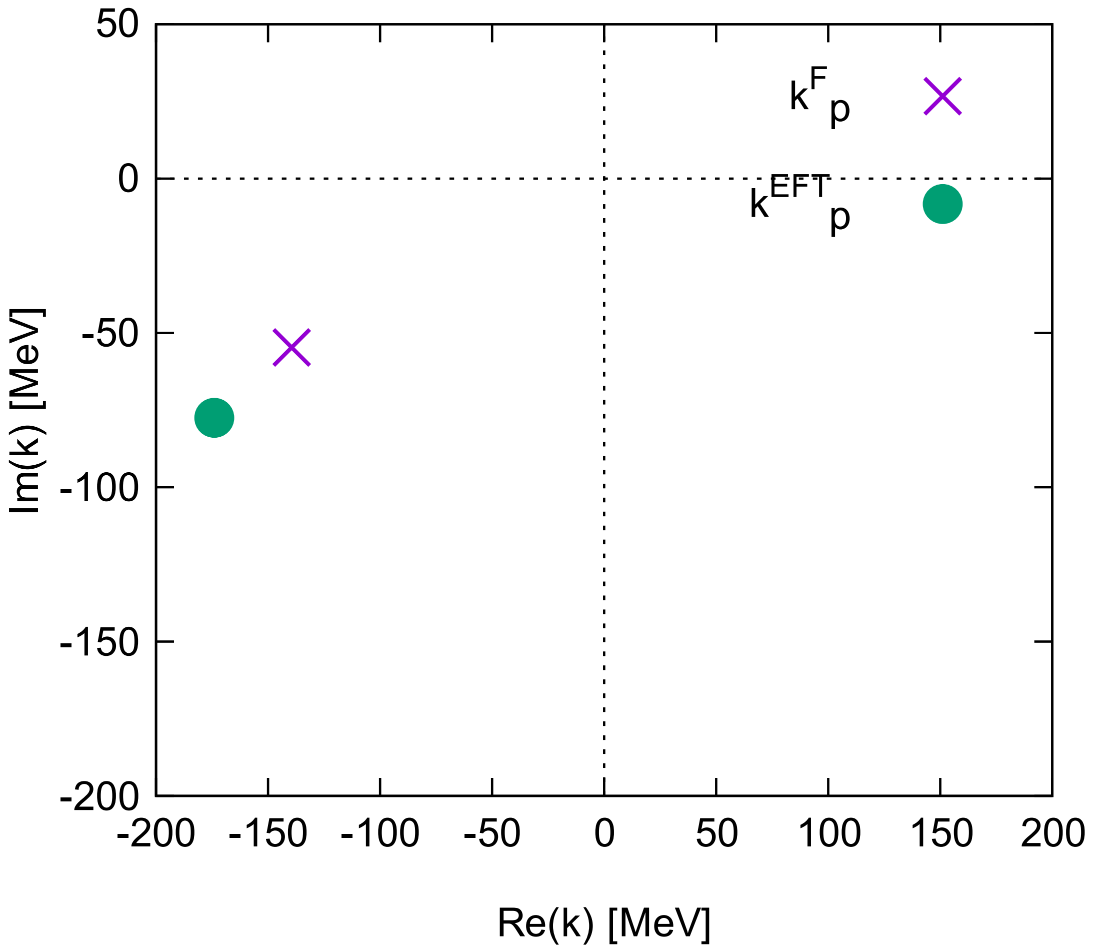
Flatté parameters $E_{BW}, \bar{g}_1, g_2^2$ corresponding to Ref.[6]

$E_{BW} = 74.5$ [MeV],
$g_1^2 = 1.51$ [dimensionless],
$g_2^2 = 0.31$ [dimensionless]

⇨

$a_{11} = 2.67$ [1/GeV],
$a_{12} = 1.22$ [1/GeV],
$a_{22} = 0.76$ [1/GeV]

Flatté pole : $k_p^F = -139 - 54.7i$ [MeV]
EFT pole : $k_p^{EFT} = -174 - 77.5i$ [MeV]



The Flatté pole position is different from the EFT pole position.

[6] R.R. Akhametshin et al., Phys. Lett B 462, 380 (1999)

Summary

We discuss determination of scattering length and effective range.

Flatté : a_F, r_F EFT : a_{EFT}, r_{EFT}

- We express a_F, r_F by the parameters in the EFT amplitude, and compare them with a_{EFT}, r_{EFT}.

- We compare a_F, r_F with a_{EFT}, r_{EFT} quantitatively in the $\pi\pi$-$K\bar{K}$ system with $f_0(980)$.

⇨ a_F and r_F are different from a_{EFT} and r_{EFT} analytically, and also in the physical system.

- We compare the pole potion of f^{EFT} with that of f^F.

⇨ The Flatté pole k_p^F is different from the EFT pole k_p^{EFT}.

We must use the scattering amplitude derived from EFT satisfying the optical theorem, in order to obtain the correct scattering length and effective range.

Pole position

Using parameters Ref.[6]

Flatté pole potision

$$k_p^{F(1)} = -98.7 - 98.7i \text{ [MeV]}$$

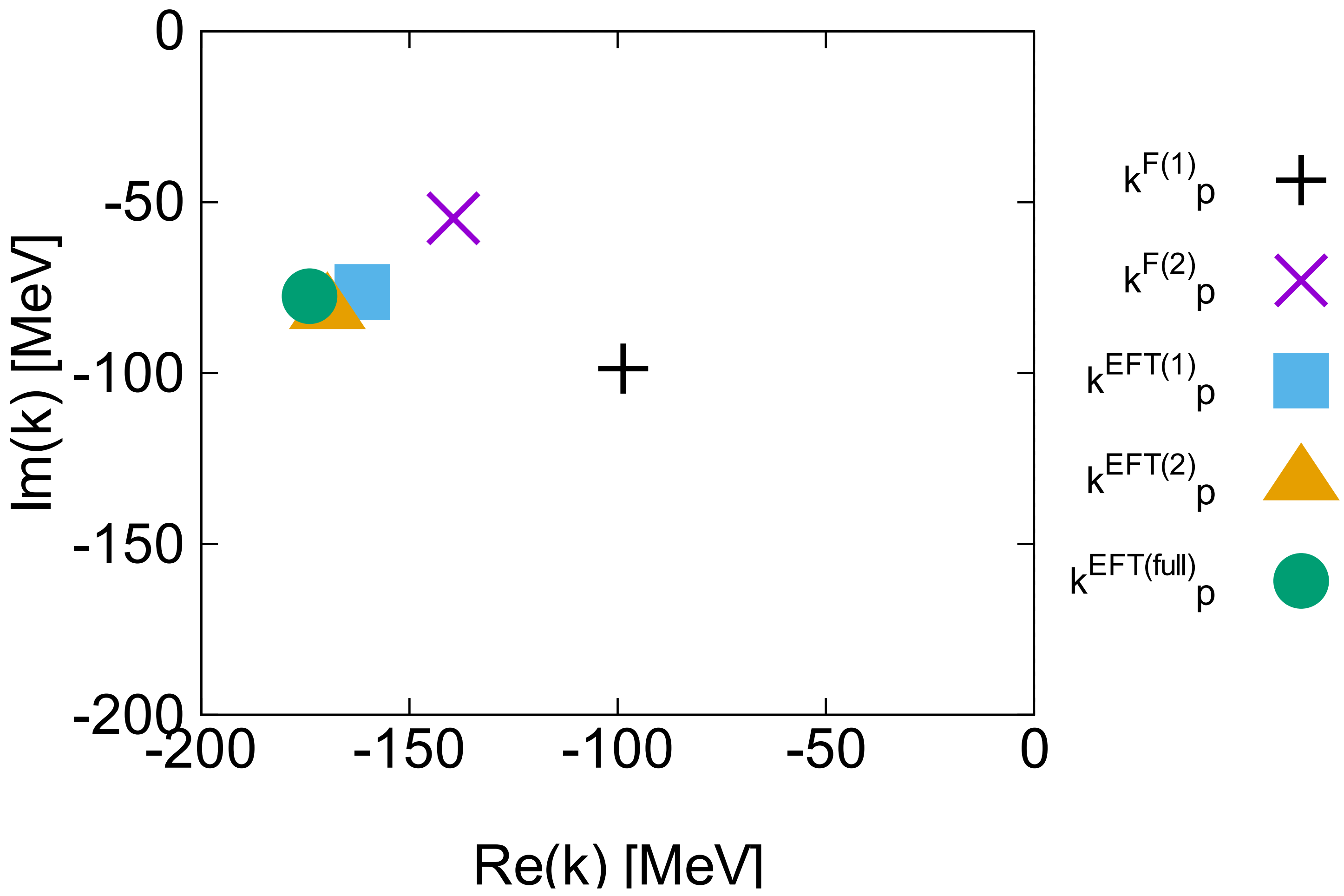
$$k_p^{F(2)} = -139 - 54.7i \text{ [MeV]}$$

EFT pole potision

$$k_p^{EFT(1)} = -161 - 76.2i \text{ [MeV]}$$

$$k_p^{EFT(2)} = -170 - 81.4i \text{ [MeV]}$$

$$k_p^{EFT(full)} = -174 - 77.5i \text{ [MeV]}$$



[6] R.R. Akhametshin et al., Phys. Lett B 462, 380 (1999)