Study for the relation of chiral symmetry breaking and $\mathrm{U}(1)_\mathrm{A}$ anomaly in instanton liquid model

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Contents

Introduction

- **Motivation**

  Isoscalar, scalar $\sigma/f_0(500)$ state is one of our most interesting objects in hadron physics.

  - Mass generation of hadrons
  - A fluctuation of chiral order parameter (quark condensate)

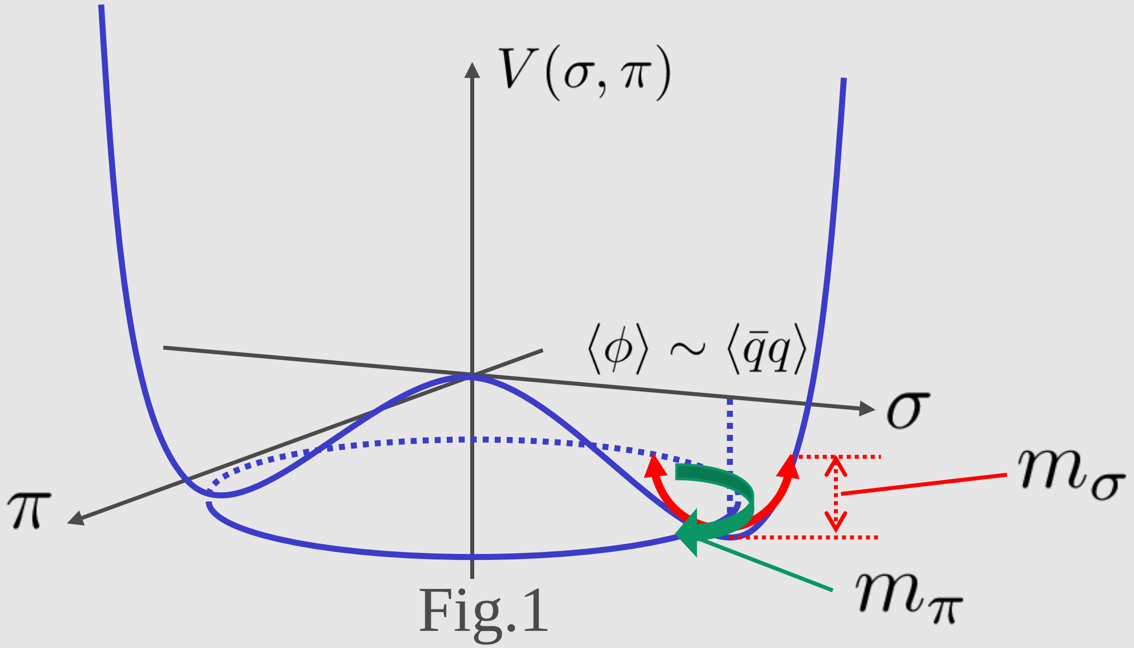

Fig.1

[1] R.L. Workman et al. (Particle Data Group), to be published in Prog. Theor. Exp. Phys. 2022, 083C01 (2022)

Introduction

- **Properties of $\sigma/f_0(500)$**
  - Small mass and large width[1] : $M_\sigma = (400\text{–}500) - i(200\text{–}350)$ MeV
    (cf. $M_{\rho(770)} = (754\text{–}764) - i(72\text{–}74)$ MeV)
  - Various candidates :
    - Chiral partner of π
    - $\bar{q}q$ composite state
    - $\pi\pi$ scattering
    - Glueball
    - $\bar{q}\bar{q}qq$ tetra quark state
    - Their mixture
  - Chiral symmetry of QCD ⇒ Existence of chiral partner of π

Recently some results are obtained along this context.

Meson masses in NJL model with anomaly [2]

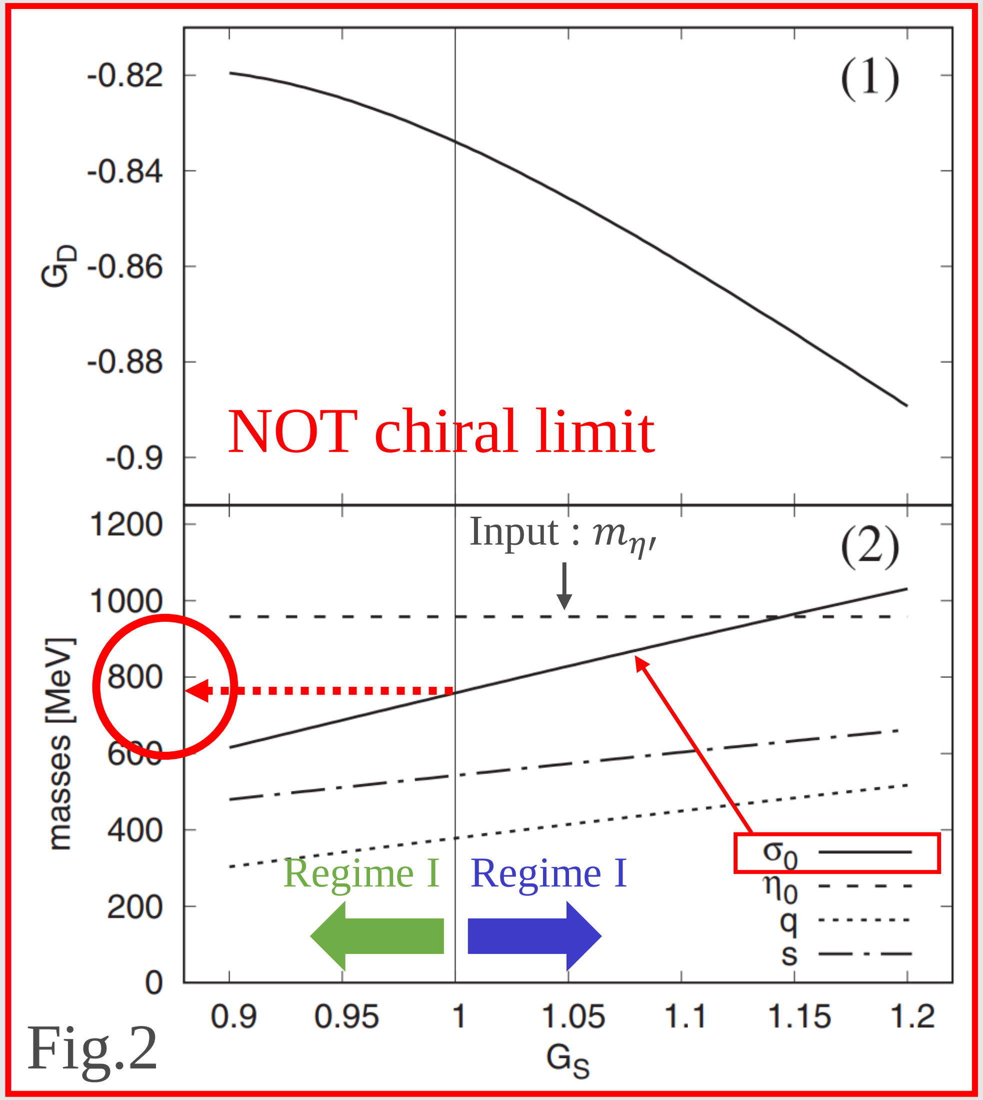

Fig.2

G_S : (Dimensionless) 4-quark coupling $\Rightarrow$ $\bar{q}q$-attraction

G_D : (Dimensionless) anomaly coupling $\Rightarrow$ anomaly strength

1. Regime I : $m_\sigma > 800$ MeV

   Regime II : $m_\sigma < 800$ MeV

   $\Rightarrow$ Next slide : definition of each regime

$\Rightarrow$ Quantitative evaluation

Cited from : S.Kono, et al. [2]

[2] S. Kono, D. Jido, Y. Kuroda and M. Harada, PTEP (2021) 093D02.

χSSB regime in NJL model with anomaly [2]

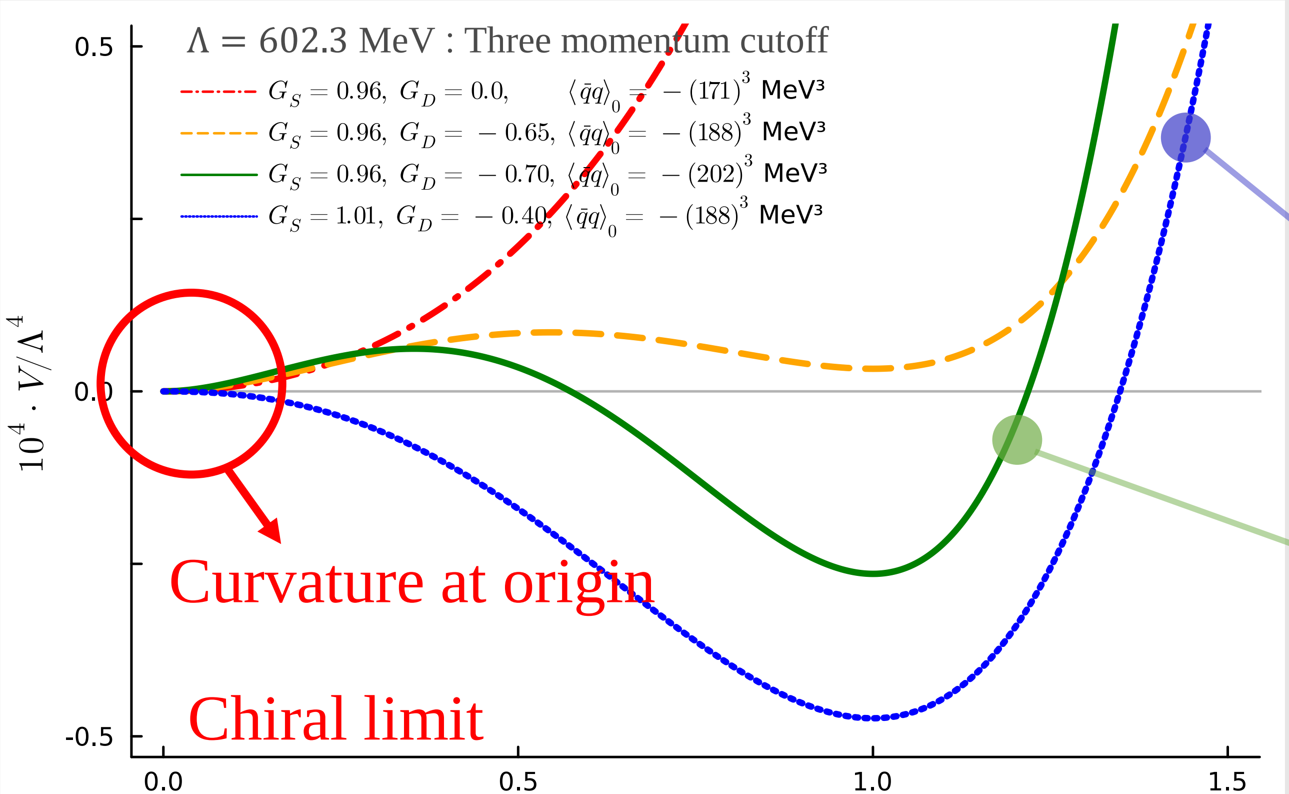

Fig.3

- - - Weak $\bar{q}$-q attraction & No anomaly

- - - Weak $\bar{q}$-q attraction & Weak anomaly

G_S : (Dimensionless) 4-quark coupling ⇒ $\bar{q}q$-attraction

G_D : (Dimensionless) anomaly coupling ⇒ anomaly strength

χSSB : **S**pontaneous **B**reaking of **Chi**ral **S**ymmetry

Regime I : Normal χSSB

$\equiv G_S > 1$ and $\langle\bar{q}q\rangle \neq 0$ in vacuum

(Strong $\bar{q}$-q attraction)

Regime II : Anomaly driven χSSB

$\equiv G_S < 1$ but $\langle\bar{q}q\rangle \neq 0$ in vacuum

(Weak $\bar{q}$-q attraction & Strong anomaly)

2. Even if $\bar{q}$-q attraction is weak,
   χSSB would be achieved by anomaly.

Correspondence in the chiral effective model

- Expected correspondences

m_σ
- large m_σ $\Leftrightarrow$ large $\left.\frac{\partial^2 V}{\partial \langle \bar{q}q \rangle^2}\right|_{\text{vacuum}}$
- small m_σ $\Leftrightarrow$ small $\left.\frac{\partial^2 V}{\partial \langle \bar{q}q \rangle^2}\right|_{\text{vacuum}}$

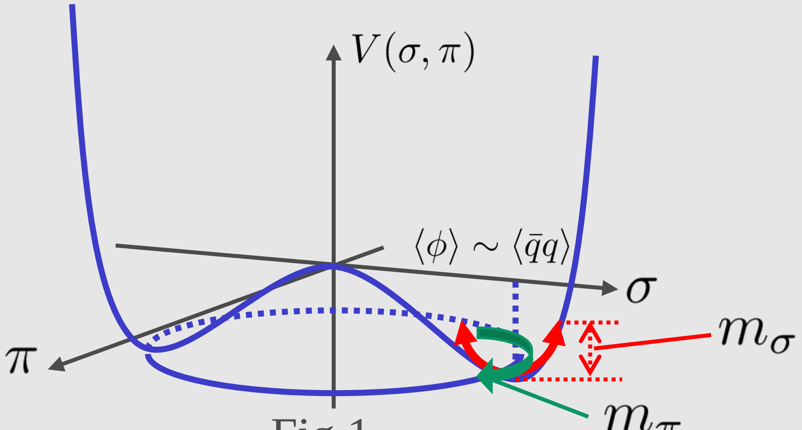

Fig.1

(also examine by direct measurement of m_σ)

χSSB regime
- normal $\Leftrightarrow$ negative $\left.\frac{\partial^2 V}{\partial \langle \bar{q}q \rangle^2}\right|_{\text{origin}}$
- anomaly driven $\Leftrightarrow$ positive $\left.\frac{\partial^2 V}{\partial \langle \bar{q}q \rangle^2}\right|_{\text{origin}}$

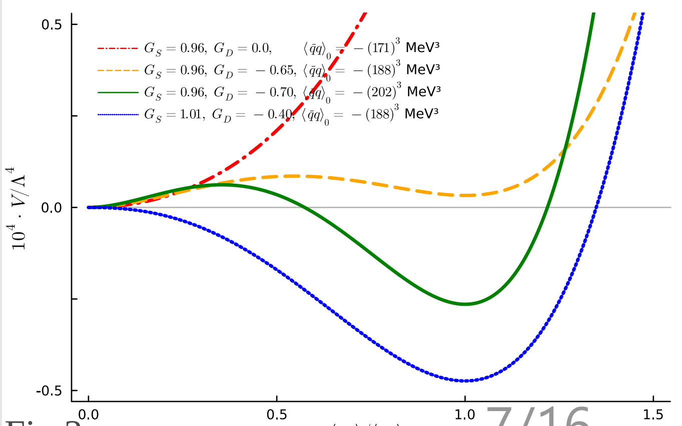

Fig.3

Our goals

[3] E. Shuryak, Nucl. Phys. **B 302** (1988), 559, 574, 599.

Q. How is a previous result interpreted in term of ***instanton*** vacuum?

- Anomaly driven regime : $m_\sigma < 800\ \mathrm{MeV}$
- Anomaly driven χSSB

To examine that, we compute the following in ***interacting instanton liquid model*** [3].

- $\langle \bar{q}q \rangle$-dependence of free energy density (corresponding to the effective potential)
- Its derivative (slope and curvature) at origin and vacuum

[3] E. Shuryak, Nucl. Phys. **B 302** (1988), 559, 574, 599.

Model : Interacting instanton liquid model [3]

- Interacting instanton liquid model (IILM) is given by the Euclidean partition function Z [3]:

Instanton interaction

Fermionic interaction

$$Z = \sum_{N_+, N_-} \frac{1}{N_+! N_-!} \int \prod_i^{N_+ + N_-} [d\Omega_i d(\rho_i)] \exp(-S_{\rm int}) \prod_f^{N_f} \det(\hat{D} + m_f)$$

Instanton distribution function

Measure of the path integral

$N = N_+ + N_-$: Number of instantons and anti-instantons

$d\Omega_i = d\rho_i dU_i d^4 z_i$: Degree of freedom of i-th instanton, size, color orientation and position

$d(\rho)$: Semiclassical instanton distribution function [4]

$$d(\rho) = \frac{4.6 \exp(-1.86 N_c)}{\pi^2 (N_c - 1)!(N_c - 2)!} \rho^{-5} \left(\frac{8\pi^2}{g^2(\rho)} \right)^{2N_c} \exp\left(-\frac{8\pi^2}{g^2(\rho)} \right)$$

$S_{\rm int}$: Instanton interaction [5,6]

[4] G. 't Hooft, Phys. Rev. **D 14** (1976) 3432.
[5] E. Shuryak and J. Verbaarschot, Phys. Rev. **D 52** (1995) 295.
[6] A.V. Yung, Nucl. Phys. **B 297** (1987) 47.

Simulation detail and observables

- Simulation : Monte Carlo method according to the weight function $p(\{\Omega_i\}) \sim e^{-S_{\text{eff}}}$

$$S_{\text{eff}} = -\sum_{i=1}^{N} \log[d(\rho_i)] + S_{\text{int}} + \sum_{f=1}^{N_f} \log\left[\det(\hat{D} + m_f)\right]$$

  - Algorithm : Hybrid Monte Carlo (HMC)
  - **Gauge & Fermion : color SU(3) & Quenched (no quark in config.)**
  - Box volume : $(1.35\ \text{fm})^4 \leq V_4 \leq (5.50\ \text{fm})^4$
  - Num. of config. : $N_{\text{conf}} = 5000$
  - Num. of $\text{I} + \bar{\text{I}}$ (fixed) : $N = 16 + 16$

- Observables
  - Free energy density : $F = -\dfrac{1}{V_4} \log Z$
  - Quark condensate :

$$\langle \bar{q}q \rangle = \frac{\int \mathcal{D}\Omega\, \bar{q}(x)q(x)e^{-S_{\text{eff}}}}{\int \mathcal{D}\Omega e^{-S_{\text{eff}}}}$$

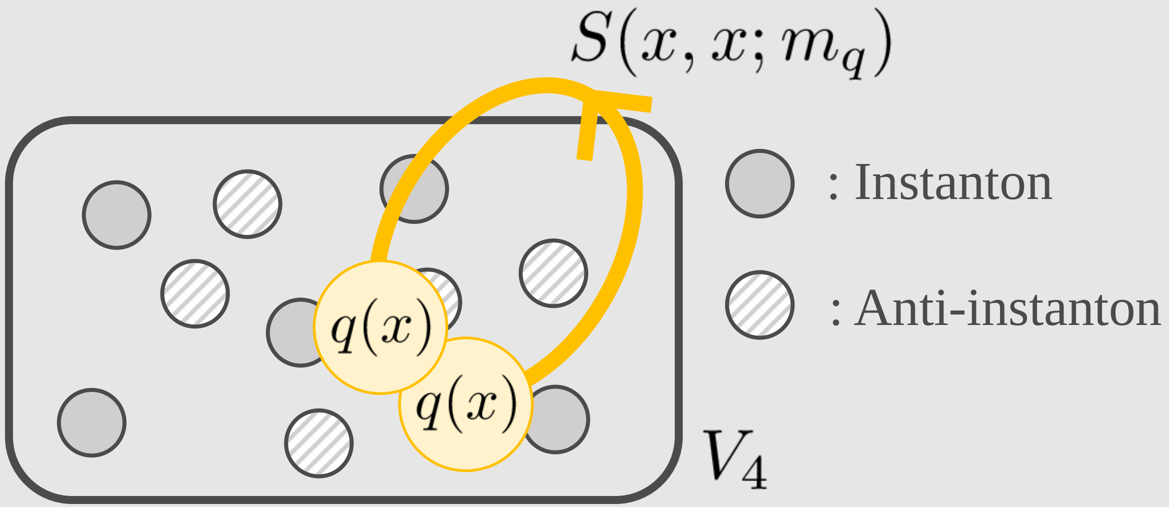

Fig.4 Image of computation of $\langle \bar{q}q \rangle$

Result : F vs instanton density in IILM

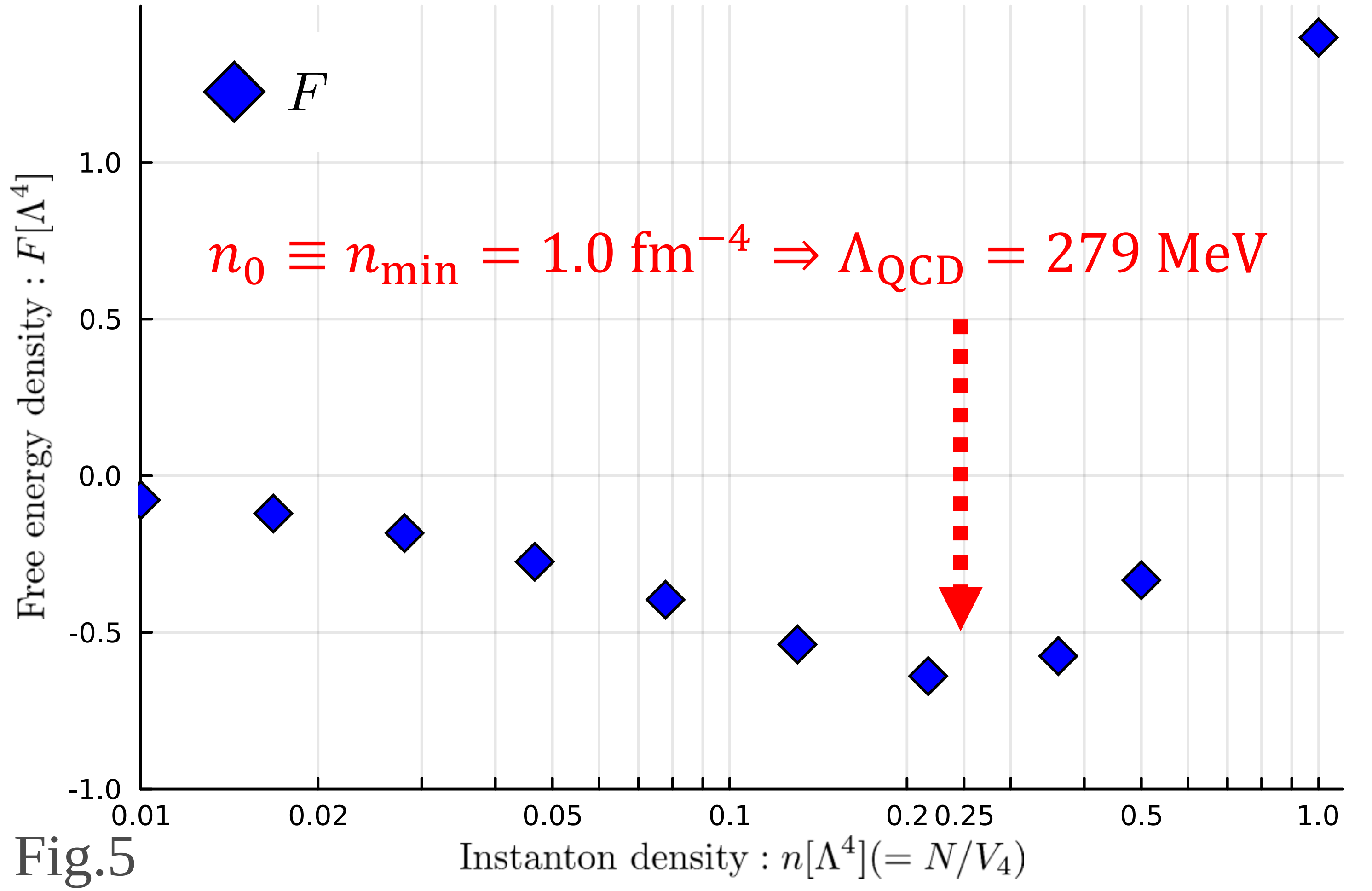

Fig.5

Result : F vs $\langle \bar{q}q \rangle$ in IILM

y-axis : Free energy density

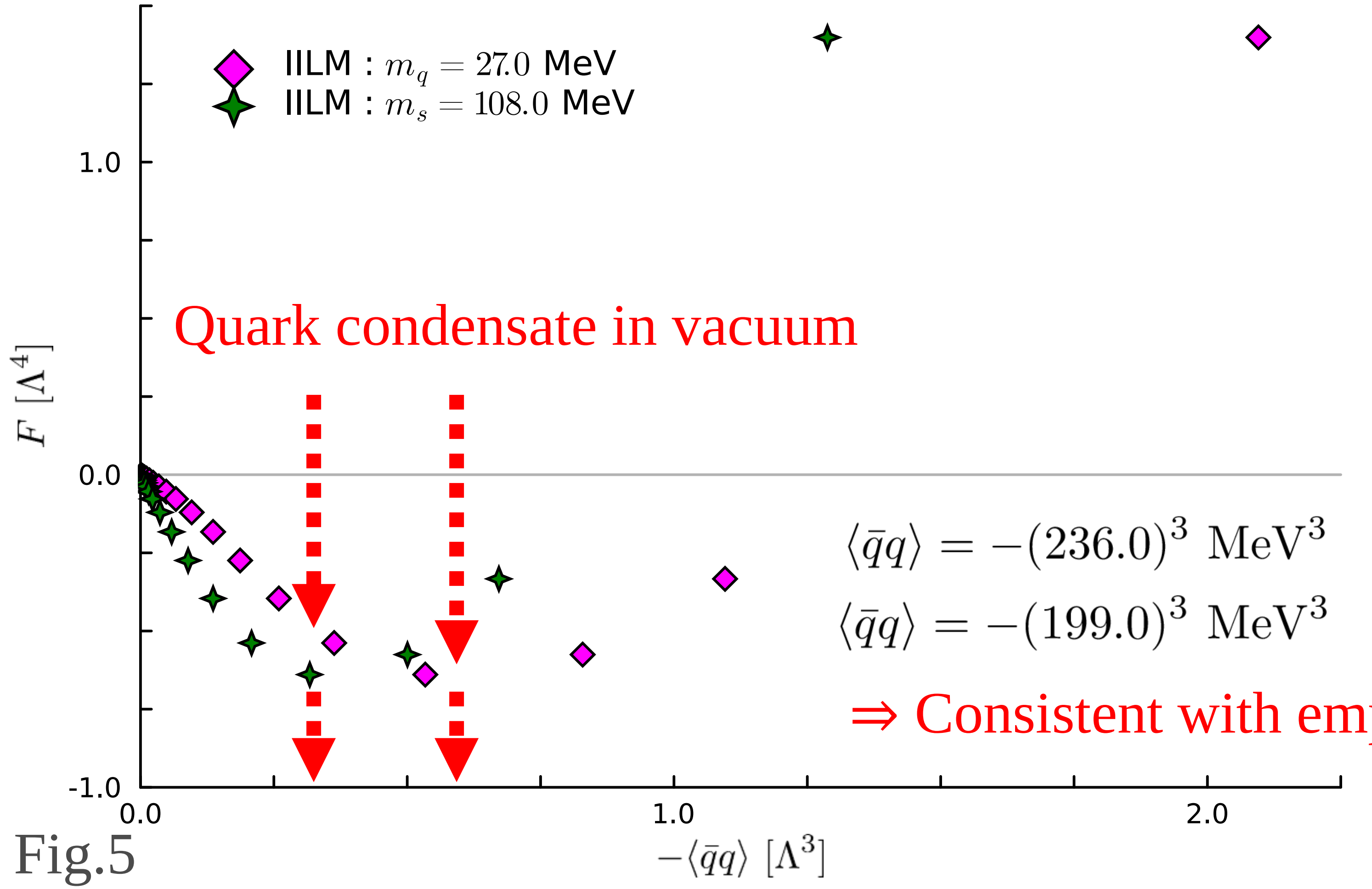

Fig.5

$$\langle \bar{q}q \rangle = -(236.0)^3\ \mathrm{MeV}^3 \quad (\text{for } m_q = 27\ \mathrm{MeV})$$

$$\langle \bar{q}q \rangle = -(199.0)^3\ \mathrm{MeV}^3 \quad (\text{for } m_s = 108\ \mathrm{MeV})$$

$\Rightarrow$ Consistent with empirical value

x-axis : quark condensate

Correspondence in the chiral effective model

- Expected correspondences

m_σ:
- large m_σ ⇔ large $\left.\frac{\partial^2 V}{\partial \langle \bar{q} q \rangle^2}\right|_{\text{vacuum}}$
- small m_σ ⇔ small $\left.\frac{\partial^2 V}{\partial \langle \bar{q} q \rangle^2}\right|_{\text{vacuum}}$

(also examine by direct measurement of m_σ)

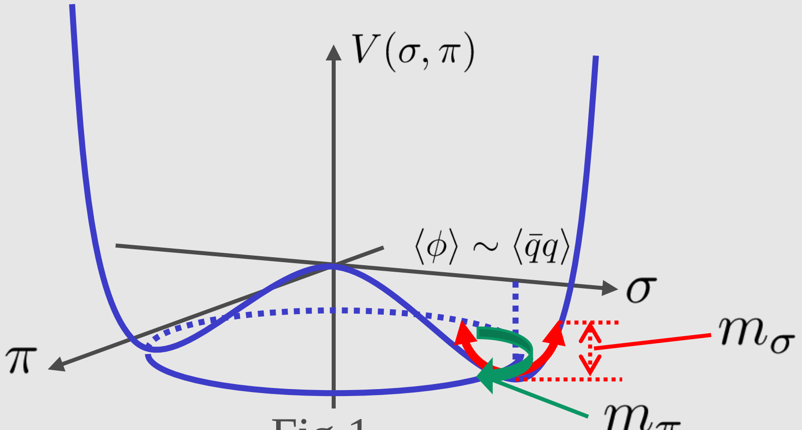

Fig.1

χSSB regime:
- normal ⇔ negative $\left.\frac{\partial^2 V}{\partial \langle \bar{q} q \rangle^2}\right|_{\text{origin}}$
- anomaly driven ⇔ positive $\left.\frac{\partial^2 V}{\partial \langle \bar{q} q \rangle^2}\right|_{\text{origin}}$

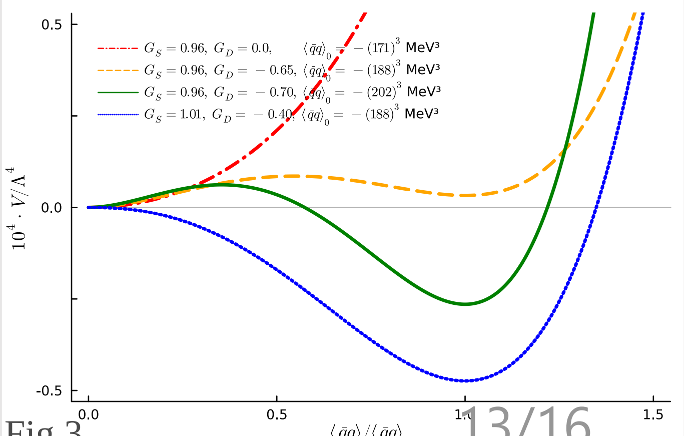

Fig.3

Result : Curvature & slope at origin in IILM

y-axis : Free energy density

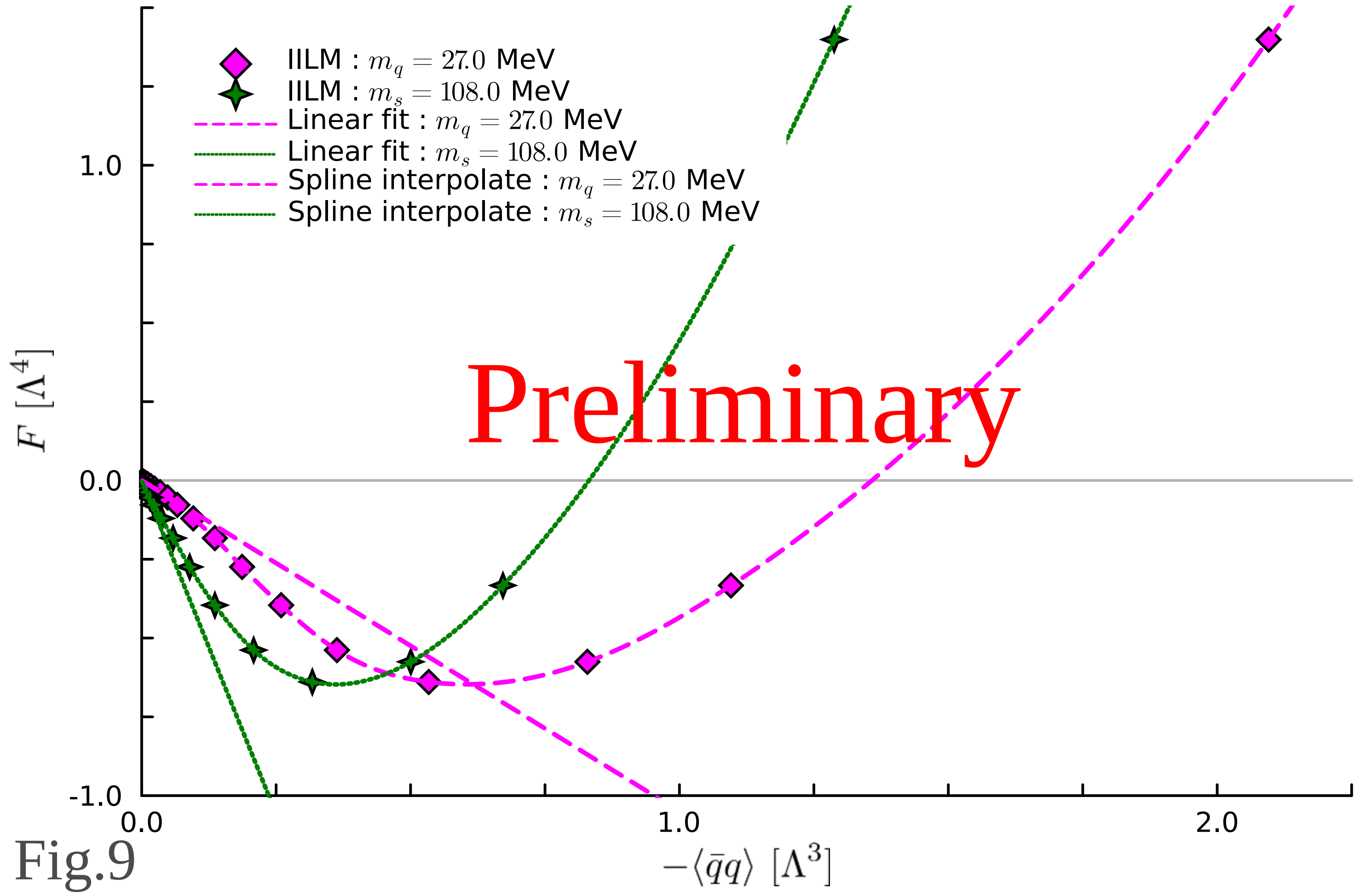

Fig.9

x-axis : quark condensate

Result : IILM & NJL model

Preliminary

- Slope and curvature in each model

$*$: Input value

Model		IILM		NJL					
m_q [MeV]		27.0*	108.0*	5.0*			15.0*		
NJL's parameter				(a)	(b)	(c)	(a)	(b)	(c)
$\partial V / \partial \langle \bar{q} q \rangle$ [MeV]	Origin	293	1182	5.00	5.00	5.00	15.00	15.00	15.00
	Vacuum	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$10^6 \cdot \partial^2 V / \partial \langle \bar{q} q \rangle^2$ [MeV^{-2}]	Origin	-	-	-0.281	0.911	0.726	-0.306	1.06	0.726
	Vacuum	3.79	10.4	7.47	4.07	2.36	11.4	8.07	5.14

(a) : $(G_S, G_D) = (1.01, -0.70)$, (b) : $(G_S, G_D) = (0.96, -0.70)$, (c) : $(G_S, G_D) = (0.96, 0)$

- In IILM, curvature at origin cannot be evaluated due to lack of the data (-).
- In IILM, slope at origin seems to be large compared to current quark masses.
- In IILM, curvature at vacuum is same order as NJL results, but we should be carefully.

Summary & Outlook

- **Summary**
  - The chiral effective models show that $\sigma/f_0(500)$ and the effective potential are indicators of compromise between χSSB and the $\mathrm{U}(1)_\mathrm{A}$ anomaly [Kono *et al*., 2021].

  - We compute $\langle\bar{q}q\rangle$-dependence of free energy density in IILM.
  - We also compute $\langle\bar{q}q\rangle$-dependence of effective potential in NJL model with anomaly.
  - Curvature and slope of $\langle\bar{q}q\rangle$-dependence of free energy density and effective potential in each model would give us some information.

- **Outlook**
  - Identify regime of χSSB in IILM and anomaly contribution by using curvature, slope and direct physical quantities (e.g. m_σ).
  - Compute m_σ in IILM
  - Compute unquench IILM