カイラル対称性とアノマリーに着目したペンタクォーク的ヘビーバリオンの性質

Pentaquark-like singly heavy baryons from chiral symmetry and anomaly

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in collaboration with Hiroto Takada (M2 in Nagoya U.),
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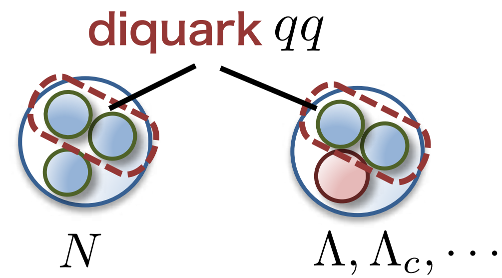
1. Introduction

- **What are diquarks?**



- Diquark is a "cluster" made of light two quarks

- Diquarks are not observable (confinement) but are useful in studying the baryon spectrum

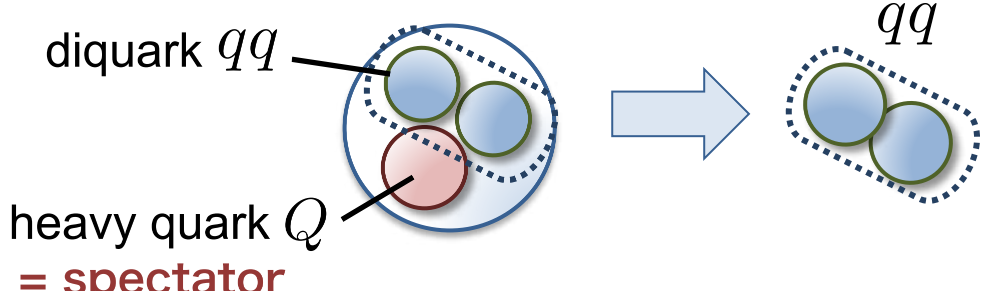
eg, Jaffe-Wilczek (2003)
Hong-Sohn-Zahed (2004)

⇔ quark model (there must be confinement but useful)

- **Singly heavy baryons (SHBs)** provide us with suitable testing ground

SHBs are useful

eg, isotope shift (λ, ρ mode shift)
eg, Yoshida-Hiyama-Hosaka-Oka-Sadato (2015)

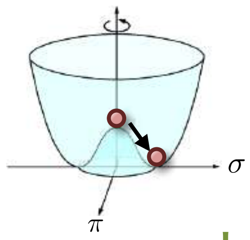


- kinetic properties?
- symmetry properties?
- coupling with pions?, etc.

1. Introduction

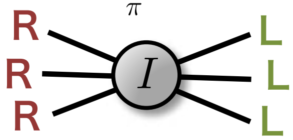
• Symmetry of diquarks

- QCD with light flavors has the following symmetry properties:



(approximate) $SU(3)_L \times SU(3)_R$ **chiral symmetry**

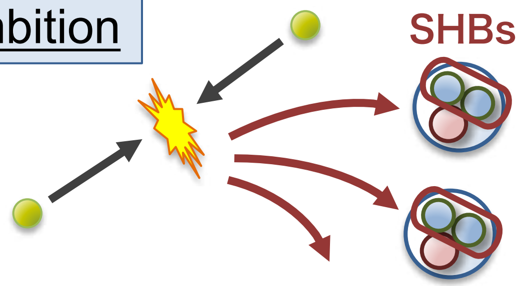
$U(1)_A$ **anomaly** (or significant instanton effects)



- Try to extract the symmetry information of diquarks from SHB spectroscopy

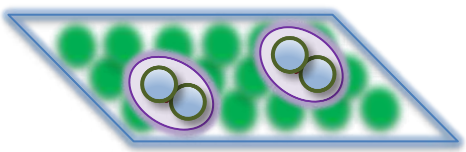
ambition

①



any indication?
eg, with chiral restoration?

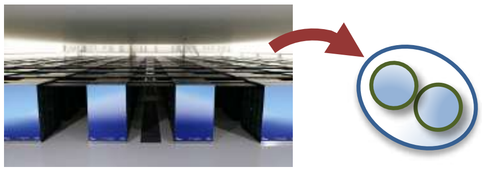
diquark condensate (color superconductor)



eg, Alford-Schmitt-Rajagopal-Schäfer (2008)

② Relation with diquark baryons in two-color QCD at density ?
(with lattice QCD)

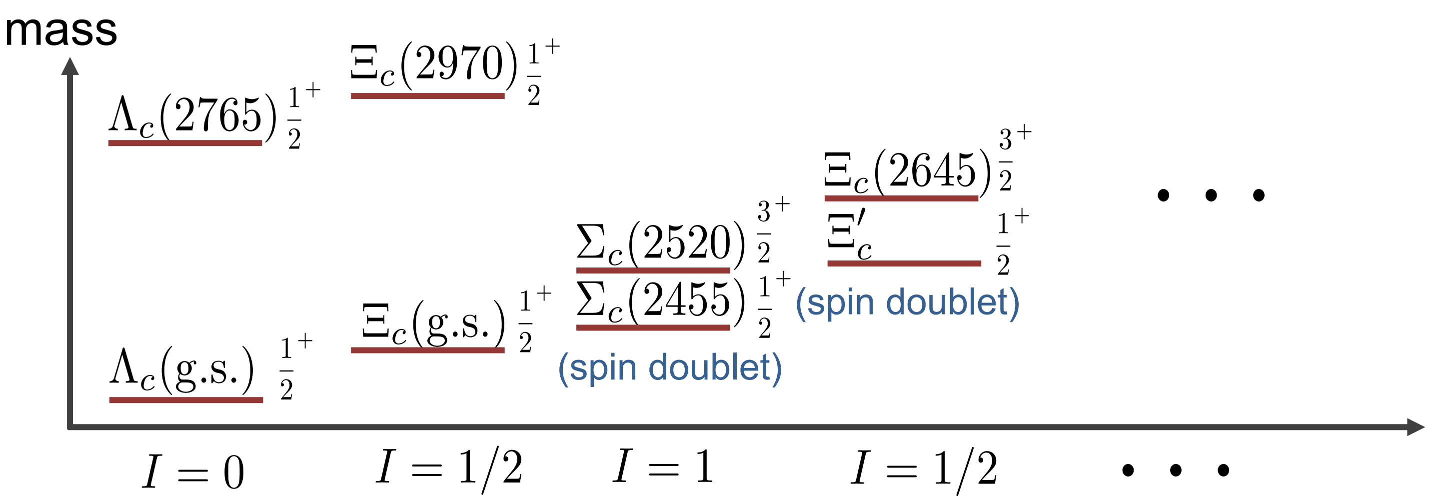
Suenaga-Murakami-Itou-Iida; 2211.01789
Murakami-Suenaga-Itou-Iida; 2211.13472



1. Introduction

- **SHB in chiral models**

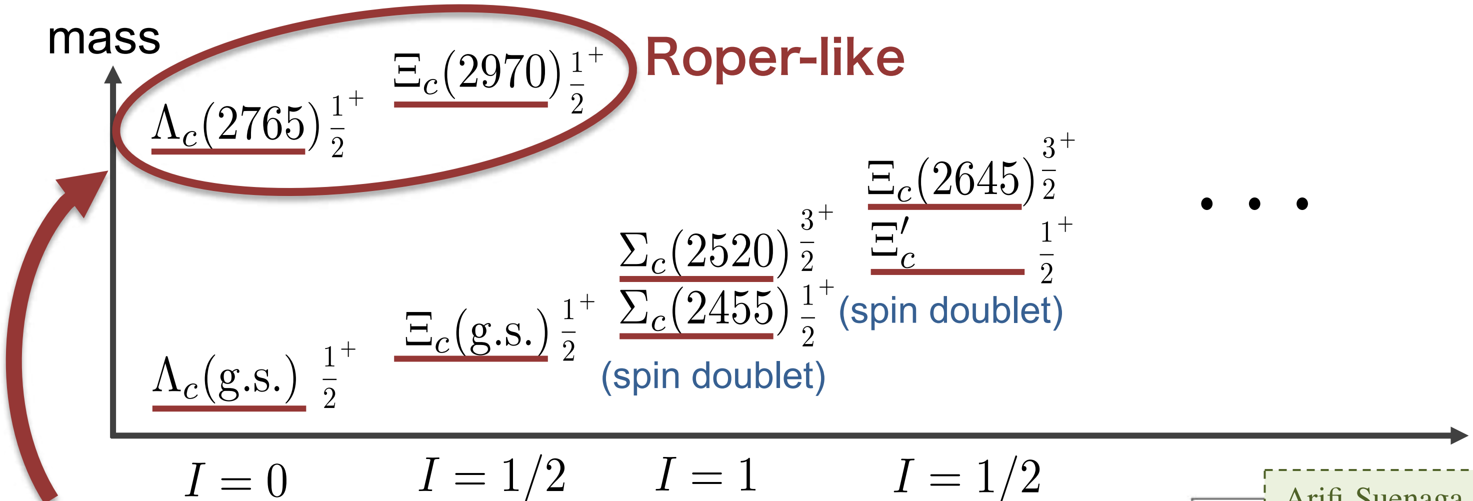
  - Mass spectrum of SHB in charmed sector



1. Introduction

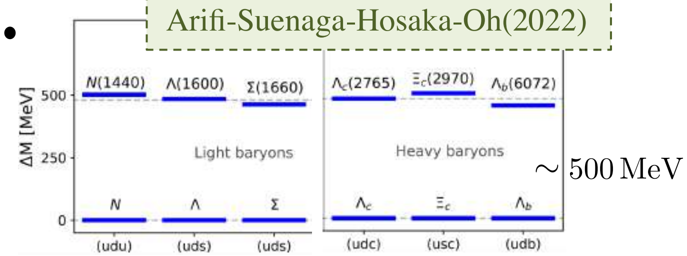
• SHB in chiral models

- Mass spectrum of SHB in charmed sector



- The Roper-like SHBs are rather mysterious

• Arifi-Suenaga-Hosaka-Oh(2022)



- Universality of mass differences

- Comparably large decay width of $\Lambda_c(2765)$, $\Xi_c(2970)$ ($\Gamma_{\text{ex}} \sim 50\,\text{MeV}$)

$\Leftrightarrow$ $\Gamma_{\text{th}} \sim 5\,\text{MeV}$ in NON-relativistic quark model, but improved by rel. corrections

Nagahiro et al (2017), Arifi-Suenaga-Hosaka (2021)

→ chiral model description

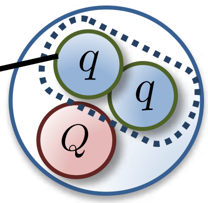
1. Introduction

- **Pentaquark picture for $\Lambda_c(2765)$ and $\Xi_c(2970)$**

- We invented pentaquark picture for the Roper-like SHBs with the tetra diquark in addition to the conventional one with chiral symmetry

Suenaga-Hosaka (2021)

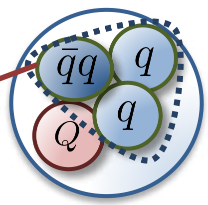
conventional diquark



$\sim \Lambda_c(2286),\ \Xi_c(2470)\ \left(\frac{1}{2}^+\right)$

ground state

tetra diquark

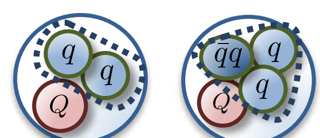


$\sim \Lambda_c(2765),\ \Xi_c(2970)\ \left(\frac{1}{2}^+\right)$

Roper-like state

- We also argued the presence of their chiral partners

$(1/2^+)$ ⟺



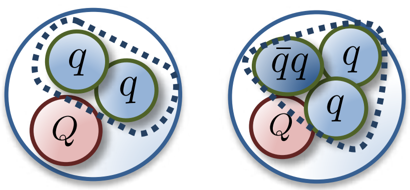
$(1/2^-)$

- Large decay width of $\Lambda_c(2765)$ and $\Xi_c(2970)$ are reasonably explained within our description Suenaga-Hosaka (2022)

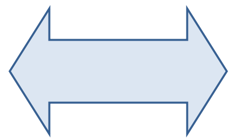
1. Introduction

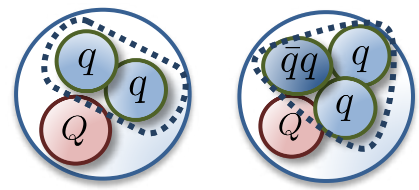
- **This work**

  - In this work, we study masses and decays of the negative-parity chiral partner SHBs for future experiments



$(1/2^+)$





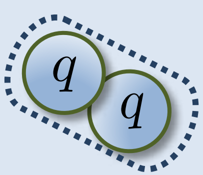
$(1/2^-)$

↑ ground state ↑ Roper-like state

chiral partners

→ <u>aim of this work</u>

- In particular, we focus on (approximate) $SU(3)_L \times SU(3)_R$ chiral symmetry and $U(1)_A$ anomaly of the following two diquarks



conventional diquark

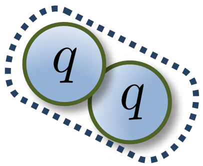
tetra diquark

2. Model

• Diquark fields

- We introduce two types of diquarks

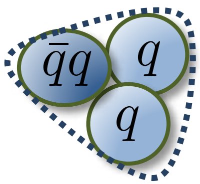
i) *conventional diquark*



$$(d_R)^a_i \sim \epsilon_{ijk}\epsilon^{abc}(q_R^T)^b_j C (q_R)^c_k$$
$$(d_L)^a_i \sim \epsilon_{ijk}\epsilon^{abc}(q_L^T)^b_j C (q_L)^c_k$$

i, j, k : flavor index
a, b, c : color index

ii) *tetra diquark*

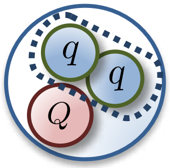


$$(d'_R)^a_i \sim \epsilon_{jkl}\epsilon^{abc}(q_R^T)^b_k C (q_R)^c_l [(\bar{q}_L)^d_i (q_R)^d_j]$$
$$(d'_L)^a_i \sim \epsilon_{jkl}\epsilon^{abc}(q_L^T)^b_k C (q_L)^c_l [(\bar{q}_R)^d_i (q_L)^d_j]$$

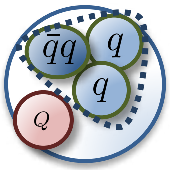
- The chiral representation of d_R, d_L, d'_R, d'_L are

$$d_R \sim (\mathbf{1}, \bar{\mathbf{3}})\ ,\quad d_L \sim (\bar{\mathbf{3}}, \mathbf{1})$$
$$d'_R \sim (\bar{\mathbf{3}}, \mathbf{1})\ ,\quad d'_L \sim (\mathbf{1}, \bar{\mathbf{3}})$$

- The baryon fields are
$$\begin{cases} B_{R(L)} \sim Q d_{R(L)} \\ B'_{R(L)} \sim Q d'_{R(L)} \end{cases}$$



3-quark



5-quark

2. Model

- **Lagrangian**

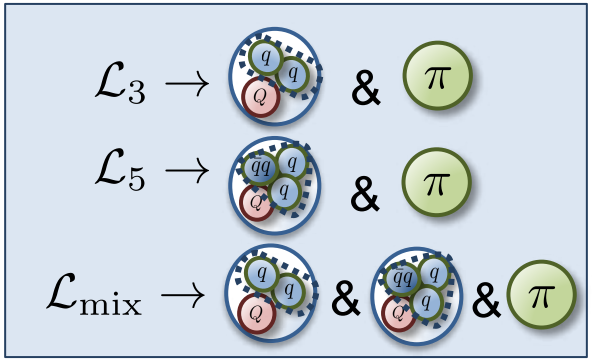
  - Our Lagrangian is invariant under $SU(3)_L \times SU(3)_R$ chiral symmetry but incorporate $U(1)_A$ anomaly: $\mathcal{L} = \mathcal{L}_3 + \mathcal{L}_5 + \mathcal{L}_{\rm mix}$

$$
\begin{aligned}
\mathcal{L}_3 = &\sum_{\chi=L,R} \bar{B}_\chi i v \cdot \partial B_\chi - \mu_1 \bar{B}_\chi B_\chi - \frac{\mu_1'}{2f_\pi^2}\epsilon_{abc}\epsilon^{dec}\left[\bar{B}^a_\chi (\Sigma\Sigma^\dagger)^b{}_e B_{\chi,d} + \text{h.c.}\right] \\
&- g_1(\bar{B}_L \Sigma^* B_R + \bar{B}_R \Sigma^T B_L) - \frac{g_4}{2f_\pi}\epsilon_{ijk}\epsilon_{lmn}(\bar{B}_{L,k}\Sigma^T_{li}\Sigma^T_{mj}B_{R,n} + \bar{B}_{R,k}\Sigma^*_{li}\Sigma^*_{mj}B_{L,n})
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_5 = &\sum_{\chi=L,R} (\bar{B}'_\chi i v B'_\chi - \mu_2 \bar{B}'_\chi B'_\chi) - \frac{\mu_2'}{f_\pi^2}\{\bar{B}'^a_L (\Sigma^\dagger\Sigma)^\alpha{}_a B'_{L,\alpha} + h.c.\} \\
&+ g_2(\bar{B}'_R \Sigma^* B'_L + \bar{B}'_L \Sigma^T B'_R) - \frac{g_5}{6f_\pi^3}\delta^\alpha{}_\beta \epsilon_{\gamma\lambda\rho}\delta^e{}_a \epsilon^{bcd}(\bar{B}'^a_R \Sigma^{\dagger\rho}{}_b \Sigma^{\dagger\lambda}{}_c \Sigma^{\dagger\gamma}{}_d \Sigma^{\dagger\beta}{}_e B'_{L,\alpha} + h.c.) \\
&- \frac{g_6}{2f_\pi^3}\delta^\alpha{}_\beta \epsilon_{\gamma\lambda\rho}\delta^b{}_a \epsilon^{cde}(\bar{B}'^a_R \Sigma^{\dagger\rho}{}_b \Sigma^{\dagger\lambda}{}_c \Sigma^{\dagger\gamma}{}_d \Sigma^{\dagger\beta}{}_e B'_{L,\alpha} + h.c.)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{\rm mix} = &-\mu_3(\bar{B}_R B'_L + \bar{B}'_L B_R + \bar{B}_L B'_R + \bar{B}'_R B_L) \\
&- g_3(\bar{B}'_R \Sigma^* B_R + \bar{B}_L \Sigma^* B'_L + h.c.)
\end{aligned}
$$

Σ is meson nonet



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$$\mathcal{L}_3 = \sum_{\chi=L,R} \bar{B}_\chi iv\cdot\partial B_\chi - \mu_1 \bar{B}_\chi B_\chi - \frac{\mu_1'}{2f_\pi^2}\epsilon_{abc}\epsilon^{dec}\left[\bar{B}^a_\chi(\Sigma\Sigma^\dagger)^b{}_e B_{\chi,d} + \text{h.c.}\right]$$
$$-\underline{g_1(\bar{B}_L\Sigma^* B_R + \bar{B}_R\Sigma^T B_L)} - \frac{g_4}{2f_\pi}\epsilon_{ijk}\epsilon_{lmn}(\bar{B}_{L,k}\Sigma^T_{li}\Sigma^T_{mj}B_{R,n} + \bar{B}_{R,k}\Sigma^*_{li}\Sigma^*_{mj}B_{L,n})$$

$$\mathcal{L}_5 = \sum_{\chi=L,R}(\bar{B}'_\chi ivB'_\chi - \mu_2\bar{B}'_\chi B'_\chi) - \frac{\mu_2'}{f_\pi^2}\{\bar{B}'^a_L(\Sigma^\dagger\Sigma)^\alpha{}_a B'_{L,\alpha} + h.c.\}$$
$$\underline{+ g_2(\bar{B}'_R\Sigma^* B'_L + \bar{B}'_L\Sigma^T B'_R)} - \frac{g_5}{6f_\pi^3}\delta^\alpha{}_\beta\epsilon_{\gamma\lambda\rho}\delta^e{}_a\epsilon^{bcd}(\bar{B}'^a_R\Sigma^{\dagger\rho}{}_b\Sigma^{\dagger\lambda}{}_c\Sigma^{\dagger\gamma}{}_d\Sigma^{\dagger\beta}{}_e B'_{L,\alpha} + h.c.)$$
$$-\frac{g_6}{2f_\pi^3}\delta^\alpha{}_\beta\epsilon_{\gamma\lambda\rho}\delta^b{}_a\epsilon^{cde}(\bar{B}'^a_R\Sigma^{\dagger\rho}{}_b\Sigma^{\dagger\lambda}{}_c\Sigma^{\dagger\gamma}{}_d\Sigma^{\dagger\beta}{}_e B'_{L,\alpha} + h.c.)$$

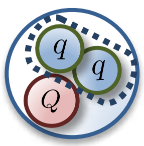
$$\mathcal{L}_{\rm mix} = \underline{-\mu_3(\bar{B}_R B'_L + \bar{B}'_L B_R + \bar{B}_L B'_R + \bar{B}'_R B_L)}$$
$$- g_3(\bar{B}'_R\Sigma^* B_R + \bar{B}_L\Sigma^* B'_L + h.c.)$$

only g_1, g_2, μ_3 terms are $U(1)_A$ NON-invariant

Σ is meson nonet

3. Analysis

- **3-quark SHBs with no mixing**
  - Under chiral symmetry breaking $\langle \Sigma \rangle = f_\pi \mathrm{diag}(1, 1, A)$ with $A = 1.38$
  - The mass of 3-quark SHBs with no mixing $\mathcal{L}_{\mathrm{mix}} = 0$ is obtained as



ground-state

$$\begin{cases} M(\Xi_c^{[\mathbf{3}]}(\pm)) = \mu_1 + \dfrac{1 + A^2}{2}\mu_1' \mp f_\pi (g_1 + A g_4) \\ M(\Lambda_c^{[\mathbf{3}]}(\pm)) = \mu_1 + \mu_1' \mp f_\pi (A g_1 + g_4) \end{cases}$$

$$B_\pm = \frac{1}{\sqrt{2}}(B_R \mp B_L)$$

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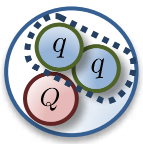

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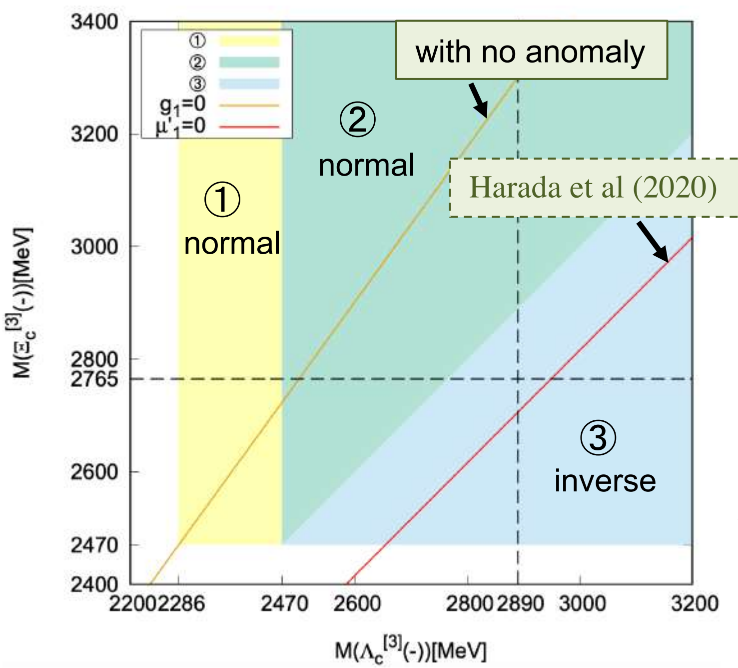


input $M(\Lambda_c^{[3]}(+)) = 2286\,\mathrm{MeV}$ $\quad M(\Xi_c^{[3]}(+)) = 2470\,\mathrm{MeV}$

- ① $M(\Lambda_c^{[3]}(+)) < M(\Lambda_c^{[3]}(-)) < M(\Xi_c^{[3]}(+)) < M(\Xi_c^{[3]}(-))$ normal
- ② $M(\Lambda_c^{[3]}(+)) < M(\Xi_c^{[3]}(+)) < M(\Lambda_c^{[3]}(-)) < M(\Xi_c^{[3]}(-))$ normal
- ③ $M(\Lambda_c^{[3]}(+)) < M(\Xi_c^{[3]}(+)) < M(\Xi_c^{[3]}(-)) < M(\Lambda_c^{[3]}(-))$ inverse

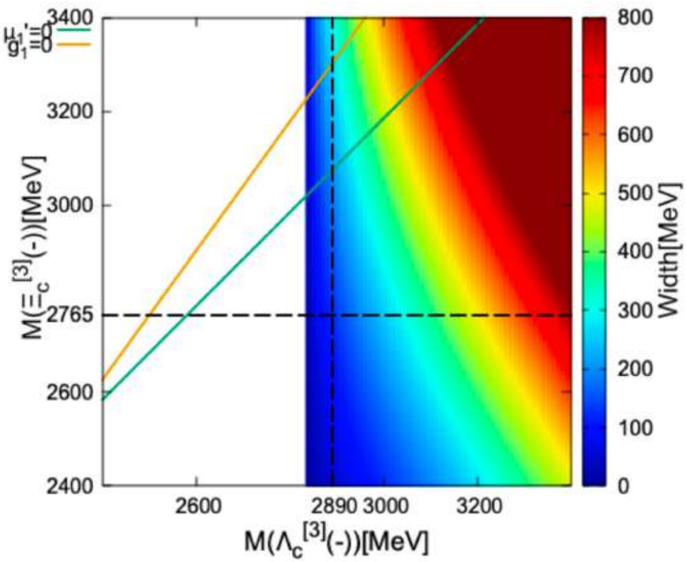
μ_1' term leads to normal hierarchy ① ②

Anomaly g_1 term leads to inverse hierarchy ③

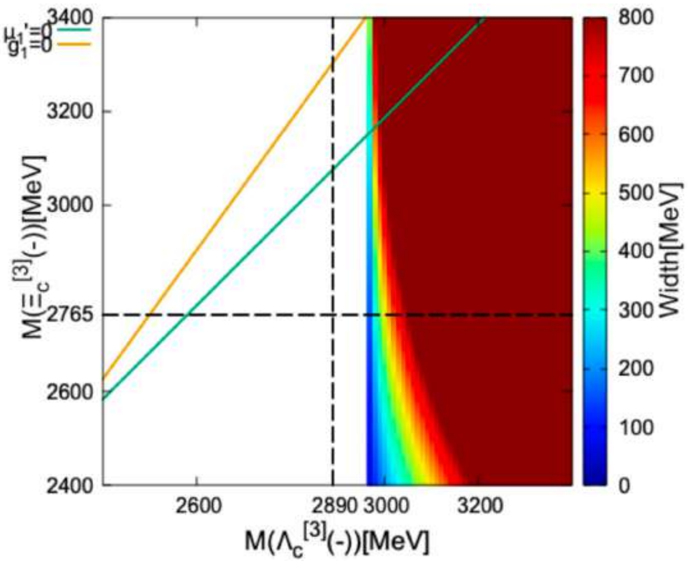
3. Analysis

- **3-quark SHBs with no mixing**
  - Decay widths of $\Lambda_c^{[3]}(-)$ are evaluated

$\Lambda_c^{[\mathbf{3}]}(-) \to \Lambda_c^{[\mathbf{3}]}(+)\eta$



$\Lambda_c^{[\mathbf{3}]}(-) \to \Xi_c^{[\mathbf{3}]}(+)K$



- $\Xi_c^{[3]}(-)$ could be assumed to be recently observed $\Xi_c(2923)$ or $\Xi_c(2930)$

- $M(\Lambda_c^{[3]}(-)) = 2890\,\mathrm{MeV}$ is quark model prediction   Yoshida-Hiyama-Hosaka-Oka-Sadato (2015)
- $M(\Xi_c^{[3]}(-)) = 2765\,\mathrm{MeV}$ is chiral-diquark model prediction   Kim-Hiyama-Oka-Suzuki (2020)

3. Analysis

- **3-quark SHBs with no mixing**

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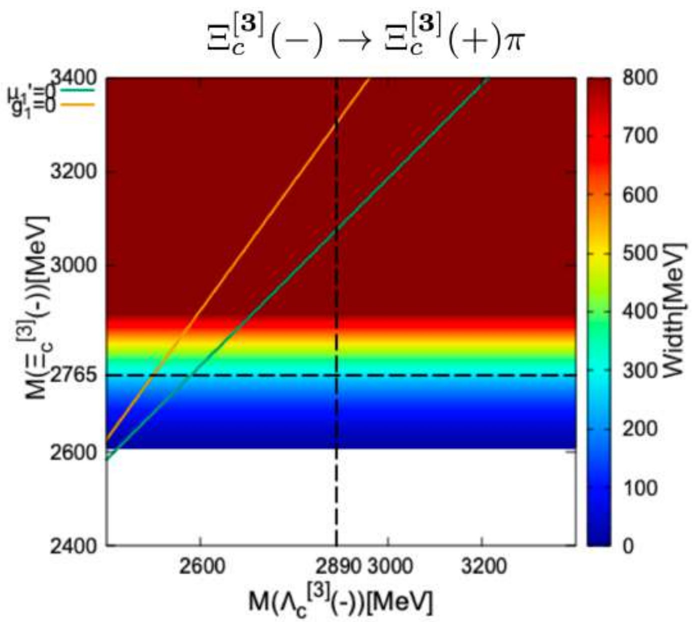
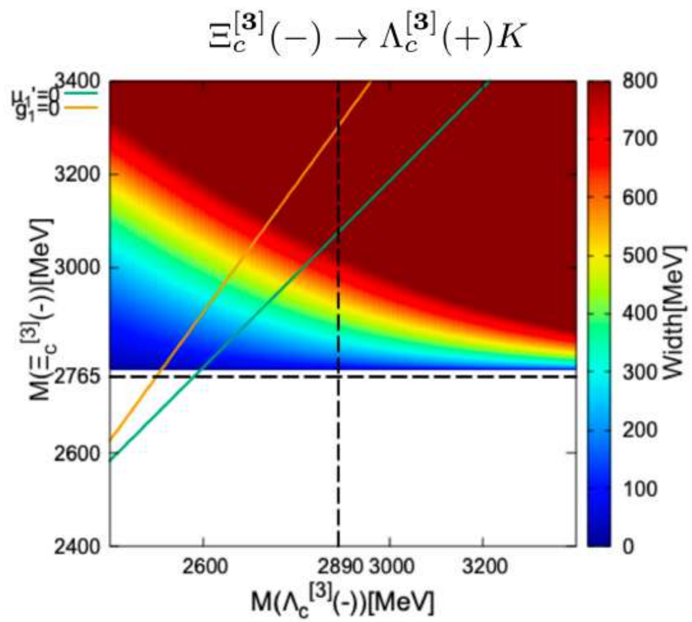




- When assuming $\Xi_c^{[3]}(-)$ to be recently observed $\Xi_c(2923)$ or $\Xi_c(2930)$, the width becomes very large ↔ cf, $\Gamma_{\rm exp} \approx 10\,{\rm MeV}$

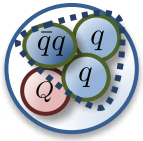
unknown J^P

⇨ $\Xi_c(2923)$ or $\Xi_c(2930)$ cannot be 3-quark (dominant) SHB

3. Analysis

- **5-quark SHBs with no mixing**

  - The mass of 5-quark SHBs with no mixing $\mathcal{L}_{\rm mix} = 0$ is obtained as



Roper-like

$$\begin{cases} M(\Xi_c^{[\mathbf{5}]}(\pm)) = \mu_2 + \mu_2' \pm f_\pi\{g_2 + A(g_5 + g_6)\} \\ M(\Lambda_c^{[\mathbf{5}]}(\pm)) = \mu_2 + A^2\mu_2' \pm A f_\pi\{g_2 + A(g_5 + g_6)\} \end{cases}$$

  - g_2 (and g_5, g_6) can be absorbed into $G \equiv g_2 + A(g_5 + g_6)$ ⇨ no anomaly effects

3. Analysis

• 5-quark SHBs with no mixing

- The mass of 5-quark SHBs with no mixing $\mathcal{L}_{\rm mix} = 0$ is obtained as

Roper-like

$$\begin{cases} M(\Xi_c^{[\mathbf{5}]}(\pm)) = \mu_2 + \mu_2' \pm f_\pi \{g_2 + A(g_5 + g_6)\} \\ M(\Lambda_c^{[\mathbf{5}]}(\pm)) = \mu_2 + A^2 \mu_2' \pm A f_\pi \{g_2 + A(g_5 + g_6)\} \end{cases}$$

- g_2 (and g_5, g_6) can be absorbed into $G \equiv g_2 + A(g_5 + g_6)$ ⇨ no anomaly effects

$$\Gamma(\Xi_c(2967) \to \Xi_c^{[\mathbf{5}]}(-)\pi) + \Gamma(\Xi_c(2967) \to \Lambda_c^{[\mathbf{5}]}(-)K) \lesssim 20.9 \text{ MeV (experiment)}$$

$$\Gamma(\Lambda_c(2765) \to \Xi_c^{[\mathbf{5}]}(-)K) + \Gamma(\Lambda_c(2765) \to \Lambda_c^{[\mathbf{5}]}(-)\eta) \lesssim 50 \text{ MeV (experiment)}$$

⇩

↓ assumption (P-wave)

$$2551\,\text{MeV} \lesssim M(\Lambda_c^{[5]}(-)) < M(\Lambda_c^{[5]}(+))$$

(= 2765 MeV)

↓ assumption (P-wave)

$$2811\,\text{MeV} \lesssim M(\Xi_c^{[5]}(-)) < M(\Xi_c^{[5]}(+))$$

(= 2967 MeV)

- $\Xi_c(2923)$ or $\Xi_c(2930)$ can be assigned to $\Xi_c^{[5]}(-)$

→ no strong decay as long as there is no mixing $\mathcal{L}_{\rm mix} = 0$: $\langle 3\mathrm{q} \,|\, 5\mathrm{q} \rangle \to 0$

3. Analysis

- **With mixing**

- When mixing between 3-quark and 5-quark is present, mass reads

$$M(B^{H/L}_{+,i}) = \frac{1}{2}\left[m_{+,i} + m'_{+,i} \pm \sqrt{(m_{+,i} - m'_{+,i})^2 + 4\tilde{m}^2_{+,i}}\right]$$

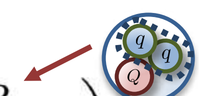
$$M(B^{H/L}_{-,i}) = \frac{1}{2}\left[m_{-,i} + m'_{-,i} \pm \sqrt{(m_{-,i} - m'_{-,i})^2 + 4\tilde{m}^2_{-,i}}\right]$$

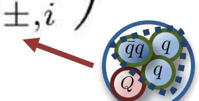
$\pm$: parity
H/L $\Leftrightarrow$ Higher/Lower

$$\begin{cases} m_{\pm,i=1,2} = \mu_1 + \frac{1+A^2}{2}\mu'_1 \mp f_\pi(g_1 + Ag_4) \\ m_{\pm,i=3} = \mu_1 + \mu'_1 \mp f_\pi(Ag_1 + g_4) \\ m'_{\pm,i=1,2} = \mu_2 + \mu'_2 \pm f_\pi\{g_2 + A(g_5 + g_6)\} \\ m'_{\pm,i=3} = \mu_2 + A^2\mu'_2 \pm Af_\pi\{g_2 + A(g_5 + g_6)\} \\ \tilde{m}_{\pm,i=1,2} = \mu_3 \mp f_\pi g_3 \\ \tilde{m}_{\pm,i=3} = \mu_3 \mp Af_\pi g_3 \end{cases}$$

# of effective parameters: 9
(except f_π, A)

- The mass eigenstates are $\begin{pmatrix} B^L_{\pm,i} \\ B^H_{\pm,i} \end{pmatrix} = \begin{pmatrix} \cos\theta_{B_{\pm,i}} & \sin\theta_{B_{\pm,i}} \\ -\sin\theta_{B_{\pm,i}} & \cos\theta_{B_{\pm,i}} \end{pmatrix} \begin{pmatrix} B_{\pm,i} \\ B'_{\pm,i} \end{pmatrix}$





3. Analysis

- **With mixing but no anomaly**

- When anomaly is absent, $g_1 = g_2 = \mu_3 = 0$ → ♯ of effective parameters: 7

input

$M(\Lambda_c^L(+)) = 2286\,\mathrm{MeV}$ $M(\Lambda_c^H(+)) = 2765\,\mathrm{MeV}$

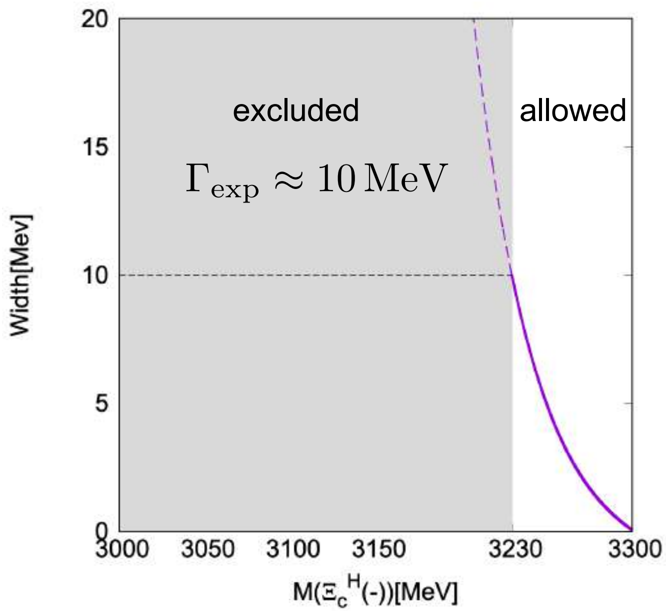
$M(\Xi_c^L(+)) = 2470\,\mathrm{MeV}$ $M(\Xi_c^H(+)) = 2967\,\mathrm{MeV}$

+

$M(\Lambda_c^H(-)) = 2890\,\mathrm{MeV}$ (quark model)

$M(\Xi_c^L(-)) = 2939\,\mathrm{MeV}$ ($\Xi_c(2930)$)

Width of $\Xi_c(2930)$ vs $M(\Xi_c^H(-))$



3. Analysis

- **With mixing but no anomaly**

  - When anomaly is absent, $g_1 = g_2 = \mu_3 = 0$ $\rightarrow$ # of effective parameters: 7

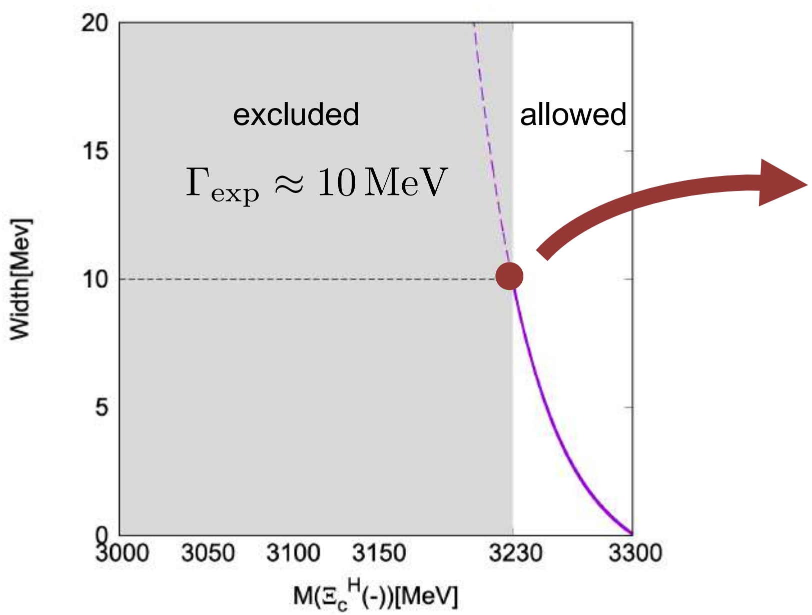
input

$$M(\Lambda_c^L(+)) = 2286\,\mathrm{MeV} \qquad M(\Lambda_c^H(+)) = 2765\,\mathrm{MeV}$$
$$M(\Xi_c^L(+)) = 2470\,\mathrm{MeV} \qquad M(\Xi_c^H(+)) = 2967\,\mathrm{MeV}$$

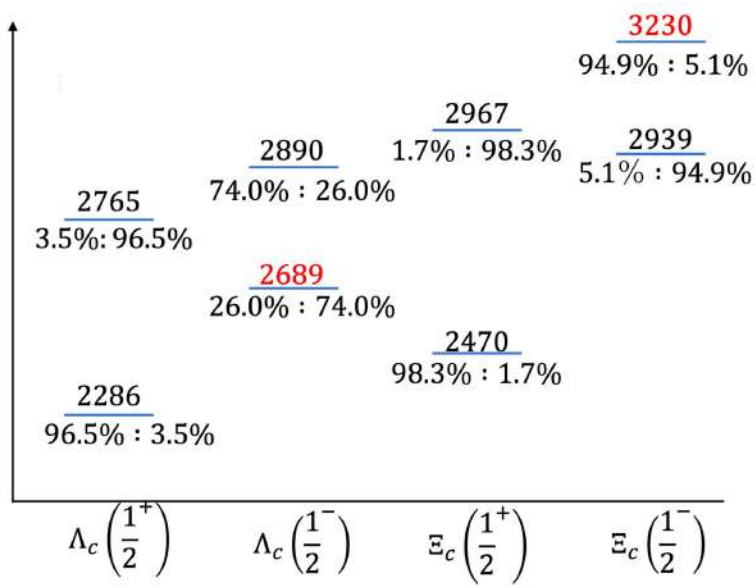
$+$

$M(\Lambda_c^H(-)) = 2890\,\mathrm{MeV}$ (quark model)
$M(\Xi_c^L(-)) = 2939\,\mathrm{MeV}$ ($\Xi_c(2930)$)

Width of $\Xi_c(2930)$ vs $M(\Xi_c^H(-))$



Mass spectrum $(Qqq : Qqq\bar{q}q)$



3. Analysis

- **With mixing but no anomaly**

  - When anomaly is absent, $g_1 = g_2 = \mu_3 = 0$ → # of effective parameters: 7

<u>input</u>

$M(\Lambda_c^L(+)) = 2286\,\mathrm{MeV}$ $\quad M(\Lambda_c^H(+)) = 2765\,\mathrm{MeV}$

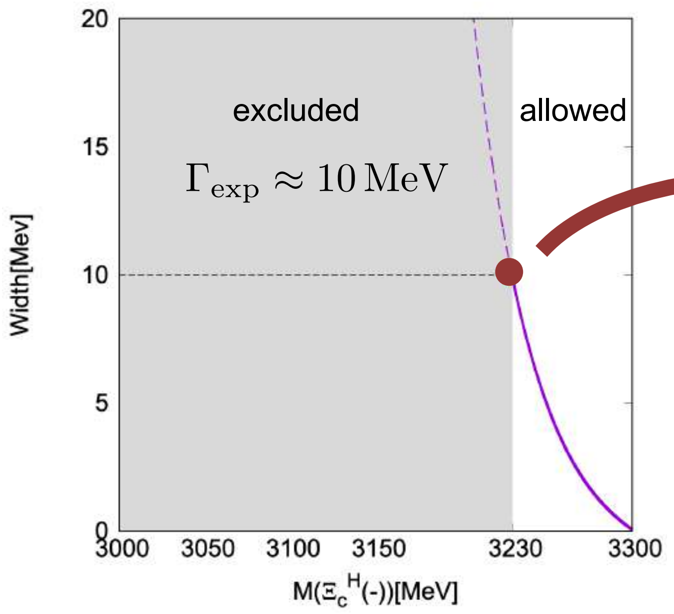
$M(\Xi_c^L(+)) = 2470\,\mathrm{MeV}$ $\quad M(\Xi_c^H(+)) = 2967\,\mathrm{MeV}$

$+$

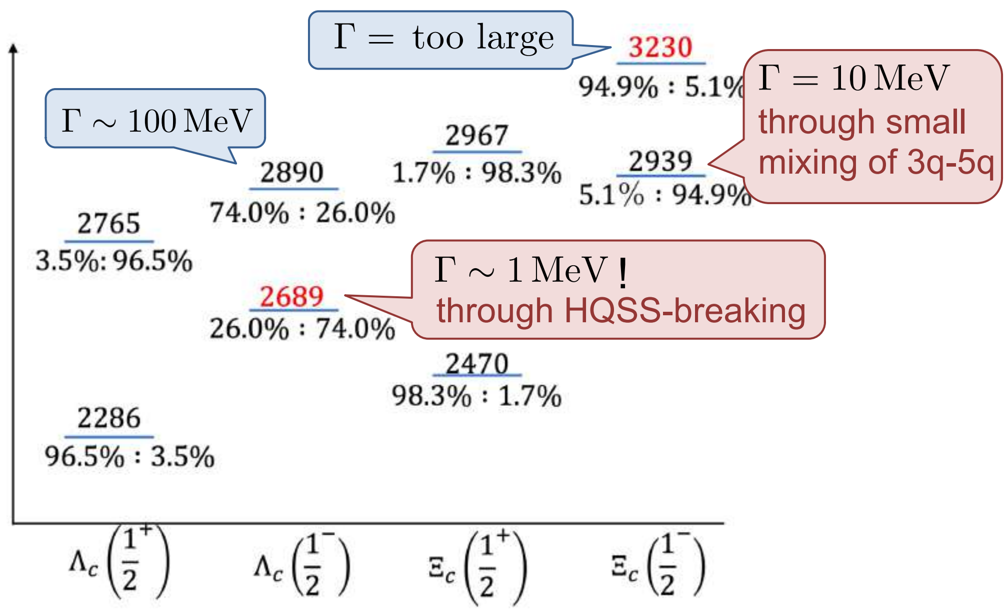
$M(\Lambda_c^H(-)) = 2890\,\mathrm{MeV}$ (quark model)

$M(\Xi_c^L(-)) = 2939\,\mathrm{MeV}$ ($\Xi_c(2930)$)

Width of $\Xi_c(2930)$ vs $M(\Xi_c^H(-))$



Mass spectrum $(Qqq : Qqq\bar{q}q)$



3. Analysis

• With mixing and anomaly

- For the case with anomaly effects, we can demonstrate allowed region of some parameters (♯ of effective parameters: 9)

input

$M(\Lambda_c^L(+)) = 2286\,\mathrm{MeV}$ $M(\Lambda_c^H(+)) = 2765\,\mathrm{MeV}$

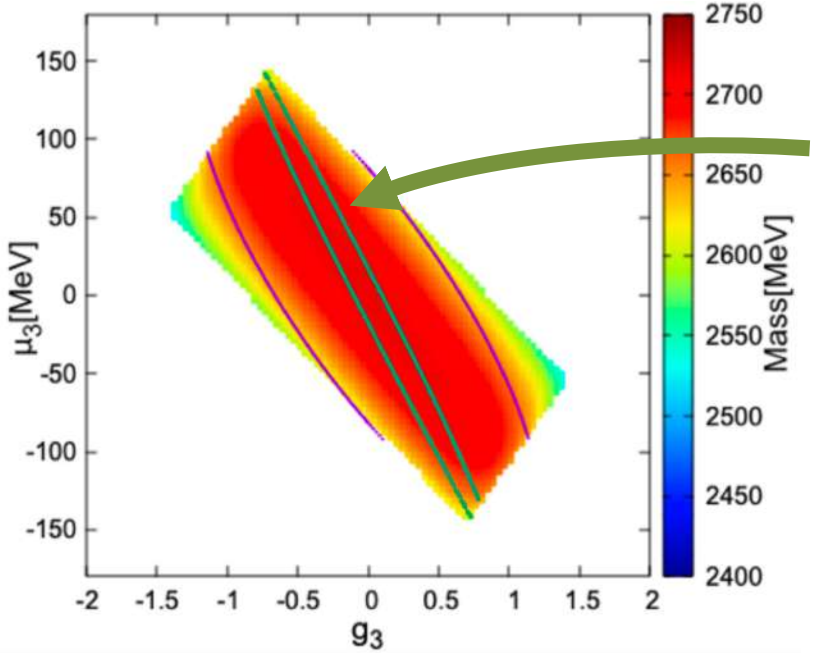
$M(\Xi_c^L(+)) = 2470\,\mathrm{MeV}$ $M(\Xi_c^H(+)) = 2967\,\mathrm{MeV}$

$+$

$M(\Xi_c^L(-)) = 2765\,\mathrm{MeV}$ (diquark model)

$M(\Lambda_c^H(-)) = 2890\,\mathrm{MeV}$ (quark model)

$M(\Xi_c^H(-)) = 2939\,\mathrm{MeV}$ ($\Xi_c(2930)$)



- Only this region is allowed by decays of

$$\begin{cases} \Gamma[\Xi_c(2930)] \approx 10\,\mathrm{MeV} \\ \Gamma[\Xi_c(2967)] \approx 20.9\,\mathrm{MeV} \end{cases}$$

- Constraint for $M(\Lambda_c^L(-))$ is also obtained

4. Conclusions

- We argued 3-quark and 5-quark picture for heavy-quark spin-singlet SHBs from chiral symmetry and anomaly

- Anomaly can lead to inverse mass hierarchy of 3-quark SHBs
- Anomaly does not affect mass of 5-quark SHBs

- When anomaly is absent, for instance we get the following mass spectrum