Compositeness for System with Energy-dependent Potential

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[1] S. Weinberg, Physical Review Vol130 No.2 (1963).
[2] S. Weinberg, Physical Review Vol137 No.3B (1965).
[3] T. Sekihara, T. Hyodo, and D. Jido, PTEP 63D04 (2015).
[4] Y. Kamiya and T. Hyodo, PTEP 023D02 (2017).
[5] T. Sekihara, Phys. Rev. C **95**, 025206

Compositeness

Idea

- **Compositeness** is an attempt to account for the structure of **exotic hadrons** in a universal way.
- Deuteron is a broadly studied system, and we will utilize it as a **verification**.
- **Composite State**: scattering state of hadrons, **Elementary State**: state outside hadron model.
- When S. Weinberg proposed compositeness, he calculated the compositeness of deuteron in weak binding limit and concluded that deuteron is at least almost composite.[1][2]
- T. Sekihara, T. Hyodo, Y. Kamiya, D. Jido developed a modern interpretation of compositeness.[3][4][5]

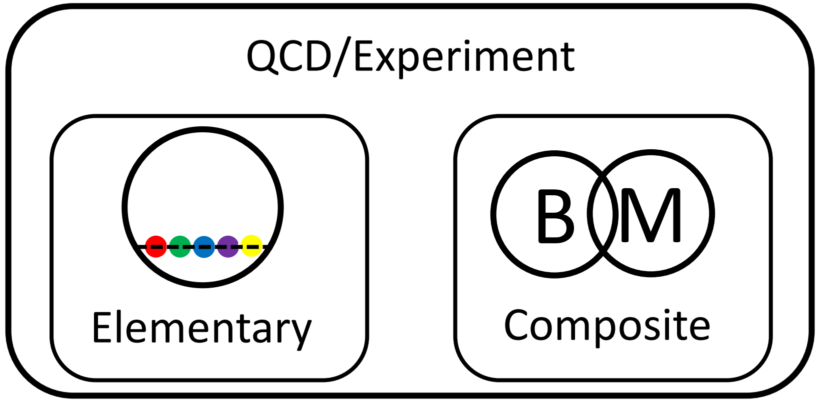
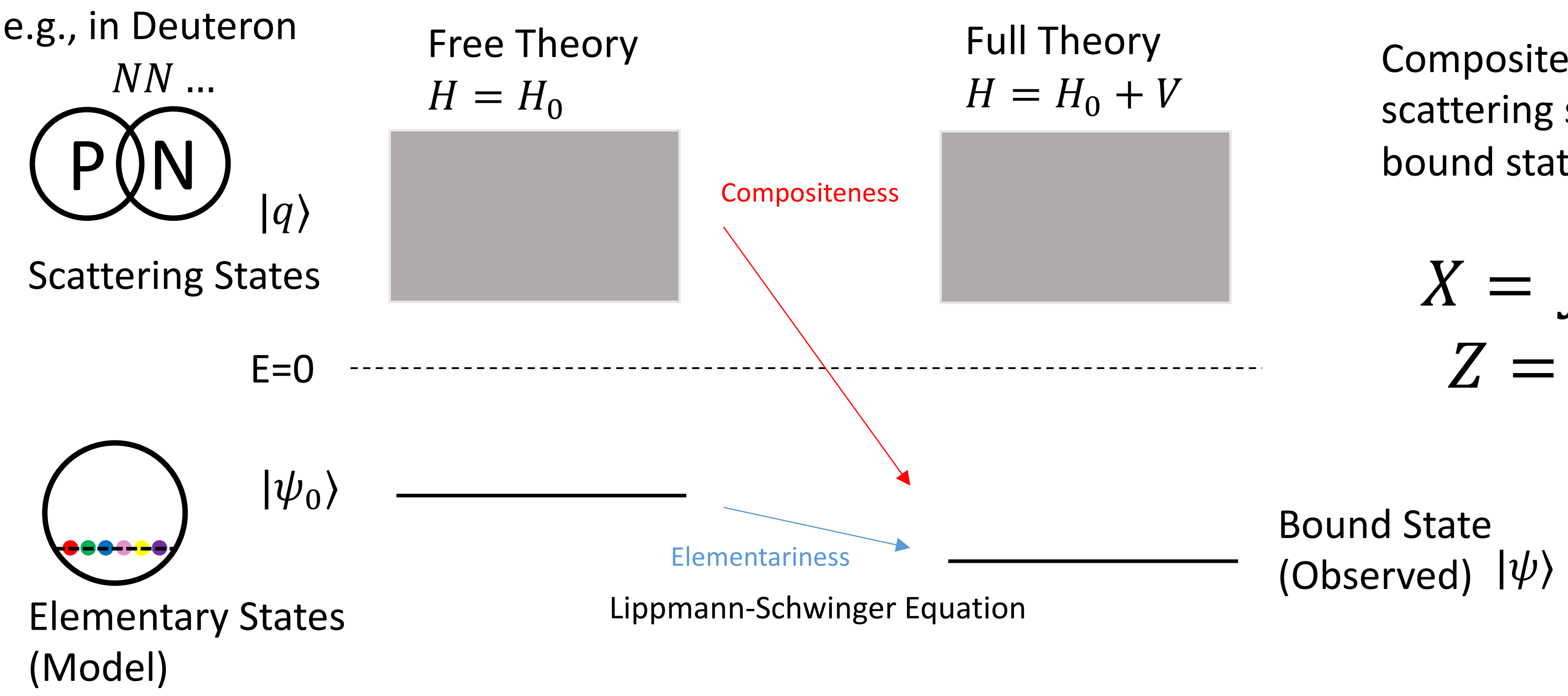


Fig.1 Model Space Analysis

Compositeness

Theory Picture



Compositeness: the contribution of scattering states wave function to bound state wave function.

$$X = \int \langle\psi|q\rangle\langle q|\psi\rangle \, dq$$

$$Z = \langle\psi|\psi_0\rangle\langle\psi_0|\psi\rangle$$

Fig.2 Theory Picture of Compositeness

[3] T. Sekihara, T. Hyodo, and D. Jido, PTEP 63D04 (2015).
[4] Y. Kamiya and T. Hyodo, PTEP 023D02 (2017).

Compositeness

Modern Interpretation

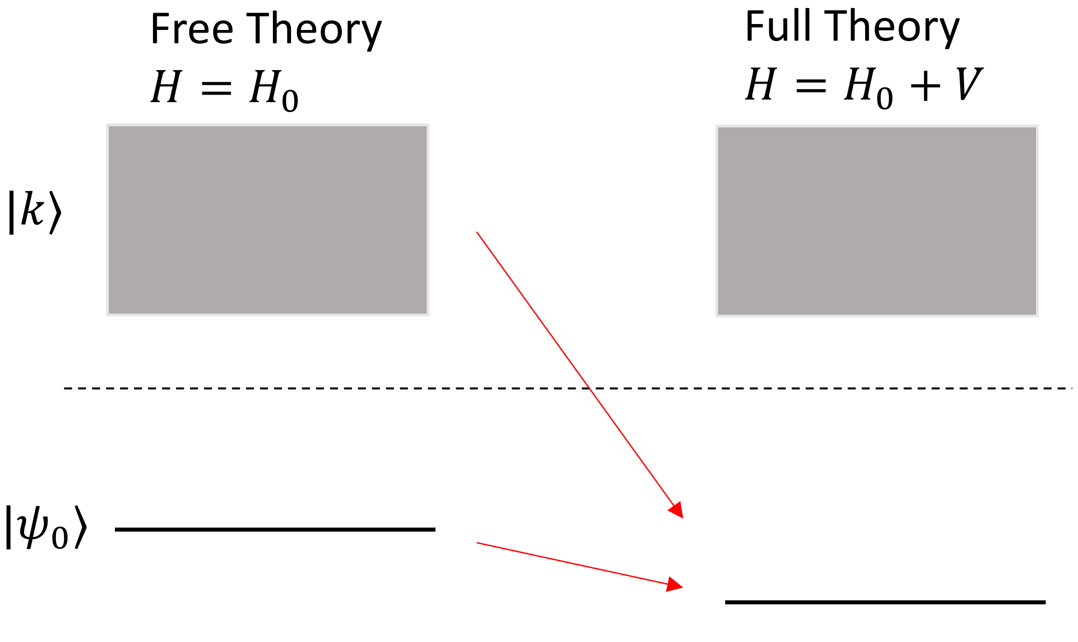


Fig.2 Theory Picture of Compositeness

$$H_0|q\rangle = (M_{th} + \frac{q^2}{2\mu})|q\rangle$$

$$H_0|\psi_0\rangle = M_0|\psi_0\rangle$$

$$\langle q'|V(E)|q\rangle = v(E)$$

$$\langle q'|V|\psi_0\rangle = g_0$$

$$\langle \psi_0|V|\psi_0\rangle = 0$$

We assume the separable form of Interaction, and real parameters as a result of time-reversal symmetry.

M_{th} is the threshold energy.

<u>Note that $v(E)$ can be energy dependent.</u>

<u>We didn't formalize form factor here.</u>

Compositeness

Calculation[3]

Compositeness $X = -|c|^2 \left[\frac{dG}{dE}\right]_{E=M_B}$

Elementariness $Z = -|c|^2 \left[G \frac{d(v^{eff} - v)}{dE} G\right]_{E=M_B}$

By definition, $X + Z = 1$

G is loop function, it is equivalent to the Green's function of H_0

c is the renormalization factor of wave function.

v^{eff} is defined through incooporating the elementary state into the the two-body channel as a potential

$(v^{eff} - v)$ is in principle only related with elementary states if we clearly know the form of $v(E)$

— Compositeness —

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This formalism is different from the formalism in [3].
Reason: the original formalism faces difficulty when trying to explain deuteron.

In the original formalism, v is a constant and $c = g$, and achieve a model independent formalism of compositeness.

If $v(E)$ is energy dependent, compositeness is purely fixed by model, especially by model space, i.e. which part we include as v and which part as $v^{eff} - v$ is arbitrary

— Compositeness —

Calculation[3]

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Due to the arbitrariness, and as well as the arbitrariness between V and G in Lippmann-Schwinger equation, we can trade form factor (or alternatively a separable form of V) for a clear understanding to the model space.

We can recover the original theory by ignoring $\frac{\mathrm{d}v}{dE}$, it is equivalent to assuming any energy dependence in v^{eff} comes from other channels or states.

Compositeness in Perturbation Theory

We can construct a non-relativistic perturbation theory as

$$V = V_0 + \lambda \tilde{V}$$

In which $V_0 = const$, and we have a pole with energy

$$M = M_0 + \lambda M_1 + \lambda^2 M_2 + \cdots$$

$$X = X_0 + \lambda X_1 + \lambda^2 X_2 + \cdots$$

Finally, we may have

$$X \simeq X_0 (1 - \lambda \frac{\tilde{V}'}{G' V_0^2})$$

Compositeness in Perturbation Theory

$$X \simeq X_0\left(1 - \lambda\frac{\tilde{V}'}{G' V_0^2}\right)$$

$G' < 0$, so any positive $\tilde{V}'$ will have a positive impact on compositeness. For a function with no 0th order contribution at threshold, which means $\tilde{V}(E_{th}) = 0$. If we require $\tilde{V}(M_0)$ to increase binding energy, $\tilde{V}'$ is likely to be positive if we impose low energy approximation. Besides, if $\tilde{V}' < 0$ at some area in $M < E < E_{th}$, it indicates that there could be two poles. However, this pole didn't appear in the theory.

In order to deepen the weakly bounded pole it is likely to increase compositeness for any potential as long as it follows analytic expansion around the threshold, e.g. deuteron.

Numerical Calculation

Interaction	Binding Energy	Compositeness	Elementariness
c_0	1.477MeV	1.000	0.000
$c_0 + c_1 q^2$	2.232MeV	1.479	-0.479
$c_0 + c_1 q^2 + c_2 q^4$	2.423MeV	1.727	-0.727
$c_0 + c_1 q^2 + OPEP$	2.200MeV(Fixed)	1.427	-0.427
D channel with c_0	2.200MeV(Fixed)	[1.133,-0.133]	[0.000,0.000]
D channel Coupling	2.2000MeV(Fixed)	[1.510.-0.068]	[-0.442,0.000]

Table.1 Outcome in different models

$B \uparrow \leftrightarrow X \uparrow$ for any potential as long as it follows analytic expansion around the threshold. It is shown both in theory and numeric.

Compositeness exhibit $X > 1$, or $Z < 0$, though unphysical.

[6] Z. Yin, D. Jido, In preparation

The Origin of Unphysical Compositeness

Assumption: v is energy independent

Explicit bare state → $X < 1, Z > 0$

Similarly, in channel N

$$\frac{\partial w_{00}}{\partial E} = v_{0N}^2 \frac{\partial}{\partial E} \frac{G_N(E)}{1 - v_{NN} G_N(E)} = v_{0N}^2 \frac{\partial G_N}{\partial E} \frac{1}{(1 - v_{NN} G_N)^2} < 0$$

Thus, other channels → $X < 1, Z > 0$

Energy dependence of some kind is required in a channel or between channels to account for the unphysical compositeness in deuteron.

The Origin of Unphysical Compositeness

v is required to be constant, in order to connect the renormalization constant c in theory to square root of residue g from experiment.

However, **if v is energy dependent**, as is indicated in deuteron, the derivative of V_{eff} doesn't totally origin from other channels, and **we can not reach the same equation for compositeness and elementariness, as well as $c \neq g$.**

$-|g|^2\left(\left[\frac{dG}{dE}\right]_{E=M_B} + \left[G\,\frac{dv^{eff}}{dE}\,G\right]_{E=M_B}\right) = 1$ is not changed as an outcome of Lippmann-Schwinger equation.

Conclusion

- An energy dependent formalism for compositeness.
- In the original formalism, $B\uparrow \leftrightarrow X\uparrow$: It is likely to have an unphysical compositeness if the binding energy is higher than what is suggested by scattering length.
- Elementary states and other channels can not solve this problem
- A well-defined model or a 3-body states can be the solution.
- Outlook: Construction of a pseudo-model-independent model incorporating 3-body states.

THANK YOU