# Impacts of $U(1)_A$ anomaly on nuclear and neutron star equation of state based on a parity doublet model

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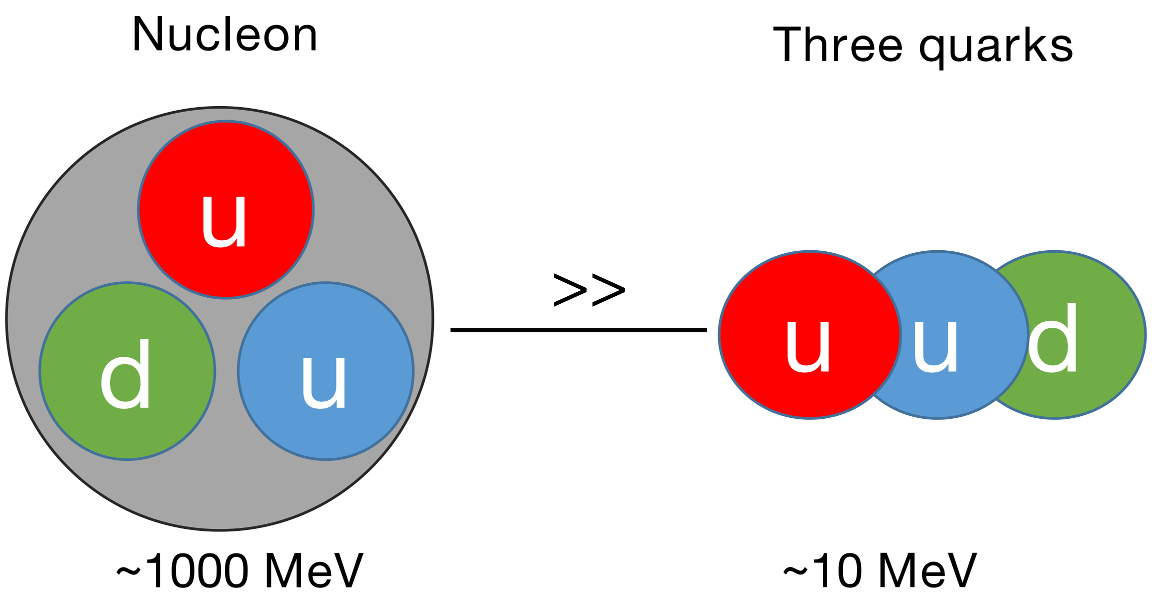
(PRC accepted)

# Introduction




Where is the origin of mass?

- The remained 99% of the nucleon mass is from the strong interaction between quarks.
- Since strong interaction is very large in the vacuum, non-perturbative effect is essential for the generation of mass.

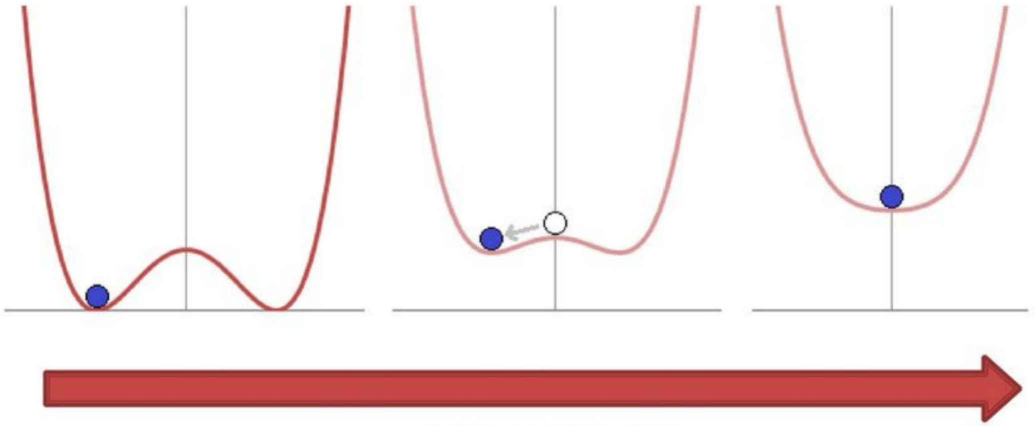
# Introduction

1. Contribution to mass from spontaneouly chiral symmetry breaking(SCSB)

order parameter
$\sigma \sim \langle \bar{q} q \rangle$

Chiral symmetry
broken phase

$\sigma \neq 0$



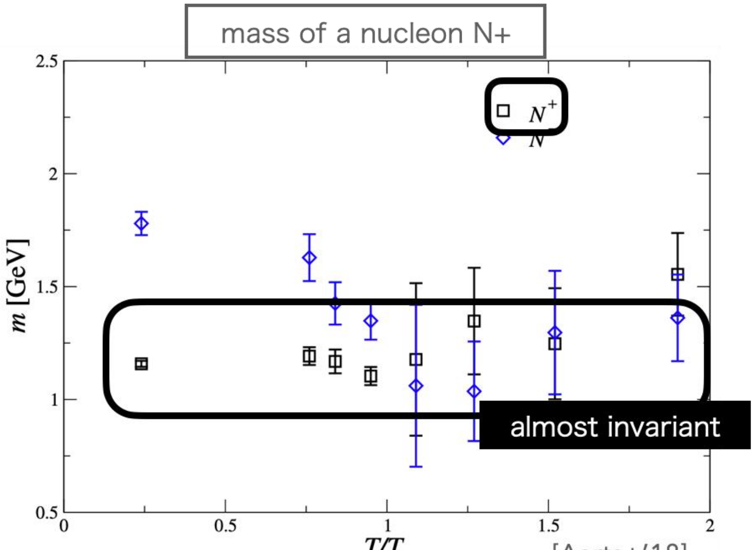
chiral symmetry
restored phase

$\sigma = 0$

density or temperature

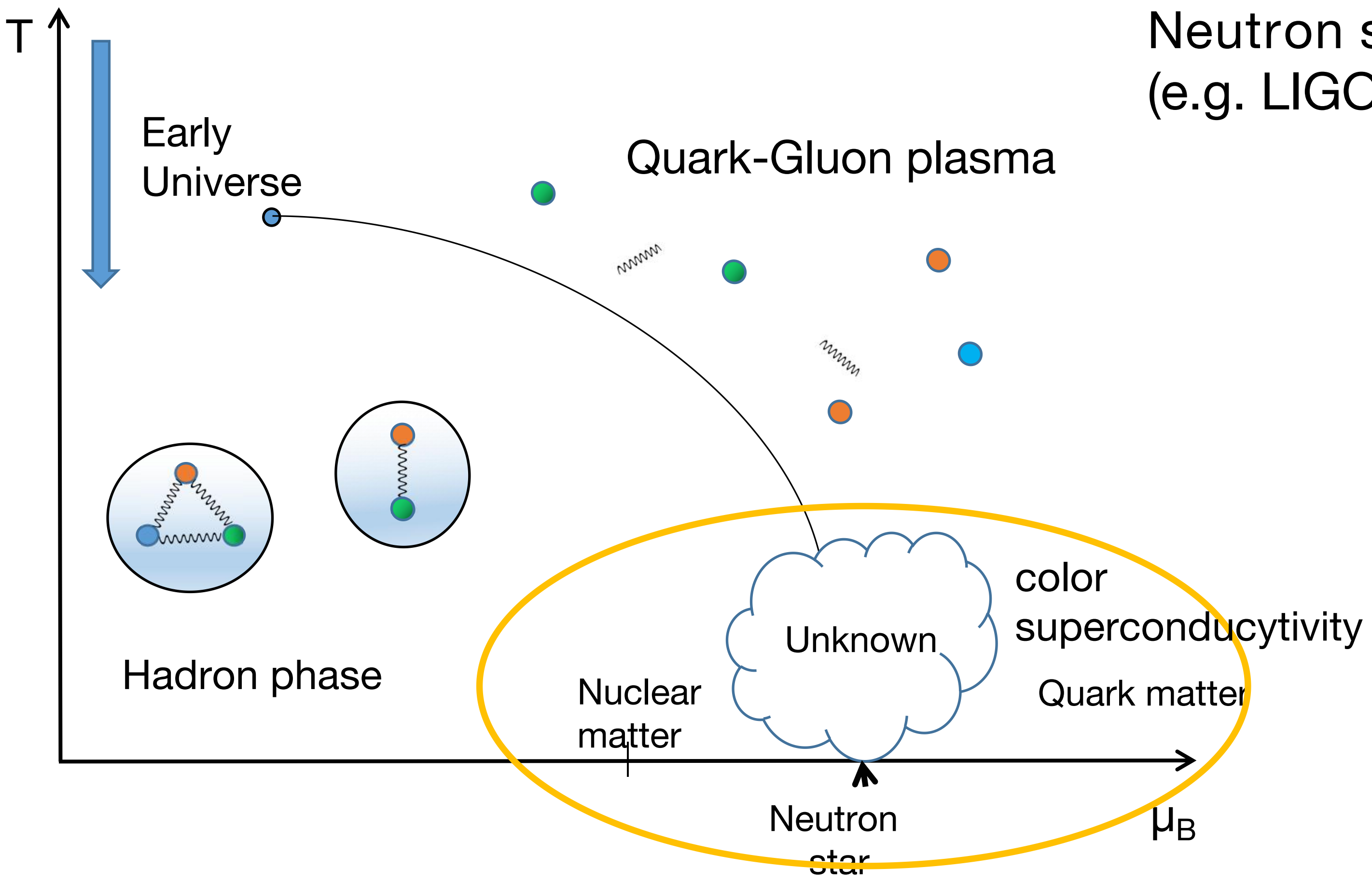
2. Contribution to mass from chiral invariant mass $m_0$

Indicated by the lattice QCD and does not depend on SCSB.[PhysRevD.92.014503]

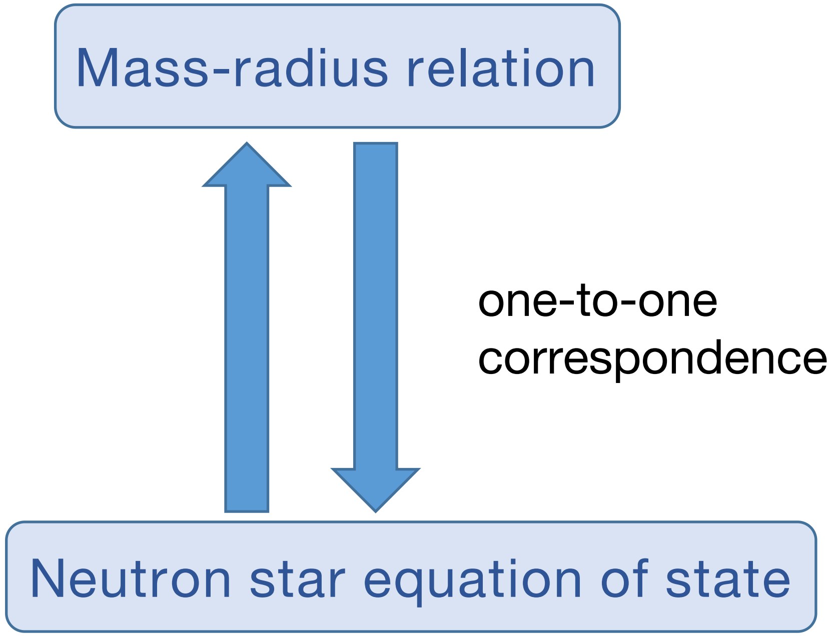



[Aarts+'18]

# Introduction



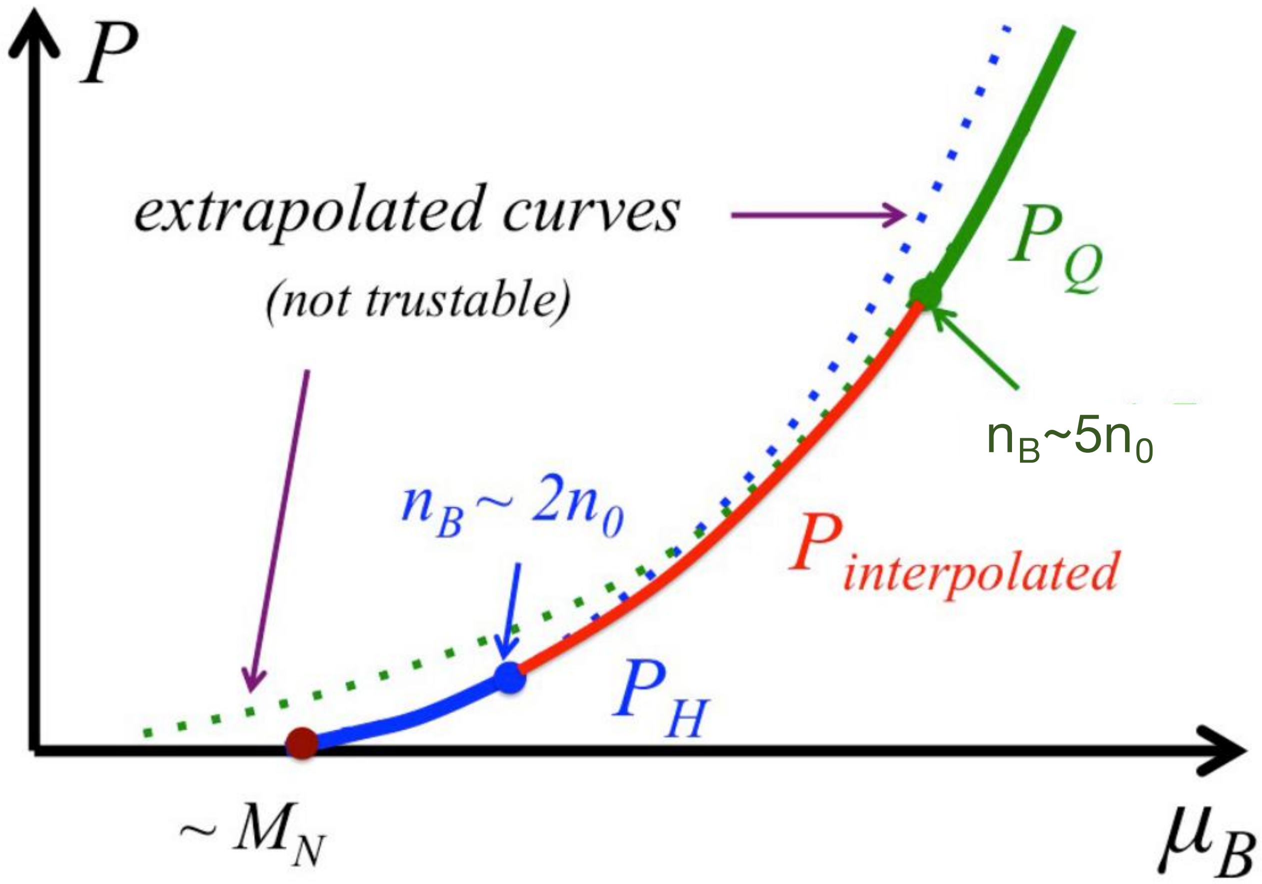

Neutron star as a natural laboratory
(e.g. LIGO&VERGO, NICER)




Chiral invariant mass

In NS, **$\sigma \to 0$, chiral invariant mass remains.**

# Introduction




$n_0$=0.16 $fm^{-3}$ (normal nuclear density)

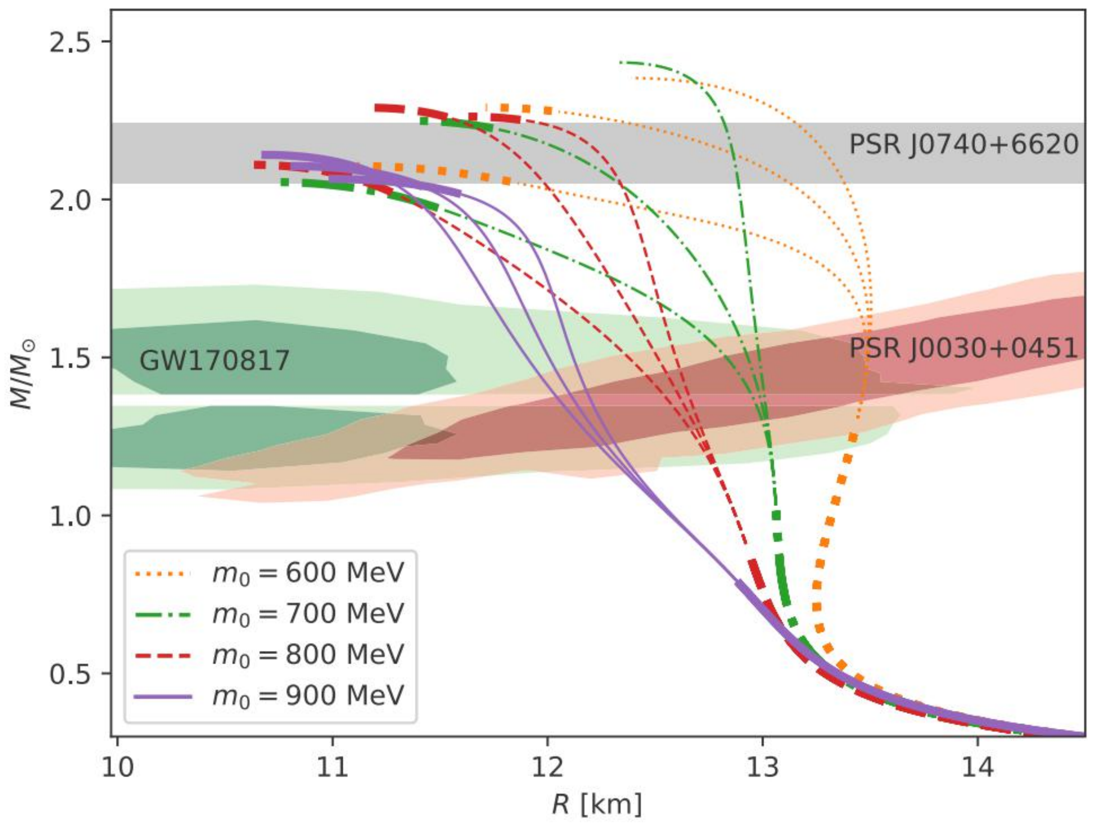
- An effective hadron model
  (Parity doublet model ) ($n_B$<=$2n_0$, blue curve)

Two baryons with positive and negative-parity are introduced. They have a degenerate chiral invariant mass when the chiral symmetry is restored.

- Interpolated(red curve)

- An effective quark model
  (Nambu–Jona-Lasinio(NJL)-type model)
  ($n_B$>=$5n_0$, green curve)

# Previous work




T.Minamikawa, et al.(2021), PhysRevC.103.045205

$$600 \lesssim m_0 \lesssim 900$$

After solving the Tolman-Oppenheimer-Volkov equation, we obtain mass-radius relation

The mass of the millisecond pulsar PSR J0740+6620

$$M_{\mathrm{TOV}}^{\mathrm{lowest}} = 2.14^{+0.10}_{-0.09}\, M_{\odot}$$

| | radius[km] | mass [$M_{\mathrm{solor}}$] |
|---|---|---|
| GW170817 (primary) | $10.8^{+2.0}_{-1.7}$ | $1.46^{+0.12}_{-0.10}$ |
| GW170817 (secondary) | $10.7^{+2.1}_{-1.5}$ | $1.27^{+0.09}_{-0.09}$ |
| J0030+0451 (NICER) | $13.02^{+1.24}_{-1.06}$ | $1.44^{+0.15}_{-0.14}$ |

Effect of strange quark condensate is included in the quark matter(NJL type model), but strange quark condensate not included in the hadron part.

# This work: Effect of anomaly

In this work, I modify the hadron model (PDM) to include the strange quark codensate through the Kobayashi-Maskawa-'t Hooft (KMT) - type interaction which reflects the U(1) axial anomaly

Non-perturbative effect of QCD

This work

- PDM

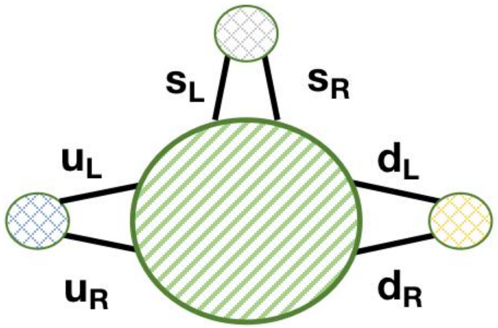
$$2B\sigma^2\sigma_s$$

Coefficient B

Anomaly term!

$$\sigma_s = \langle \bar{s}s \rangle$$

$$\sigma = \langle \bar{u}u \rangle$$




$$\sigma = \langle \bar{d}d \rangle$$

- NJL-type model

$$2K\sigma^2\sigma_s$$

Coefficient K

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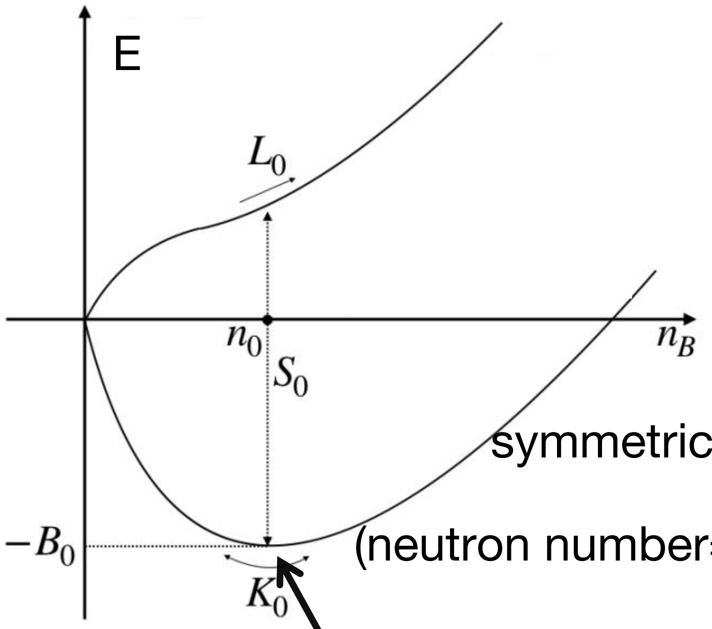
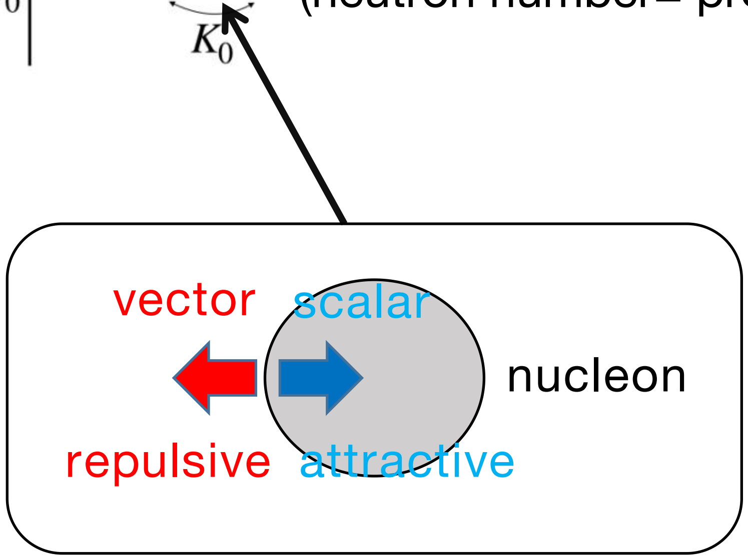
# Physical inputs



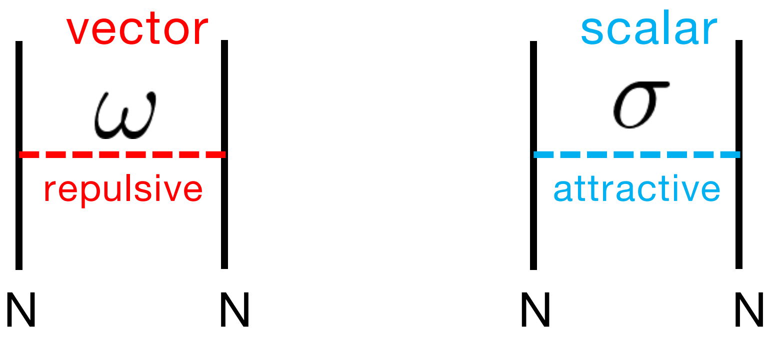

$n_0$=0.16 fm$^{-3}$ (normal nuclear density),

$K_0$=240 MeV (incompressibility)

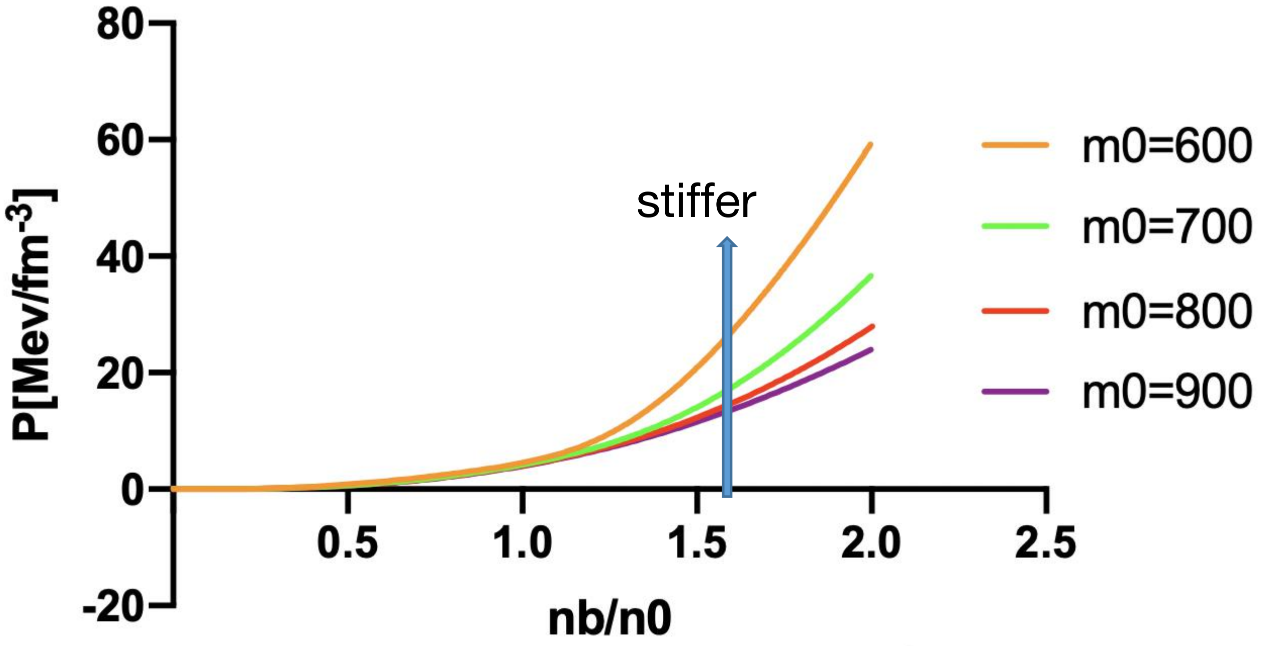
$S_0$=31 MeV (symmetry energy),

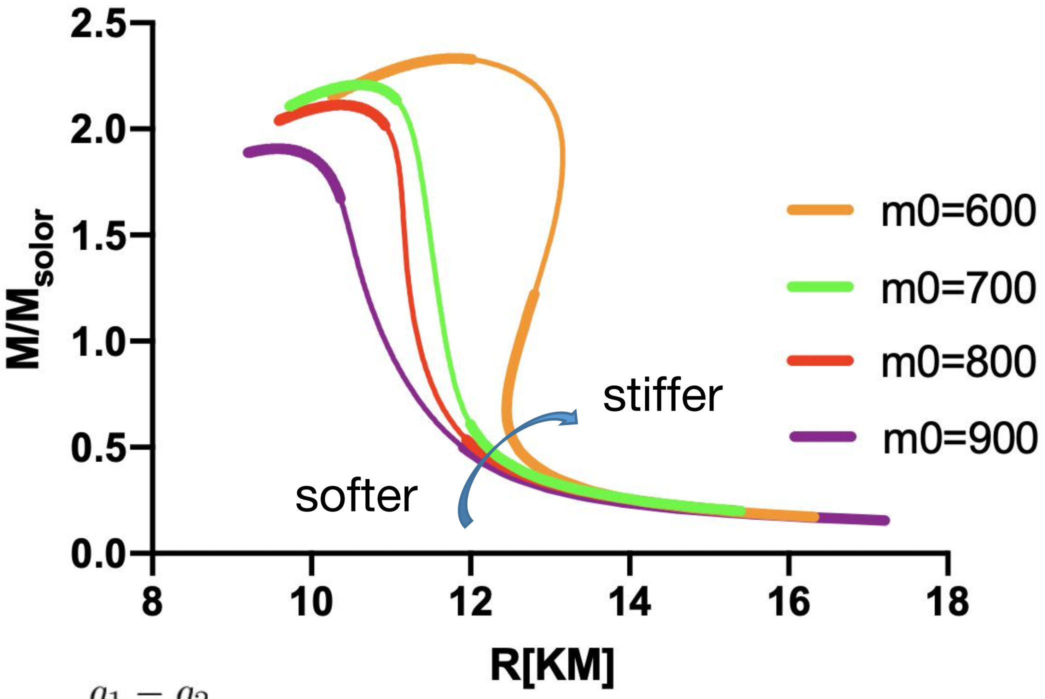
$B_0$=-16 MeV (bounding energy)

We determine model parameters through above inputs.



# Result without anomaly term



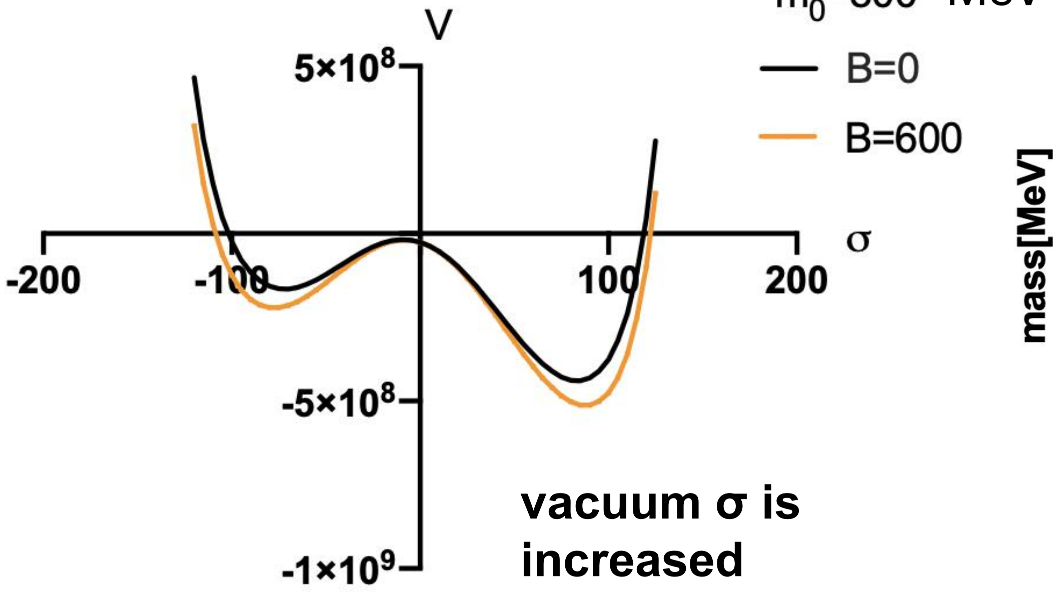




Mass fomula in PDM: $m_{\pm} = \sqrt{m_0^2 + \left(\frac{g_1+g_2}{2}\right)^2 \sigma^2} \mp \frac{g_1-g_2}{2}\sigma$

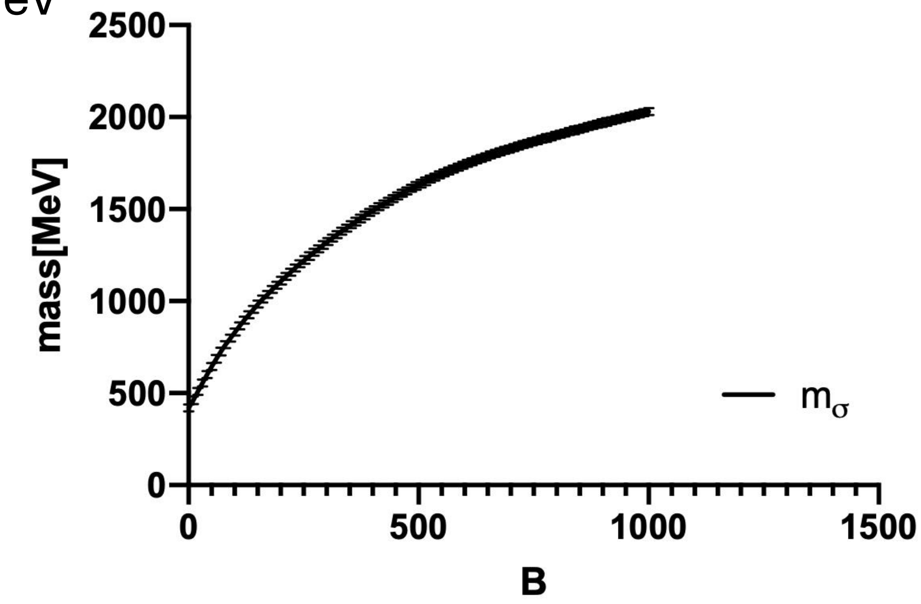
| $m_0$ | interaction | Attractive force | Repulsive force | EOS |
|---|---|---|---|---|
| small | strong | strong | strong | stiff |
| large | weak | weak | weak | soft |

# Effect of anomaly term in PDM

Vacuum potential




**vacuum σ is increased**




scalar potential between two nucleons

$$V_{\text{scalar}}(r) = -g^2 \frac{e^{-m_\sigma r}}{r}$$

$$r \approx \frac{1}{m_\sigma}$$

is the approximate range

B=600 is determined from $\eta, \eta'$ mass

| | $m_\sigma$ | $r_{eff}$ | $F_{attractive}$(at $n_0$) | $F_{repulsive}$(at $n_0$) | EOS($n_B$>$n_0$) |
|---|---|---|---|---|---|
| B=0 | small | large | large | large | stiffer |
| B=600 [MeV] | large | small | small | small | softer |

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# NJL-type model

- Original NJL-type model(Hatsuda and Kunihiro) includes four point interaction

$$+G(\bar{\psi}\psi)^2$$

HK parameter: $G\Lambda^2 = 1.835, \quad K\Lambda^5 = 9.29$

$\Lambda = 631.4\mathrm{MeV}$

- U(1) axial anomaly

$$-K\det(\bar{\psi}\psi)$$

- Vector interaction for repulsive force[Rept.Prog.Phys. 81 (2018) 5, 056902]

$$-g_V(\bar{\psi}\gamma_\mu\psi)^2$$

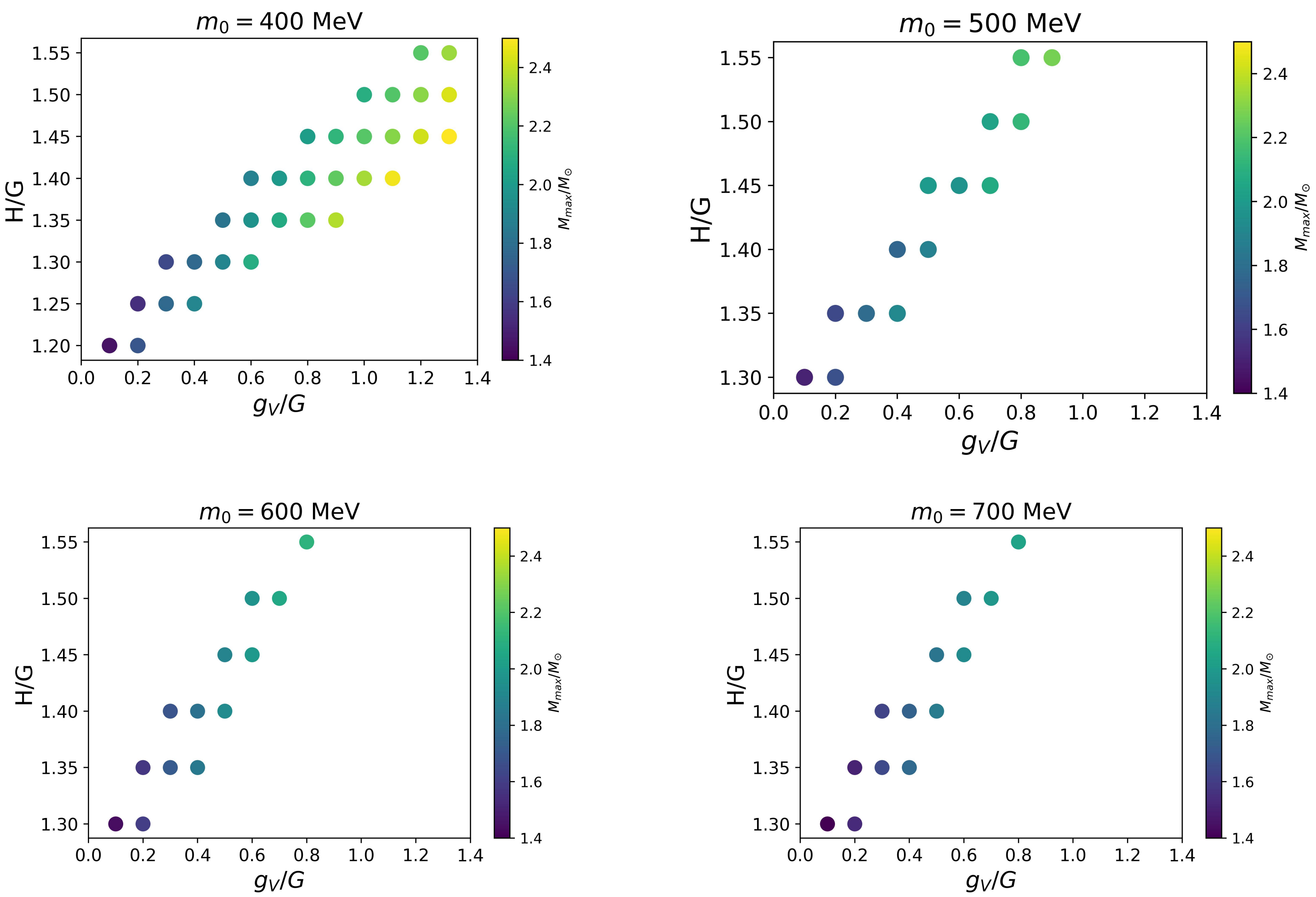
- Four-Fermi interaction lead to color superconductivity

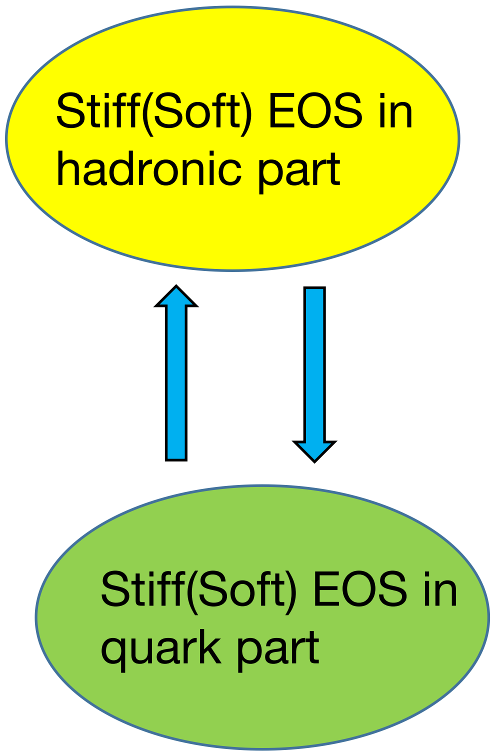
$$H(\bar{\psi}\bar{\psi})(\psi\psi)$$

- $g_V$ and H are two parameters to be adjusted

In NJL-type model, EOS also becomes softer after including anomaly effect

# Effect of anomaly in NJL-type model



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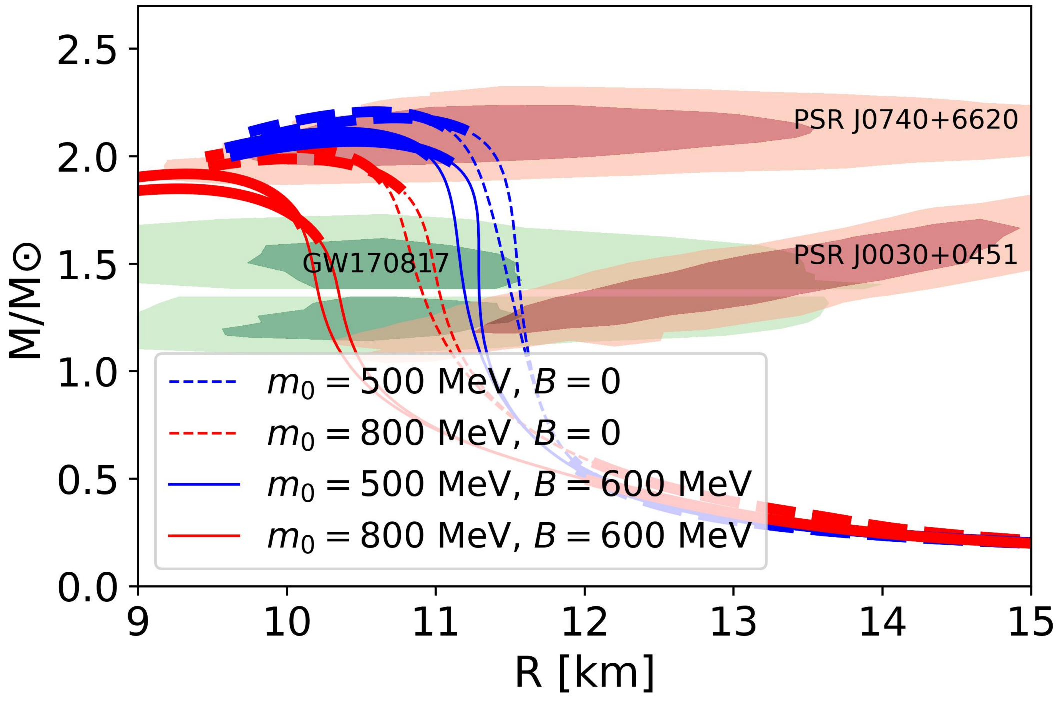
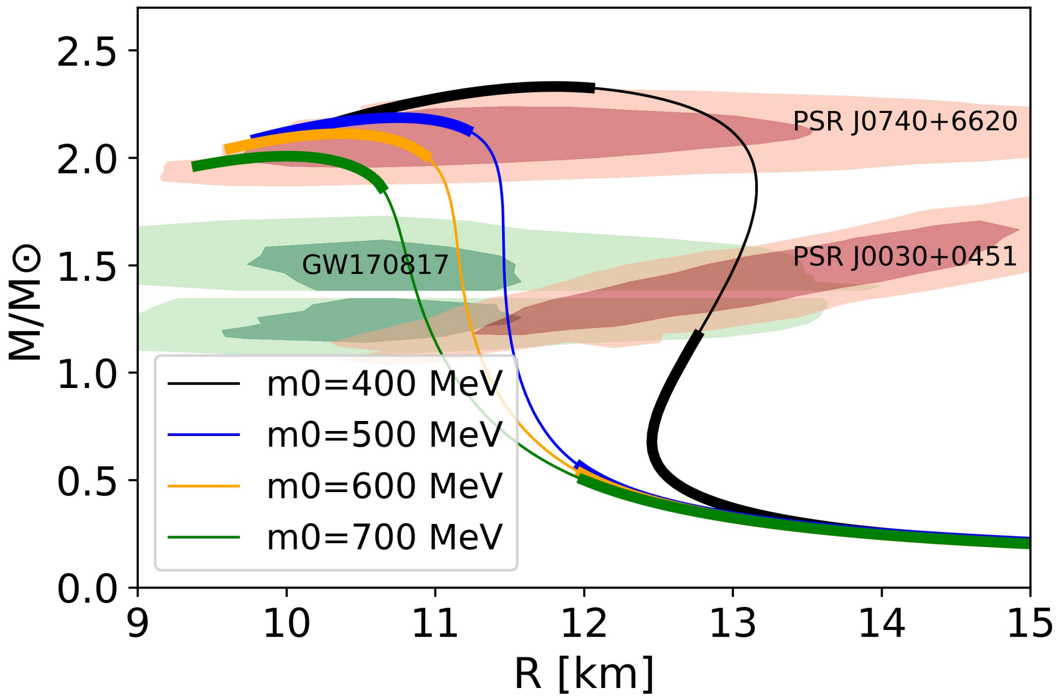
# Constrain chiral invariant mass




Maximum mass is not satisfied for $m_0$=800 MeV after including anomaly.

After including anomaly effect, chiral invariant mass is constrained to be

$$400 \lesssim m_0 \lesssim 700 \text{ MeV}$$

# Summary

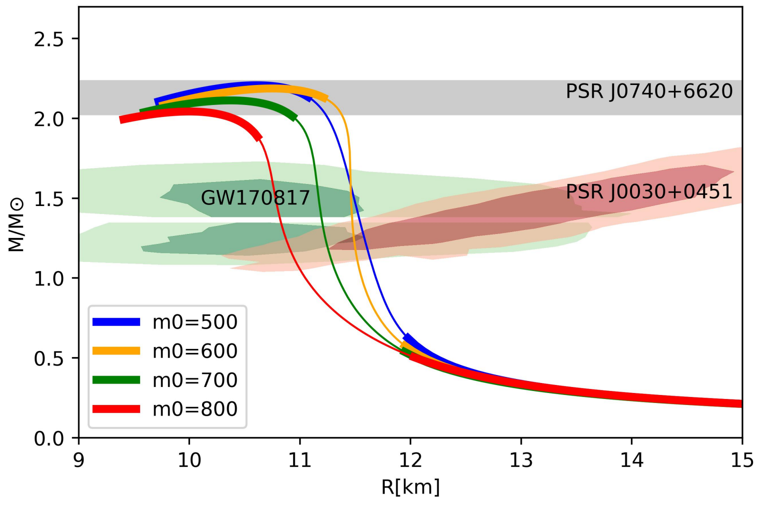
- I constructed a new model including strange quark condensate through the anomaly term.
- I studied the effect of anomaly term.

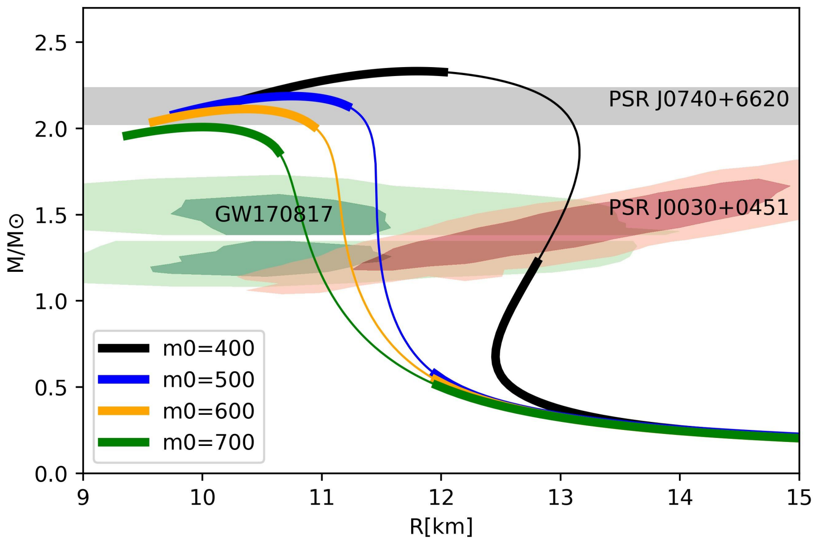
  In the PDM, effect of anomaly term softens the neutron star EOS.

- I obtained new constraints to the chiral invariant mass

1.B=0 and K=9.29/Λ[5] : $500 \lesssim m_0 \lesssim 800$

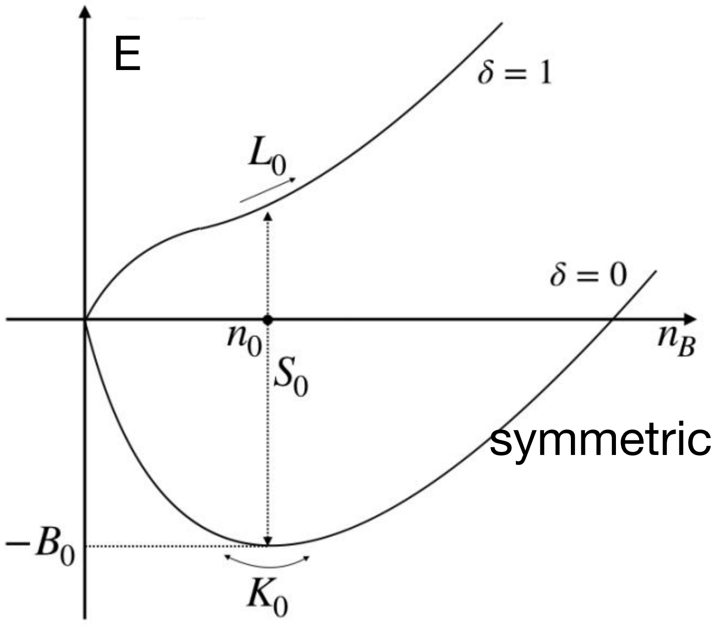
2. B=600 and K=9.29/Λ[5] $400 \lesssim m_0 \lesssim 700$



back up

# Physical inputs




$n_0$=0.16 $fm^{-3}$ (normal nuclear density),

$K_0$=240 MeV (incompressibility)

$S_0$=31 MeV (symmetry energy),

$B_0$=-16 MeV (bounding energy)

We determine all the parameters through saturation properties.

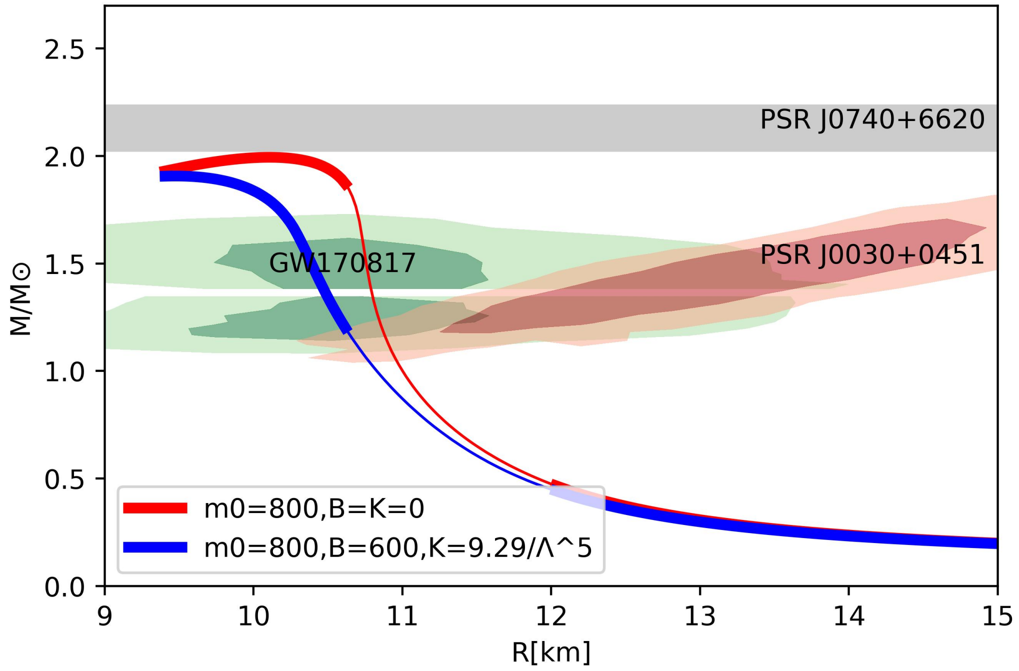
| $m_0$[MeV] | 500 | 600 | 700 | 800 | 900 |
|---|---|---|---|---|---|
| $L_0$ [MeV] | 84.7 | 83.4 | 81.6 | 80.9 | 79.9 |

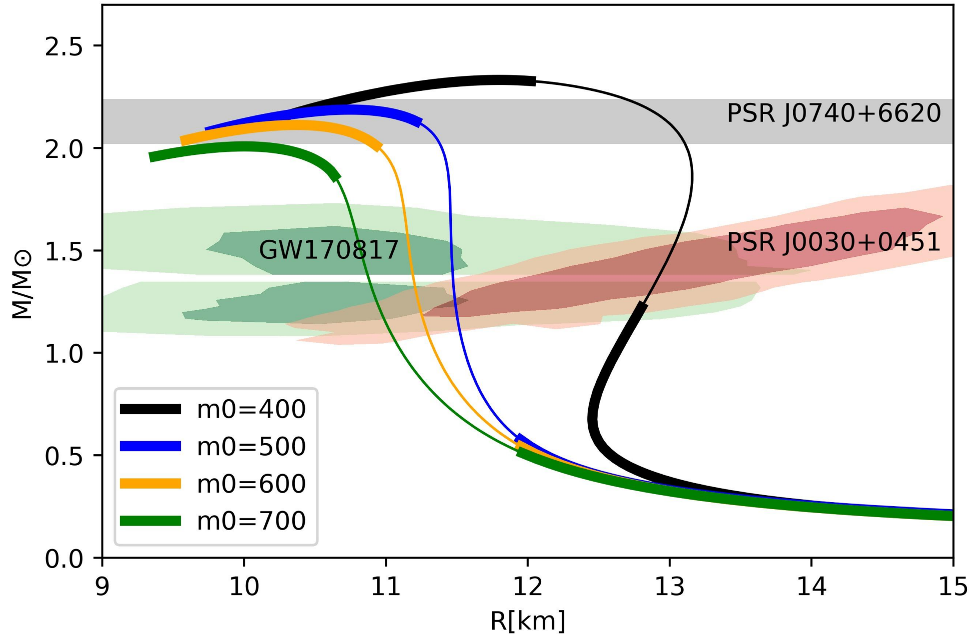
Slope parameter too large!

$\Lambda_{\omega\rho}\omega^2\rho^2$ term is introduced to lower the slope parameter

$L_0$=57.7 MeV [arXiv:2105.04629]

# Constrain chiral invariant mass

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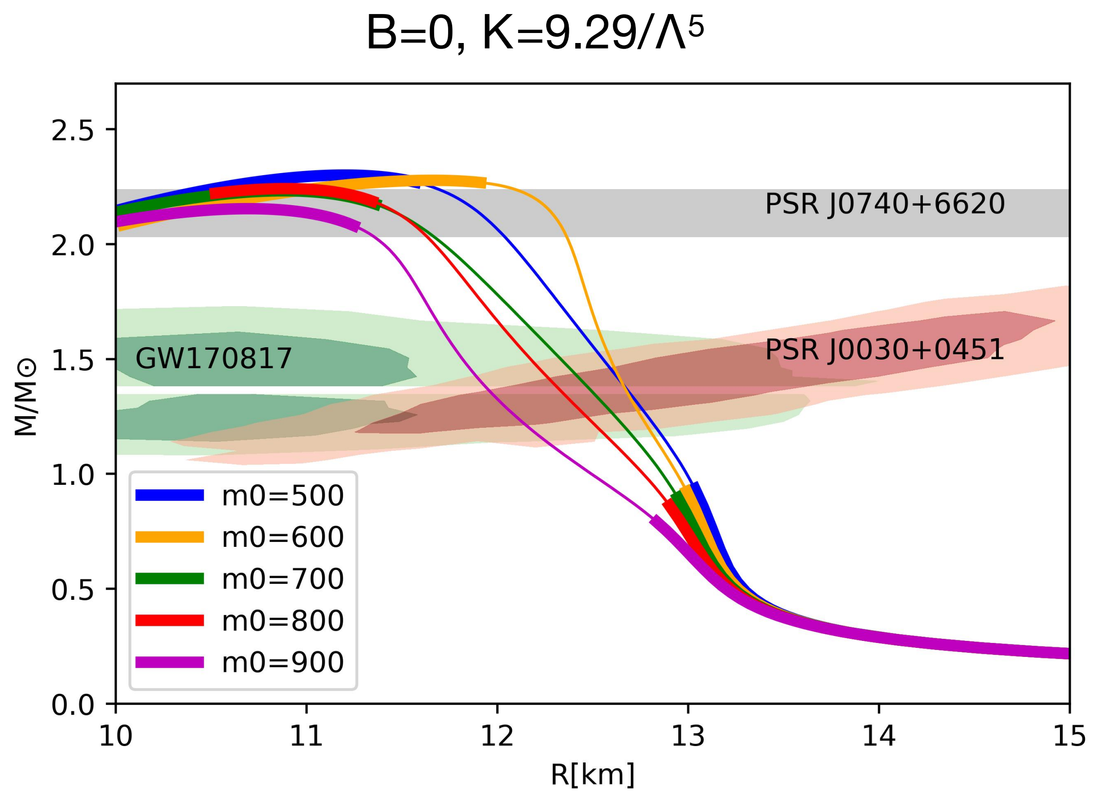
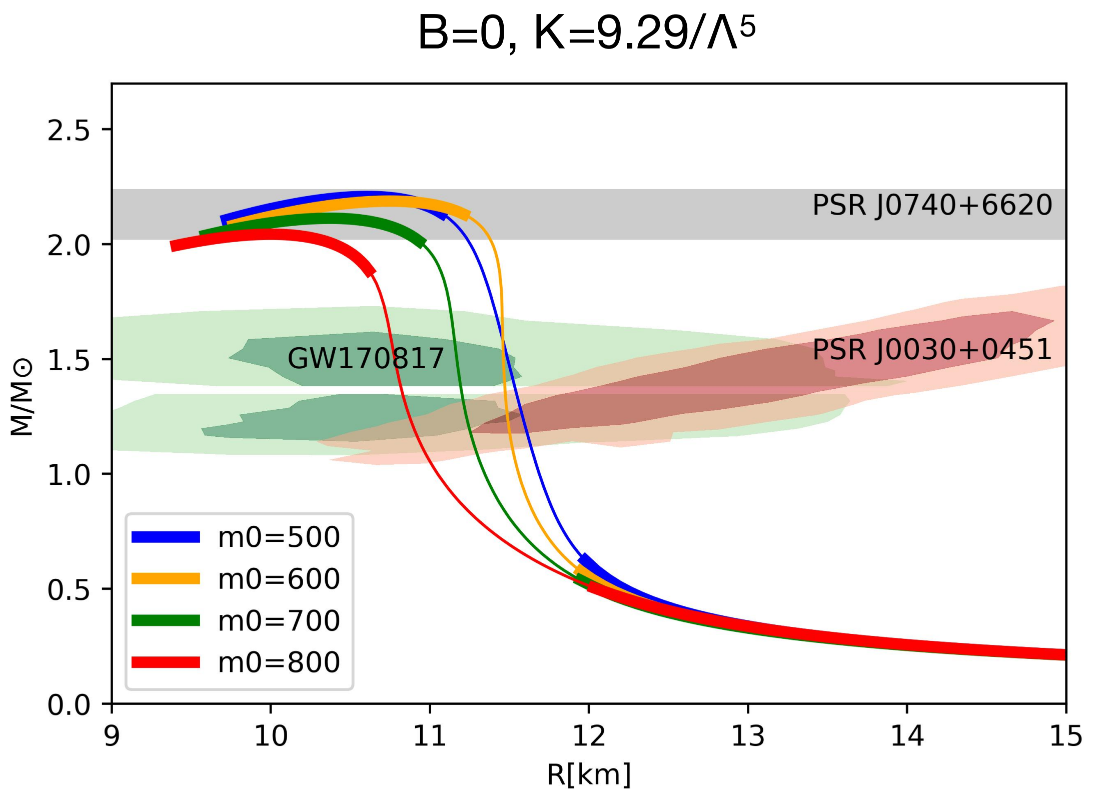
# Constrain chiral invariant mass




Small $L_0$ lead to softer M-R relation

The allowed range of $m_0$ is changed

# Summary

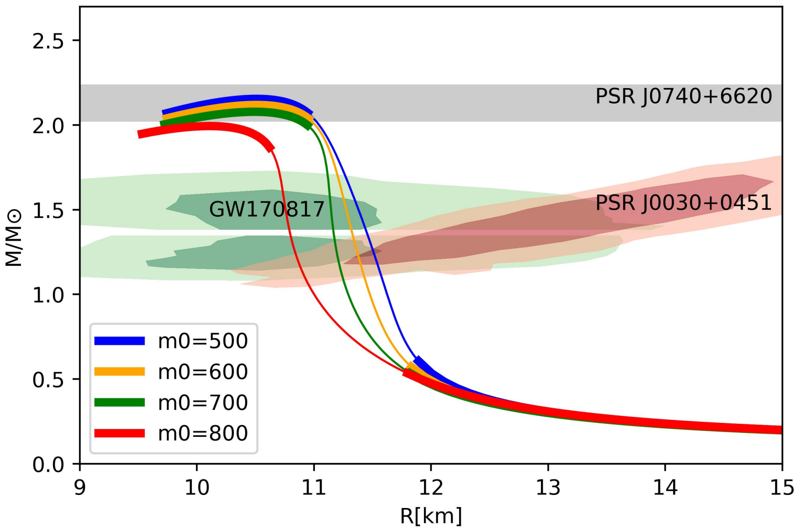
- I constructed a new model including strange quark condensate.
- I also clarified the effect of anomaly term.

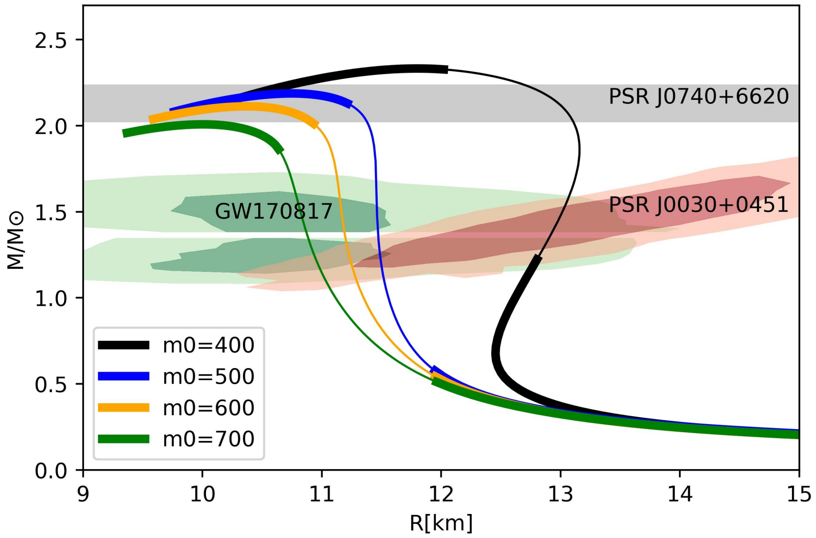
  In both PDM model and NJL-type model, anomaly effect softens the neutron star EOS.

- I constrained chiral invariant mass with anomaly effect

Without anomaly: $500 \lesssim m_0 \lesssim 800$

With anomaly: $400 \lesssim m_0 \lesssim 700$

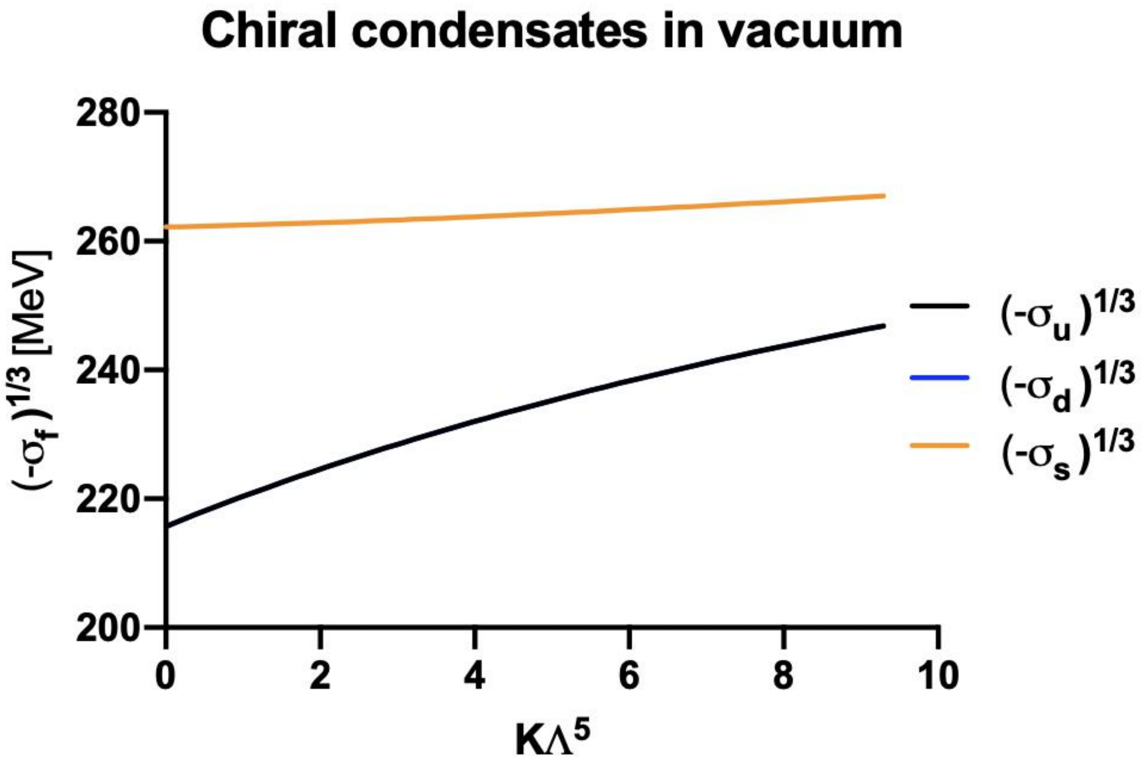


# Effect of anomaly in NJL-type model

In NJL-type model, H and $g_v$ are two parameters which should be adjusted

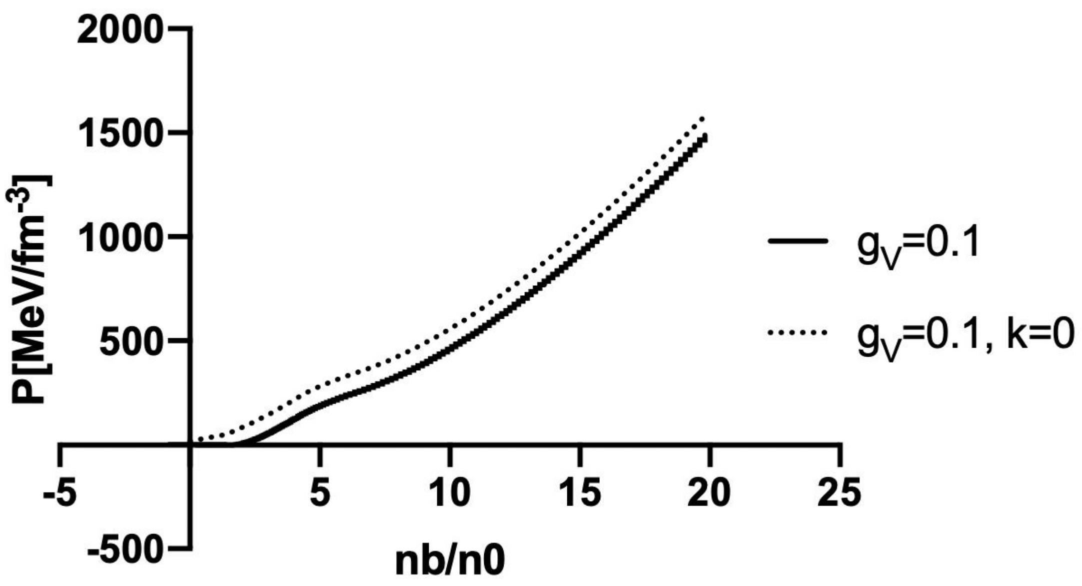
- chiral condensates

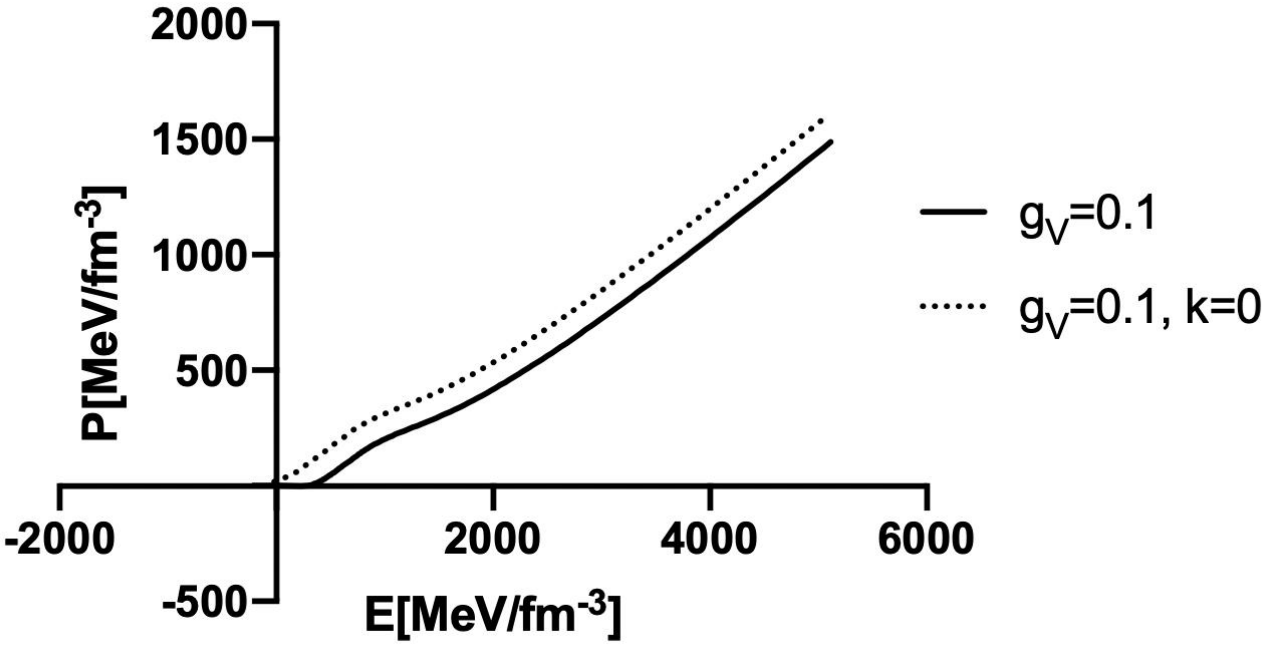



- Chiral symmetry breaking is enhanced with anomaly effect
- Ground state energy is decreased

# Effect of anomaly in NJL-type model

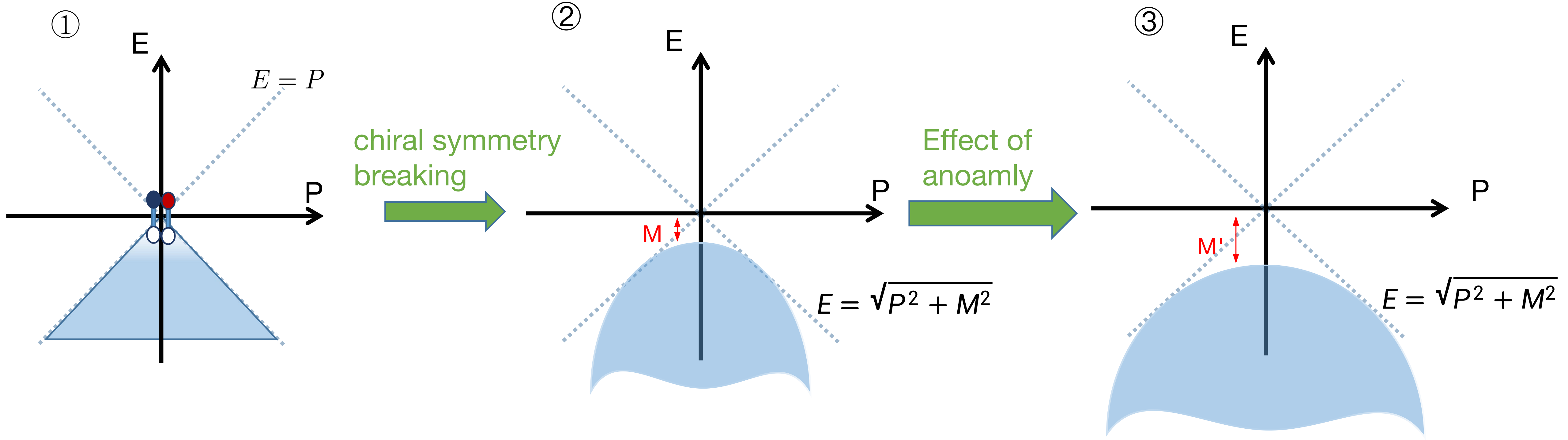
- Equation of state




In NJL-type model, EOS become softer with enhancing anomaly effect. (similar with PDM)

# Effect of anomaly in NJL-type model




① Chiral symmetry breaking via quark-antiquark paring.

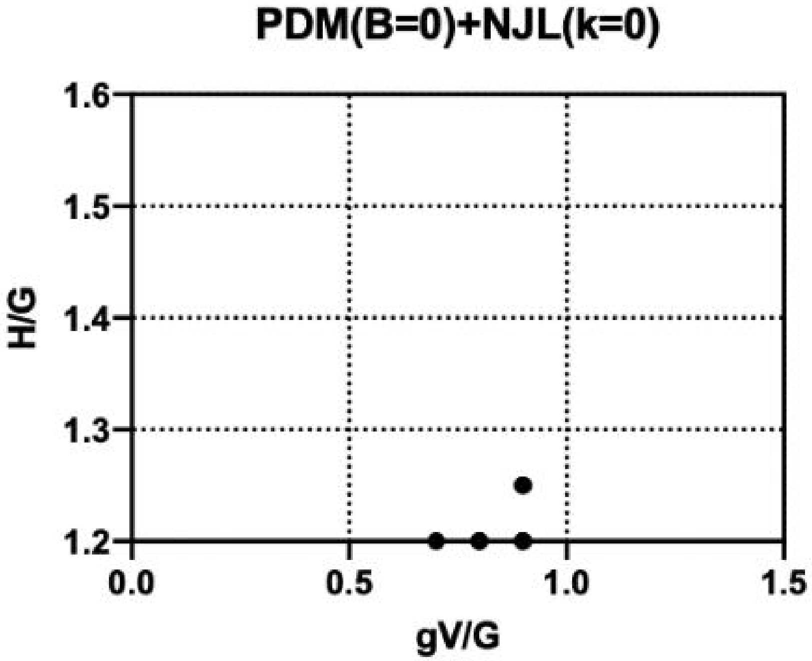
② Condensation of pairs opens a gap M in the quark dispersion relation, changing the structure if Dirac sea

③ After including anomaly, M' > M

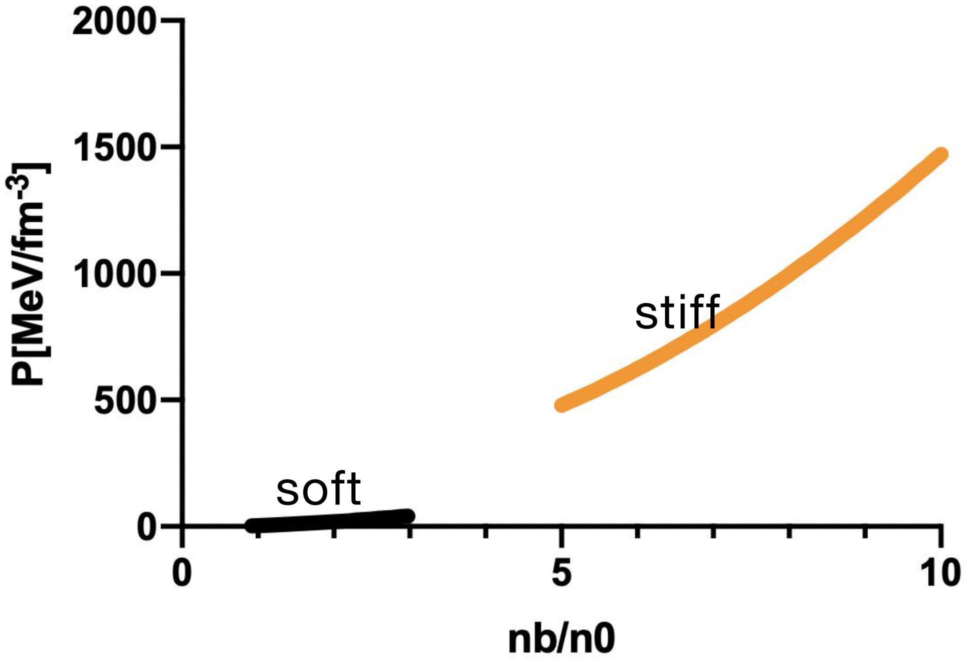
# Effect of anomaly in NJL-type model

$m_0$ is fixed to be 800 MeV as a typical example

1. B=0 and K=0



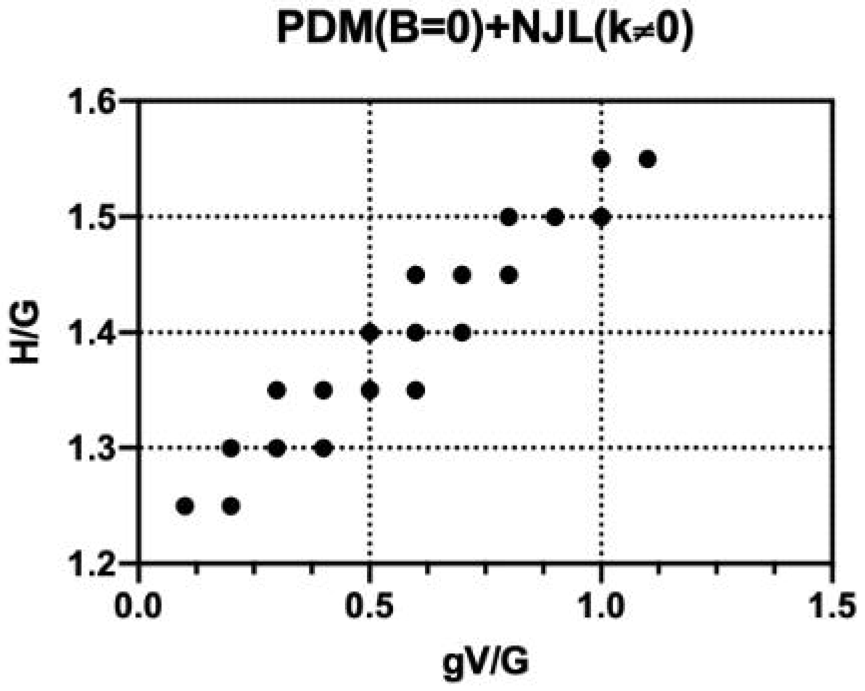

2. B=600 and K=0



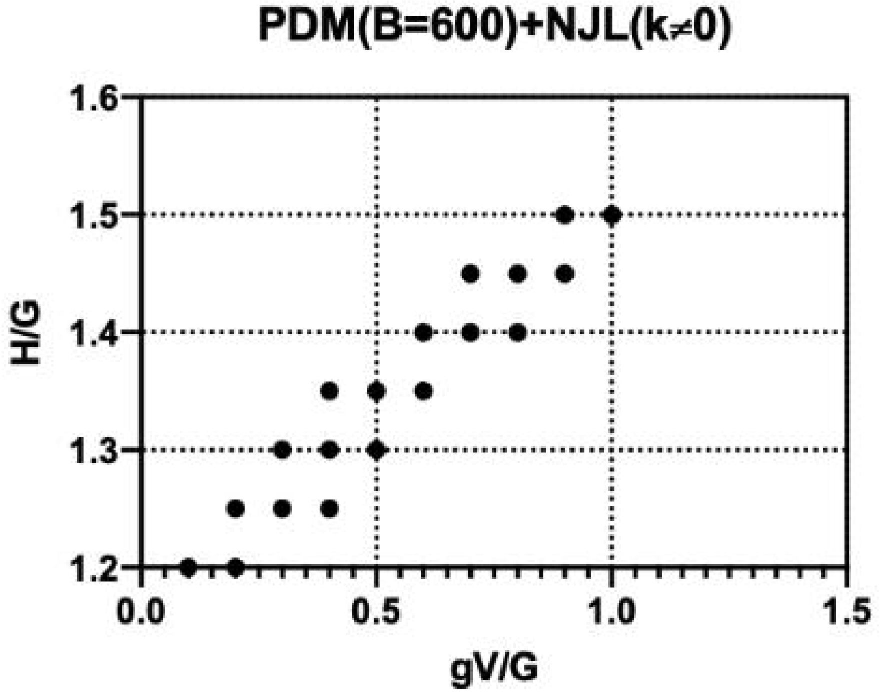

There is no smooth connection in this case

# Effect of anomaly in NJL-type model

3. B=0 and $K=9.29/\Lambda^5$




4. B=600 and $K=9.29/\Lambda^5$




Our final result is to constrain chiral invariant mass with B=600 and $K=9.29/\Lambda^5$

Constrained region is changed after including anomaly